

# Solitary and double-layer structures in quantum bi-ion plasma

Mehran Shahmansouri<sup>1</sup> · Mouloud Tribeche<sup>2</sup>

Received: 15 August 2015 / Accepted: 9 February 2016 / Published online: 22 February 2016  
© The Author(s) 2016. This article is published with open access at Springerlink.com

**Abstract** Weak ion-acoustic solitary waves (IASWs) in an unmagnetized quantum plasmas having two-fluid ions and fluid electrons are considered. Using the one-dimensional quantum hydrodynamics model and then the reductive perturbation technique, a generalized form of nonlinear quantum Korteweg-de Vries (KdV) equation governing the dynamics of weak ion acoustic solitary waves is derived. The effects of ion population, warm ion temperature, quantum diffraction, and polarity of ions on the nonlinear properties of these IASWs are analyzed. It is found that our present plasma model may support compressive as well as rarefactive solitary structures. Furthermore, formation and characteristics properties of IA double layers in the present bi-ion plasma model are investigated. The results of this work should be useful and applicable in understanding the wide relevance of nonlinear features of localized electro-acoustic structures in laboratory and space plasma, such as in super-dense astrophysical objects [24] and in the Earth's magnetotail region (Parks [43]). The implications of our results in some space plasma situations are discussed.

**Keywords** Bi-ion plasma · Ion acoustic wave · Quantum hydrodynamics

## Introduction

Ion acoustic solitary waves (IASWs), as one of the most important nonlinear excitations in plasma physics, have been studied by a number of authors in various models of plasma physics in the last few decades [2, 3, 8, 23, 30, 40, 78, 82, 86]. They arise due to a delicate balance between the dispersive and nonlinear properties of the medium. In the absence of dissipative effects and geometry distortion, a Korteweg-de Vries (KdV) equation describes the propagation of IA waves in a weakly nonlinear regime as shown by Washimi et al. [82]. The IA solitons have been observed experimentally by Ikezi et al. [23] (also see Refs. [3, 8, 86]). The IA waves are highly Landau damped unless one considers the case  $T_i \ll T_e$  [2]. Properties of IA waves may be useful to explain and understand the wave characteristics observed in the Earth's ionosphere [44, 70, 71], interplanetary space [12] and transport phenomena in the solar wind, corona, and chromospheres [50]. Formation of IA solitons [1, 10, 16, 23, 54, 56, 57, 60, 63, 78], double layers or shocks [8, 12, 59, 79, 80, 86] in plasma have been studied by a number of authors.

In the presence of a second ion species in a plasma, the basic properties of ion acoustic wave would be significantly modified in the linear [17, 39, 77] and nonlinear regime [78, 85] (see also Refs. [9, 13, 14, 17, 20, 35, 36, 45, 47–49, 61, 62, 64, 72] studied linear IA waves in a plasma composed of two ion species. They found that their model supports the existence of two ion acoustic modes, called the light and heavy ion modes. The experimental investigations [39, 77] verify the existence of these two ion wave modes. Song et al. [72] investigated propagation and damping of ion acoustic waves in a Q-machine plasma consisting of  $K^+$  positive ions,  $SF_6^-$  negative ions, and

✉ Mehran Shahmansouri  
mshmansouri@gmail.com

<sup>1</sup> Department of Physics, Faculty of Science, Arak University, Arak 38156-8 8349, Iran

<sup>2</sup> Theoretical Physics Laboratory, Faculty of Physics, Plasma Physics Group, University of Bab-Ezzouar, USTHB, B.P. 32, El Alia, Algiers 16111, Algeria

electrons. McKenzie et al. [35] have investigated propagation properties of IASWs in a bi-ion plasma. They found that the compressive as well as rarefactive IA solitons may be excited in their plasma model. Drift instability has been studied by Rosenberg and Merlino [45] in a magnetized plasma comprising of positive ions and negative ions.

On the other hand, investigation of the propagation properties of waves in quantum plasmas is of fundamental importance due to their vital role for understanding the collective behaviors in super-dense astrophysical objects [24], in intense laser-solid density plasma experiments [6, 33], in ultra-cold plasma [27], in microelectronic devices [34], in nanowires [67], carbon nanotubes [84], and quantum diodes [7]. New pressure laws and new quantum forces are included in a quantum plasma, such that in the presence of these new forces the collective behavior of quantum plasmas affects significantly. During the last few decades, the different electrostatics and electromagnetic plasma modes [19, 68, 69, 73]; Ali and Shukla [4, 43, 53] have investigated in quantum plasmas using the QHD and quantum magneto-hydrodynamic models. Haas et al. [19] have presented a detailed study of linear and nonlinear IA waves in an unmagnetized quantum plasma using QHD model. They reported several new quantum features of the IA waves referred to as quantum IA waves. Ion-acoustic shocks in quantum dusty pair-ion plasma were investigated by Misra [38]. The linear and nonlinear quantum IA waves were investigated by Rehman [51] in a plasma with positive and negative ions and Fermi electron gas. A theoretical study on the non-planar electrostatic solitary waves in a negative ion degenerate plasma is presented by Hussain et al. [22]. Wang et al. [83] investigated dressed IA solitons in a quantum pair-ion plasma in the presence of a relativistic electron beam. Sahu and Singha [52] investigated the arbitrary amplitude ion acoustic solitary waves as well as double-layer structures for an ultra-relativistic degenerate dense plasma comprising cold and hot electrons and inertial ultra-cold ions. Formation of IA solitons [5], double layers or shocks [46] and vortices [21] has been studied in quantum plasmas.

The spacecraft observations indicate that the Earth's plasma sheet boundary layer is consist of the back ground electrons, the cold electron beams (with energies of the order of a few eV—a few 100 s of eV), the back ground cold ions and the ion beams (having energies of the order of a few eV –10 s of keV) [42, 25, 26, 41]. Here, our aim is the study of solitary as well as double-layer structures in a bi-ion plasma, consisting of warm ions, cool ions and Maxwellian electrons.

The manuscript is organized as follows. In the next section, we present the basic equations of our theoretical model followed by discussion on Soliton solution; The

m-KdV Equation is derived in the subsequent section and our findings and conclusion are provided in the last section.

## Theoretical Model

Let us consider an unmagnetized plasma consisting of electrons with quantum effects and two-temperature fluid ions. In one dimension, the quantum hydrodynamic (QHD) governing the dynamics of nonlinear IASWs, in normalized form, reads

$$\frac{\partial n_{ic}}{\partial t} + \frac{\partial(n_{ic}u_{ic})}{\partial x} = 0, \quad (1)$$

$$\frac{\partial u_{ic}}{\partial t} + u_{ic} \frac{\partial u_{ic}}{\partial x} = -s_c \frac{\partial \phi}{\partial x} - 3\sigma_c n_{ic} \frac{\partial n_{ic}}{\partial x}, \quad (2)$$

$$\frac{\partial n_{ih}}{\partial t} + \frac{\partial(n_{ih}u_{ih})}{\partial x} = 0, \quad (3)$$

$$\frac{\partial u_{ih}}{\partial t} + u_{ih} \frac{\partial u_{ih}}{\partial x} = -s_h \frac{\partial \phi}{\partial x} - 3\sigma_h n_{ih} \frac{\partial n_{ih}}{\partial x}, \quad (4)$$

