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Optimal Trajectory Planning of a Box Transporter Mobile Robot

Hossein Barghi Jond[1], Adel Akbarimajd[2], Nurhan Gürsel Özmen[3], Sonia Gharibzadeh[4]

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Abstract - **This paper aims to discuss the requirements of safe and smooth trajectory planning of transporter mobile robots to perform non-prehensile object manipulation task. In non-prehensile approach, the robot and the object must keep their grasp-less contact during manipulation task. To this end, dynamic grasp concept is employed for a box manipulation task and corresponding conditions are obtained and are represented as a bound on robot acceleration. A trajectory optimization problem is defined for general motion where dynamic grasp conditions are regarded as constraint on acceleration. The optimal trajectory planning for linear, circular and curve motions are discussed. Optimization problems for linear and circular trajectories were analytically solved by previous studies and here we focused with curve trajectory where Genetic Algorithm is employed as a solver tool. Motion simulations showed that the resulted trajectories satisfy the acceleration constraint as well as velocity boundary condition that is needed to accomplish non-prehensile box manipulation task .**

Index Terms - **Non-prehensile Manipulation, Mobile Robots, Trajectory Planning, Dynamic Grasp, Genetic Algorithm.**

1,4 - Ahar Branch, Islamic Azad University, Ahar, IRAN.
(h-barghi@iau-ahar.ac.ir)
2- Department of Electrical and Computer Engineering, University of Mohaghegh Ardabili, Ardabil, IRAN.
3- Department of Mechanical Engineering, Karadeniz Technical University, Trabzon, TURKEY.

I. INTRODUCTION

MANIPULATOR hands encounter with high complexities to accomplishing stable object grasping. Recently a new manipulation mode has launched known as non-prehensile manipulation. This mode mentions to manipulation without grasping object and facilitates using simpler mechanisms increases flexibility and workspace [1]. In non-prehensile manipulation task both robot's and the object's geometries used to model the dynamics of the task and then, there is no need to dexterous mechanisms in manipulation system [2]. The geometry of the object and the kinematics of the robot are vital to design appropriate motion controller [3]. In quasi-static non-prehensile manipulation the object should keep its contact with the manipulation surface all time [4].

In one of the earliest works, a mobile manipulator proposed where the desired task was to manipulate a paper or a cylinder using robot wheels [5]. Pushing and pulling objects by means of a stick and string [6], picking up a box from the floor in a corner [7], cooperatively carrying an object without grasping [8], presenting a tracked mobile robot where it has the ability of box-pushing [9], and moving an object by lifting and pulling against vertical horizontal edges [10] are some distinct studies have concurrently published in the non-prehensile manipulation field.

Among the non-prehensile manipulation tasks, one can refer to carrying a box or generally an object on a surface. For long distance manipulations, where manipulators are inapplicable, conveyors and vehicles conventionally employed. In transport of goods where objects loaded on a vehicle, it is necessary that the motion of the vehicle is appropriately planned to preserve immovability of the object relative to the surface. This referrers to dynamic grasp, which is a concept in the quasi-static manipulation category. To perform the manipulation task precisely, dynamic grasp should be considered in motion planning of the conveyors or material transporter mobile robots.

In motion planning, the robot follows a planned path from its initial point to the target. Trajectory planning deals with robot position, velocity and acceleration in time. However, when a mobile robot should traverse along a given path, there are infinite possible trajectories that the robot can run. Although, only finite numbers of them are appropriate to run in applications. Generally these suitable trajectories are generated based on optimality criteria related with time and vehicle dynamic-kinematic constraints. Namely some related works include time minimizing in the spline curve path [11], polynomial s-curve motion planning [12], straight-line, circular segments, and continuous-curvature path planning [13], optimizing trajectory based on dynamic potential function [14]. In [15] time-optimal trajectories for car-like robots are obtained after solving a formulated dynamic optimization problem. In [16] to steer agricultural machinery automatically, the paper applied continuous-curvature path planning known from mobile robotics to generate feasible headland turn manoeuvres trajectories. In [17] motion planning of autonomous on-road driving is considered in order to determining the most feasible trajectory in motion time. Among the various types of geometric curves are usually employed in mobile robots path planning, Bezier curves are known methods to generate trajectories for curvature path [18-21].

