



About dark matter as an emerging entity from elementary energy density fluctuations of a three-dimensional quantum vacuum

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Abstract

A new suggestive explanation of dark matter as an emerging phenomenon determined by the polarization of a three-dimensional quantum vacuum, i.e., by opportune quantum vacuum energy density fluctuations, associated with processes of manifestation/demanifestation of virtual pairs of particles-antiparticles, in a background characterized by a fluctuating viscosity, is proposed. It is shown that the observed flattening of the orbital speeds of the arms of spiral galaxies is generated by an appropriate perturbative fluctuation of energy density of the three-dimensional quantum vacuum, which corresponds to a degree of viscosity, on the ultra-low frequencies. In this approach, a unifying re-reading of ordinary matter, dark matter, and dark energy as phenomena deriving from the energy density fluctuations of the same three-dimensional quantum vacuum is obtained.

Keywords Standard model · Timeless three-dimensional quantum vacuum · Quantum vacuum energy density · Ordinary matter · Dark energy · Dark matter

Introduction

Our understanding of the universe, within the so-called Standard Model of particle physics, is limited to 5% of the things that actually exist, i.e., we can say that, through the theories which we have, physics is able to account for only 5% of all matter/energy contained in the universe. Although the Standard Model successfully describes all the interactions produced by particle accelerators, it is not actually able to explain the 95% of all that exists: these “things” are made up of mass and energy that are not visible and escape our observation instruments, more precisely 26% is made up of an unknown form of matter (referred to as “dark matter”), which is invoked to explain the rotation curves of galaxies and the mass of galaxy clusters (as well as the anisotropies of the cosmic microwave background and distribution of galaxies on a large scale) and 69% of an elusive form of energy (called “dark energy”), invoked to account for the accelerated expansion of the universe.

How to get out of this impasse? Despite the extraordinary results to which the scientific revolutions of the XX century (in particular relativity and quantum physics) have brought us, it is evident that in our description of the universe, there is something to be adjusted, in the sense that the theories we have need at least to be extended. In other words, we have to find a new, more fulfilling synthesis that accounts for all the things that exist and all the large-scale observations. Where to start, then, if we want to provide a more adequate description, a more satisfactory image, of all that exists and all the large-scale observations?

In some recent works [1–10], the author of this article has formulated a suggestive hypothesis according to which dark energy is not a primary physical reality, but actually derives from elementary fluctuations of the energy density of a timeless three-dimensional quantum vacuum defined by elementary reduction-state (RS) process of creation/annihilation (i.e., of manifestation/demanifestation) of quanta. In this paper, we plan to address in detail, within our model of a timeless three-dimensional quantum vacuum, the problem of dark matter, starting from the foundations, and to compare the results of our model with other current models of explanation of dark matter. This article is structured as follows. In chapter 2, we will briefly summarize the foundations of our model of timeless

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three-dimensional quantum vacuum and its explanation of ordinary matter of the Standard Model, dark energy, and space curvature. In chapter 3, we will propose our key of interpretation of dark matter. Finally, in chapter 4, we will make some considerations about how our model stands compared to other attempts to explain dark matter.

The fundamental features of three-dimensional quantum vacuum energy density model: explanation of ordinary matter of the Standard Model, curvature of space, and dark energy

According to a model developed by the author in [1–10], our everyday physical world emerges from a more fundamental background, a timeless three-dimensional (3D) quantum vacuum defined by a Planckian metric and a variable (i.e., characterized by fluctuations) quantum vacuum energy density, where time is not a primary physical reality, but exists only as a mathematical parameter measuring the sequential numerical order of material changes.

In summary, the foundational postulates of this model of 3D quantum vacuum can be so synthesized:

- 1- The medium of space is an isotropic, granular, three-dimensional (3D) “quantum vacuum” constituted by energetic packets having the size of Planck’s volume and whose most universal physical property is the energy density characterized by fluctuations with respect to the Planck energy density:

$$\rho_{pE} = \frac{m_p c^2}{l_p^3} = 4,641266 \cdot 10^{113} J/m^3, \tag{1}$$

(where m_p is Planck’s mass, c is the light speed, and l_p is Planck’s length).

- 2- In the free space, without the presence of massive particles, the quantum vacuum energy density is at its maximum and corresponds to the Planck energy density (1) which defines the so-called “ground state” of the 3D quantum vacuum.
- 3- The appearance of ordinary matter derives from an opportune excited state of the 3D quantum vacuum defined by an opportune diminishing of the quantum vacuum energy density corresponding to elementary reduction-state (RS) processes of creation/annihilation of virtual particle/antiparticle pairs. The excited state of the quantum vacuum corresponding to the appearance of a material particle is defined (in the centre of that particle) by the energy density:

$$\rho = \rho_{pE} - \frac{m \cdot c^2}{V}, \tag{2}$$

(and by the change of the energy density:

$$\Delta\rho_{qvE} \equiv \rho_{pE} - \rho = \frac{mc^2}{V} \tag{3}$$

with respect to the ground state), depending on the amount of mass m and the volume V of the particle. The curvature of space–time can be seen as an emergent phenomenon generated by the variable energy density of the 3D quantum vacuum.

The postulates here enunciated, which imply that the properties of physical space can be described by a variable energy density of a 3D quantum vacuum, find their physical justification in the considerations which follow below.

Let us consider, before all, the postulates 1 and 2. In this regard, if, on the basis of the assumption that dark energy is owed to an interplay between quantum mechanics and gravity, the Planck energy density (1) is usually considered as the origin of the dark energy and, thus, of a cosmological constant, and this generates a big problem in the sense that the observed cosmological energy density turns out to be 123 orders of magnitude lower than the Planck energy density; in some recent papers, Santos proposed an explanation for the actual value of dark energy which invokes the fluctuations of the quantum vacuum [11–13]. In Santos’ approach, the quantum vacuum fluctuations can be associated with a curvature of space–time similar to the curvature generated by a “dark energy” density, on the basis of equation:

$$\rho_{DE} \cong 70G \int_0^\infty C(s)ds, \tag{4}$$

which states that the possible value of the “dark energy” density is the product of Newton’s gravitational constant, G , times the integral of the two-point correlation function of the vacuum fluctuations defined by:

$$C(|\vec{r}_1 - \vec{r}_2|) = \frac{1}{2} \langle vac | \hat{\rho}(\vec{r}_1, t) \hat{\rho}(\vec{r}_2, t) + \hat{\rho}(\vec{r}_2, t) \hat{\rho}(\vec{r}_1, t) | vac \rangle, \tag{5}$$

$\hat{\rho}$ being an energy density operator, such that its vacuum expectation is zero, while the vacuum expectation of the square of it is not zero. Santos showed that the quantum vacuum fluctuations give rise to a curvature of space–time similar to the curvature produced by a “dark energy” density by invoking, in the picture of the Friedmann equations:

$$\left[\frac{\dot{a}}{a} \right]^2 = \frac{8\pi G}{3} (\rho_{mat} + \rho_{DE}), \quad \frac{\ddot{a}}{a} = \frac{8\pi G}{3} \left(\frac{1}{2} \rho_{mat} t + \rho_{DE} \right), \tag{6}$$

a quantum metric of the form:

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu, \tag{7}$$

where the quantum coefficients (in polar coordinates) are:

$$\begin{aligned} \hat{g}_{00} &= -1 + \hat{h}_{00}, \hat{g}_{11} = 1 + \hat{h}_{11}, \hat{g}_{22} = r^2(1 + \hat{h}_{22}), \\ \hat{g}_{33} &= r^2 \sin^2 \vartheta(1 + \hat{h}_{33}), \hat{g}_{\mu\nu} = \hat{h}_{\mu\nu} \text{ for } \mu \neq \nu, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \langle \hat{h}_{\mu\nu} \rangle &= 0 \text{ except } \langle \hat{h}_{00} \rangle = \frac{8\pi G}{3}(\rho_{mat} + \rho_{DE})r^2 \text{ and} \\ \langle \hat{h}_{11} \rangle &= \frac{8\pi G}{3}\left(\rho_{DE} - \frac{1}{2}\rho_{mat}\right)r^2. \end{aligned} \tag{9}$$

In Eq.(9), $\langle \hat{h}_{\mu\nu} \rangle$ stands for $\langle \Psi | \hat{h}_{\mu\nu} | \Psi \rangle$, ρ_{DE} is the dark energy density, ρ_{mat} is the matter density, and Ψ is the quantum state of the universe for which the expectation of the stress-energy tensor operator of the quantum fields satisfies equations:

$$\langle \Psi | \hat{T}_4^4 | \Psi \rangle = \frac{\Delta\rho_{qvE}}{c^2}; \langle \Psi | \hat{T}_\mu^\nu | \Psi \rangle \approx 0 \text{ for } \mu\nu \neq 00, \tag{10}$$

to obtain the correct Friedmann–Robertson–Walker metric.

Now, in our approach of the 3D quantum vacuum whose ontologically primary reality is represented by a variable quantum vacuum energy density, both the dark energy and the matter density appearing in 4) and (7) can be seen as different aspects of the same energy of the 3D quantum vacuum and thus Santos’ results can receive a new re-reading into a more general and unifying approach. In our model, in the light of Santos’ results, we can consider a quantized metric of the quantum vacuum condensate given by relation:

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu, \tag{11}$$

where here the (quantum operators) coefficients of the metric are defined (in polar coordinates) as:

$$\begin{aligned} \hat{g}_{00} &= -1 + \hat{h}_{00}, \hat{g}_{11} = 1 + \hat{h}_{11}, \hat{g}_{22} = r^2(1 + \hat{h}_{22}), \\ \hat{g}_{33} &= r^2 \sin^2 \vartheta(1 + \hat{h}_{33}), \hat{g}_{\mu\nu} = \hat{h}_{\mu\nu} \text{ for } \mu \neq \nu \end{aligned} \tag{12}$$

As regards the coefficients (10), multiplication of every term times the unit operator is implicit and, at the order $O(r^2)$, by following treatment in [2, 5, 6, 9, 10], one obtains:

$$\langle \hat{h}_{\mu\nu} \rangle = 0 \text{ except } \langle \hat{h}_{00} \rangle = \frac{8\pi G}{3} \left(\frac{\Delta\rho_{qvE}}{c^2} + \frac{35Gc^2}{2\pi\hbar^4 V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 \right) r^2 \text{ and } \langle \hat{h}_{11} \rangle = \frac{8\pi G}{3} \left(-\frac{\Delta\rho_{qvE}}{2c^2} + \frac{35Gc^2}{2\pi\hbar^4 V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 \right) r^2 \tag{13}$$

Moreover, it is assumed that the metric (11) is close to the Minkowski metric (namely that, when the distance $r \rightarrow \infty$, one has $\hat{g}_{\mu\nu} \rightarrow \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric) and, in Eq. (13), $\Delta\rho_{qvE}^{DE}$ are opportune fluctuations

of the quantum vacuum energy density which determine the dark energy density on the basis of relation:

$$\rho_{DE} \cong \frac{35Gc^2}{2\pi\hbar^4 V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6. \tag{14}$$

Here, therefore, the variable energy density of the vacuum (producing dark energy) acts as a two-point correlation function according to relation:

$$\frac{c^4}{4\pi\hbar^4} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 \cong \int_0^\infty C(s) ds, \tag{15}$$

where $C(s)$ is the two-point correlation function of the quantum vacuum fluctuations which depends only on the distance between the two points. In other words, in our model, on the basis of Eqs. (12)–(15), the 3D quantum vacuum defined by the quantized metric (11), which is determined by the changes and fluctuations of the energy density of space, can be considered as the ultimate visiting card of general relativity, namely as the fundamental origin of the curvature of space–time characteristic of general relativity [2, 5, 6, 9, 10].