$$\frac{\partial n_e}{\partial t} + \frac{\partial(n_e u_e)}{\partial x} = 0, \quad (5)$$

$$\frac{m_e}{m_c} \frac{\partial u_e}{\partial t} + \frac{m_e}{m_c} u_e \frac{\partial u_e}{\partial x} = \frac{\partial \phi}{\partial x} - n_e \frac{\partial n_e}{\partial x} + \frac{H^2}{2} \frac{\partial}{\partial x} \left( \frac{1}{\sqrt{n_e}} \frac{\partial^2 \sqrt{n_e}}{\partial x^2} \right), \quad (6)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - s_c f n_{ic} - (1 - s_c f) n_{ih}, \quad (7)$$

where the following normalized variables  $x \rightarrow \omega_{pi}x/c_s$ ,  $t \rightarrow \omega_{pi}t$ ,  $n_j \rightarrow n_j/n_{j0}$ ,  $u_{ic} \rightarrow u_{ic}/c_s$ ,  $u_{ih} \rightarrow u_{ih}/c_s$ ,  $\phi \rightarrow e\phi/2T_{Fe}$ , have been used. Here,  $\omega_{pi} = \sqrt{4\pi n_0 e^2/m_c}$  is the ion plasma frequency,  $c_s = \sqrt{2T_{Fe}/m_c}$  is the acoustic speed,  $H = \hbar\omega_{pe}/2T_{Fe}$  is quantum diffraction parameter,  $T_{Fe}$  is the electron Fermi temperature,  $\sigma_c = T_c/T_{Fe}$ ,  $\sigma_h = T_h/T_{Fe}$ , and the subscript  $c$  ( $h$ ) stands for the cold (hot) ions. At equilibrium, the charge neutrality reads as  $n_{e0} = s_c n_{ic0} + s_h n_{ih0} = n_0$ , from which we define  $f = n_{ic0}/n_0 = (1 - s_h n_{ih0}/n_0)/s_c$ . In the following sections we neglect the left hand side of Eq. (6), due to  $m_e/m_c \ll 1$ .

## Linear dispersion relation

To obtain the linear dispersion relation of the IA waves in quantum bi-ion plasma we linearize Eqs. (1)–(7) to a first-order approximation. The zeroth-order fields and velocities are also neglected and linear dispersion relation takes the form

$$\frac{s_c^2 f}{\omega^2 - 3\sigma_c k^2} + \frac{s_h(1 - s_c f)}{\omega^2 - 3\sigma_h k^2} = \frac{1 + k^2(1 + H^2 k^2/4)}{k^2(1 + H^2 k^2/4)} \quad (8)$$

In the above linear dispersion relation of IA waves the quantum effects, ion polarity and ions temperature are included. This equation gives rise to two IA modes propagating with different phase velocities, similar to that observed theoretically [17] and experimentally [39, 77]. The mode with larger phase velocity is called the fast mode whereas the mode with smaller phase velocity is known as the slow mode. Note that in the absence of thermal effects the slow mode does not appear in the present plasma model.

Substituting Eqs. (10a)–(10c) into Eq. (10d), one can obtain the following relation

$$\frac{s_c^2 f}{U_0^2 - 3\sigma_c} + \frac{(1 - s_c f)s_h}{U_0^2 - 3\sigma_h} - 1 = 0. \tag{11}$$

The above equation is quadratic in  $U_0^2$ , and shows that the present plasma system supports two distinct IA modes. In this case, the phase speeds of the fast (positive sign) and slow (minus sign) IA modes are given by

$$U_{0\pm}^2 = \frac{3}{2}(\sigma_c + \sigma_h) + \frac{s_c^2 f + (1 - s_c f)s_h}{2} \pm \sqrt{\left(\frac{3}{2}(\sigma_c + \sigma_h) + \frac{s_c^2 f + (1 - s_c f)s_h}{2}\right)^2 - 9\sigma_c\sigma_h - 3\sigma_h s_c^2 f - 3\sigma_c(1 - s_c f)s_h}. \tag{12}$$

### Soliton solution

To study weakly nonlinear IASWs that may develop in our present plasma model, we employ the standard reductive perturbation technique. The stretched coordinates are defined by  $\xi = \varepsilon^{1/2}(x - U_0 t)$ ,  $\tau = \varepsilon^{3/2}t$ , where  $\varepsilon$  is a small parameter measuring the strength of nonlinearity or dispersion, and  $U_0$  the normalized phase velocity. The variables in (1)–(7) are expanded in power series of  $\varepsilon$  as [75, 76]

$$n_{ic} = 1 + \varepsilon n_{ic1} + \varepsilon^2 n_{ic2} + \dots, \tag{9a}$$

$$u_{ic} = \varepsilon u_{ic1} + \varepsilon^2 u_{ic2} + \dots, \tag{9b}$$

$$n_{ih} = 1 + \varepsilon n_{ih1} + \varepsilon^2 n_{ih2} + \dots, \tag{9c}$$

$$u_{ih} = \varepsilon u_{ih1} + \varepsilon^2 u_{ih2} + \dots, \tag{9d}$$

$$n_e = 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \dots, \tag{9e}$$

$$u_e = \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \dots, \tag{9f}$$

$$\phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots. \tag{9g}$$

Substituting the power series (9a–9g) along with the stretching coordinates into Eqs. (1)–(7), and collecting the terms in different powers of  $\varepsilon$ , we obtain at the lowest order

$$n_{ic1} = \frac{u_{ic1}}{U_0} = \frac{s_c \phi_1}{U_0^2 - 3\sigma_c}, \tag{10a}$$

$$n_{ih1} = \frac{u_{ih1}}{U_0} = \frac{s_h \phi_1}{U_0^2 - 3\sigma_h}, \tag{10b}$$

$$n_{e1} = \phi_1, \tag{10c}$$

$$n_{e1} - s_c f n_{ic1} - (1 - s_c f)n_{ih1} = 0. \tag{10d}$$

Gill et al. [18] reported the similar results in their study. For  $\sigma_c \rightarrow 0$  and  $\sigma_h \rightarrow 0$ , the present plasma model does not support slow IA modes, as already reported by Mishra and Chhabra [37].

At the next higher order of  $\varepsilon$ , the condition for annihilation of secular terms Eq. (11), leads to the following Korteweg de Vries (KdV) equation for IA waves

$$\frac{\partial \phi_1}{\partial \tau} + A \phi_1 \frac{\partial \phi_1}{\partial \xi} + B \frac{\partial^3 \phi_1}{\partial \xi^3} = 0 \tag{13}$$

