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Modified Goal Programming Approach for Improving the Discrimination Power and Weights Dispersion

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Abstract

Data envelopment analysis (DEA) is a technique based on linear programming (LP) to measure the relative efficiency of homogeneous units by considering inputs and outputs. The lack of discrimination among efficient decision making units (DMUs) and unrealistic input-outputs weights have been known as the drawback of DEA. In this paper the new scheme based on a goal programming data envelopment analysis (GPDEA) are developed to moderate the homogeneity and reasonability of weights distribution by using of facet analysis On GPDEA (GPDEA-CCR and GPDEA-BCC) models. These modifications are done by considering the lower bounds for each individual inputs and outputs weights in standard CCR model and an upper bound just for free variable of standard BCC model. In the both of the cases the mentioned modification preserved the inputs and outputs weights from zero value. The modified GPDEA models also improve the discrimination power of DEA. The advantages of each modified GPDEA-CCR and GPDEA-BCC models are shown by some examples.

Keywords: Data Envelopment Analysis, Discrimination Power, Facet Analysis, Goal Programming, Weight Dispersion.

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1. Introduction

Data envelopment analysis (DEA), developed by Charnes et al. (1978), is a powerful tool based on a fractional mathematical programming technique. It is used to measure the productive efficiency of decision making units (DMUs) and evaluate their relative efficiency.

Two inter-related problems that have long been recognized are weak discriminating power and incommensurate weight distribution. The lack of discriminating power problem occurs when there are not enough DMUs or number of inputs and outputs is too high compared to the number of DMUs under evaluation. In this situation, classical DEA models often yield solutions that identify too many DMUs as efficient. The problem of unfitness weight dispersion for DEA occurs when some DMUs are rated as efficient because of input -out put weights have the extreme or zero values. In some cases we meet the unfitness of weights, i.e., a solution giving a big weight to variables with less importance or zero weight to important variables. In particular, in the zero cases, weights of input-output do not contribute to interpret results of analysis. To improve discrimination power and overcome to unrealistic weight, restriction techniques has been discussed in several papers. Thompson et al. (1986, 1990) developed the assurance region approach to help choosing a best site for the location of high energy physics laboratory in Texas. Charnes et al. (1990) developed cone-ratio envelopment to restrict weight flexibility directly in the weight space. By adjusting bounds on the proportions of individual inputs (or outputs) to total inputs (or outputs), Wong and Beasley (1990) have been developed a weight restriction. Doyle and Green (1994, 1995) proposed cross-efficiency evaluation technique and equated cross-efficiency with a process of peer-appraisal. By extending single criterion-based conventional approach, Li and Reeves (1999) have been developed multiple criteria data envelopment analysis (MCDEA). Jahanshahloo et al. (2005) suggested a feasible interval for weights by using goal programming and big-M method their approach adjusts the bounds weights, which have been considered by decision maker. Bal et .al (2008) tried to improve discrimination power and dispersion of weight by the minimization of coefficient of variation (CV) for input-output weights. Wang and Leo (2009) presented that the introduced method by Bal et .al is questionable and pointed out the DEA model with the inclusion of CVs of input-output weights, makes no sense if input and output data are not normalized to eliminate their dimensions and units. Then Bal et .al (2010) are suggested GPDEA models which yield more balanced input-output weight dispersions on compared to basic DEA model. Also by reduction of efficient DMUs without any addition constraints on weights, cause to improve the discrimination power.

In this paper a new scheme are proposed where by bounding of weights in GPDEA-CCR model and bounding free variable in GPDEA-BCC model, i.e. with modification of GPDEA models based on facet analysis, useful results are obtained with respect to the GPDEA models.

The paper is organized as follows: In section 2, we present the basic DEA models, the multi-criteria data envelopment analysis (MCDEA) models and goal programming DEA models (GPDEA-CCR and GPDEA-BCC). In section 3, we develop our method to improve the discrimination power and weights dispersion. An example is considered in section 4 which illustrate the proposed method. Conclusions are given in section 5.