In our model, in the light of the treatment made in [2, 5, 6, 9, 10], the fundamental point is that the quantized metric of the 3D quantum vacuum (11) with the above-mentioned features allows us to obtain the quantum Einstein equations:

$$\hat{G}_{\mu\nu} = \frac{8\pi G}{c^4} \hat{T}_{\mu\nu}, \tag{16}$$

where the right-hand side, as a consequence of the variable quantum vacuum energy density, can be seen as a stochastic inhomogeneous field that acts as a source for these inhomogeneities, thus avoiding the overwhelmingly large prediction for cosmic acceleration.

In the traditional formulations of the proposals of solution of the cosmological constant problem, the following assumptions are usually made:

1. The total effective cosmological constant is on at least the order of magnitude of the vacuum energy density generated by zero-point fluctuations of particle fields.
2. Quantum field theory is an effective field theory description of a more fundamental, discrete theory, which becomes significant at some high-energy scale Λ .

3. The vacuum energy–momentum tensor is Lorentz invariant.

4. The Moller–Rosenfeld approach to semiclassical gravity (using an expectation value for the energy–momentum tensor) is sound.
5. The Einstein equations for the homogeneous Friedmann–Robertson–Walker metric accurately describes the large-scale evolution of the Universe.

Now, in [14] S.S. Cree, T.M. Davis, T.C. Ralph, Q. Wang, Z. Zhu, and W.G. Unruh, Cree and his co-authors have shown that there is an inconsistency between assumptions 2 and 3: the vacuum state cannot be Lorentz invariant if modes are ignored above some high-energy cut-off Λ , because a mode that is high energy in one reference frame will be low energy in another appropriately boosted frame. Therefore, Cree and his co-authors have showed that the contradiction can be resolved by removing assumption 3, so that equation

$$\langle 0 | \hat{T}_{\mu\nu} | 0 \rangle = -\rho_{vac} g_{\mu\nu} \quad (17)$$

no longer is valid. To resolve this issue, therefore, they have assumed an explicit cut-off in frequency as well as an explicitly non-Lorentz-invariant form of the metric, thus construing a picture where both the energy density and the pressure are large and positive, and the matter gravitates attractively.

Taking account of these results, also in our model, we assume that the 3D quantum vacuum is not Lorentz-invariant and, in analogy to Cree’s approach, here, the right-hand side of the Einstein is a stochastic inhomogeneous field that acts as a source for these inhomogeneities generated by the variable quantum vacuum energy density. In this way, in our approach, the standard Friedmann–Robertson–Walker metric can be replaced by a metric of the form:

$$ds^2 = -dt^2 + a(t, x)^2 (dx^2 + dy^2 + dz^2), \quad (18)$$

where the scale factor $a(t, x)^2$, because of the variable quantum vacuum energy density acting as a stochastic inhomogeneous field, is now inhomogeneous representing the relative size of spacetime at each point. In other words, we can say that, as a consequence of the action of the variable quantum vacuum energy density as a stochastic inhomogeneous field, produces a Friedmann–Robertson–Walker metric which has not the standard form, but has the form (18). Taking account of Cree’s results, this scale factor satisfies a dynamical equation of the form:

$$\ddot{a}(t, \vec{x}) + \Omega^2(t, \vec{x})a(t, \vec{x}) = 0 \quad (19)$$

which is a harmonic oscillator equation for each \vec{x} , with Ω playing the role of frequency which depends on the variable quantum vacuum energy density on the basis of relation:

$$\Omega^2(t, \vec{x}) = \frac{4\pi}{3} \left(-\frac{\Delta\rho_{qvE}}{c^2} + \frac{35Gc^2}{2\pi\hbar^4V} \left(\frac{V}{c^2} \Delta\rho_{qvE}^{DE} \right)^6 + \sum_{i=1}^3 P_i(t, \vec{x}) \right) \quad (20)$$

where $P_i(t, \vec{x}) = a(t, x)^{-2} T_{ii}(t, \vec{x})$.

Because of the linearity and symmetry of Eq. (19), it turns out that decaying solutions of this equation, known also a parametric resonance, will be suppressed unless the initial conditions are fine-tuned, so that the long-term solution will either grow exponentially or remain steady. This means that the general solution can be written as follows:

$$a(t, x) \approx e^{Ht} P(x, \Delta\rho_{qvE}) \quad (21)$$

where $H \geq 0$ is a constant and P is a quasiperiodic function depending on the variable quantum vacuum energy density. Taking account of Cree’s results, parametric resonance is usually strongest (i.e., growth or decay is most rapid) when the timescale of frequency oscillation and amplitude oscillation are similar, and one expects an exponential decrease of H with respect to Λ , thus providing a mechanism which leads to $H^2 \approx 10^{-120}$, which indicates that in this way, our model predicts an appropriate order of magnitude for the acceleration, and has the potential to resolve the cosmological constant problem. Therefore, in conclusion, we can say that the postulates 1 and 2 of our 3D quantum vacuum model find their physical justification in the results of Santos, Cree, and his co-authors, by invoking the concept of a variable energy density of the 3D quantum vacuum which acts as an inhomogeneous field, thus avoiding the overwhelmingly large prediction for cosmic acceleration, breaking the Lorentz-invariance, and, therefore, clearing in what sense the Planck energy density (1) defines the ground state of the 3D quantum vacuum.

As regards the postulate 3, regarding the fact that, in our approach, the appearance of ordinary matter derives from an opportune excited state of the 3D quantum vacuum defined by an opportune diminishing of the quantum vacuum energy density corresponding to elementary reduction-state (RS) processes of creation/annihilation of virtual particle/antiparticle pairs, the physical justification comes from the considerations of the results obtained by Sbitnev [15–18] and Fedi [19–21] as regards the interpretation of the physical vacuum as a superfluid medium which contains pairs of particles–antiparticles which make up a Bose–Einstein condensate (in the picture, respectively of a Navier–Stokes equation and of a Gross–Pitaevskii equation), which leads to obtain the spin 1/2 particles of the Standard Model as the quantized vortices of the superfluid medium where superfluidity breaks down). In epistemological affinity with the models of Sbitnev and Fedi, in the approach developed by the author in [1, 3–5, 7–10], the 3D quantum vacuum can be characterized as an organized timeless Bose–Einstein condensate of virtual particle/antiparticle

pairs, where the real elementary particles of the Standard Model emerge from opportune reduction-state (RS) processes of creation/annihilation of these virtual particle/antiparticle pairs, which give rise to an opportune excited state of the 3D quantum vacuum. The RS processes of creation/annihilation of quanta of the 3D quantum vacuum are somewhat similar to the transactional processes, corresponding to a peculiar reduction of a state vector, invoked by Chiatti and Licata [22–27] in their interpretation of an archaic, a temporal vacuum as fundamental arena in which the only truly existent “things” in the physical world are the events of creation and destruction (or, in other words, physical manifestation and demanifestation) of certain qualities and all the other constructions of physics are “emergent” with respect to the network of events.

In virtue of the physical correspondence between opportune diminishing of the quantum vacuum energy density and opportune RS processes of creation/annihilation of particle/antiparticle pairs, the excited state of the quantum vacuum can be described by a wave function $C = \begin{pmatrix} \psi \\ \phi \end{pmatrix}$ at two components satisfying a time-symmetric extension of the Klein–Gordon quantum relativistic equation:

$$\begin{pmatrix} H & 0 \\ 0 & -H \end{pmatrix} C = 0, \tag{22}$$

where $H = \left(-\hbar^2 \partial^\mu \partial_\mu + \frac{V^2}{c^2} (\Delta \rho_{qvE})^2 \right)$ and $\Delta \rho_{qvE} = (\rho_{PE} - \rho_{qvE})$ is the change of the quantum vacuum energy density. Equation (22) corresponds to the following two equations:

$$\left(-\hbar^2 \partial^\mu \partial_\mu + \frac{V^2}{c^2} (\Delta \rho_{qvE})^2 \right) \psi_{Q,i}(x) = 0, \tag{23}$$

for creation events and

$$\left(\hbar^2 \partial^\mu \partial_\mu - \frac{V^2}{c^2} (\Delta \rho_{qvE})^2 \right) \phi_{Q,i}(x) = 0 \tag{24}$$

for destruction events.

In our model, a crucial aspect of the 3D quantum vacuum lies in its fundamental non-locality. In fact, if one decomposes the real and imaginary parts of the generalized Klein–Gordon equation [Eq. (22)] after writing the two components of the wave function in polar form, for the real part, one obtains a couple of quantum Hamilton–Jacobi equations that leads to define a time-symmetric quantum potential of the vacuum at two components of the form:

$$Q_{Q,i} = \frac{\hbar^2 c^2}{V^2 (\Delta \rho_{qvE})^2} \left(\begin{array}{l} \left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) |\psi_{Q,i}| \\ - \frac{|\psi_{Q,i}|}{\left(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) |\phi_{Q,i}|} \end{array} \right), \tag{25}$$

which emerges as the ultimate entity guiding the occurring of the processes of creation or annihilation events in space. In other words, the RS processes turn out to be choreographed by the quantum potential of the vacuum (25). As a consequence of the primary physical reality of the processes of creation and annihilation and of the non-local action of the quantum potential and in analogy to the epistemology of a recent model proposed by Licata and Chiatti where quantum jumps are processes of entry and exit from the usual temporal domain to a timeless vacuum [26, 27], inside our approach, the behaviour of the ordinary matter existing in the universe can be seen as an undivided network of RS processes that take place in a 3D timeless non-local quantum vacuum.