where

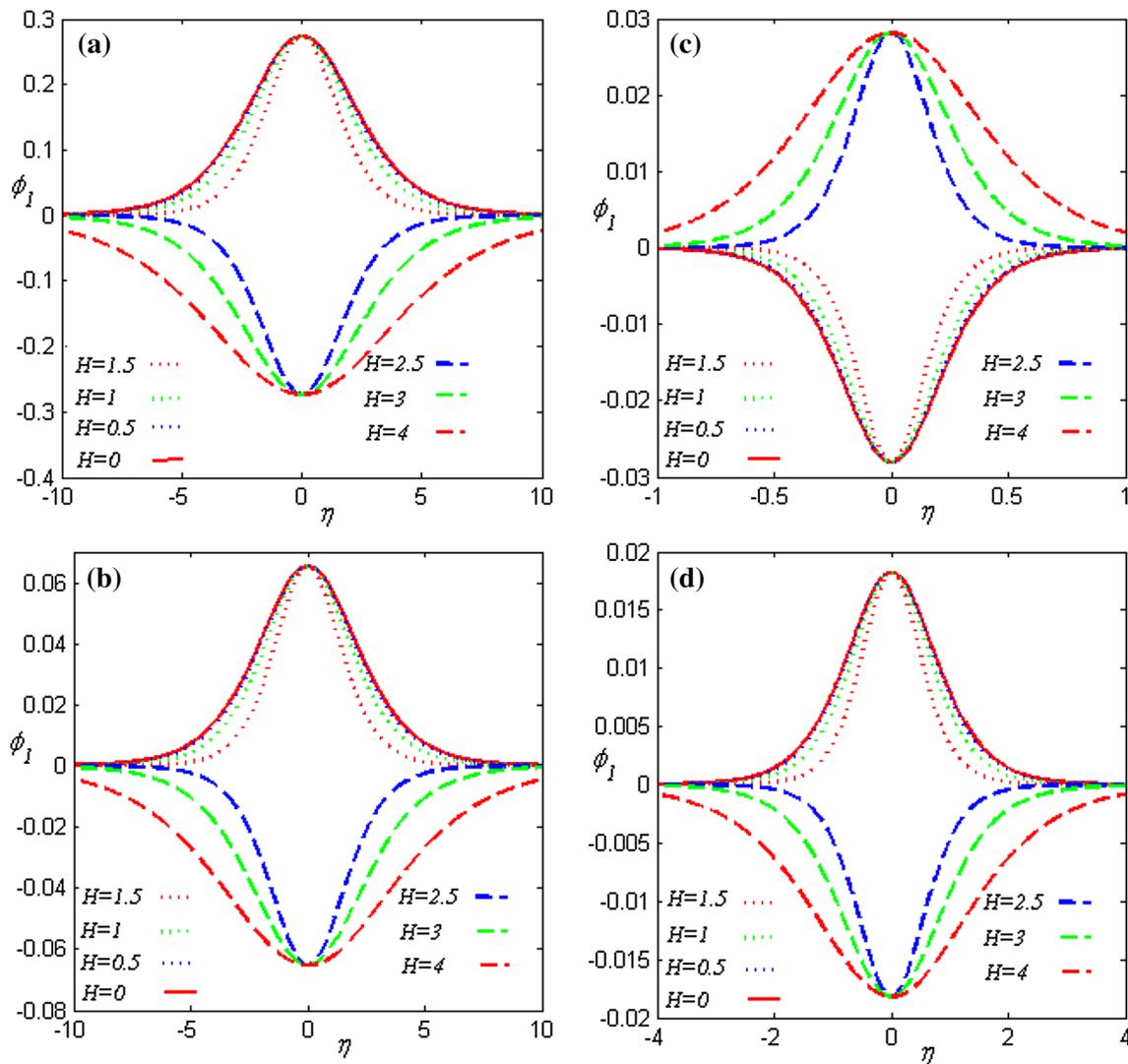
$$A = \left[ 1 + \frac{3s_c^3 f (U_0^2 + \sigma_c)}{(U_0^2 - 3\sigma_c)^3} + \frac{3(1 - s_c f)s_h^2 (U_0^2 + \sigma_h)}{(U_0^2 - 3\sigma_h)^3} \right] / \left[ \frac{2fU_0 s_c^2}{(U_0^2 - 3\sigma_c)^2} + \frac{2(1 - s_c f)s_h U_0}{(U_0^2 - 3\sigma_h)^2} \right],$$

$$B = (1 - H^2/4) / \left[ \frac{2fU_0 s_c^2}{(U_0^2 - 3\sigma_c)^2} + \frac{2(1 - s_c f)s_h U_0}{(U_0^2 - 3\sigma_h)^2} \right].$$

The nonlinear and dispersive coefficients in Eq. (13), both are depend on the relevant parameters such as ions population, ions temperature, and polarity of ions. The latter is also depends on the quantum diffraction.

To solve the KdV Eq. (13), we use the transformation  $\eta = \varepsilon^{1/2}[x - (U_0 + \varepsilon V_0)t]$ , where  $V_0$  is the normalized solitary wave speed in the stationary frame. We can then define the Mach number as  $M = U_0 + \varepsilon V_0$  (or normalized solitary wave speed in laboratory frame). The stationary solution of Eq. (13) can then be obtain as [32, 81]

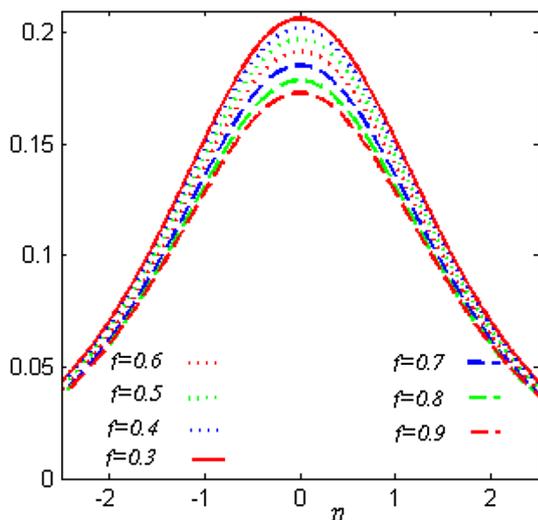
$$\phi_1(\eta) = \Phi_{\max} \sec h^2 \left( \frac{\eta}{\Delta} \right) \tag{14}$$



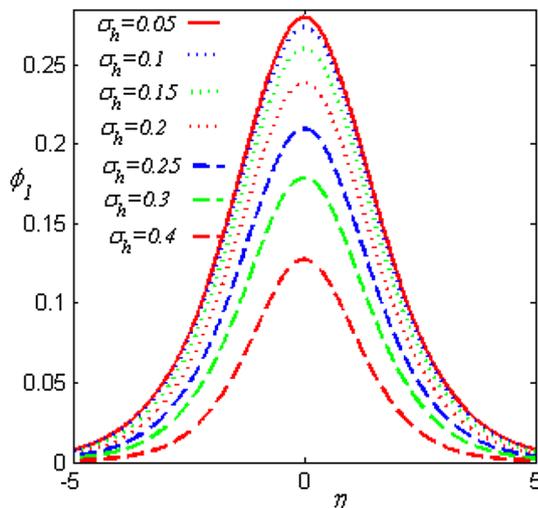
**Fig. 1** The IA solitary wave profile for different values of quantum parameter in the cases of fast mode with **a**  $s_h = +1$  and **b**  $s_h = -1$ , and slow mode with **c**  $s_h = +1$  and **d**  $s_h = -1$ , where the other parameters are:  $f = 0.8$ ,  $\sigma_c = 0.05$ ,  $\sigma_h = 0.1$ , and  $s_c = +1$