2. Background

2.1. Data envelopment analysis

Suppose that there are n DMUs, where each DMUj (j=1,...,n) consumes m inputs x_{ij} (i=1,...,m) to produce s outputs y_{rj} (r=1,…,s). Taking into assumption, the efficiency rating for each DMUo can be computed using the CCR ratio model as follows:

$$\begin{aligned} \max\quad & \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ s.t.\quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad \forall j, \\ & u_r \ge 0, v_i \ge 0 \quad \forall r, i. \end{aligned} \tag{1}$$

The ratio form of model (1) can be transferred to the multiplier form by applying Charnes–Cooper transformation and rewritten as follows:

$$\begin{aligned} \max\quad & \sum_{r=1}^{s} u_r y_{ro} \\ s.t.\quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad \forall j, \\ & u_r \ge 0, v_i \ge 0 \quad \forall r, i. \end{aligned} \tag{2}$$

A DMUo is said to be CCR efficient if and only if the optimal value of model (2), equal to unity. Associated with the CCR model, we have

Proposition1. At least one DMU is CCR efficient.

Another DEA model, which is usually referred to as the BCC model, can be expressed as:

$$\begin{aligned} max \quad & \sum_{r=1}^{s} u_r y_{ro} + u_o \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \le 0 \quad \forall j, \\ & u_r \ge 0, v_i \ge 0 \quad \forall r, i, \\ & u_o \ free. \end{aligned} \tag{3}$$

A DMUo is said to be BCC efficient if and only if the optimal value of model (3), equal to unity. The model (2) and (3) are called the basic DEA models. The super efficiency concept is proposed for all efficient DMUs when there are more than one efficient DMU. One of the super efficiency models for ranking efficient DMUs in DEA was introduced by Andersen and Petersen (1993). This method enables an extreme efficient unit *o* to achieve an efficiency score greater than one by removing the *o*th constraint in the envelopment linear programming formulation.

2.2. Multiple criteria DEA model

The CCR and BCC multiplier models have been discussed most frequently when dealing with problems of discriminating power and weight restriction. Based on model (2) Li and Reeves (1999) developed the following model:

$$\begin{aligned} \min \quad & d_o \left(or \ \max \sum_{r=1}^{s} u_r y_{ro} \right) \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad \forall j, \\ & u_r, v_i, d_j \ge 0 \quad \forall r, i, j. \end{aligned} \tag{4}$$

Where d_o is the deviation variable for DMU_o and d_j ($j=1,...,n$) is the deviation variable of DMU_j ($j=1,...,n$). The quantity d_o ($0<d_o<1$) point out the inefficiency score. Under model (4) DMU_o is efficient if and if $d_o=0$ or $\sum_{r=1}^{s} u_r y_{ro} = 1$. If DMU_o is not efficient, it efficiency is $1-d_o$. Therefore the inefficiency score for basic DEA models under the constraint that the weighted sum of outputs is less than or equal to weighted sum of the inputs for each DMU.

The lack of discriminating power of the basic DEA can be overcome by using a single objective function in place of multiple. A multiple criteria data envelopment model formulation with the minmax and minsum criteria, which minimizes a deviation variable, *do*, rather than maximizing the efficiency score, max $\sum_{r=1}^{s} u_r y_{ro}$, is shown below:

$$
\begin{aligned}
&\min \quad d_o \left(or \quad \max \sum_{r=1}^{s} u_r y_{ro} \right) \\
&\min \quad M \\
&\min \quad \sum_{j=1}^{n} d_j \\
&s.t. \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad \forall j, \\
&\qquad M - d_j \geq 0 \quad \forall j, \\
&\qquad u_r, v_i, d_j \geq 0 \quad \forall r, i, j.
\end{aligned} \tag{5}
$$

where dj (j=1,…,n) is a deviation variable for DMUj (j=1,…,n), M is a maximum deviation variable (max {dj}). Also the quantity M is bounded by interval (0,1]. The first objective function, min do, is the classical DEA objective. Under the objective min do, DMU is efficient if and only if do=0. The second objective function, min M, (minmax) minimizing the maximum deviation variable. The third objective function, min $\sum_{j=1}^{n} d_j$ (minsum) minimizing the sum of the deviation variables. The constraints $M - d_j \geq 0$, j=1,...,n that define the maximum deviation M, do not change the feasible region of decision variables.