Moreover, always following the results obtained in [1, 3, 5–7, 10], in the presence of ordinary matter, the superfluid features of the 3D quantum vacuum can be characterized by defining the following Einstein energy–momentum tensor:

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} \tag{26}$$

where ε and p are functions per unit volume expressed in units of pressure and the metric tensor $\eta^{\mu\nu}$ has the space-like signature $(-, +, +, +)$, which leads directly to obtain the quantum behaviour of matter described by the usual Hamilton–Jacobi equations—associated with Klein–Gordon and Schrödinger equations, respectively—which emerge here as a result of the pressure due to the collisions of the virtual particles of the medium, which are linked with the fluctuations of the quantum vacuum energy density. In particular, if one considers the RS processes of creation, the quantum potential associated with the virtual particles of the RS processes of the 3D quantum vacuum may be expressed as:

$$Q = V \frac{p_1 + p_2}{n} = - \frac{\hbar^2 c^2 n^2}{4 \Delta \rho_{qvE}^2 V^2} \left[\nabla^2 \Delta \rho_{qvE} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \Delta \rho_{qvE} \right] + \frac{\hbar^2 c^2 n^2}{8 \Delta \rho_{qvE}^3 V^2} \left[(\nabla \Delta \rho_{qvE})^2 - \frac{1}{c^2} \left(\frac{\partial}{\partial t} \Delta \rho_{qvE} \right)^2 \right], \tag{27}$$

where n is the number of virtual particles-antiparticles of the RS processes in the volume V of the 3D quantum vacuum into consideration. Equation (27) shows that the quantum potential of the vacuum describes the geometry via the pressures p_1 and p_2 that arise by the collisions between the virtual particles-antiparticles populating the vacuum and corresponding to the RS processes. And in this approach, it can be easily shown that the quantum potential of ordinary quantum mechanics is an effect of the more fundamental quantum potential of the quantum vacuum (27).

It must also be emphasized that, as a consequence of the motion of the virtual particles corresponding to the

elementary fluctuations of the quantum vacuum energy density, space–time is filled with virtual radiation with frequency:

$$\omega = \frac{2\Delta\rho_{qvE}V}{\hbar n} \quad (28)$$

In the light of Eqs. (28), the timeless 3D quantum vacuum constituted by virtual particle/antiparticle pairs giving rise to elementary quantum vacuum energy density fluctuations turns out to be characterized by vibratory states, and thus, each material ordinary particle of the Standard Model indeed emerges from opportune oscillations of the vacuum.

Moreover, in the light of the formalism (26)–(28), we can say that the action of the variable quantum vacuum energy density as a stochastic inhomogeneous field is owed to the collisions between the virtual particles–antiparticles populating the vacuum and corresponding to the RS processes. Elementary processes of collision of the virtual particles of the superfluid vacuum, and, therefore, the pressures appearing in the quantum potential of the vacuum (27), can be considered as the real ultimate origin of the action of the variable quantum vacuum energy density as a stochastic inhomogeneous field, and the microscopic counterpart of the action of the variable quantum vacuum energy density as a stochastic inhomogeneous field.

After seeing the physical justifications of the postulates of our model of 3D quantum vacuum and the fundamental mathematical formalism which emerges from them, let us see in major detail the link between the virtual particles of the superfluid vacuum and the ordinary real particles of our manifest world. Our model of 3D timeless quantum vacuum suggests in fact a link, inside a unifying approach, between the virtual particles of the 3D timeless superfluid vacuum (ruled by a mathematical formalism which is summarized in the previous pages) on one hand, and the real particles of the Standard Model on the other hand. In this regard, in our model, a compatibility with the description of Standard Model elementary particles can be obtained, by considering fruitful considerations made by Chiatti and Licata in [26, 27] about the background of transactional processes in reference to quantum jumps of elementary particles. In the light of the mathematical formalism developed by Licata and Chiatti in [26, 27], in our model of 3D timeless quantum vacuum, one introduces an internal wave function factor (inaccessible by direct observation) $\phi(\tau')$, which is real and harmonic in an internal time variable τ' of the vacuum background, null at the boundary, and outside of the interval $\left[-\frac{\vartheta_0}{2}, \frac{\vartheta_0}{2}\right]$, where $c\vartheta_0 \approx 10^{-13} \text{ cm}$, and which obeys the following general equation:

$$\begin{cases} -\hbar^2 \frac{\partial^2}{[(2\pi\tau')]^2} \phi(\tau') = (mc^2)^2 \phi(\tau') \text{ for } \tau' \in [-\vartheta_0/2, \vartheta_0/2] \\ \phi(\tau') = 0 \text{ otherwise} \end{cases} \quad (29)$$

where $m = \frac{V \cdot \Delta\rho}{c^2}$ is the mass of the virtual sub-particles of the 3D quantum vacuum, which is determining the skeleton, namely the “bare” state of the mass of the real particle. From Eq. (29), it follows that the virtual particles of the 3D quantum vacuum generate the usual real elementary particles when their mass satisfies the following relation:

$$mc^2 = n' \frac{\hbar c^2}{V^2 \vartheta_0} \quad (30)$$

Equation (30), where $n' = 0, 1/2, 1, 3/2, \dots$, is an integer for odd solutions, a half-integer for even solutions, provides the condition that links the geometry of the 3D quantum vacuum characterized by virtual particle/antiparticle pairs with the quantum jumps characteristic of an elementary particle of the Standard Model described by the scale $c\vartheta_0 \approx 10^{-13} \text{ cm}$ and governed by the ordinary quantum laws.

Here, in epistemological affinity with the Licata–Chiatti model, the crucial idea is that a real quantum massive particle of the Standard Model is given by the sum of the nascent “bare” mass produced by the virtual particles of the 3D quantum vacuum, and a term ε/c^2 associated with the self-interaction, which, if the particle created at the interaction vertex is subjected to gauge fields, is given by relation:

$$\frac{\varepsilon}{c^2} = -\frac{e}{c^2} \int \bar{\phi} \gamma^\mu A_\mu \phi dV \quad (31)$$

where A_μ is the self-field, ϕ is the spinor satisfying the ordinary Dirac equation:

$$i\hbar\gamma^\mu \partial_\mu \phi = mc\phi \quad (32)$$

By computing the integral appearing in Eq. (31) into a volume of diameter \hbar/mc around the vertex, one finds that in the particle rest frame of reference, this turns out to be a Coulomb integral whose order of magnitude is about αmc^2 , where α is the fine structure constant. More precisely, the appearance of a subatomic particle of the Standard Model corresponds to the following sequence of processes:

The particle is initially massless (namely corresponds to the ground state of the 3D quantum vacuum).

2. Its localization in an interaction event, for a duration of ϑ_0/n' , requires an amount of energy equal to the ratio of \hbar and such duration; this ratio is the rest energy in Eq. (30).

Thus, the fluctuations of the quantum vacuum associated with the appearance of a virtual particle take place.

3. The particle self-interacts for a duration of \hbar/mc^2 , and, therefore, on a scale of lengths equal to \hbar/mc . The total mass of the real particle is, therefore, the sum of the “bare” mass associated with the virtual particles of the 3D quantum vacuum and the ϵ/c^2 mass derived from this self-interaction.

If $n' = 0$, only the term of self-interaction survives. As regards electrons, the self-interaction turns out to be essentially electrostatic with energy $e^2/c\vartheta_0$ where e is the elementary electric charge. If one equates $e^2/c\vartheta_0$ to the rest energy of the electron, one finds $c\vartheta_0$ is the classical radius of the electron and, thus, ϑ_0 becomes a new constant of nature which corresponds to the time taken by light to travel a distance equal to the classical radius of the electron and this constant is a fundamental property of the 3D quantum vacuum that manifests itself in external time only as a minimum duration ϑ_0/n' (and coincides, at least by a factor of 2/3, with the chronon of Caldirola’s model of electron [28, 29]).

In summary, we can, therefore, conclude that, inside the 3D quantum vacuum model developed by the author, the RS processes of virtual particles of the 3D quantum vacuum ruled by the laws (22)–(28) generate the usual real elementary particles when their mass satisfies Eq. (30) inside the length scale $c\vartheta_0 \approx 10^{-13} \text{ cm}$. In other words, a real subatomic particle of the Standard Model can be seen as a result of the sum of the “bare” mass produced by the virtual sub-particles of the 3D quantum vacuum and a term ϵ/c^2 associated with the self-interaction given by Eq. (31)).

Finally, as regards the interpretation of the ordinary matter predicted by the Standard Model inside our model of 3D timeless quantum vacuum, another relevant result lies in the fact that here Higgs boson cannot be considered a primary physical entity, a primary physical concept, but is an emerging entity from the interplay of opportune fluctuations of the quantum vacuum energy density. In other words, Higgs boson does not exist as an irreducible entity of reality, but is a structure derived from something more fundamental, that is to say from appropriate specific fluctuations of the quantum vacuum energy density.

By following the treatment of [8], one starts by considering a general scalar potential in the picture of a 3D timeless quantum vacuum, invariant under the Standard Model gauge group, having the following form:

$$V = \lambda_C C^4 + \lambda_I s_I^4 + \lambda_{RI} s_I^2 s_R^2 + \lambda_R s_R^4 + \lambda_{IC} C^2 s_I^2 + \lambda_{RC} C^2 s_R^2, \tag{33}$$

where $C = \begin{pmatrix} \psi_{Q,i} \\ \varphi_{Q,i} \end{pmatrix}$ is the wave function at two components describing the probability of the occurrence of a creation/destruction event for a quantum particle (determined by an opportune change $\Delta\rho_{qvE}$ of the quantum vacuum energy density) in a point event x , s_R and s_I are the real and

imaginary parts of a singlet field S which is a function of the fluctuations of the quantum vacuum energy density, λ_C is the coupling associated with the wave function C , λ_R is the coupling associated with the real part s_R of the singlet field S , λ_I is the coupling associated with the imaginary part s_I of the singlet field S , and the following relations hold regarding the various couplings:

$$\lambda_R = \lambda_S + \lambda'_S + \lambda''_S \tag{34}$$

$$\lambda_I = \lambda_S + \lambda'_S - \lambda''_S \tag{35}$$

$$\lambda_{RI} = 2(\lambda_S - 3\lambda'_S) \tag{36}$$

$$\lambda_{RC} = \lambda_{SC} + \lambda'_{SC} \tag{37}$$

$$\lambda_{IC} = \lambda_{SC} - \lambda'_{SC}. \tag{38}$$

Here, by approximating λ_R as:

$$\lambda_R = \beta_{\lambda_R} \ln \frac{|s_R|}{s_0} \tag{39}$$

where β_{λ_R} is the always positive beta function of λ_R , and s_0 is the scale at which λ_R becomes negative, in the basis (C, s_R) , one finds that if $\frac{\lambda_{RC}}{\beta_{\lambda_R}} \ll 1$, the square quantum vacuum energy density matrix for CP-even fields given by:

$$\begin{pmatrix} 2v^2 \lambda_C & -\sqrt{2}v^2 \sqrt{\lambda_C |\lambda_{RC}|} \\ -\sqrt{2}v^2 \sqrt{\lambda_C |\lambda_{RC}|} & |\lambda_{RC}|v^2 + \frac{2\beta_{\lambda_R} \lambda_C v^2}{|\lambda_{RC}|} \end{pmatrix}, \tag{40}$$

Where:

$$v = \frac{s_0}{e^{1/4}} \sqrt{\frac{|\lambda_{RC}|}{2\lambda_C}} \tag{41}$$

leads to the following eigenvalues for the energy density of the quantum vacuum:

$$\rho_h^2 \cong v^2 \left(2\lambda_C - \frac{\lambda_{RC}^2}{\beta_{\lambda_R}} \right), \tag{42}$$

$$\rho_s^2 \cong v^2 \left(2 \frac{\beta_{\lambda_R} \lambda_C}{|\lambda_{RC}|} + \frac{\lambda_C^2}{\beta_{\lambda_R}} + |\lambda_{RC}| \right), \tag{43}$$

while the CP-odd quantum vacuum energy density is:

$$\rho_s^2 \cong v^2 \left(2 \frac{\lambda_C \lambda_{RI}}{|\lambda_{RC}|} + \frac{\lambda_{IC}}{2} \right) \tag{44}$$