where  $\Phi_{\max} = 3V_0/A$  and  $\Delta = (4B/V_0)^{1/2}$  represent, respectively, the amplitude and width of the solitary wave. The IA solitary structures are depicted in Figs. 1, 2 and 3 for different values of the plasma parameters. It can be seen from Fig. 1 that both the compressive and rarefactive IA solitons will be observe in this plasma model, due to the quantum parameter value. Also an increase of the quantum parameter leads to decrease (increase) of the fast compressive (rarefactive) IA solitons width whereas the amplitude of IA solitons remains unchanged. This means that the bi ion plasma in the presence of strong quantum effects supports narrower (wider) compressive (rarefactive) IA solitons. A comparison between Fig. 1a, b shows that the structure of IA solitary waves depends sensitively on the polarity of warm ions, as for negative warm ions IA solitons have smaller amplitudes. Similar figures for slow

mode are depicted in Fig. 1c, d. Note that the effect of quantum parameter on the slow IA solitons width is reverse in comparison with the fast one. Coexistence of compressive and rarefactive solitary structures in the present model (in the presence as well as in the absence of quantum effects) may be applicable to describe such behavior in the magnetospheric regions, which is consisting of multi-ion plasma. Lui et al [31] observed the presence of earthward and anti-earthward streaming of ions having density  $\leq 0.1 \text{ cm}^{-3}$  with temperatures of the order of  $0.1 - 1 \text{ keV}$  (or  $50 \text{ eV}$  [65, 66]) in the plasma sheet boundary layer. Dusenbery and Lyons [15] discussed the effect of two population ion species on the formation of electrostatic structures in the plasma sheet boundary layer. Here, Fig. 1 is depicted for parameters  $T_c/T_h = 0.5$ , cold ion density of the order of 0.2 for different values of quantum diffraction



**Fig. 2** The fast IA solitary wave profile for different values of cold ion concentration with parameters:  $H = 1.5$ ,  $\sigma_c = 0.05$ ,  $\sigma_h = 0.1$  and  $s_c = s_h = +1$



**Fig. 3** The fast IA solitary wave profile for different values of  $\sigma_h$  with parameters:  $f = 0.8$ ,  $H = 1.5$ ,  $\sigma_c = 0.05$  and  $s_c = s_h = +1$

parameter  $0 \leq H \leq 4$ , in which we found the electrostatic potentials which support the coexistence of compressive and rarefactive solitons. It is easy to show that such compressive and rarefactive structures have bipolar electric fields. The effect of cold ions concentration  $f$  on the fast IA solitons is shown in Fig. 2. It can be seen that the amplitude of IA solitons shows a decreasing behavior with  $f$  while the width has an inverse behavior. Thus, a decrease in  $f$  leads to appearance of larger and narrower IA solitons, as in the presence of more warm ions the solitary structures become more spiky.

Figure 3 indicates the influence of warm ion temperature on the fast IA solitary wave. An increase of the warm

ion temperature decreases the amplitude of fast IA solitons. This shows that the warm ion temperature has a destructive effect for the formation of IA solitary structures in such a quantum bi-ion plasma. Similar result was also observed for dust acoustic solitons in dusty plasma as shown in the work of Shahmanouri and Alinejad [58].

### Derivation of m-KdV equation

In order to study the double-layer structures in the present plasma model, we consider the higher order effects to obtain the evolution equation. Thus, we introduce the new stretching coordinates as follows:

$$\xi = \varepsilon(x - U_0t), \quad \tau = \varepsilon^3t, \tag{15}$$

Then, we substitute the above stretching coordinates along with the set of Eqs. (9a–9g) into the Eqs. (1)–(7), and again collect the terms in different powers of  $\varepsilon$ . The second power of  $\varepsilon$  in Poisson’s equation leads to

$$A\phi^{(1)2} = 0, \tag{16}$$

We have assumed that  $\phi^{(1)} \neq 0$  and to derive the modified KdV (m-KdV) equation we also consider  $A \neq 0$ , then the magnitude of  $A$  should be of the first order of  $\varepsilon$  [18, 55]. Therefore,  $A\phi^{(1)2}$  becomes of the order of  $\varepsilon^3$ , and will appear in the third order of the Poisson’s equation. After a long algebraic but straightforward manipulation we eliminate the third-order variables to derive the following m-KdV equation

$$\frac{\partial\phi^{(1)}}{\partial\tau} + A\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial\xi} + D(\phi^{(1)})^2\frac{\partial\phi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi^{(1)}}{\partial\xi^3} = 0, \tag{17}$$

where

$$D = \left[ -\frac{3}{2} + \frac{3s_c^4f}{2(U_0^2 - 3\sigma_c)^5} (3U_0^2(U_0^2 + \sigma_c) + 2U_0^4 - 18\sigma_c^2 + 27\sigma_c(U_0^2 + \sigma_c)) + \frac{3(1 - s_c f)s_h^3}{2(U_0^2 - 3\sigma_h)^5} \times (2U_0^4 + 3U_0^2(U_0^2 + \sigma_h) + 2U_0^4 - 18\sigma_h^2 + 27\sigma_h(U_0^2 + \sigma_h)) \right] / \left[ \frac{2fs_c^2U_0}{(U_0^2 - 3\sigma_c)^2} + \frac{2(1 - s_c f)s_hU_0}{(U_0^2 - 3\sigma_h)^2} \right]$$

By introducing a new variable  $\eta = \xi - u_0\tau$  in a stationary frame, we can write Eq. (17) in the form of an energy-like equation as follows

$$\frac{1}{2} \left( \frac{\partial\phi}{\partial\eta} \right)^2 + \psi(\phi) = 0, \tag{18}$$

where  $\phi = \phi^{(1)}$ , and the Sagdeev pseudo-potential is given by

$$\psi(\phi) = -\frac{u_0}{2B}\phi^2 + \frac{A}{6B}\phi^3 + \frac{D}{12B}\phi^4 \tag{19}$$

Equation (18) is similar to the “energy-integral” of an oscillating particle of a unit mass, with speed  $d\phi/d\xi$  and position  $\phi$ , in a potential  $\psi(\phi)$ . The first term of (18) can be considered as the kinetic energy of the unit mass, and  $\psi(\phi)$  is the potential energy. Since the kinetic energy is a positive quantity, thus it requires that  $\psi(\phi) \leq 0$  for the entire of motion. The existence condition of electrostatic structures could be determined by the behavior of Sagdeev potential. The existence of localized solution requires that the Sagdeev potential have a maximum value at the origin, viz.  $\psi(\phi = 0) = 0$  and  $(\partial\psi/\partial\xi)|_{\phi=0} = 0$ ,  $(\partial^2\psi/\partial\xi^2)|_{\phi=0} < 0$ . Also it needs that  $\psi(\phi)$  be negative in the interval  $0 < \phi < \phi_{\max}$  ( $\phi_{\min} < \phi < 0$ ) for the compressive (rarefactive) solitary waves. Where  $\phi_{\max}$  ( $\phi_{\min}$ ) is the maximum (minimum) value of  $\phi$  for which  $\psi(\phi) = 0$ . Additional conditions that the double-layers need to existence are as follows  $\psi(\phi = \phi_m) = 0$  and  $(\partial\psi/\partial\xi)|_{\phi=\phi_m} = 0$ ,  $(\partial^2\psi/\partial\xi^2)|_{\phi=\phi_m} < 0$ . The first two boundary condition is required to satisfy the global charge neutrality, and lead to the following conditions

$$\phi_m = -A/D, \quad u_0 = -D\phi_m^2/6, \tag{20}$$

Substituting the above expressions into Eq. (18), we obtain the energy integral in the following form

$$\left(\frac{\partial\phi}{\partial\eta}\right)^2 + \frac{D\phi^2}{6B}(\phi - \phi_m)^2 = 0. \tag{21}$$

The above equation is analytically solvable, and thus one can obtain a stationary solution for  $\phi$  of the form [32]

$$\phi(\xi, \tau) = \frac{\phi_m}{2} \left[ 1 - \tanh\left(\frac{\xi - u_0\tau}{L}\right) \right], \tag{22}$$

where  $L$  is the thickness of the IA double-layer, and it is given by  $L = \sqrt{-24B/D}/|\phi_m|$ . The definition of  $L$  provided that  $B/D < 0$ . It appears appropriate here to add that the nature of the IA double layer depends on the sign of  $A/D$ . Positive sign of  $A/D$  supports appearance of rarefactive double layers whereas negative sign of  $A/D$  supports compressive one.