2.3. Goal programming DEA models: GPDEA-CCR and GPDEA-BCC

Because of the complexity of multiple objectives for MCDEA models Bal et. al (2010) suggested GPDEA model and Converted the multiple objective programming problems into a single objective problem. the converted models were solved easily than priori models. Goal programming simultaneously considers all goals in a composite objective function minimizing the deviations between goals and aspiration levels. However, the MCDEA model (5) can be adapted to the weighted goal programming in the form given by Bal et al.

$$
\begin{aligned}
&\min \quad a = \left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\} \\
&s.t. \quad \sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1, \\
&\qquad \sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1, \\
&\qquad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad \forall j, \\
&\qquad M - d_j + d_{3j}^- - d_{3j}^+ = 0 \quad \forall j, \\
&\qquad u_r, v_i, d_j, d_{3j}^-, d_{3j}^+ \geq 0 \quad \forall r, i, j, \\
&\qquad d_1^-, d_1^+, d_2^-, d_2^+ \geq 0.
\end{aligned} \tag{6}
$$

where for the DMU under evaluation, d_1^- and d_1^+ are the unwanted deviation variables for the goal which constraints the weighted sum of inputs to unity, d_2^- is the wanted deviation variable for the goal which makes the weighted sum of outputs less than or equal to unity, d_2^+ is the unwanted deviation variable for the goal which makes the weighted sum of outputs less than or equal to unity. d_{3j}^-'s are the unwanted deviation variables for the goal (i.e., $M - d_j \geq 0$, j=1,..., n) which realizes M as the maximum deviation, and d_{3j}^{+}'s are the wanted deviation variables for the same goal (i.e., $M - d_j \geq 0$, j= 1,..., n), where all d_j deviation variables are also unwanted.

Similarly, based on the BCC model, GPDEA-BCC model can be formulated as:

$$\begin{aligned} \min \quad & a = \left\{ d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j \right\} \\ s.t. \quad & \sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1, \\ & \sum_{r=1}^{s} u_r y_{ro} + u_o + d_2^- - d_2^+ = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o + d_j = 0 \quad \forall j, \\ & M - d_j + d_{3j}^- - d_{3j}^+ = 0 \quad \forall j, \\ & u_r, v_i, d_j, d_{3j}^-, d_{3j}^+ \geq 0 \quad \forall r, i, j, \\ & d_1^-, d_1^+, d_2^-, d_2^+ \geq 0 \\ & u_o \quad free. \end{aligned} \tag{7}$$

Where $d_1^-, d_1^+, d_2^-, d_2^+, d_{3j}^-$ and d_{3j}^+ are defined as in the GPDEA-CCR model.

3. Modified GPDEA models

3.1. Modified GPDEA-CCR model

Since the feasibility space of CCR model and GPDEA-CCR model are same, therefore the modification of GPDEA-CCR model can be formulated similar to the CCR model as follows:

Based on approach that is given by Daneshvar (2002), consider the following model for all CCR efficient DMUs :

$$\begin{aligned} \max \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\ s.t. \quad & x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} - s_i^- = 0, \quad i = 1,\dots,m, \\ & y_{ro} - \sum_{j=1}^{n} \lambda_j y_{rj} + s_r^+ = 0, \quad r = 1,\dots,s, \\ & \lambda_j \geq \circ, \quad j = 1,\dots,n, \\ & s_i^- \geq \circ, \quad i = 1,\dots,m, \\ & s_r^+ \geq \circ, \quad r = 1,\dots,s. \end{aligned} \tag{8}$$