Instead, when $\frac{\lambda_{RC}^2}{\beta_{\lambda_R}} \ll 1$ is not true, the eigenvalues of the density matrix (22) for the energy density of quantum vacuum are:

$$\rho_h^2 \cong v^2 (2\lambda_C + |\lambda_{RC}| + \beta_{\lambda_R}) \tag{45}$$

$$\rho_s^2 \cong v^2 \left(2 \frac{\beta_{\lambda_R} \lambda_C}{|\lambda_{RC}|} + \beta_{\lambda_R} \right) \tag{46}$$

which imply that the real singlet s_R derives from a quantum vacuum energy density which is associated with a mass lighter than the Higgs boson. In the light of Eqs. (42)–(46), the mixing action of the Higgs boson in the production of the mass of Standard Model particles cannot be considered as a fundamental physical entity, but derives from more fundamental entities, represented by opportune physical values of the quantum vacuum energy density, given just by Eqs. (42)–(46). In other words, in this picture, at a fundamental level, the Higgs boson does not exist as a physical reality: the action of the Higgs boson is only an emerging reality in the sense that is determined by the interplay of opportune fluctuations of the quantum vacuum energy density. Moreover, the treatment of particle physics based on the general scalar potential (33) of the timeless 3D quantum vacuum described by creation/annihilation events of quantum particles occurring in correspondence to quantum vacuum energy density fluctuations, on the basis of the results obtained by the author and Sorli in [8], leads to define a new effective lagrangian, which can be defined as “beyond Standard Model lagrangian”, which, on one hand, is invariant under gauge transformations, which guarantees the renormalizability of the associated quantum field theory (in agreement with the original Weinberg–Glashow–Salam theory [30–33]) and, on the other hand, allows electroweak symmetry breaking to emerge dynamically via dimensional transmutation determined by the singlet couplings associated with the singlet field S which is a function of the changes and fluctuations of the energy density of the timeless 3D quantum vacuum, thus removing the global minimum of the Standard Model Higgs potential. This “beyond Standard Model lagrangian” is defined as:

$$\begin{aligned} L_{effective}^{beyondSM} = & -\frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \\ & - g \bar{C}_1 \gamma^\mu \bar{W}_\mu C_1 - g \sin \theta_W A_\mu \sum_j \bar{C}_j \gamma^\mu Q_j C_j \\ & - \frac{g}{2 \cos \theta_W} J_\mu^Z Z^\mu + \frac{1}{2} \partial_\mu s_R \partial^\mu s_R \\ & + (\lambda_R + s_R)^2 \left\{ \frac{g^2}{4} W_\mu^+ W^\mu + \frac{g^2}{8 \cos^2 \theta_W} Z_\mu Z^\mu \right\} \\ & - (\lambda_C C^4 + \lambda_I s_I^4 + \lambda_{RI} s_I^2 s_R^2 + \lambda_R s_R^4 + \lambda_{IC} C^2 s_I^2 + \lambda_{RC} C^2 s_R^2), \end{aligned} \tag{47}$$

where C_1 is the wave function of the quantum vacuum determining the appearance of left-handed electron and neutrino in the state $\psi_1 = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$:

$$C = \exp \left\{ i \frac{\sigma_i}{2} \theta_i(x) \right\} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \lambda_R + s_R(x) \end{pmatrix} \tag{48}$$

is the wave function of the quantum vacuum associated with the occurrence of creation/destruction events which trigger the electroweak symmetry breaking,

$$\bar{W}_{\mu\nu} = \frac{\sigma_i}{2} W_{\mu\nu}^i W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \epsilon^{ijk} W_\mu^i W_\nu^k \tag{49}$$

Here, \bar{W}_μ being the mass less isovector triplet (for $SU(2)_L$), g being the coupling of the isospin current \bar{J}_μ of the fermions to \bar{W}_μ , B_μ being the mass isosinglet (for $U(1)_Y$), g' being the coupling of the hypercharge current of the same fermions to B_μ , θ_W being the weak mixing angle that in the electroweak symmetry breaking regime is determined by the coupling λ_R as

sociated with the real part of the singlet field S on the basis of equation:

$$\lambda_R = \frac{75}{g \sin \theta_W} \tag{50}$$

In the light of the effective lagrangian (47) which determines the electroweak Symmetry Breaking dynamically via dimensional transmutation by means of the couplings associated with the singlet field S of the timeless 3D quantum vacuum, the Vacuum Expectation Value of the scalar sector then induces the Standard Model Higgs Vacuum Expectation Value through a portal coupling in agreement with the results obtained by Gabrielli and collaborators in [34].

A relevant merit of the beyond Standard Model lagrangian (47) lies in the fact that it allows us to face in a significant and coherent way different problems encountered by the Standard Model (in connection to the electroweak symmetry breaking and the possibility to remove the global minimum at energies $\approx 10^{26} GeV$) in the picture of the same 3D timeless quantum vacuum characterized fluctuations of its energy density corresponding to elementary processes of annihilation/creation of quanta. However, as we have seen, these significant results are obtained at the price of the introduction of several physically important parameters, namely the couplings associated with the wave function of the quantum vacuum (but it is also worth to mention that, among these, as regards the generation of masses associated with the spontaneous symmetry breaking, at the end, the most fundamental parameter is, indeed, the coupling associated with the real component of the singlet field S).

Dark matter in the three-dimensional quantum vacuum energy density model

The next step regards the interpretation and explanation of the dark matter. How and in what sense, in our model of timeless 3D quantum vacuum, does dark matter emerge from more fundamental quantum vacuum energy density fluctuations? This is the issue which we face in this chapter.

As well known, to “reproduce” the anomalous dynamics of galaxies and of galaxy clusters, modern cosmology assumes that there is a large deficit of matter at different astronomical scales in the form of unknown particles, which are completely outside the rigid schemes of the Standard Model of the strong and electroweak interactions, and are referred to as “dark matter” particles. According to the common interpretation, the most important contribution to dark matter comes from particles which are heavy and are subjected to gravity, have not got electric charge and interact weakly, are stable, and were generated in the interactions which followed the Big-Bang and survived till today.

To provide a satisfactory treatment of dark matter, an appropriate approach could be to build models which provide a natural extension of the Standard Model. The final theory of dark matter will not only have to introduce new entities which can reproduce the phenomena on a cosmological scale that we have mentioned above, such as the rotation of galaxies, the mass of galaxy clusters, the orbital velocity of stars within a galaxy, the anisotropy of the cosmic microwave background, and the distribution of galaxies on a large scale, but also at the same time will have to describe the ordinary baryonic matter and all its complex interactions (in a way compatible with experimental data and the Standard Model).

In current physics, in particular, a specific class of theories, the so-called supersymmetric extensions of the Standard Model, would seem to predict the existence of particles that have the features of dark matter. The supersymmetric extensions of the Standard Model are based on the introduction of a peculiar geometry, the idea that each elementary particle of physics has a “superpartner” with similar characteristics but different geometry and different quantum properties: to each particle with a half-integer spin of the Standard Model, a new particle is associated with an integer spin and vice versa. In other words, one has an exotic symmetry which makes possible the exchange of fermions and bosons. Superpartners have a spin decreased by $\frac{1}{2}$ with respect to the original particle. The introduction of this symmetry has an immediate practical effect of considerable interest: in every supersymmetric theory, electrically neutral, stable, long-lived, and weakly interacting particles with ordinary matter are born naturally, just the kind of particles that could be dark matter. However, supersymmetry requires

as a counterpart the introduction of a very large number of completely undefined and arbitrary parameters and this implies the existence of specific supersymmetric models which have extremely distant and sometimes inconsistent predictions [35].

In fact, it must be remarked that, although supersymmetry introduces interesting perspectives in the solution of the hierarchy problem, if, in nature, it was valid and rigorously satisfied, one would exactly know what the masses of all the superpartners are, but, until now, no superparticle has been observed. And another important problem of supersymmetry lies in its difficulty to find a model devoid of contradictions which contemplates supersymmetry and is in agreement with experimental results. In fact, to reproduce the experimental results, in this kind of theories a breaking of supersymmetry, in particular at the electroweak scale, must be invoked but, in this picture, when a breaking of supersymmetry is reached, an anarchy occurs in the sense that the most part of these supersymmetric models predict decays which have not been seen in nature or which turn out to be very improbable. Some of the results also seem to suggest that the electroweak scale turns out to be not actually protected by supersymmetry, that, since the scale of supersymmetry breaking is high, supersymmetry plays no role in stabilising the electroweak hierarchy. In the light of considerations of this kind, we can conclude that, if it is true that the search for satisfactory supersymmetric extensions of the Standard Model has been one of the primary objectives of the last generation of accelerators, really there is currently no real evidence that supersymmetry is the path chosen by Nature to be able to explain and reproduce the existence of dark matter [35].

In the theory, we are examining, in the present paper, the symmetry postulated by the Standard Model between fermions and bosons with equal mass is replaced with the idea that the energy E and the mass m of a given particle are made out of the same “stuff”, represented by the timeless 3D quantum vacuum: energy and mass of each material particle are symmetric to the diminishing of energy density of quantum vacuum ρ_{qvE} in the centre of a given particle. In other words, in our model of timeless 3D quantum vacuum, only a supersymmetry exists in nature, namely between mass of material objects and the variable quantum vacuum energy density, which is expressed by equation:

$$E = mc^2 = \Delta\rho_{qvE}V, \quad (51)$$

which directly derives from Eq. (2). Therefore, in our model of timeless 3D quantum vacuum, both ordinary matter and dark energy and dark matter can be interpreted as different aspects of the same 3D quantum vacuum, more precisely as different specific fluctuations of the variable quantum vacuum energy density.

