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RESEARCH

Effect of external applied electric field on the silicon solar cell's thermodynamic efficiency

R. Zieba Falama[1,2] · Justin Mibaile[3] · E. Guemene Dountio[2] · Noël Djongyang[1] · Serge Y. Doka[4] · Timoleon C. Kofane[5]

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Abstract This paper presents a possible solution to improve the efficiency of photovoltaic solar cells. An external electric field is applied on a silicon photovoltaic solar cell, inducing band-trap ionization of charge carriers. Output current is then monitored and the thermodynamic efficiency is calculated. Results show on the one hand a significant increase in efficiency for a certain margin of applied electric field, and on the another hand the instabilities of efficiency. A simple approach is then suggested for the implementation of these results. An efficiency of 67% has been reached for an applied electric of 1586 V/Cm.

Keywords Improve efficiency · External applied electric field · Band-trap ionization of charge carriers · Silicon solar cell

Abbreviation

h	Planck's constant (6.626×10^{-34} Js)
c	Photons propagation speed (3×10^{8} m/s)
q	Charge of electron (1.6×10^{-19} C)
V	Voltage (V)
k	Boltzmann's constant: (1.38×10^{-23} J/K)
T_c	Cell's temperature (K)
T_s	Sun's temperature (6000 °K)
μ_p	Hole mobility
E_G	Gap energy (for silicon $E_G = 1.12$ eV)
τ_n, τ_p	Recombination life time of electrons and holes, respectively (s)
μ_n	Electron mobility
L_n, L_p	Electron diffusion length and hole diffusion length, respectively
S	Area of surface subject to the radiation (Cm^2)
n_i	Intrinsic concentration of electrons and holes ($n_i = 1.45 \times 10^{10}$ Cm^{-3} for silicon)
f_ω	Geometrical factor
Q_s	Number of quanta
t_s	Probability that incident photon will produce a hole-electron pair
E_o	External applied electric field (V/Cm)
C.B	Conduction band
V.B	Valence band

✉ R. Zieba Falama
rubenziebafalama@gmail.com

[1] Department of Renewable Energy, The Higher Institute of the Sahel, University of Maroua, P.O Box 46, Maroua, Cameroon

[2] Laboratory of Energy Research, Institute of Geological and Mining Research, P.O Box 4110, Yaoundé, Cameroon

[3] Department of Physics, Higher Teachers' Training College, University of Maroua, P.O Box 55, Maroua, Cameroon

[4] Department of Physics, Faculty of Science, University of Ngaoundéré, P.O Box 454, Ngaoundéré, Cameroon

[5] Department of Physics, Faculty of Science, University of Yaoundé I, P.O. Box. 812, Yaoundé, Cameroon

Introduction

Developing new concepts to improve the efficiency of photovoltaic solar cells is a well-known challenge for the scientific community. In 1961 Shockley and Queisser [1] brought out the theoretical limit of a photovoltaic solar cell. The results of their study are worldwide recognized as theoretical limit of efficiency for a single *pn*-junction solar cell. After them, many studies have been carried out to explore the possibilities of exceeding this limit. Different technologies and methods were used for that purpose [2–15]. Among these approaches, we could cite tandem cells, concentrator cell, carrier multiplication, down conversion, hot carriers, etc.

In 1997, by considering the impact ionization phenomenon to generate hot electrons, Würfel [11] found a maximum efficiency of 85% for a vanishing band gap of the solar cell. In 1993, Landsberg et al. [12] reported an efficiency of 60.3% at $E_G = 0.8$ eV for a solar cell submitted to band–band impact ionization effect. By considering the impact ionization effects on the efficiency of intermediate band solar cells, Gorji [16] has obtained a thermodynamic efficiency of 81.2%, which was higher than the maximum efficiency of 63.2% for an intermediate band without impact ionization mechanism. All these results show that the impact ionization phenomenon is very promising for the improvement of solar cell efficiency.