It is clear that for the inefficient DMUs the optimal solution of above model is infeasible, then, for all efficient DMUs, which the optimal value of above problem is nonzero, let the set of these DMUs be called B. For DMUs that are belonged to B, model (9) and model (10) are solved.

$$\begin{aligned} \max \quad & u_r \\ s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \forall j, \\ & u_r \geq 0, v_i \geq 0 \quad \forall r, i. \end{aligned} \tag{9}$$

$$\begin{aligned} \max \quad & v_i \\ s.t \quad & \sum_{i=1}^{m} v_i x_{io} = 1, \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad \forall j, \\ & u_r \geq 0, v_i \geq 0 \quad \forall r, i. \end{aligned} \tag{10}$$

Suppose that the optimal values for model (9) and model (10) are represented by u_r^+ and v_i^+ respectively. By placing ε_r and ε_i according to

$$\varepsilon_r = min \quad \left\{ u_r^+ \middle| \text{DMU} \in B, \quad r = 1,\dots,s \right\}$$
$$\varepsilon_i = min \quad \left\{ v_i^+ \middle| \text{DMU} \in B, \quad i = 1,\dots,m \right\},$$

As lower bounds for $u_r\,(r=1,\ldots,s)$ and $v_i\,(i=1,\ldots,m)$, the GPDEA-CCR model is modified as follow:

$$
\begin{aligned}
\min \quad & a=\left\{d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j\right\} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1, \\
& \sum_{r=1}^{s} u_r y_{ro} + d_2^- - d_2^+ = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0 \quad \forall j, \\
& M - d_j + d_{3j}^- - d_{3j}^+ = 0 \quad \forall j, \\
& u_r \ge \varepsilon_r, \\
& v_i \ge \varepsilon_i, \quad \forall r,i, \\
& d_j, d_{3j}^-, d_{3j}^+ \ge 0 \quad \forall j, \\
& d_1^-, d_1^+, d_2^-, d_2^+ \ge 0.
\end{aligned} \tag{11}
$$

Where $d_1^-, d_1^+, d_2^+, d_{3j}^-$ and $d_j\ (j=1,\ldots,n)$ are unwanted deviation variables and d_2^- and d_{3j}^+ are wanted deviation variables.

3.2. Modified GPDEA-BCC model

Since the feasibility space of BCC and GPDEA-BCC are same, therefore the modification of GPDEA-BCC can be formulated similar to BCC. Based on approach that is given by Daneshvar (2002), consider the following model for all efficient DMUs:

$$
\begin{aligned}
\max \quad & u_o \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{ro} + u_o = 1, \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{1}^{s} u_r y_{rj} - \sum_{1}^{m} v_i x_{ij} + u_o \le 0 \quad \forall j, \\
& u_r \ge 0,\ v_i \ge 0 \quad \forall r,i, \\
& u_o \quad free.
\end{aligned} \tag{12}
$$

The optimal value for model (12) is u_O^+. Then for all efficient DMUs where is $u_O^+ = 1$, solve the following model:

$$
\begin{aligned}
\min \quad & u_o \\
s.t. \quad & \sum_{r=1}^{s} u_r y_{ro} + u_o = 1, \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o \le 0 \quad \forall j, \\
& u_r \ge 0,\ v_i \ge 0 \quad \forall r,i, \\
& u_o \quad free.
\end{aligned} \tag{13}
$$

The optimal value for model (13) is u_O^- and take ε as follow:

$\varepsilon = max\,\{u_o^- \mid u_o^- \ne 1$, for efficient DMUs$\}$

By placing ε as upper bound for free variable of GPDEA-BCC model, this model is modified as follow:

$$
\begin{aligned}
\min \quad & a=\left\{d_1^- + d_1^+ + d_2^+ + \sum_j d_{3j}^- + \sum_j d_j\right\} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{io} + d_1^- - d_1^+ = 1, \\
& \sum_{r=1}^{s} u_r y_{ro} + u_o + d_2^- - d_2^+ = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + u_o + d_j = 0 \quad \forall j, \\
& M - d_j + d_{3j}^- - d_{3j}^+ = 0 \quad \forall j, \\
& u_r, v_i, d_j, d_{3j}^-, d_{3j}^+ \ge 0 \quad \forall r,i,j, \\
& d_1^-, d_1^+, d_2^-, d_2^+ \ge 0 \\
& u_o \le \varepsilon.
\end{aligned} \tag{14}
$$