If ordinary matter represented by the material particles of the Standard Model emerges as opportune diminutions of the quantum vacuum energy density corresponding to opportune elementary RS processes of creation/annihilation of virtual particles with mass satisfying Eq. (29) inside the length scale $c\theta_0 \approx 10^{-13} \text{ cm}$, thus being given by the sum of the “bare” mass produced by the virtual sub-particles of the 3D quantum vacuum and a term ϵ/c^2 associated with the self-interaction given by Eq. (30), in a picture where the fundamental arena is a perfect superfluid medium described by the Einstein energy–momentum tensor (26), and if dark energy represents itself structured energy of 3D quantum vacuum on the basis of Eq. (14) which produces the curvature of space in agreement with the results of general relativity, in analogous way, here also dark matter is not a primary physical reality but emerges as the result of more fundamental fluctuations of the quantum vacuum energy density, in particular in the range of the ultra-low frequencies. As we will show in the next pages, our model of 3D quantum vacuum predicts that opportune fluctuations of the quantum vacuum energy density, in the range of the ultra-low frequencies, can explain the observed flattening of the orbital speeds of the spiral galaxies in agreement with experimental results, which show that the orbital speeds of the spiral arms of the galaxies tends to be almost constant as distance increases from the galactic core (see, for example, the references [36–40] for a review of these experimental results). Here, the difference between ordinary matter and dark matter lies in the fact that, while ordinary matter subjected to electromagnetic interactions derives from a variable energy density giving rise to excited states of the 3D physical vacuum which act as a perfect fluid medium and their mass the sum of the “bare” mass produced by the virtual sub-particles of the 3D quantum vacuum and a term ϵ/c^2 associated with the self-interaction given by Eq. (31); instead, dark matter derives from a variable energy density giving rise to excited states of the 3D quantum vacuum which cannot act as a perfect superfluid medium, in the sense that is characterized by a certain viscosity, and thus correspond to RS processes which produce only the “bare” mass of the virtual particles (namely without a self-interaction which is able to make the particles visible). The viscosity of the vacuum makes dark matter as an emerging reality from a sort of polarization of the 3D quantum vacuum.

To formulate a mathematical treatment of these concepts regarding the interpretation of the dark matter, and to check how they can reproduce the flattening of the orbital speeds of the spiral galaxies in agreement with experimental results, taking account of the results of Sbitnev in [17], we start from the generalization of the Einstein energy–momentum tensor (8) given by the following relation:

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu + p\eta^{\mu\nu} + \Pi^{\mu\nu} \tag{52}$$

where $\Pi^{\mu\nu}$ is a term describing the viscosity of the vacuum.

By considering the case of an incompressible vacuum, the conservation law of the energy–momentum tensor (52) takes the form:

$$\partial_\mu (VT^{\mu\nu}/n) = \partial_\mu \left(\frac{V(\epsilon + p)}{n} \gamma u^\mu u^\nu \right) + \partial^\nu Q + \partial_\mu \left(\frac{V\mu(t)}{n} \right) \pi^{\mu\nu} = 0 \tag{53}$$

where

$$\pi^{\mu\nu} = c(\partial^\mu u^\nu + \partial^\nu u^\mu) - c\frac{2}{3}\partial^\mu u_\mu \eta^{\mu\nu} \tag{54}$$

Here, Q corresponds to the quantum potential (27) which describes the geometry via the pressures that arise by the collisions between the virtual particles–antiparticles populating the vacuum and corresponding to the RS processes of creation occurring in the 3D quantum vacuum, and we have introduced the parameter $\mu(t)$, a fluctuating quantity about zero, which describes the dispersion of the viscosity average. In this picture, essentially fluctuations of the viscosity about zero describe exchanging energy of the orbital rotation with the zero-point fluctuations of the physical vacuum on the ultra-low frequencies. In the light of the results obtained by Sbitnev in [17, 41], one has the possibility to consider here a relativistic form of the Navier–Stokes-type equation for the excited states of the 3D quantum vacuum:

$$\frac{V(\epsilon + p)}{n} \gamma \left(\frac{1}{c} \frac{\partial v}{\partial t} + (\vec{v} \cdot \nabla \vec{v}) \right) + \nabla Q - \frac{v}{c} \frac{\partial Q}{\partial t} + \frac{\partial(V\mu(t)/n)}{\partial t} \cdot (\pi^{0,i} - v\pi^{0,0}) + \frac{V\mu(t)}{n} (\partial_\mu \pi^{\mu,i} - \vec{v} \partial_\mu \pi^{\mu,0}) = 0 \tag{55}$$

whose non-relativistic limit is:

$$\frac{\Delta \rho_{qvE_0} V}{c^2 n} \left(\frac{\partial \vec{V}}{\partial t} + \frac{\nabla V^2}{2} + \vec{\omega} \times \vec{V} \right) = \nabla Q + \frac{V\mu(t)}{n} \nabla^2 \vec{V}, s \tag{56}$$

where $\Delta \rho_{qvE_0}$ is the change of the quantum vacuum energy generating the appearance of matter at a rest mass, $\vec{V} = c\vec{v}$ is the real velocity of the non-relativistic fluid associated with the fluctuations of the quantum vacuum, $\gamma = \left(1 - \frac{v^2}{c^2} \right)^{-1/2}$, and we have defined the vorticity $\vec{\omega} = \nabla \times \vec{V}$ of the vacuum. By making the curl operator of Eq. (56), one finds:

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{\omega} \cdot \nabla) \vec{V} = \frac{\mu(t)c^2}{n\Delta \rho_{qvE_0}} \nabla^2 \vec{\omega} \tag{57}$$

where the kinetic viscosity $\frac{\mu(t)c^2}{n\Delta \rho_{qvE_0}}$ has the physical dimension of *length²/time*.

The physical meaning of the viscosity of the vacuum is the following. The viscosity of the 3D quantum vacuum produces a polarization of the vacuum in the sense that it corresponds to RS processes which can be associated to the appearance of virtual particles of the “bare” mass $m = \frac{V\Delta \rho}{c^2}$

in a background of elementary “perturbative” mass $\frac{\mu\hbar}{n_p^2\Delta\rho_{qvE_0}}$ determined by a “perturbative” fluctuation of the quantum vacuum energy density given by relation.

$$\Delta\rho_{perturbative} = \frac{\mu\hbar c^2}{nV_p^2\Delta\rho_{qvE_0}} \tag{58}$$

This means that the regions of the vacuum characterized by the viscosity μ can be described by an internal wave function factor (inaccessible by direct observation) $\phi(\tau')$, which is real and harmonic in an internal time variable τ' of the vacuum background, which obeys the following general equation:

$$-\hbar^2 \frac{\partial^2}{[2\pi\tau']^2} \phi(\tau') = \left(V\Delta\rho_{qvE_0} + \frac{\mu\hbar c^2}{n_p^2\Delta\rho_{qvE_0}} \right)^2 \phi(\tau') \tag{59}$$

Equation (59) ruling the appearance of the skeleton mass of virtual particles of the vacuum characterized by viscosity can be considered a sort of generalization of Eq. (29) for the excited states of the vacuum which do not behave as a perfect fluid.

Now, in the light of Eqs. (52)–(59), the excited states of the 3D quantum vacuum in the presence of viscosity are characterized by the formation of vortices. In particular, by considering a vortex tube having the cross section oriented along the z-axis and with its centre placed in the coordinate origin of the plane (x;y), Eq. (57) may be rewritten as:

$$\frac{\partial\omega}{\partial t} = \frac{\mu(t)c^2}{n\Delta\rho_{qvE_0}} \left(\frac{\partial^2\omega}{\partial r^2} + \frac{1}{r} \frac{\partial\omega}{\partial r} \right) \tag{60}$$

Namely:

$$\frac{\partial\omega}{\partial t} = \frac{\Delta\rho_{perturbative} V_p^2}{\hbar} \left(\frac{\partial^2\omega}{\partial r^2} + \frac{1}{r} \frac{\partial\omega}{\partial r} \right) \tag{61}$$

whose solution reads:

$$\omega(r, t) = \frac{\Gamma}{\Sigma(t)} \cdot \exp\left(-\frac{r^2}{\Sigma(t)}\right), \tag{62}$$

and thus, the orbital speed becomes:

$$V(r, t) = \frac{\Gamma}{2r} \left(1 - \exp\left[-\frac{r^2}{\Sigma(t)}\right] \right), \tag{63}$$

where Γ is an integration constant and:

$$\Sigma(t) = 4 \left(\int_0^t \frac{\mu(t)c^2}{n\Delta\rho_{qvE_0}} dt' + \sigma^2 \right) \tag{64}$$

namely

$$\Sigma(t) = 4 \left(\int_0^t \frac{\Delta\rho_{perturbative} V_p^2}{\hbar} dt' + \sigma^2 \right) \tag{65}$$

σ being an arbitrary constant, such that the denominator is always positive.

In particular, if, for sake of simplicity, one makes the choice:

$$\mu(t) = \mu \cos(\Omega t), \tag{66}$$

one obtains:

$$\Sigma(t) = 4 \left(\frac{\mu(t)c^2}{n\Omega\Delta\rho_{qvE_0}} \right) \sin(\Omega t) + \sigma^2, \tag{67}$$

namely:

$$\Sigma(t) = 4 \left(\frac{\Delta\rho_{perturbative} V_p^2}{\hbar\Omega} \right) \sin(\Omega t) + \sigma^2, \tag{68}$$

where Ω is an oscillation frequency and μ is chosen equal to $n\hbar/2V$. The vorticity (62) and the orbital speed (63) oscillate about some average values limited by the constant σ [15–18, 41]. It looks as if the vortex is pulsating in time, as a consequence of the annihilation-creation processes of the virtual particle–antiparticle pairs. Moreover, because of the fact that averaged in time the viscosity coefficient vanishes, but its dispersion is not zero, the vortex can live infinitely long and the vortex radius trembles. The trembling is caused by the exchange of the vortex energy with the quantum vacuum fluctuations and provides just an infinite lifetime of the vortex.

The formalism of the excited states of the quantum vacuum, based on Eqs. (52)–(68), allows us to provide a suggestive description of the motion of the spiral galaxies in a picture where dark matter does not exist as a primary physical reality, but can be assimilated to the polarization of the virtual particles of the 3D quantum vacuum produced by its viscosity, which acts through creation of self-organized vortices in the vacuum. In this picture, it is the dance of the vortices produced by the polarization of the 3D quantum vacuum that determines a stabilization of the orbital rotation of the arms of spiral galaxies in agreement with experimental results (which show that, if the distance from the core of the galaxy increases, the orbital speeds of galaxies remains at a constant level or even increases). In fact, if one makes a simulation of the vortex one can get that it exhibits a spiral form which looks almost like a galaxy, which has a core that does not exceed the radius $r^* \approx 5,29 \cdot 10^{-11} m$ where the point of inflection of the orbital velocity takes place and an agglomeration of the slowly rotating matter emerges. The stars in the galaxy rotate around the galactic core with almost constant speed

even being located far from the galactic core. The orbital speed of stars and gas, given by (63), is rising inside the core and reach almost constant level outside the core in agreement with the results obtained by Sbitnev in [17, 41].