To examine the possibility of double layer excitation in the present plasma model, we must check the necessary conditions, for which  $D < 0$ ,  $B > 0$  and  $\sqrt{-6u_0/D} = -A/D$  are satisfied simultaneously. The result of a careful study of these cases is illustrated for different values of the relevant plasma parameters in Table 1. In this table, we have listed the corresponding values of  $B/D$ ,  $-6u_0/D$ ,  $-A/D$  and also the double layer amplitude  $\phi_m$ . Note that the essential conditions for the formation of double layers are confined to the simultaneous satisfaction of the following conditions: (1)  $D < 0$ , (2)  $B > 0$  and (3)  $-\sqrt{-6u_0/D} = -A/D$  (for rarefactive double layers) or  $\sqrt{-6u_0/D} = -A/D$  (for compressive double layers). First, we have listed various values of the cold ion concentration

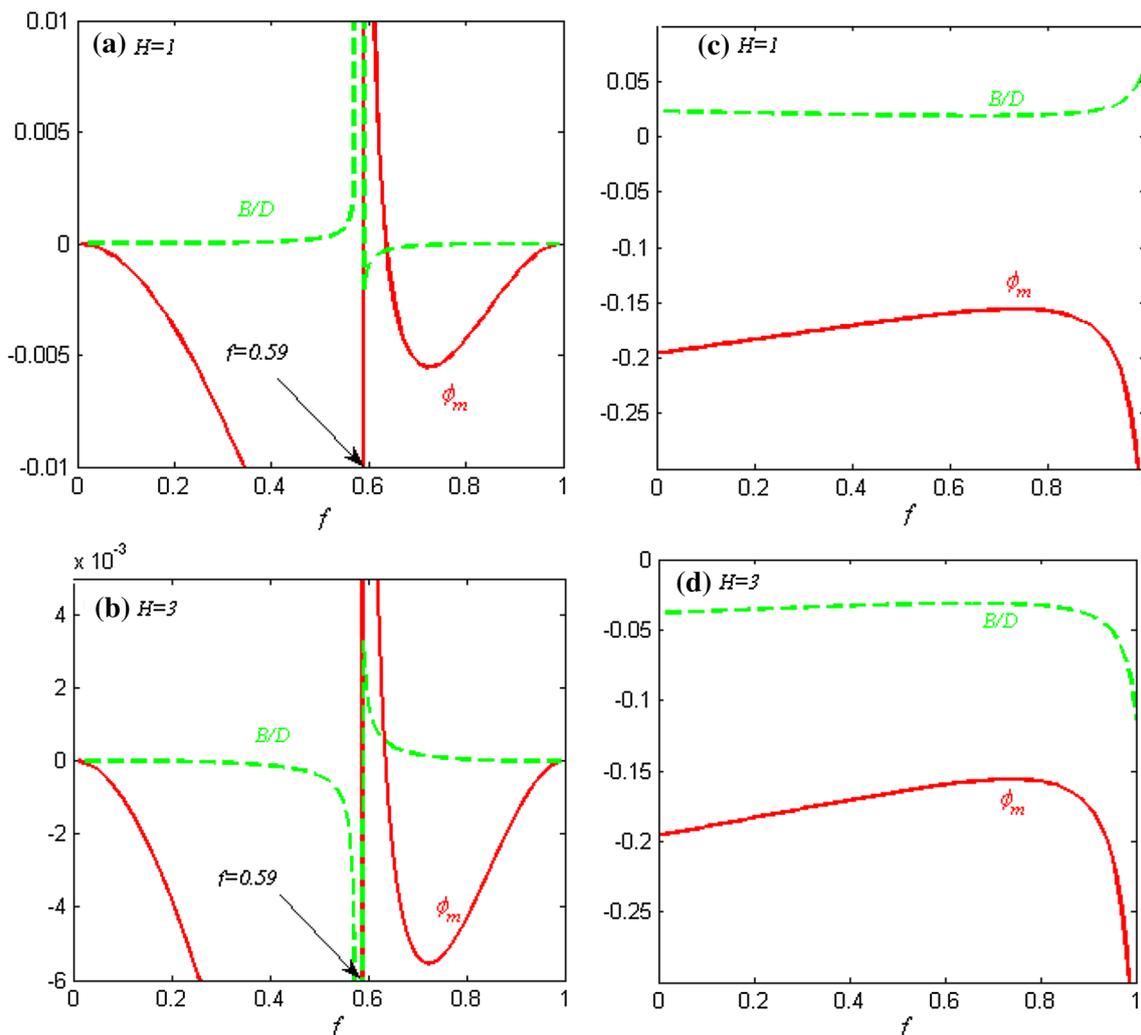
**Table 1** The numerical results of the double layer conditions for different relevant physical parameters

	$H = 1, u_0 = 0.0013, \sigma_c = 0.05, \sigma_h = 0.2$				$f = 0.6, u_0 = 0.0025, \sigma_c = 0.05, \sigma_h = 0.2$			
	$f = 0.5$	$f = 0.6$	$f = 0.7$	$f = 0.8$	$H = 0$	$H = 1$	$H = 3$	$H = 4$
$B/D$	0.0002	-0.0001	-0.0001	-0.0001	-0.0013	-0.0010	0.0016	0.0039
$\frac{-6u_0}{D}$	-0.0001	0.0002	0.0000	0.0000	0.0005	0.0005	0.0005	0.0005
$\frac{-A}{D}$	-0.0205	0.0217	-0.0053	-0.0053	0.0217	0.0217	0.0217	0.0217
$ \phi_m  = \sqrt{\frac{-6u_0}{D}}$	0.008i	0.0157	0.0053	0.0053	0.0217	0.0217	0.0217	0.0217
$\phi_m = \frac{-A}{D}$	-0.0205	0.0217	-0.0053	-0.0053	0.0217	0.0217	0.0217	0.0217
	$f = 0.8, H = 1, \sigma_c = 0.05, \sigma_h = 0.2$				$f = 0.8, H = 1, u_0 = 0.0036, \sigma_c = 0.05$			
	$u_0 = 0.001$	$u_0 = 0.002$	$u_0 = 0.003$	$u_0 = 0.0036$	$\sigma_h = 0.1$	$\sigma_h = 0.15$	$\sigma_h = 0.2$	$\sigma_h = 0.25$
$B/D$	-0.0000	-0.0000	-0.0000	-0.0000	$-1e - 7$	-0.0000	-0.0000	-0.0001
$\frac{-6u_0}{D}$	0.0001	0.0000	0.0000	0.0000	$6e - 7$	0.0000	0.0000	0.0001
$\frac{-A}{D}$	-0.0043	-0.0043	-0.0043	-0.0043	$-6.24e - 4$	-0.0021	-0.0043	-0.0066
$ \phi_m  = \sqrt{\frac{-6u_0}{D}}$	0.0023	0.0032	0.0039	0.0043	$-7.94e - 4$	0.0022	0.0043	0.0076
$\phi_m = \frac{-A}{D}$	-0.0043	-0.0043	-0.0043	-0.0043	$-6.24e - 4$	-0.0021	-0.0043	-0.0066