Where $d_1^-, d_1^+, d_2^-, d_2^+, d_{3j}^-$ and d_{3j}^+ are defined in the model (7). By bounding u_r and (12) v_i variables in modified GPDEA-CCR model and by bounding free variable, u_o, in modified GPDEA-BCC model, the useful results are obtained with respect to GPDEA-CCR and GPDEA-BCC models i.e. by reducing the number of efficient DMUs, it causes to improve the discrimination power.

Table1. Inputs and output of 10 DMUs

DMU	Y_1	Y_2	Y_3	Y_4	X_1	X_2	X_3	X_4
1	47	93	54	65	32	50	82	46
2	88	56	92	80	61	56	68	37
3	94	65	80	80	42	58	45	34
4	50	53	93	97	73	39	88	81
5	47	42	70	52	45	38	68	41
6	86	45	100	47	86	62	44	32
7	83	91	62	74	38	74	71	74
8	79	60	72	98	61	54	70	62
9	85	68	51	41	84	52	38	47
10	78	95	70	92	87	47	31	52

Table 2. Results of the basic CCR and GPDEA-CCR for the data set.

	DEA-CCR				GPDEA-CCR			
DMU	Eff.	SuperEff.	Rank	CVsof weight	Eff.	Super Eff.	Rank	Cvs of weight
1	1	1.481	3	1.354	1	1.014	4	1.012
2	1	1.098	7	1.626	0.948	0.948	6	1.069
3	1	1.541	2	1.832	1	1.067	2	1.070
4	1	1.456	4	1.269	1	1.001	5	1.015
5	1	1.042	8	1.343	1	1.031	3	1.013
6	1	1.321	5	1.292	0.789	0.789	8	1.070
7	1	1.137	6	1.354	0.747	0.747	10	1.013
8	1	1.025	9	1.238	0.824	0.824	7	1.013
9	0.994	0.994	10	1.796	0.770	0.770	9	1.070
10	1	1.951	1	1.400	1	1.285	1	1.069

Table 3. Optimal solution of model (9) and model (10) for the data set.

DMU	u_1^+	u_2^+	u_3^+	u_4^+	v_1^+	v_2^+	v_3^+	v_4^+
1	0.8441	1.0215	1.4521	1.5083	2.7174	1.4110	0.7502	1.7606
2	0.8660	0.6380	1.0870	1.0604	0.6196	1.3061	0.3438	2.0818
3	1	1.4148	1.2500	1.2255	2.0704	1.2552	1.7570	2.3810
4	1.0475	0.7758	1.0753	1.0101	0.8615	1.8939	0.5117	0.5688
5	0.3294	0.6414	1.4286	0.2496	1.2549	1.3198	0.2236	0.5231
6	0.9824	0.8638	1	0.6095	0.4639	1.0651	2	2.5316
7	1.0588	1.0438	0.8333	1.2299	2.2883	0.4772	0.8442	0.2923
8	0.5804	0.2521	0.1080	1	0.9428	1.1041	0.3650	0.5047
10	1.2048	1	1.4164	1.0661	0.7413	1.5748	2.8409	1.4843

Table 4. Results of the modified GPDEA-CCR model.

DMU	Eff.	Super Eff.	Rank	CVs of weight
1	0.993	0.993	4	0.564
2	0.975	0.975	5	0.564
3	1	1.196	2	0.544
4	0.867	0.867	7	0.564
5	0.727	0.727	10	0.550
6	0.822	0.822	9	0.564
7	0.849	0.849	8	0.520
8	0.994	0.994	3	0.564
9	0.899	0.899	6	0.564
10	1	1.211	1	0.531

Also, the mentioned models result homogeneous dispersion input-output weights. In order to illustrate the proposed modified GP-DEA models, we have used the following example.