This paper aims to discuss about suitable and possibly optimal trajectories to run in the mentioned box manipulation task. A safe and suitable trajectory requires fulfilling dynamic grasp related acceleration constraint at all time of motion [22]. The desired trajectory can be determined via an optimization problem. A third degree polynomial is used as the objective function to formulate of a trajectory optimization problem. The problem constraint is dynamic grasp required limitation on acceleration. Also, the boundary conditions are defined as zero velocity at initial and final time of motion. Three trajectory optimization problems are established for linear, circular and curve motions such that the constraints are related to observe acceleration limitation due to non-prehensile box manipulation task. The problems are under velocity boundary conditions as making zero velocity at starting and ending. Optimization problems for linear and circular trajectories were analytically solved by previous studies and here we presented a solution for curve trajectory using Genetic Algorithm (GA).

The rest of this paper organized as the sequel. The next section describes dynamic grasp requirements and contact modelling. In the third section, optimization problems are formulated and the solutions of them are discussed to find suitable trajectories to the transporter robot. The simulation results and discussions are provided in the Fourth Section. The last section includes conclusions.

II. CONTACT MODELING AND MANIPULATION

A contact model is required to describe how relative motions of the contacting bodies can be avoided. It could be determined by the geometry of the contacting surfaces and the friction. We assume a simple Coulomb friction model with a single friction coefficient μ as it is usual in robotics for hard and dry materials [23]. Assume that an irregularly shaped object with mass m rested on the upper flat surface of a mobile robot as shown in Fig.1. It assumed that the center of gravity (CoG) of the object is located in height l from the surface. As it is shown in Fig.1, dl and dr are distances between the vertices of the resting edge and the projection point of the CoG.

The robot moves in a planned linear trajectory from an initial position to a goal position. The friction between the robot's wheels and the ground is neglected. During motion, dynamic grasp should be preserved i.e. there should be no relative motion between the robot and the object [1]. Therefore no slippage and no rotating around vertices should be occurred. The friction force must be bounded by ±μg ([11], [23]); however the free diagram of the contacting bodies (Fig. 1) implies that the robot horizontal acceleration must be bounded by same value as next.

$$|a_{\max}| \leq \mu g. \tag{1}$$

where g is earth's gravitational acceleration, and μ is the static friction coefficient.

To obtain no rotation condition, we borrow the Zero Moment Point (ZMP) concept from the literature related to the balance of the robots. ZMP defined as a point on the support surface where the resultant tipping moment is zero [24]. In the case considered here, where the object has only one segment, ZMP defined is the intersection point of $\vec{a}+\vec{g}$ vector and the upper surface of the mobile robot (Fig. 1 (b)). To prevent the object from the rotation, the ZMP point should locate between the left and right vertices of the resting edge of the object. To this aim, from Fig. 1, for the right vertex, we can write

$$\tan\left(\frac{|a_{\max}|}{g}\right) \le \frac{d_r}{l} \rightarrow |a_{\max}| \le g\tan^{-1}\left(\frac{d_r}{l}\right). \quad (2)$$

Similarly, for the left vertex, we obtain as follows

$$|a_{\max}| \le g\tan^{-1}\left(\frac{d_l}{l}\right). \quad (3)$$

In summary, mentioned limited acceleration constraints are the requirements of dynamic grasp condition. Therefore, these conditions can be unified as follows

$$|a_{\max}| \le \Delta = g\min\left\{\begin{matrix}\mu, \tan^{-1}\left(\frac{d_r}{l}\right)\left(\frac{1-\operatorname{sgn}(a)}{2}\right) \\ +\tan^{-1}\left(\frac{d_l}{l}\right)\left(\frac{1+\operatorname{sgn}(a)}{2}\right)\end{matrix}\right\}. \quad (4)$$

where the direction of the acceleration vector at motion time is taken in account. Now, if the absolute value of the acceleration of the robot is less than the positive threshold Δ, the stability of the object (i.e. dynamic grasp) is guaranteed for whole motion.

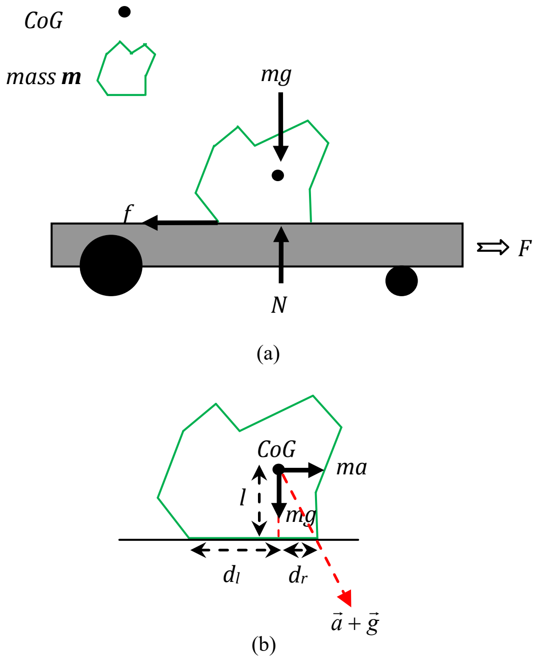


Fig. 1. (a) Manipulation task, (b) free diagram of the object.