Taking account of the results in [17], in our model, one can assume a wide spectrum of the viscosity coefficients exists which is discrete with equidistant position of each component and is condensed in the point $\Omega = 0$:

$$\Sigma_n(t) = 4 \left(\frac{\mu(t)c^2}{n\Omega_n^2 \Delta\rho_{qvE_0}} \right) \sin(\Omega_n t) + \sigma_n^2, \quad (69)$$

Namely:

$$\Sigma_n(t) = 4 \left(\frac{\Delta\rho_{perturbative} V_p^2}{\hbar\Omega_n^2} \right) \sin(\Omega_n t) + \sigma_n^2 \quad (70)$$

which means that the strongest contribution to the vorticity generates modes with frequencies close to zero. On the basis of Eq. (70), one obtains the following relations for the vorticity and the orbital speed:

$$\omega(r, t) = \frac{\Gamma}{N} \sum_{n=1}^N \frac{1}{\Sigma_n(t)} \exp\left(-\frac{r^2}{\Sigma_n(t)}\right) \quad (71)$$

$$V(r, t) = \frac{\Gamma}{2rN} \sum_{i=1}^N \left(1 - \exp\left[-\frac{r^2}{\Sigma_n(t)}\right] \right) \quad (72)$$

Here, to reproduce in the correct way the rotation curves of the spiral galaxies, the crucial point is the following. If one chooses $\Gamma = 10^{27} m^2/s$, $\Omega_n = 10^{-11} s^{-1}$, and $N = 25$ (in such a way that the parameter $\sigma_n = 4c/\Omega_n$ ranges from 10,000 to 300,000 light years, which covers the diameter of the ordinary spiral galaxies), through relations (71) and (72), one can explain in a satisfactory way the rotating motion of the spiral galaxies reproducing the observed flattening of the orbital speeds. More precisely, one can describe thermalization of the vorticity and angular velocities of spiral galaxies as a result of the energy interplay of the 3D quantum vacuum characterized by viscosity, which produces a polarization in terms of “bare” skeleton mass which is not subjected to electromagnetic interactions, in regime of ultra-low frequencies (which, in terms of wavelengths, cover just almost entirely the diameter of the ordinary spiral galaxies $\lambda = \frac{c}{\omega} \approx 5 \cdot 10^5 \text{ lightyears}$). In other words, fluctuations of the ω viscosity about zero leads to a polarization of the 3D quantum vacuum which describes exchanging energy of the orbital rotation with the zero-point fluctuations of the 3D quantum vacuum on the ultra-low frequencies. In fact, in the light of Eq. (72), the evolution of the orbital speed of the spiral galaxies turns out to keep its own general form for the whole time

interval ranging from 100 to 110 light years, but exhibits small fluctuations in time, resembling the breathing of the galaxy. The ultimate cause of this breathing is represented just by the interplay with the “perturbative” fluctuation of the quantum vacuum energy density (58), determined by the viscosity $\mu(t)$ of the vacuum, in regime of ultra-low frequencies.

In summary, we can say that the polarization of the vacuum associated with the “perturbative” mass $\frac{\mu\hbar}{n_l^2 \Delta\rho_{qvE_0}}$, in other words with the “perturbative” fluctuation of the quantum vacuum energy density (58), determined by the viscosity $\mu(t)$ given by Eq. (66), provokes an exchange of the energy of the rotating galactic matter with the quantum vacuum fluctuations, which generates, in regime ultra-low frequencies, by virtue of the term $\Sigma_n(t)$, an effect that looks as a breathing of the galaxy. It is the presence of the “perturbative” mass $\frac{\mu\hbar}{n_l^2 \Delta\rho_{qvE_0}}$ originated by the “perturbative” fluctuation of the quantum vacuum energy density (58) which makes stars in the galaxy rotate around the galactic core with almost constant speed even being located far from the galactic core, in agreement with experimental observations. It is this sort of breathing of the galaxy associated with the polarization of the 3D quantum vacuum which is generated by the “perturbative” mass $\frac{\mu\hbar}{n_l^2 \Delta\rho_{qvE_0}}$, corresponding to the “perturbative” fluctuation of the quantum vacuum energy density $\frac{\mu\hbar c^2}{n V_p^2 \Delta\rho_{qvE_0}}$, which allows us to explain the stabilized behaviour of the speed of the arms of spiral galaxies, with increasing distance from the core of the galaxy, compatibly with the experimental observations. And, in this picture, the breathing of the galaxy can be itself associated with the small fluctuations in time of the orbital speed (72), namely manifests itself in variations of the vorticity. This breathing is produced by exchange of the rotation energy of the galaxy with the fluctuations of virtual particles of the vacuum, namely by the polarization of the vacuum generated by its viscosity and associated with the the “perturbative” fluctuation of the quantum vacuum energy density (58), at very low frequencies, that is, at wavelengths commensurate with sizes of the galaxies.

This physically means that here, as regards the explanation of the motion of the spiral arms of galaxies, dark matter does not exist as a primary physical reality, but can be itself considered as an emerging entity which indeed corresponds to the polarization of the vacuum generated by its viscosity on the ultra-low frequencies, and thus, by the “perturbative” mass $\frac{\mu\hbar}{n_l^2 \Delta\rho_{qvE_0}}$, and ultimately by the “perturbative” fluctuation of the quantum vacuum energy (58). In this way, in our approach—although the parameters $\Gamma = 10^{27} m^2/s$, $\Omega_n = 10^{-11} s^{-1}$, and $N = 25$ are introduced ad hoc to obtain the diameter of the ordinary spiral galaxies—specific quantum vacuum fluctuations on the ultra-low frequencies can

explain the observed flattening of the orbital speeds of the spiral galaxies (on radii far exceeding the radius of the galactic core) without invoking the idea that dark matter constitutes a fundamental primary reality in the universe. Our 3D quantum vacuum model introduces, therefore, the relevant perspective that dark matter does not exist as a primary physical reality, but derives just from opportune quantum vacuum energy density fluctuations, associated to virtual pairs of particles–antiparticles, in a polarized 3D quantum vacuum as a consequence of its viscosity. Of course, the price to pay, to provide this interpretation of dark matter, lies in the phenomenological nature of the parameters $\Gamma = 10^{27} m^2/s$, $\Omega_n = 10^{-11} s^{-1}$, and $N=25$ which have to be introduced ad hoc.

We conclude this chapter by observing that the polarization of the 3D quantum vacuum caused by the “perturbative” fluctuation of the quantum vacuum energy density (58) which corresponds to its viscosity may be somewhat compared to what occurs in the vacuum of QED. In fact, if in QED in situations where the excitations of the electromagnetic field are absent, the background of frenetic activity of the vacuum is always present and the properties of the QED vacuum can be interpreted in a heuristic way using concepts such as fluctuations of the vacuum, creation of electron–positron pairs, polarization of the vacuum, similarly also in our theory of 3D timeless quantum vacuum in absence of visible matter, the background of frenetic activity of the vacuum is always present in the form of “perturbative” fluctuations of the energy density of the quantum vacuum, associated with its viscosity, which are able to determine real and observable effects represented by the stabilization of the orbital rotation of the arms of spiral galaxies (the fact that the orbital speeds of galaxies, if the distance from the core of the galaxy increases, remain at a constant level or even increases).

About parallelisms of the here presented model with other attempts to explain dark matter

In this chapter, we propose to frame our key to the explanation of dark matter as a phenomenon emerging from more fundamental processes concerning the energy density of the 3D quantum vacuum within the state of the art on the subject, by making some parallels with current theories on dark matter. Here, we certainly do not pretend to exhaust, or even to deal with due carefulness, all the questions related to the possible links between the theory examined in this article and other attempts to explain dark matter, a fraction moreover than those that could be taken into consideration. The aim is rather to suggest a set of considerations that hopefully can open new perspectives in the debate on the subject.

According to much current research, weakly interacting massive particles (WIMPs) seem to be the best-motivated and the most compelling candidates for cold dark matter, intended as a key ingredient required to explain the formation of large-scale structures (as regards recent developments as regards the theoretical foundations of the WIMPs approach to dark matter, the interested reader may refer, for example, to the papers [42–45]). The paradigm of thermal decoupling, based on applications to cosmology of statistical mechanics and particle and nuclear physics, is enormously successful at making detailed predictions for observables in the early universe, including the abundances of light elements and the cosmic microwave background [46]. The idea of WIMPs means to invoke a paradigm of thermal decoupling where the abundance of dark matter as a thermal relic from the early universe derives uniquely from underlying cosmologically stable dark matter particle properties, whose interactions with Standard Model particles have the effects that for a high enough temperature T , the dark matter is in thermal equilibrium with the primordial thermal bath. In the WIMPs paradigm, the cosmological evolution of the dark matter particle obeys the following Boltzmann equation:

$$\frac{dn_\chi}{dt} + 3H(T)n_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi,eq}^2), \quad (73)$$

where χ is the generic dark matter particle, $\langle\sigma v\rangle$ is the thermally averaged pair annihilation cross section associated with the process of conversion of two dark matter particles into two particles of the Standard Model, and $H(T)$ is the Hubble rate:

$$n_\chi(T) = \int \frac{d^3p}{(2\pi)^3} f_\chi(p, T) \quad (74)$$

is the dark matter number density, and

$$n_{\chi,eq} = g_\chi \frac{M_\chi^2 T}{2\pi} K_2\left(\frac{m_\chi}{T}\right) \quad (75)$$

where f_χ is the dark matter distribution function, g_χ is the number of the internal degrees of freedom of the dark matter particle, m_χ is its mass, and K_2 is the modified Bessel function of the second type. Here, it must be emphasized that the thermally pair averaged cross section, as well as the freeze-out temperature that results from it, practically contain all the operational information about the particle physics framework connected to this specific dark matter particle candidate represented by WIMPs and is expressed as:

$$\langle\sigma v\rangle = \frac{1}{8m_\chi^4 T K_2\left(\frac{m_\chi}{T}\right)^2} \int_{4m_\chi^2}^{\infty} ds \sigma(s) \sqrt{s} (s - 4m_\chi^2) K_1\left(\frac{\sqrt{s}}{T}\right) \quad (76)$$

where s is the entropy density of the universe and K_1 is the modified Bessel function of the first type [47].

If one assumes that during expansion of the universe, where the scale factor increases and the temperature decreases, dark matter continues to be in thermal equilibrium, eventually the temperature tends to drop below the dark matter mass; in this situation, the annihilation rate for dark matter particles, which is linked linearly with the number equilibrium density $n_{\chi,eq}$, enters the so-called “Boltzmann-tail”, namely $n_{\chi,eq} \propto \exp\left(-\frac{m_\chi}{T}\right) m_\chi \ll T$; then, the annihilation rate eventually fall below the universe expansion rate, $H(T)$, leading to the thermal freeze-out of this “cold” relic. Thereafter, the dark matter comoving number density $Y_\chi = \frac{n_\chi}{s}$, s being the entropy density of the universe, is approximately constant.

Moreover, by expressing the dark matter abundance as a fraction of the critical density of the universe, in this paradigm, the thermal relic density may be evaluated as:

$$\Omega_{DM} h^2 \approx 8,76 \cdot 10^{-11} GeV^{-2} \left[\int_{T_0}^{T_f} g_*^{\frac{1}{2}} \langle \sigma v \rangle \frac{dT}{m_\chi} \right]^{-1}, \quad (77)$$

where h is the Hubble expansion rate today, T_f is the freeze-out temperature, and $g_*^{1/2}$ is a function related to the relativistic degrees of freedom of the primordial thermal bath [42].