4. Numerical Examples

Because of unit-invariance property in DEA models and the structure of objective functions for the GPDEA models, normalizing of weights has no effect on DEA efficient hence, in these examples, we have used a normalized data obtained by dividing each input-output with their highest value.

Example 1

Table 1 shows data set with four inputs and four outputs. Table 2 shows the efficiency, super efficiency and rank values of DMUs by the super efficiency scores and the coefficient of variation of input–output weight values for the basic CCR and the GPDEA-CCR models. As seen in Table 2, variation of input–output weight in GPDEA-CCR model for peer DMU is less than variation of input-output weight value for CCR model. In addition, in Table 2, while 9 of 10 DMUs are found as efficient by the basic CCR model, the GPDEA-CCR model is reduced the number of efficient units to 5. According to the results, GPDEA-CCR models are generally more homogeneous than those of the basic CCR model in DEA. Also GPDEA-CCR model reduce the number of efficient DMUs and improve the discrimination power.

In Table 3 results of model (9) and model (10) are given. By placing ε_r and ε_{i} as lower bounds $u_r (r = 1,...,s)$ and $v_i (i = 1,...,m)$ respectively, the modified GPDEA-CCR model for data set is solved. Table 4 shows these results. As seen in Table 2 and Table 4 it is clear that model (11) reduces the number of efficient DMUs. Therefore, improves the discrimination power and according to the results of these tables, the dispersion (variation) of input-output weights assigned to DMUs by the modified GPDEA-CRR model is less than, i.e., more homogeneous than that of GPDEA-CCR model. The results of BCC and GPDEA-BCC models summarized in Table 5.Obviously, GPDEA-BCC model reduced the number of efficient units to 4 and, variation of input–output weight in GPDEA-BCC model for peer DMU is less than variation of input-output weight value for BCC model. The optimal solution of model (12) for BCC efficient DMUs equal to 1, 0.6798, 1, 0.9387, 1, 1, 0.7452, 0.2181 and 1. The optimal value of model (13) for DMU_1 and DMU_5 equal to -79.63 and -0.2108 and for DMU_3, DMU_6 and DMU_{10} are unbounded. Replace ε as upper bound for free variable u_0 of GPDEA-BCC model and solve the model (14). The results are summarized in Table 6. By comparing Table 5 and Table 6, we realize that the number of efficient DMUs is reduced to 3 by the modified GPDEA-BCC. modified GPDEA-BCC model for peer DMU is less than variation of input-output weight value for GPDEA-BCC and BCC models.

Table 5. Results of the basic BCC and GPDEA-BCC for the data set.

DMU	DEA-BCC				GPDEA-BCC			
	Eff.	SuperEff.	Rank	CVs of weight	Eff.	Super Eff.	Rank	CVs of weight
1	1	1.953	8	1.731	0.762	0.762	9	1.555
2	1	3.081	6	1.603	0.945	0.945	5	1.157
3	1	4.179	4	1.964	1	1.245	1	1.157
4	1	5.283	2	1.070	1	1.006	4	1.099
5	1	1.270	9	1.303	1	1.214	3	1.157
6	1	4.612	3	1.411	0.789	0.789	8	1.157
7	1	2.848	7	1.392	0.727	0.727	10	1.333
8	1	3.465	5	1.355	0.889	0.889	6	1.157
9	0.996	0.996	10	2.128	0.821	0.821	7	1.157
10	1	7.525	1	1.522	1	1.227	2	1.156

Table 6. Results of the modified GPDEA-BCC model.