III.OPTIMAL TRAJECTORY PLANNING

Using polynomial trajectories are common in robotics. Given initial and final positions and velocities, the trajectory equation at least has four parameters. To achieve smooth and continuous motions, we use a third-order polynomial. Here, the trajectory problem can be determine the polynomial coefficients to get optimal motions consider the velocity and acceleration limits.

In this section, three trajectory optimization problems will be defined by taking into account acceleration constraint and velocity boundary conditions. Acceleration limit caused by the contact friction force between the robot and the object should be considered to keep dynamic grasp manipulation requirements. In other word our aim is to find motion strategies for the robot that would give suitable and possibly optimal trajectories to move from a start point and rest state to an end point and into rest state again, where the motion should stay inside the acceleration limit all the time.

To devise trajectory of the manipulation system, it is assumed that the robot travels along a predefined linear, circular or curve path. The follow third-order polynomial can be used to describe trajectory.

$$q(t) = \lambda_1 t^3 + \lambda_2 t^2 + \lambda_3 t + \lambda_4. \quad (5)$$

To obtain trajectories based on the polynomial shown in (5) such that fulfilling the velocity boundaries and the acceleration constraint, an optimization problem can be defined and mathematically established as the following.

Optimization Problem: Assume that the final time of the motion tf is fixed, it is desired to find suitable ‘s such that the robot will cover possible maximum distance when the acceleration constraint as well as the velocities boundary conditions are fulfilled. Therefore, this problem can be formulated as next.

$$\max_{\lambda} q(t) = \lambda_1 t^3 + \lambda_2 t^2. \quad (6)$$

subjected to

$$q(0) = 0. \quad (6a)$$

$$\dot{q}(0) = v(0) = 0. \quad (6b)$$

$$v(t_f)=0. \quad (6c)$$

$$|a(t)| \le \Delta, t \in [0, t_f] \quad (6d)$$

where q(t) is the objective function that shows travelled distance at time t, v is velocity and tf is a given final time for motion. (6a) shows that the robot initially is located in the origin, (6b) and (6c) show the boundary conditions which imply zero velocity at the beginning and end of the trajectory, respectively. Inequality (6d) specifies the acceleration constraint or dynamic grasp required condition. Here, the trajectory planning is reduced to the problem of determining the polynomial coefficients to get suitable motions satisfying velocity boundary and acceleration constraint.
With deriving of q(t) with respect to time variable, the velocity function of v(t) can be obtained. However, second derivation of q(t) will not leads to acceleration function a(t) at non-straight paths. From (6a) and (6b), λ_3 and λ_4 obtain as zero.

Then, the optimization problem can be expressed as below in compact form.

$$\max_{\lambda} q(t)=\lambda_1 t^3+\lambda_2 t^2. \quad (7)$$

subjected to

$$v(t_f)=0 \rightarrow 3\lambda_1 t_f+2\lambda_2=0. \quad (7a)$$

$$|a(t)| \le \Delta, t \in [0, t_f] \quad (7b)$$

[25] declares an analytically solving procedure for the polynomial trajectory optimization problem, where the closed form solutions for linear and circular arc trajectories are obtained as (8) and (9), respectively.

$$\lambda_1=-\frac{1}{3t_f}\Delta, \lambda_2=\frac{1}{2}\Delta. \quad (8)$$

$$\lambda_1=-\frac{2}{3t_f}\lambda_2, \lambda_2=\min\left\{\left|\frac{2\Delta^{\frac{1}{2}}c^{-\frac{1}{2}}}{t_f}\right|, \left|-\frac{\Delta c^{-1}}{2}\right|\right\}. \quad (9)$$