The columns which are included in highlight indicate the occurrence of slow double layer structures, where the essential conditions would be satisfied simultaneously

**Table 2** The slow double layer conditions for different values of  $u_0$ , with  $f = 0.7$ ,  $H = 1$ ,  $\sigma_c = 0.05$  and  $\sigma_h = 0.2$

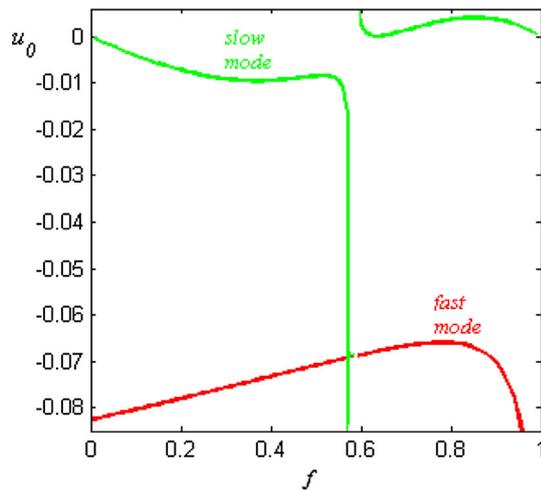
$\frac{d\psi}{d\phi}$									
$u_0 = 0.002$	0.00921	0.00161	0.00002	-0.00247	0.00003	0	-0.00066	-0.00247	-0.01169
$u_0 = 0.002005$	0.00923	0.00162	0.0000	-0.00248	0.0000	0	-0.00066	-0.00248	-0.01170
$u_0 = 0.00205$	0.00924	0.00164	0.00004	-0.00248	0.00003	0	-0.00066	-0.00248	-0.01171
$\psi$									
$u_0 = 0.002$	-5.21e-6	-4.7e-7	0.0000	-8.5e-7	-2e-8	0	-1.3e-7	-8.5e-7	-7.19e-6
$u_0 = 0.002005$	-5.22e-6	-4.8e-7	-1e-8	-8.5e-7	-2e-8	0	-1.3e-7	-8.5e-7	-7.10e-6
$u_0 = 0.00205$	-5.27e-6	-5e-7	-1e-8	-8.6e-7	-2e-8	0	-1.3e-7	-8.6e-7	-7.21e-6
$\phi$									
	-0.003	-0.002	-0.00122	-0.001	-0.0005	0	0.0005	0.001	0.002



**Fig. 4** Variation of  $B/D$  and  $\phi_m$  with respect to cold ion concentration for slow mode with **a**  $H = 1$  and **b**  $H = 3$ , and for fast mode with **c**  $H = 1$  and **d**  $H = 3$ , where  $\sigma_c = 0.05$ ,  $\sigma_h = 0.2$

lying in the range  $0.6 < f < 0.9$ . For each such value, we have obtained the corresponding values of  $B/D$ ,  $-6u_0/D$ ,  $-A/D$ . The table shows that only for  $f = 0.7$  (the third

column) both double layer conditions satisfy and a rarefactive IA double layer can be form in the system. In the second four columns we have listed various values of the



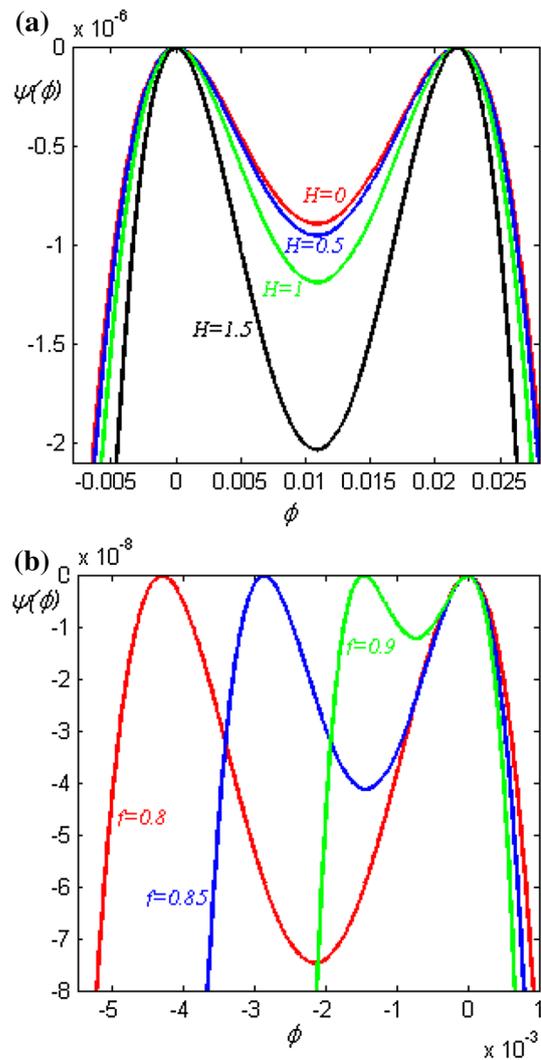
**Fig. 5** The primitive values of  $u_0$  which are corresponding to the essential condition for the formation of IA double layers, where  $\sigma_c = 0.05$ ,  $\sigma_h = 0.2$ ,  $H = 3$

quantum parameter. In this case we see that only for  $H = 1$  (the 6th column) the double layer conditions satisfy, and system supports excitation of compressive IA double layer. A similar treatment for other two cases, i.e., different values of  $u_0$  and  $\sigma_h$ , are shown in another columns as in the 12th and 15th columns double layers may occur.

The formation of double layers can be also investigated via another simple fashion, in which we have obtained the relevant value of  $\psi(\phi)$  and  $d\psi(\phi)/d\phi$  for various values of  $u_0$  in Table 2. The existence of double layers need to simultaneous zero in  $\psi(\phi)$  and  $d\psi(\phi)/d\phi$  for  $\phi_m \neq 0$ . Such this condition satisfies at  $\phi = -0.00122$ , which is in accordance with a rarefactive IA double layer.