DMU	Eff.	Super Eff.	Rank	CVs of weight
1	0.055	0.055	9	1.107
2	0.952	0.952	4	1.120
3	1	1.063	3	1.131
4	1	1.136	2	0.871
5	0.010	0.010	10	1.107
6	0.789	0.789	6	1.114
7	0.752	0.752	7	1.139
8	0.797	0.797	5	1.145
9	0.706	0.706	8	1.087
10	1	1.337	1	1.138

Example 2. Efficiency evaluation of seven departments in a university

Table7 shows Li and Revees (1999) data set for 7 DMUs with three inputs and three outputs. Similarly, Table8 shows the efficiency, super efficiency and rank values of DMUs and the coefficient of variation of input–output weight values for the basic CCR and the GPDEA-CCR models.

It is clear, GPDEA-CCR model reduced the number of efficient DMUs and, variation of input–output weight in GPDEA-CCR model for peer DMU is less than variation of input-output weight value for CCR model. The results of model (9) and model (10) and modified GPDEA-CCR summarized in Table 9 and Table 10. By comparing Table 8 and Table 10, we observe that the modified GPDEA-CCR model specify three DMUs as efficient and hence improves the discrimination power. Also GPDEA-CCR results homogeneity of input-output weights.

Table 7. The data of seven departments in a university.

DMU	Y1	Y2	Y3	X1	X2	X3
1	60	35	17	12	400	20
2	139	41	40	19	750	70
3	225	68	75	42	1500	70
4	90	12	17	15	600	100
5	253	145	130	45	2000	250
6	132	45	45	19	730	50
7	305	159	97	41	2350	600

Table 8. Results of the basic CCR and GPDEA-CCR for the university data set.

DMU	DEA-CCR				GPDEA-CCR			
	Eff.	SuperEff.	Rank	CVsof weight	Eff.	Super Eff.	Rank	CVs of weight
1	1	1.834	1	1.309	1	1.196	1	0.735
2	1	1.049	6	1.528	0.957	0.957	5	0.735
3	1	1.199	4	1.650	0.764	0.766	6	0.735
4	0.819	0.819	7	1.495	0.576	0.577	7	0.735
5	1	1.220	3	1.549	1	1.109	3	0.735
6	1	1.190	5	1.184	1	1.036	4	0.735
7	1	1.266	2	0.837	1	1.128	2	0.735

Table 9. Result of model (9) and model (10) for the university data set.

DMU	u_1^+	u_2^+	u_3^+	v_1^+	v_2^+	v_3^+
1	4.7859	4.5455	5.2016	3.5973	5.8824	30.3030
2	2.1930	1.0079	0.8954	2.3571	3.1348	1.5284
3	1.3350	0.4697	1.7331	0.5245	0.8852	8.6207
5	0.4966	1.0977	1	1	1.1751	1.0153
6	2.3095	1.7390	2.8902	2.3514	3.2154	8.5474
7	1	1	0.7029	1.0977	0.6974	0.3145

Table 10. Results of the modified GPDEA-CCR model for the university data set.

DMU	Eff.	Super Eff.	Rank	CVs of weight
1	0.557	0.557	6	0.566
2	0.958	0.958	5	0.566
3	0.973	0.973	4	0.566
4	0.520	0.520	7	0.566
5	1.542	1.542	1	0566
6	1.121	1.121	3	0.566
7	1.491	1.491	2	0.566

Table 11. Results of the basic BCC and GPDEA-BCC for the university data set.

	DEA-BCC				GPDEA-BCC			
DMU	Eff.	Super Eff.	Rank	CVs of weight	Eff.	Super Eff.	Rank	CVs of weight
1	1	2.515	3	2.071	0.592	0.892	6	1.432
2	1	1.059	6	1.590	1	1.012	4	1.475
3	1	2.921	1	1.736	0.954	0.954	5	1.562
4	0.976	0.976	7	1.588	0.483	0.483	7	1.505
5	1	2.902	2	1.705	1	1.117	2	1.620
6	1	1.227	5	1.472	1	1.029	3	1.287
7	1	1.817	4	2.109	1	1.270	1	1.760

Table 12. Results of the modified GPDEA-BCC model for the university data set.