Here we focus on the trajectory optimization problem for curve path. Considering optimization problem of (7) for curve path, acceleration constraint of (7b) here should be expressed. Total acceleration in a curvature path is composed of two vectors which are centripetal (a_c) and tangential (a_t). Therefore, the acceleration constraint in inequality (7b) for non-straight motion must be written as

$$|\vec{a}_c+\vec{a}_t| \le \Delta. \quad (10)$$

For circular and curve motions, inequality (10) can be rearranging in terms of vehicle velocity v and radius of curvature c as the following

$$\sqrt{\left(\frac{v^2}{c}\right)^2+\left(\frac{dv}{dt}\right)^2} \le \Delta. \quad (11)$$

Inequality (11) can be expressed in terms of q(t) as bellows

$$\sqrt{\frac{\dot{q}^4(t)}{c^2}+\ddot{q}^2(t)} \le \Delta. \quad (12)$$

In circular motion, c is constant value. For the third-order polynomial curve, the radius of curvature can be defined as next [26].

$$c=\left|\frac{\sqrt{(1+\dot{q}^2(t))^3}}{\ddot{q}(t)}\right|. \quad (13)$$

Substituting c in inequality (12), the acceleration constraint can be rewritten in terms of first and second derivate of q(t) as bellows

$$\ddot{q}(t)\sqrt{\frac{\dot{q}^4(t)}{(1+\dot{q}^2(t))^3}+1} \le \Delta. \quad (14)$$

with substituting polynomial forms of first and second derivate of q(t) into (14), the acceleration constraint finally can be obtained as inequality (15) for curve motion.

$$(6\lambda_1 t + 2\lambda_2)^2 \left| \frac{\left(3\lambda_1 t^2 + 2\lambda_2 t\right)^4}{\left(1 + \left(3\lambda_1 t^2 + 2\lambda_2 t\right)^4\right)^3} + 1 \right| \leq \Delta^2 . \quad (15)$$

Therefore, curve path trajectory optimization problem can be rewrite as next.

$$\max_{\lambda} q(t) = \lambda_1 t^3 + \lambda_2 t^2 . \quad (16)$$

subjected to (7a) and (15).

This constrained problem is highly complex and cannot be tracked analytically. Here, we use GA to optimize the parameters of the trajectory optimization (16), i.e. the objective polynomial's coefficients (λ1, λ2). An outline of GA algorithm is as below [27][28]. A random initial population is created. For each individual of the current population, a fitness value is assigned. A selection method is used to selects individual with better fitness values, called parents. Childs of these parents are produced either by mutation or crossover operators. Current population is replaced with the children. The procedure runs until a termination criterion is reached.

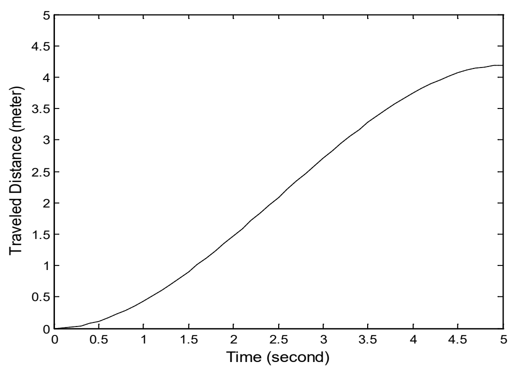
GA starts with randomly generation of individual's population represented as a chromosome that encodes a binary value for (λ1, λ2) as candidate solution. To produce the next generation, each chromosome is rated by the problem fitness function q(t) in (16). Chromosomes are subject to crossover and mutation as basic operators. The GA used for solving (16) is summarized as the sequel:

1. Create an initial population for (λ1, λ2) randomly,
2. Compute the fitness value q(t) of all individuals of population,
3. Scale the individual's fitness values and choose the best subset of the population of (λ1, λ2) as parents,
4. Generate children of the parents by crossover and mutation operators,
5. Cross out the genomes those do not satisfy constrains (16b) and replace them by new strings re-generated using step 4,
6. Verify the fitness value of the new population individuals,
7. Repeat 3 to 6 until a fixed amount of fitness is attained.