In order to study the nature of double layer (i.e., compressive or rarefactive double layers), we analyzed numerically the essential conditions for the existence of double layer, for typical parameters  $\sigma_c = 0.05$ ,  $\sigma_h = 0.2$  and  $s_c = s_h = +1$ . Variation of  $\phi_m = -A/D$  and  $B/D$  as a function of cold ion concentration are plotted in Fig. 4. From the sign of  $\phi_m = -A/D$  and  $B/D$  we can show that for  $H = 1$  in slow mode, both compressive and rarefactive double layers are obtained respectively for  $0.59 < f < 0.636$  and  $0.636 < f < 1$  while for  $H = 3$  only negative double layers are obtained for  $f < 0.59$ . The amplitude of double layers tend to zero when system goes to two-component plasma, i.e., in the cases of  $f \rightarrow 0$  and  $f \rightarrow 1$ . On the other hand, for the fast mode with  $H = 1$  the double layer structures do not exist in the system due to the positive sign of  $B/D$ . While for the fast mode with  $H = 3$  only the rarefactive double layers will exist.

One of the necessary conditions for the formation of double layers bounds  $u_0$ , as it needs to satisfy  $u_0 = -A^2/6D$ . It can be seen that the primitive values of  $u_0$



**Fig. 6** The slow IA double layers for **a** different values of quantum parameter and  $f = 0.6$ , and **b** different values of cold ions concentration and  $H = 1$ , with parameters:  $\sigma_c = 0.05$ ,  $\sigma_h = 0.2$ ,  $s_c = s_h = +1$

depends sensitively on the cold ion concentration. The behavior of  $u_0$  with respect to  $f$  for both slow and fast modes is depicted in Fig. 5. We found that for the fast mode,  $u_0$  is always negative whereas for the slow mode in the region of  $0 < f < 0.58$ ,  $u_0$  is negative and in the region of  $0.58 < f < 1$ ,  $u_0$  is positive.

The effect of quantum parameter on the slow IA double layers is depicted in Fig. 6a. It can be seen that the double layer structure is highly sensitive to the quantum parameter. This shows that an increase in the quantum parameter leads to increase of the depth of potential well. Indeed, this is equivalent with a decrease in the width of double layers, while the amplitude of the produced double layer remains unchanged. Furthermore, the double layer structure for different values of the cold

ion concentration is depicted in Fig. 6b. Both characteristics properties of double layers, i.e., depth of potential well and amplitude of double layer  $\phi_m$ , decrease with an increasing  $f$ . Similar to that observed in Fig. 4a, when  $f$  tends to the unit value the amplitude of double layer tends to zero.

## Conclusions

Ion acoustic solitary waves and double layer structures are studied in a bi-ion plasma consisting of cold and warm fluid ions and quantum electrons. The existence of solitary waves and double layers is shown to be dependent on the different parameters such as quantum parameter, ion concentration and ion temperature in detail. The main results are as follows:

1. We found that the present bi-ion plasma model supports two distinct IA modes which propagates with different phase speed, these modes are known as the fast and slow IA modes. When both ion species are cold, i.e.,  $\sigma_c = \sigma_h = 0$ , the slow mode disappears and the fast mode reduces to the usual IA mode.
2. We found that the system supports the compressive as well as rarefactive excitations (solitary and double-layer structures), due to the particular set of plasma parameters.
3. We found that the IA solitary waves depend sensitively on the plasma parameters such as the quantum effects, ion concentration, ion polarity and ion temperature. The present quantum bi-ion plasma model supports formation of both the compressive and rarefactive IA solitons, due to the quantum parameter value. Also an increase of the quantum parameter leads to decrease (increase) of the fast compressive (rarefactive) IA solitons width whereas the amplitude of IA solitons remains unchanged. This means that the quantum bi-ion plasma in the presence of strong quantum effects supports narrower (wider) compressive (rarefactive) IA solitons. Note that the effect of quantum parameter on the slow IA solitons width is reverse in comparison with the fast one. The effect of cold ions concentration  $f$  on the fast IA solitons is decrease of their amplitude. Also we found that the warm ion temperature has a destructive effect on the fast IA solitary wave, as an increase of the warm ion temperature decreases the amplitude of fast IA solitons.
4. We determined numerically the existence domain of compressive and rarefactive IA double layers. As for  $H = 1$  in slow mode, both compressive and rarefactive double layers are obtained respectively for  $0.59 < f < 0.636$  and  $0.636 < f < 1$  while for  $H = 3$

only negative double layers are obtained for  $f < 0.59$ . The amplitude of double layers tend to zero when system goes to two-component plasma, i.e., in the cases of  $f \rightarrow 0$  and  $f \rightarrow 1$ . On the other hand, for the fast mode with  $H = 1$  the double layer structures do not exist in the system due to the positive sign of  $B/D$ . While for the fast mode with  $H = 3$  only the rarefactive double layers will exist.

The numerical analysis in the present study is a systematic investigation in a wide range of physical parameters. Therefore, our results should be useful and applicable in understanding the wide relevance of nonlinear features of localized electro-acoustic structures in laboratory and space plasma, such as in super-dense astrophysical objects [24] and in the Earth's magnetotail region [42].

**Open Access** This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.

## References

1. Alinejad, H.: Phys. Scr. **81**, 025501 (2010)
2. Agrimson, E., D'Angelo, N., Merlino, R.L.: Phys. Rev. Lett. **86**, 5282 (2001)
3. Akimoto, K., Papadopoulos, K., Winske, D.: J. Plasma Phys. **34**, 467 (1985)
4. Ali, S., Shukla, P.K.: Phys. Plasmas **13**, 052113 (2006)
5. Ali, S., Moslem, W.M., Shukla, P.K., Kourakis, I.: Phys. Lett. A **366**, 606 (2007)
6. Andreev, A.V.: JETP Lett. **72**, 238 (2000)
7. Ang, L.K., Kwan, T.J.T., Lau, Y.Y.: Phys. Rev. Lett. **91**, 208303 (2003)
8. Bharuthram, R., Shukla, P.K.: Phys. Fluids **20**, 3214 (1986)
9. Bharuthram, R., Shukla, P.K.: Planet. Space Sci. **40**, 973 (1992)
10. Boubakour, N., Tribeche, M., Aoutou, K.: Phys. Scr. **79**, 065503 (2009)
11. Cranmer, S.R., Ballegoijen, A.A., Edgar, R.J.: Astrophys. J. **171**, 520 (2007)
12. Das, G.C., Sen, K.M.: Earth Moon Planets **64**, 47 (1994)
13. Dezfily, S.Gh, Dorranean, D.: Cont. Plasma Phys. **53**, 564 (2013)
14. Dorranean, D., Sabetkar, A.: Phys. Plasmas **19**, 013702 (2012)
15. Dusenbery, P.B., Lyons, L.R.: J. Geophys. Res. **90**, 10935 (1985)
16. El-Awady, E.I., El-Tantawy, S.A., Moslem, W.M., Shukla, P.K.: Phys. Lett. A **374**, 3216 (2010)
17. Fried, B.D., White, R.B., Samec, ThK: Phys. Fluids **14**, 2388 (1971)
18. Gill, T.S., Bala, P., Kaur, H., Saini, N.S., Bansal, S., Kaur, J.: Eur. Phys. J. D **31**, 91 (2004)
19. Haas, F., Garcia, L.G., Goedert, J., Manfredi, G.: Phys. Plasmas **10**, 3858 (2003)
20. Haidar, M.M., Frdous, T., Duha, S.S.: J. Theor. Appl. Phys. **9**, 159 (2015)
21. Haquea, Q., Mahmood, S.: Phys. Plasmas **15**, 034501 (2008)