DMU	Eff.	Super Eff.	Rank	CVs of weight
1	0.543	0.543	4	1.319
2	0.508	0.508	5	1.306
3	1	1.176	1	1.370
4	0.005	0.005	7	1.499
5	1	1.103	2	1.584
6	0.484	0.484	6	1.222
7	1	1.090	3	1.542

The results of BCC and GPDEA-BCC models summarized in Table 11. As seen in Table 11, it is clear that GPDEA-BCC reduce the number of efficient DMUs. Therefore, improves the discrimination power and the dispersion of input-output weights assigned to DMUs by the GPDEA-BCC model are more homogeneous than that of BCC model. Optimal values of model (12) for BCC efficient DMUs equal to 1, 0.3514, 0.0746, 0.1279, 0.4501 and 0.1200. The optimal solution of model (13) for DMU_1 equal to -1.7577 .Replace $\varepsilon = Max\{-1.758\} = -1.758$ as upper bound for free variable u_0 of GPDEA-BCC model and solve the model (14). The results are summary in Table 12.

By comparing Table 11 and Table 12 we

observe the number of efficient DMUs is reduced to 3 by the modified GPDEA-BCC model.

Therefore, modified GPDEA-BCC model improves the discrimination power and the dispersion (variation) of input-output weights assigned to DMUs by the modified GPDEA-BCC model is less than, i.e., more homogeneous than GPDEA-BCC model.

5. Conclusion

It has been realized that DMUs may have dispersion of non-homogeneous input-output weight. Hence yield many numbers of efficient DMUs. In this paper we have investigated GPDEA model and proposed modified GPDEA. Our research findings have clearly show that modified GPDEA model imply more homogeneous input-output weight.

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References

Andersen, P., & Petersen, N. (1993). A procedure for ranking efficient units in data envelopment analysis. *Management Science*, 39(10), 1261–1264.

Bal, H., Örkcü, H.H., & Çelebioğlu, S. (2008). A New Method Based on the Dispersion of Weights in Data Envelopment Analysis. *Computers and Industrial Engineering*, 54, 502-512.

Bal, H., Örkcü, H.H., & Çelebioğlu, S. (2010). Improving the Discrimination Power and Weights Dispersion in the Data Envelopment Analysis. *Computers and Operations Research*, 37, 99-107.

Charnes, A., Cooper, W. W., Huang, Z. M., & Sun, D. B. (1990). Polyhedral cone-ratio models with an illustrative application to large commercial banks. *Journal of Econometrics*, 46, 73–91.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research*, 2, 429–444.

Daneshvar, S. (2010). The modification of BCC model using facet analysis. *Recent Advances in Applied Mathematics*, 635-641.

Doyle, J. R., & Green, R. (1994). Efficiency and cross-efficiency in data envelopment analysis: Derivatives, meanings and uses. *Journal of Operational Research Society*, 45(5), 567–578. Doyle , J. R., & Green, R. H. (1995). Cross-evaluation in DEA: Improving discrimination among DMUs. *INFOR*, 33, 205–222.

Jahanshahloo, G. R., Memariani, A., Hosseinzadeh, F., & Shoja, N. (2005). A feasible interval for weights in data envelopment analysis. *Applied Mathematics and Computation*, 160, 155–168.

Li, X. B., & Reeves, G. R. (1999). A multiple criteria approach to data envelopment analysis. *European Journal of Operational Research, 115, 507–517.*

Thompson, R. G., Langemeier, L. N., Lee, C. T., & Thrall, R. M. (1990). The role of multiplier bounds in efficiency analysis with application to Kansas farming. *Journal of Econometrics*, 46, 93–108.

Thompson, R. G., Singleton, F. G., Thrall, R. M., & Smith, B. A. (1986). Comparative site evaluations for locating a high-energy physics lab in Texas. *Interfaces*, 16, 35–49.

Wong, Y. H. B., & Beasley, J. E. (1990). Restricting weight flexibility in data envelopment analysis. Journal of the Operational Research Society, 41(9), 829–835.

Wong, Y.M., & Luo, Y. (2009). *A* Note ona New Method Based on the Dispersion of Weights in Data Envelopment