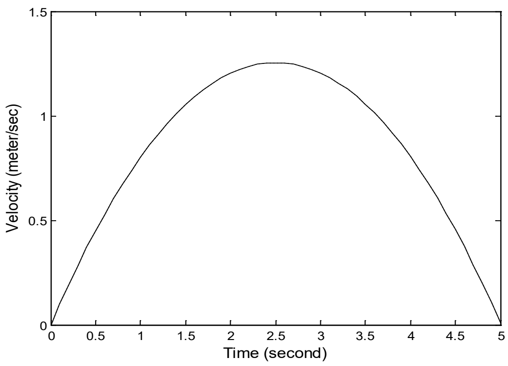
IV. RESULTS AND DISCUSSIONS

To see how the trajectories precisely observes the acceleration constraint and velocity boundaries, a manipulation task with follow parameters is considered. Without loss of generality, for a square object model parameters chosen as μ=0.1, m=0.1kg. Earth's gravitational acceleration is set to g=10m/s2. In the object, CoG parameters computed as dr=d1=l=5cm. Therefore, the acceleration safety range for guaranteeing dynamic grasp manipulation based equation (1) should be bounded by (-1, 1) m/s2 for avoiding the slippage. Also to avoid the rotation, by (3) the acceleration must be bounded to (-7.85, 7.85)m/s2. In fact, due to the object symmetrical geometry absolute limit values for not rotating around right and left vertices are the same. It is easy to argue that in this experimental setup if the acceleration constraint is satisfied for avoiding the slippage, then the rotation is avoided too. The trajectory optimization problem for curve path defined in (16). Here, the acceleration constraint is nonlinear and highly complicated. After multiple running of the GA for tf=5 seconds, the polynomial's coefficients optimal values are obtained as λ1=−0.0667, λ2=0.5010. Maximum value for the fitness function obtains as q(5)=4.1833.

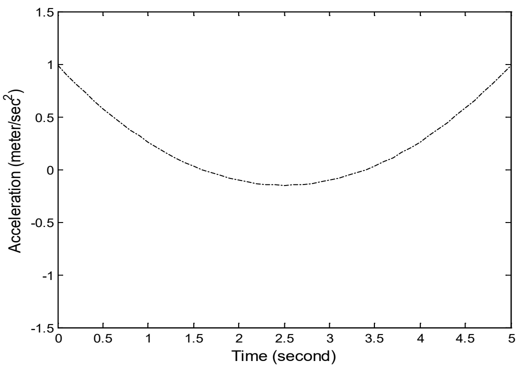
The GA solution for the curve problem is near to the closed-form solution of the linear problem. Trajectory plots for curve path example shown in Fig. 2. The robot travelled distance, its velocity and its acceleration during of motion is illustrated in this figure. It is clear that the problem boundary conditions for velocity and the problem constraint for acceleration is satisfied. In other words, the trajectory guarantees a safe manipulation.



(a)



(b)



(c)

Fig. 2. (a) Traveled curve-length, (b) Robot's velocity, (c) Robot's total acceleration.

V. CONCLUSION

In this paper, a box manipulation task with a mobile robot is introduced. It was shown that to successfully carry out the manipulation task and preserving dynamic grasp between the object and the robot, acceleration of the robot has to remain in specified bounds. This fact was explained theoretically as constraint and optimal motion of the robot was described by an optimization problem. The simulation showed that the obtained trajectories can be used in proposed quasi-static box manipulation model to achieve a reliable motion towards in indoor environments.

REFERENCE

[1] Lynch, K. M. and Mason, M. T., 1999. Dynamic Nonprehensile Manipulation: Controllability, Planning, and Experiments. International Journal of Robotics Research, 18 (1), pp. 64-92.

[2] Lynch, K. M., Northrop, M. and Pan, P., 2002. Stable limit sets in a dynamic parts feeder. IEEE Trans Robotics and Automation, 18 (4), pp. 608-615.

[3] Lynch, K. M. and Murphey, T. D., 2003. Control of nonprehensile manipulation. Control Problems in Robotics-Springer Tracts in Advanced Robotics, 4, pp. 39-57.

[4] Li, Q. and Payande, S., 2004. Planning Velocities of Free Sliding Objects as a Free Boundary Value Problem. International Journal of Robotics Research, 23 (1), pp. 69-87.

[5] Mason, M. T., Pai, D., Rus D., Taylor, R. L. et. al., 1999. A Mobile Manipulator. Proc. IEEE International Conf. on Robotics and Automation, Detroit, Michigan, pp. 2322 – 2327.

[6] Yamashita, A., Kawano, K., Ota, J., Arai, T. et. al., 1999. Planning method for cooperative manipulation by multiple mobile robots using tools with motion errors. Proc. IEEE/RSJ International Conf. on Intelligent Robots and Systems, Kyongju, pp. 978 – 983.

[7] Huang, W. H. and Holden G. F., 2001. Nonprehensile Palmar Manipulation with a Mobile Robot. Proc. IEEE/RSJ International Conf. on .Intelligent Robots and Systems, Maui, HI, USA.