22. Hussain, S., Akhtar, N., Saeed-ur-Rehman, Chin: Phys. Lett. **28**, 045202 (2011)
23. Ikezi, H., Taylor, R., Baker, D.: Phys. Rev. Lett. **25**, 11 (1970)
24. Jung, Y.D.: Phys. Plasmas **8**, 3842 (2001)
25. Kakad, A.P., Singh, S.V., Reddy, R.V., Lakhina, G.S., Tagare, S.G.: Adv. Space Res. **43**, 1945 (2009)
26. Kakad, A.P., Singh, S.V., Reddy, R.V., Lakhina, G.S., Tagare, S.G., Verheest, F.: Phys. Plasmas **14**, 052305 (2007)
27. Killian, T.C.: Nature (Lond.) **441**, 298 (2006)
28. Kim, K.Y.: Phys. Lett. A **97**, 45 (1983)
29. Koepke, M.E.: Phys. Plasmas **9**, 2420 (2002)
30. Lonngren, K.E.: Plasma Phys. **25**, 943 (1983)
31. Lui, A.T.Y., Eastman, D.T.E., Williamsa, D.J., Frank, L.A.: J. Geophys. Res. **88**, 7753 (1983)
32. Mace, R.L., Baboolal, S., Bharuthram, R., Hellberg, M.A.: J. Plasma Phys. **45**, 323 (1991)
33. Marklund, M., Shukla, P.K.: Rev. Mod. Phys. **78**, 591 (2006)
34. Markowich, P.A., Ringhofer, C.A., Schmeiser, C.: Semiconductor equations. Springer, New York (1990)
35. McKenzie, J.F., Verheest, F., Doyle, T.B., Hellberg, M.A.: Phys. Plasmas **11**, 1762 (2004)
36. McKenzie, J.F., Verheest, F., Doyle, T.B., Hellberg, M.A.: Phys. Plasmas **12**, 102305 (2005)
37. Mishra, K., Chhabra, R.S.: Phys. Plasmas **3**, 4446 (1996)
38. Misra, A.P.: Phys. Plasmas **16**, 033702 (2009)
39. Nakamura, N., Nakamura, M., Itoh, T.: Phys. Rev. Lett. **37**, 209 (1976)
40. Nakamura, Y.: IEEE Trans. Plasma Sci. **PS-7**, 232 (1982)
41. Onsager, T.G., Thomsen, M.F., Elphic, R.C., Gosling, J.T., Anderson, R.R., Kettmann, G.: J. Geophys. Res. **98**, 15509 (1993)
42. Parks, G.K.: J. Geophys. Res. **89**, 8885 (1984)
43. Ren, H., Wu, Z., Chu, P.: Phys. Plasmas **14**, 062102 (2007)
44. Rosenberg, M., Merlino, R.L.: Planet. Space Sci. **55**, 1464 (2007)
45. Rosenberg, M., Merlino, R.L.: J. Plasma Phys. **79**, 949 (2013)
46. Roy, K., Mishra, A.P., Chatterjee, P.: Phys. Plasmas **15**, 032310 (2008)
47. Sabetkar, A., Dorrnanian, D.: J. Plasma Phys. **80**, 565 (2014)
48. Sabetkar, A., Dorrnanian, D.: J. Theor. Appl. Phys. **9**, 141 (2015)
49. Sabetkar, A., Dorrnanian, D.: Phys. Scr. **90**, 035603 (2015)
50. Sabry, R., Moslem, W.M., Shukla, P.K.: Phys. Plasmas **16**, 032302 (2009)
51. Rehman, S.U.: Phys. Plasmas **17**, 062303 (2010)
52. Sahu, B., Singha, P.: Earth Moon Planets **110**, 165 (2013)
53. Saleem, H., Ahmed, A., Khan, S.A.: Phys. Plasmas **15**, 014503 (2008)
54. Shahmansouri, M.: Chinese Phys. Lett. **29**, 105201 (2012)
55. Shahmansouri, M.: Phys. Plasmas **20**, 102104 (2013)
56. Shahmansouri, M., Alinejad, H.: Astrophys. Space Sci. **344**, 463 (2013)
57. Shahmansouri, M., Alinejad, H.: Phys. Plasmas **20**, 082130 (2013)
58. Shahmansouri, M., Alinejad, H.: Phys. Plasmas **20**, 033704 (2013)
59. Shahmansouri, M., Mamun, A.A.: Phys. Plasmas **20**, 082122 (2013)
60. Shahmansouri, M., Shahmansouri, B., Darabi, D.: Ind. J. Phys. **87**, 711 (2013)
61. Shahmansouri, M., Tribche, M.: Astrophys. Space Sci. **350**, 623 (2014)
62. Shahmansouri, M., Tribche, M.: Astrophys. Space Sci. **349**, 781 (2014)
63. Shahmansouri, M., Astaraki, E.: J. Theor. Appl. Phys. **8**, 189 (2014)
64. Shahmoradi, N., Dorrnanian, D.: Phys. Scr. **89**, 065602 (2014)
65. Sharp, R.D., Carr, D.L., Peterson, W.K., Shelley, E.G.: J. Geophys. Res. **86**, 4639 (1981)
66. Sharp, R.D., Lennartsson, W., Peterson, W.K., Shelley, E.G.: J. Geophys. Res. **87**, 10420 (1982)
67. Shpatakovskaya, G.V.: J. Exp. Theor. Phys. **102**, 466 (2006)
68. Shukla, P.K., Stenflo, L.: Phys. Lett. A **355**, 378 (2006)
69. Shukla, P.K., Stenflo, L.: Phys. Lett. A **357**, 229 (2006)
70. Singh, D.K., Narayan, D., Singh, R.P.: Earth Moon Planets **77**, 75 (1996)
71. Singh, A.K., Narayan, D., Singh, R.P.: Earth Moon Planets **91**, 161 (2002)
72. Song, B., D'Angelo, N., Merlino, R.L.: Phys. Fluids B **3**, 284 (1991)
73. Stenflo, L., Shukla, P.K., Marklund, M.: Europhys. Lett. **74**, 844 (2006)
74. Stix, T.H.: Waves in plasmas. AIP, New York (1992)
75. Taniuti, T., Yajima, M.: J. Math. Phys. **10**, 1369 (1969)
76. Taniuti, T.: Prog. Theor. Phys. Suppl. **55**, 1 (1974)
77. Tran, M.Q., Coquerand, S.: Phys. Rev. A **14**, 2301 (1976)
78. Tran, M.Q.: Phys. Scr. **20**, 317 (1979)
79. Tribeche, M., Boubakour, N.: Phys. Plasmas **16**, 084502 (2009)
80. Tribeche, M., Mayout, S., Amour, R.: Phys. Plasmas **16**, 043706 (2009)
81. Verheest, F.: Waves in dusty space plasmas. Kluwer Academic, Dordrecht (2000)
82. Washimi, H., Taniuti, T.: T. Phys. Rev. Lett. **17**, 996 (1966)
83. Wang, Y., Dong, Y., Eliasson, B.: Phys. Lett. A **377**, 2604 (2013)
84. Wei, L., Wang, Y.N.: Phys. Rev. B **75**, 193407 (2007)
85. White, R.B., Fried, B.D., Coroniti, F.V.: Phys. Fluids **15**, 1484 (1972)
86. Yadav, L.L., Sharma, S.R.: Phys. Scr. **43**, 106 (1991)