The WIMP paradigm seems to be an attractive solution of the dark matter issues in the sense that it predicts that for typical weak-scale pair annihilation cross sections, say $\sigma \sim G_F^2 T^2$, with G_F the Fermi constant, and $T \sim m_\chi/20$ the typical freeze-out temperature, and for electroweak-scale mass scales, $m_\chi \sim E_{EW} \sim 200 GeV$, the thermal relic density matches the observed cosmological density. In the WIMP paradigm, the dark matter abundance is set to the observed value by a new physics scale that is well motivated, and by interactions mediated by one of the Standard Model gauge interactions. This implies that concrete realizations of WIMP models have the perspective to be developed in different beyond Standard Model frameworks, accessible to several different search strategies [42].

Now, inside the model of the 3D quantum vacuum proposed in this paper, our key of explanation of the orbital flattening of the arms of spiral galaxies implies that dark matter is not a primary physical reality, but emerges as an effect of the the polarization of the vacuum generated by its viscosity on the ultra-low frequencies, namely by the “perturbative” mass $\frac{\mu \hbar}{n_p^2 \Delta \rho_{qvE_0}}$, which is ultimately generated by the “perturbative” fluctuation of the quantum vacuum energy density (58). Therefore, here, the fundamental perspective is opened that the generic dark matter (WIMP) particle χ is indeed an emerging reality from more

fundamental fluctuations of the quantum vacuum energy density, namely the “perturbative” fluctuation of the quantum vacuum energy density (58) associated with the viscosity of the vacuum and thus to its polarization on ultra-low frequencies. More precisely, we can say that the WIMP particles of number density (74) can be associated with the “bare” skeleton virtual particles of the RS processes of the 3D vacuum, when the vacuum is characterized by a polarization determined by its viscosity and, thus, by the “perturbative” fluctuation of the quantum vacuum energy density (58). In other words, in the expression of the “perturbative” fluctuation of the quantum vacuum energy density (58), the quantity n representing the number of virtual particles in the generic volume V of the 3D vacuum may be assimilated to the dark matter number density (74). The physical correspondence between the number density of WIMP particles and the virtual particles of the 3D quantum vacuum physically means that the former emerges from the “perturbative” fluctuation of the quantum vacuum energy density (58) on the basis of relation:

$$n_\chi(T) = \frac{\mu \hbar c^2}{V l_p^2 \Delta \rho_{perturbative} \Delta \rho_{qvE_0}} \quad (78)$$

Namely:

$$\int \frac{d^3 p}{(2\pi)^3} f_\chi(p, T) = \frac{\mu \hbar c^2}{V l_p^2 \Delta \rho_{perturbative} \Delta \rho_{qvE_0}} \quad (79)$$

As a consequence, by substituting (74) into the fundamental Eq. (59) ruling the behaviour of the 3D quantum vacuum characterized by a degree of viscosity, and thus, by a polarization, this latest equation reads:

$$-\hbar^2 \frac{\partial^2}{[2\pi \tau']^2} \phi(\tau') = \left(V \Delta \rho_{qvE_0} + \frac{\mu \hbar c^2}{l_p^2 \Delta \rho_{qvE_0} \int \frac{d^3 p}{(2\pi)^3} f_\chi(p, T)} \right)^2 \phi(\tau') \quad (80)$$

Moreover, by substituting the number density of WIMP particles (74) into Eq. (60), one finds:

$$\frac{\partial \omega}{\partial t} = \frac{\mu(t) c^2}{\Delta \rho_{qvE_0} \int \frac{d^3 p}{(2\pi)^3} f_\chi(p, T)} \left(\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} \right) \quad (81)$$

which physically means that WIMP particles are strictly related with, and ultimately are generated by, the vortices of the 3D vacuum characterized by a degree of viscosity. In this way, one finds that, in virtue of the solutions (62) and (63) regarding vorticity and orbital velocity, WIMP particles can be considered as possible upper manifestations of more fundamental processes of the 3D quantum

vacuum because of the dependence of the quantity $\Sigma(t)$ on (65) expressed by equation:

$$\Sigma(t) = 4 \left(\int_0^t \frac{\mu(t')c^2}{\Delta\rho_{qvE_0} \int \frac{d^3p}{(2\pi)^3} f_\chi(p, T)} dt' + \sigma^2 \right) \quad (82)$$

In particular, in the regime in which viscosity of the vacuum satisfies (66) and the spectrum of the viscosity coefficients is discrete with equidistant position of each component and is condensed in the point $\Omega = 0$, one can say that WIMP particles are themselves strictly associated with the oscillations of the 3D quantum vacuum as well as fluctuations of the quantum vacuum energy density in terms of relation:

$$\Sigma_n(t) = 4 \left(\frac{\mu(t)c^2}{\Omega_n^2 \Delta\rho_{qvE_0} \int \frac{d^3p}{(2\pi)^3} f_\chi(p, T)} \right) \sin(\Omega_n t) + \sigma_n^2 \quad (83)$$

Equations (78)–(83) show that WIMP particles can be interpreted as upper manifestations of the excited states of the quantum vacuum, which emerge from the polarization of the virtual particles of the 3D quantum vacuum produced by its viscosity, which acts through creation of self-organized vortices in the vacuum. Therefore, if WIMP particles provide a coherent explanation of the stabilization of the orbital rotation of the arms of spiral galaxies, this is simply owed to the fact that, at a fundamental level, the observed flattening of the orbital speeds in agreement with experimental results is ultimately determined by the energy interplay of the 3D quantum vacuum characterized by viscosity, which produces a polarization in terms of a perturbative quantum vacuum energy density which is not subjected to electromagnetic interactions, in regime of ultra-low frequencies.

As a consequence, this re-reading of WIMP particles as an emergence of deepest processes of a 3D quantum vacuum characterized by a viscosity and thus by a perturbative quantum vacuum energy density fluctuation, implies that, in our model of 3D quantum vacuum, the perspective is opened that the so-called “Boltzmann-tail” during the expansion of the universe, as well as the thermal freeze-out of the “cold” relic where Eq. (77) holds, are ultimately determined by the polarization of the 3D quantum vacuum corresponding to its grade of viscosity. In summary, one can say that the theory of WIMPs particles as potential candidates for cold dark matter responsible of the formation of large-scale structures may here be seen as an emerging approach from a more fundamental picture of a 3D quantum vacuum characterized by RS processes creation/annihilation of virtual sub-particles corresponding to elementary fluctuations of the quantum vacuum energy density, when the vacuum itself is polarized as a consequence of its viscosity. According to the author of

this paper, the fact that the WIMPs approach may be interpreted as emergent from a more fundamental model of a 3D quantum vacuum is justified also by the manner in which these two theories treat the weak scale. In fact, the WIMPs approach uses the weak scale as a guiding principle in the sense that it features a weak-scale pair annihilation cross section that can naturally lead to the correct relic density [44]. In analogous way, also our approach of 3D quantum vacuum somewhat uses the weak scale as a guide principle in the sense that it allows electroweak symmetry breaking to emerge dynamically via dimensional transmutation determined by the singlet couplings associated with the singlet field S which is a function of the changes and fluctuations of the energy density of the timeless 3D quantum vacuum, as we have seen in chapter 2.

Moreover, also as regards the WIMPs approaches, it must be underlined that, in the light of the data which we have got, the structure and evolution of a star, such as the increase of energy production or, conversely, the dissipation of the internal energy at a faster rate, are altered by WIMPs, in the sense that an high concentration of WIMPs accumulated in the core of a star activates self-annihilation processes which provide an additional source of energy, provoke the decreasing of temperature and pressure in the stellar core, and increase the life of all the stellar spectral classes, with a more significant contribution to lower mass stars (see, for example, the references [48, 49]). Therefore, in the model proposed in this paper, the perspective is opened that also all these facts regarding the effects of accumulation of WIMPs in the core of a star indeed occur in virtue of more fundamental processes of polarization of the 3D quantum vacuum associated to its viscosity and determined by the perturbative quantum vacuum energy density fluctuation (58).

On the other hand, it must be emphasized that, according to current studies, another important component of dark matter particles, besides WIMPs, is represented by ultra-light fields, such as axions of mass $50 - 1500 \mu\text{eV}/c^2$ [50, 51], which have the effect to perturb the stability of the structure of a star by transporting away the energy generated by the nuclear reactions in a very efficient way and by inducing ultrafast oscillations of stellar configurations which can be described by a perfect fluid and a bosonic condensate [52, 53]. In [54, 55], Brito and collaborators have shown that axions present in the centre of stars can generate matter/dark matter configurations with oscillating geometries at a very rapid frequency, which is a multiple of the axion mass. In [56], Tamburini and Licata have suggested that the ultrafast oscillations of the axion components of dark matter, which occur at a precise multiple of a fundamental frequency that depends on the mass of the axion on the basis of relation:

$$f_m = 2,5 \cdot 10^{14} \frac{m_B c^2}{eV} \text{Hz} \quad (84)$$

could be the possible explanation of the ultrafast periodic spectral modulations found recently by Borra and Trottier [57] in 236 main sequence stars in the sample of 2.5 million spectra of galactic halo stars of the Sloan Digital Sky Survey. In [56], Tamburini and Licata have found that, for the frequency range $0,6070\text{THz} < f < 0,6077\text{THz}$ derived from the spectral modulations, the mass m_B of the axion associated with a precise multiple of the main frequency (84) is in the range $50\mu\text{eV} < m_B < 2,4 \cdot 10^3 \mu\text{eV}$.

In chapter 3, we have explained the observable effects regarding the stabilization of the orbital rotation of the arms of spiral galaxies in terms of elementary fluctuations of virtual particles of the 3D quantum vacuum, namely by the polarization of the vacuum generated by its viscosity and associated with the “perturbative” fluctuation of the quantum vacuum energy (49), at very low frequencies $\Omega_n = 10^{-11} \text{s}^{-1}$ present in Eq. (66). The results of Borra and Trottier, Tamburini, and Licata about the existence of ultrafast

periodic spectral modulations in 236 main sequence stars in the sample of 2.5 million spectra of galactic halo stars of the Sloan Digital Sky Survey, if one does not want to invoke the presence of extraterrestrial civilizations, seem to suggest that the “perturbative” fluctuation of the quantum vacuum energy (58) associated with the viscosity of the vacuum does not determine in this case the very low frequencies $\Omega_n = 10^{-11} \text{s}^{-1}$, and thus, in this regime, Eqs. (66)–(72) do not seem to be valid and applicable. Instead, to reproduce these ultrafast periodic oscillations, one can consider the possibility to extend our model of 3D quantum vacuum characterized by viscosity in such a way that the “perturbative” fluctuation of the quantum vacuum energy (58) associated with the viscosity of the vacuum in this regime produces the fundamental frequency (84), namely:

$$f_m = 2,5 \cdot 10^{14} \frac{\mu \hbar}{n_p^2 \Delta \rho_{qvE_0}} \frac{c^2}{eV} \text{Hz} \quad (85)$$

In the light of Eq. (85), we can say that the axion-like components of dark matter are themselves emerging entities from more fundamental “perturbative” fluctuations of the quantum vacuum energy density which thus have the effect to generate ultrafast oscillations. In other words, while the rotation curves of galaxies may be explained in terms of a perturbative fluctuation of the quantum vacuum energy density in a regime where Eq. (66) is valid, and thus, for the very low frequencies $\Omega_n = 10^{-11} \text{s}^{-1}$, instead, there is the possibility that the ultrafast periodic spectral modulations of 236 main sequence stars in the sample of 2.5 million spectra of galactic halo stars of the Sloan Digital Sky Survey found by Borra and Trottier may be explained in terms of

“perturbative” fluctuations of the quantum vacuum energy density having a different behaviour and thus generating the ultrafast frequencies (85).