[8] Gupta, A. and Huang, W. H., 2003. A carrying task for nonprehensile mobile manipulators. Proc. IEEE/RSJ International Conf. on Intelligent Robots and Systems, Las Vegas, USA.

[9] Liu, Y. and Liu, G., 2009. Mobile manipulation using tracks of a tracked mobile robot. Proc. IEEE/RSJ International Conf. on Intelligent Robots and Systems, St. Louis, MO.

[10] Kolhe, P., Dantam, N., Stilman, M., 2010. Dynamic Pushing Strategies for Dynamically Stable Mobile Manipulators. Proc. IEEE International Conf. on Robotics and Automation, Alaska, USA.

[11] Lepetic, M., Klancar, G. Skrjanc, I., Matko, D. et. al., 2003. Time optimal path planning considering acceleration limits. Robotics and Autonomous Systems, 45, pp. 199–210.

[12] Naguyen, K. D., Chen, I. M., Ng, T. C., 2007. Planning algorithms for s-curve trajectories. Proc. IEEE/ASME International conf. on Advanced Intelligent Mechatronics, Zurich.

[13] Kardos, E. S. and Kiss, B., 2009. Continuous-curvature paths for mobile robots. Periodica polytechnic Electrical Engineering, 53 (1-2), pp. 63-72.

[14] Korayem, M. H., Nazemizadeh, M., Rahimi, H. N., 2013. Trajectory optimization of nonholonomic mobile manipulators departing to a moving target amidst moving obstacles. Acta Mechanica, 224 (5), pp. 995-1008.

[15] Li, B. and Shao, Z., 2015. Simultaneous dynamic optimization: A trajectory planning method for nonholonomic car-like robots. Advances in Engineering Software, 87, pp. 30–42.

[16] Sabelhaus, D., Lammers, P. S., Helligen, L. P., Röben, M. F., 2015. Path planning of headland turn manoeuvres. LANDTECHNIK ,70(4), pp. 123–131.

[17] Katrakazas, C., Quddus, M., Chen, W., Deka, L., 2015. Real-time motion planning methods for autonomous on-road driving: State-of-the-art and future research directions. Transportation Research Part C: Emerging Technologies, 60, pp. 416-442.

[18] Yang, G. J., Delgado, R., Choi, B. W., 2016. A Practical Joint-space Trajectory Generation Method Based on Convolution in Real-time Control. International Journal of Advanced Robotic Systems, International Journal of Advanced Robotic Systems, 13(56), pp. 1-12.

[19] Yang, K., Jung, D., Sukkarieh, S., 2013. Continuous curvature path-smoothing algorithm using cubic Bezier spiral curves for non-holonomic robots. Advanced Robotics, 27(4): pp. 247-258.

[20] Jolly, K. G., Sreerama-Kumar, R., Vijayakumar, R., 2009. A Bezier curve based path planning in a multi-agent robot soccer system without violating the acceleration limits. Robotics and Automation Systems, 57(1), pp. 23-33.

[21] Yang, G. J. and Choi, B. W., 2013. Smooth trajectory planning along Bezier curve for mobile robots with velocity constraints. International Journal of Control and Automation, 6(2), pp. 225-234.

[22] Barghijand, H., Akbarimajd, A., Keighobadi, J., 2011. Quasi-Static Object Manipulation by Mobile Robot: Optimal Motion Planning Using GA. International Conference on Intelligent Systems Design and Applications, Spain, pp. 202-207.

[23] Siciliano, B. and Khatib, O. (Eds.), 2008. Springer Handbook of Robotics, Springer, Springer-Verlag Berlin Heidelberg, Chaps. 5, 27.

[24] Sardain, P. and Bessonnet, G., 2004. Forces Acting on a Biped Robot. Center of Pressure—Zero Moment Point. IEEE Transactions on Systems, Man, and Cybernetics, 34 (5), pp. 630-637.

[25] Jond, H. B., Akbarimajd, A. and Ozmen, N. G., 2014. Time-Distance Optimal Trajectory Planning For Mobile Robots On Straight And Circular Paths. Journal of Advances in Computer Research, 5 (2), pp. 23-36.

[26] Gray, A., 1997. Modern Differential Geometry of Curves and Surfaces with Mathematica, 2nd ed. Boca Raton, FL: CRC Press.

[27] Melanie, M., 1996. An Introduction to Genetic Algorithms, Fifth printing, Cambridge, MA: MIT Press., Chap. 2.

[28] Haupt, R. L. and Haupt, S. E., 2004. Practical Genetic Algorithms. 2nd ed. New York, USA, Wiley.