Finally, let us see to analyse the possible parallelisms that can be found between our model of dark matter as emerging reality from a “perturbative” fluctuation of the quantum vacuum energy density $\frac{\mu \hbar c^2}{n_p^2 \Delta \rho_{qvE_0}}$ associated with the viscosity of the 3D quantum vacuum and thus to its polarization (which generate ultra-low frequencies oscillations in the case of the orbital rotation of the arms of spiral galaxies), with a recent alternative model suggested by Santos in [58]. In [58], Santos suggested that the flat rotation curves of spiral galaxies are owed to the interaction of gravitational fields with the vacuum quantum fields in the form of a gravitational repulsion between particles and antiparticles, which can be ascribed to a negative gravitational mass (but positive inertial mass) of antiparticles, and has the effect to originate a gravitationally polarizable vacuum via vacuum fluctuations [59]. As regards the interaction between a weak external gravitational field and the QED vacuum, the fundamental equation of Santos’ model is the following:

$$(1 - aK)\nabla^2 \phi(\vec{r}) = \rho_B(\vec{r}) + K \left| \nabla \phi(\vec{r}) \right|^2, \quad (86)$$

where K and a are positive parameters in units $c = 4\pi G = 1$, $\phi(\vec{r})$ is the Newtonian potential, $\rho_B(\vec{r})$ is the baryonic density, and

$$\nabla^2 \phi(\vec{r}) = \rho_B(\vec{r}) + \rho_g(\vec{r}), \quad (87)$$

where $\rho_g(\vec{r})$ is the gravitational vacuum interaction. For a spherically symmetric system, such as a galaxy or cluster of galaxies, Eq. (86) becomes:

$$\frac{d\phi'}{dr} + 2 \frac{\phi'}{r} = (1 - aK)^{-1} \left[\rho_B(\vec{r}) + K\phi'^2 \right] \quad (88)$$

where $-\phi'$ is the gravitational field. The model based on Eq. (88) turns out to be in agreement with experimental observations that “dark matter” appears only near regions with baryonic matter, because it implies that there is no regular solution (both at the origin and at infinity) if $\rho_B(\vec{r}) = 0$ everywhere. In the external region, $r > R$, where $\rho_B(\vec{r}) = 0$, there is a solution regular at infinity, namely:

$$\phi'(r) = \frac{1 - aK}{Kr} \quad (89)$$

which predicts rotation curves which are in agreement with observations. Equation (89) implies that the rotation velocity satisfies relation:

$$v^2 = \frac{1 - aK}{Kr} c^2, \quad (90)$$

and therefore, to obtain a compatibility with rotation speed in the halo of a galaxy which is $v \sim 10^{-3}c$, one has:

$$\frac{1 - aK}{Kr} = 10^{-6} \quad (91)$$

Moreover, the dark mass density in a region without baryonic matter is:

$$\rho_g(\vec{r}) = \frac{1 - aK}{Kr^2}. \quad (92)$$

Santos' formalism flat rotation curves in the haloes of galaxies and a maximum of the dark density at the centre of the galaxy, but does not seem valid for gravitational fields much stronger than those of galaxy haloes such as near individual stars, where the density (92) predicted by the model turns out to be too big [58].

Now, in the model of 3D timeless quantum vacuum suggested in this paper, instead, Eqs. (89)–(92) of Santos' model can be embedded into a more general formalism, in the sense that the gravitational field and the dark mass density are not primary physical realities, but derive from more fundamental fluctuations of the 3D quantum vacuum, namely by a fundamental polarization of the vacuum determined by the perturbative quantum vacuum energy density fluctuations linked with the vacuum viscosity. According to my view, it is the behaviour of the vacuum viscosity which makes, in haloes of galaxies, the gravitational field equal to (89) and the dark mass density without baryonic matter equal to (92), and, at the same time, makes a different behaviour of the dark mass density without baryonic matter with respect to (92) near individual stars.

Indeed, to solve the problem of the too big amount of dark mass near stars like the Sun, in Santos' model, one has to introduce a modification of the dark mass density associated with an ad hoc decreasing function $f(x)$, namely:

$$\rho_g(\vec{r}) = \frac{1 - aK}{Kr^2} \times f(\nabla^2 \phi(\vec{r})) \quad (93)$$

Instead, our model of 3D quantum vacuum has not the need to introduce ad hoc parameters, in the sense that all the behaviours in the different regions of space are linked with the specific deep properties of the vacuum, in particular its features of polarization produced by the grade of viscosity. In this sense, our model of 3D quantum vacuum can be considered more general than Santos' model.

Conclusions

During the first phase of development of general relativity, the rejection of the ether as an ubiquitous special substance has led to assumptions about the existence of until now

unknown substances, called dark matter and dark energy, which have been introduced to reproduce, on one hand, stabilization of the orbital speeds of the spiral arms of galaxies (as well as other phenomena on a cosmological scale, such as the mass of galaxy clusters, the anisotropy of the cosmic microwave background, and the distribution of galaxies on a large scale), and, on the other hand, the discrepancies found in the Hubble's law, namely the law of expanding the universe.

In this paper, we have developed a model of a fundamental background of physical processes in terms of a 3D timeless quantum vacuum based on Planck energy density as universal property defining the ground state of the same flat-space background, which somewhat allows a resurrection of a new form of physical ether endowed with real physical properties where all physical constructions and entities somewhat are emergent from the vacuum. Today, we are assisting to a renowned interest towards the idea of an emergent physics from the same flat-space background in a picture which involves processes of quantum gravity. For example, in a series of recent works [60–63], Consoli has suggested the idea of a physical vacuum represented by a Bose condensate of elementary spinless quanta, where gravity is determined by the density fluctuations of these elementary spinless quanta (and of their scattering length) characterizing the Bose condensate. In our model analysed in this paper, we have made a step ahead by linking this medium to a 3D granular structure characterized by an energy density corresponding to the Planck energy density (intended as a universal property of nature) and, differently from Consoli's model, our approach suggests the fundamental insight that a 3D physical vacuum is postulated without the need to postulate the consideration of other more fundamental processes, involving more fundamental particles (such as the Higgs mechanism that is at the base of spontaneous symmetry breaking in the Standard Model). In our model, gravitational phenomena have origin primarily in the changes of the energy density of the same flat three-dimensional background space and the Planck energy density can be considered as the ground state of the same physical flat-space background; the appearance of material objects and subatomic particles corresponds to changes of the quantum vacuum energy density and, thus, can be considered as the excited states of the same physical flat-space background.

In summary, our model of 3D timeless quantum vacuum, indeed, introduces the perspective of a new fundamental ether to build theoretical physics, which can lead to significant satisfactory results regarding the interpretation of the things existing in universe. On one hand, in our model, ordinary matter represented by the material particles of the Standard Model emerges as a result of opportune diminutions of the quantum vacuum energy density corresponding to opportune elementary RS processes of creation/

annihilation of virtual particles with mass satisfying Eq. (29) inside the length scale $c\vartheta_0 \approx 10^{-13} \text{ cm}$, thus being given by the sum of the “bare” mass produced by the virtual sub-particles of the 3D quantum vacuum and a term ε/c^2 associated with the self-interaction given by Eq. (31), in a picture where the fundamental arena is a perfect superfluid medium described by the Einstein energy–momentum tensor (26). On the other hand, dark energy does not exist as a primary physical reality, but represents itself structured energy of 3D quantum vacuum on the basis of Eq. (14) which produces the curvature of space in agreement with the results of general relativity. And, above all, inside our model, we have demonstrated that dark matter is not a primary physical reality, but emerges as the result of more fundamental fluctuations of the quantum vacuum energy density. Dark matter is not a primary physical reality, but emerges as an effect of the polarization of the quantum vacuum, of opportune quantum vacuum energy density fluctuations, associated with virtual pairs of particles-antiparticles, in a background characterized by a fluctuating viscosity. The perturbative fluctuation of the quantum vacuum energy density (58) of the excited states of the 3D quantum vacuum characterized by a degree of viscosity, on the ultra-low frequencies, can explain the observed flattening of the orbital speeds of the spiral galaxies in agreement with experimental results. Finally, the interesting perspective is opened that the current theories of WIMPs particles as potential candidates for cold dark matter responsible of the formation of large-scale structures, as well as alternative attempts of explanation such as the one developed by Santos in [58], may here be seen as emerging approaches from a more fundamental picture of a 3D quantum vacuum characterized by RS processes creation/annihilation of virtual sub-particles corresponding to elementary fluctuations of the quantum vacuum energy density, when the vacuum itself is polarized as a consequence of its viscosity associated with the perturbative fluctuation of the quantum vacuum energy density (58).

According to the author of this paper, from an epistemological point of view, a positive element of this model lies in the fact that this model contains the potentiality to obtain all the complexity of the physical world as emerging by the same fundamental background, and in this sense, it seems to satisfy a general idea of simplicity, in line with the epistemological attitude of the eminent American physicist J.A. Wheeler, who, in the course of his life, always claimed that our complex world would be composed by simpler constituents, since the pioneering research with Feynman which seemed to suggest that all would derive from electrons and positrons, to arrive at his model of quantum foam of quantum fluctuations at Planck scale and to his programme of the *It from Bit (or QBit)*, namely the possibility of describing the emerging characteristics of space–time matter as expressions, constrained and conveyed, of an informational matrix

“at the bottom of the world”, that is to say of all dynamic phenomena as emerging realities starting from a fundamental background endowed with physical properties [64–66].

On the other hand, it must be emphasized that, to reproduce the observed flattening of the orbital speeds of the arms of spiral galaxies, our model of 3D quantum vacuum involves supplementary conditions regarding, in particular, the behaviour of the viscosity, represented by Eq. (66), which then lead then to Eqs. (66)–(72), which imply that the stabilization of the orbital speeds of the arms of galaxies is caused by a perturbative fluctuation of the quantum vacuum energy density (58) of the excited states of the 3D quantum vacuum characterized by a degree of viscosity, in regime of the ultra-low frequencies. As a consequence, the formalism (66)–(72) meets difficulties to reproduce recent results regarding the ultrafast spectral modulations found recently by Borra and Trotter [57] in 236 main sequence stars in the sample of 2,5 million spectra of galactic halo stars of the Sloan Digital Sky Survey, which could be, instead, explained by invoking a supplementary condition regarding the behaviour of the perturbative fluctuation of the quantum vacuum energy density (58) in this regime, represented by Eq. (84). As regards new possible extensions of this 3D quantum vacuum model which can account for dark matter contributions coming also from ultrafast ultralight oscillations, further research will give you more information [67, 68].

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