Impact ionization is a process in which a charge carrier with high kinetic energy collides with a second charge carrier transferring its kinetic energy to the latter which is hereby lifted to higher energy level [17]. This process increases the number of charge carriers. There are many impact ionization models. One can have: the one carrier model and the two carrier models which are often classified as band–band and band-trap impact ionization [18]. The current study focuses on the band trap impact ionization of the solar cell to reach a high efficiency. Free carriers are subject to trapping [19]. The default and the presence of some impurities in the solar cell material introduce trap levels into the band gap. These levels can emit an electron towards conduction band (case A in Fig. 1), receive an electron from valence band (case B in Fig. 1), receive an electron from conduction band (case C in Fig. 1) or loss an electron towards valence band (case D in Fig. 1). The cases A and B are those on which this paper focused because they permit to generate additional free charge carriers. These cases can easily been obtained through the process of impact ionization induced by an external source of energy. The current study considers an external applied electric field as the parameter which induces the impact ionization. The models of generation–recombination mechanism with band trap impact ionization involving electrons and holes are presented in Refs. [18, 20–23].

A single *pn*-junction of the solar cell is considered in this work. In Ref. [17] it has been shown that outside an electron diffusion length L_n to the right or a hole diffusion length L_p to the left of the *pn*-junction, the charge current through a *pn*-junction is a pure electron current in the *n*-region and a pure hole current in the *p*-region. This charge current is then given by integrating over the contributions to the electron current (alternatively, the contributions to the hole current). Knowing the number of free electrons in *n*-region (or free holes in *p*-region) could be sufficient to evaluate the charge current through a *pn*-junction and thus the open circuit voltage. In this paper, the number of free electrons in *n*-region of the *pn*-junction is evaluated by solving the reaction diffusion equation. This equation is solved with the factorization method. The total hole-electron generation rate due to the solar radiation is evaluated by following the Shockley–Queisser approach [1].

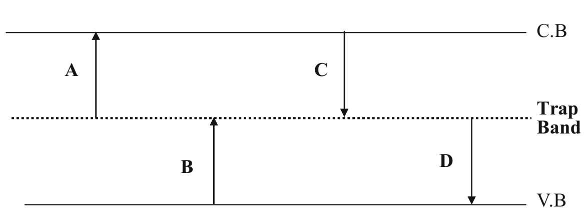

Fig. 1 Transition of charge carriers via trap level

Studied model

The current delivered by a photovoltaic solar cell is based on the generation–recombination mechanism. The generation–recombination model in this paper is based on the band trap impact ionization phenomenon involving two carriers (holes and electrons). The rate equations of the model are:

$$f_n(n,p) = Y + \left[X_1 N_D^* - X_1 n - (B - X_1)p\right]n \quad (1a)$$

$$f_p(n,p) = Y + [X_2 P_D - X_2 p - (B - X_2)n]p \quad (1b)$$

where n and p are the number of free electrons and free holes, respectively. The variables X_1 and X_2 are band-trap impact ionization coefficients (to generate additional electrons and holes, respectively) which depend on the applied electric field. Y and B are the band–band generation coefficient and the band–band recombination coefficient, respectively. Y is a photo-generation parameter due to the illumination of solar cell. This variable is evaluated through Eq. (43). The constants N_D^* and N_t are the effective donor density and the trap density, respectively. $P_D = N_t - N_D^*$.

Considering the fact that the total charge current through a *pn*-junction is a pure electron current in the *n*-region, the knowledge of the free electrons number in the *n*-region is requested to determine the total charge current through the *pn*-junction. Considering only the transverse direction and neglecting the transverse electric field, the number of free electrons can be determined using the reaction–diffusion equation presented in Eq. (2).

$$\frac{\partial n}{\partial t} - D_n \frac{\partial^2 n}{\partial x^2} = f_n(n,p) \quad (2)$$

In steady state regime, $\frac{\partial n}{\partial t} = 0$. Thus Eq. (2) turns to:

$$\frac{\partial^2 n}{\partial x^2} + \frac{f_n(n,p)}{D_n} = 0 \quad (3)$$

the constant D_n is the electron diffusion coefficient defined by:

$$D_n = \mu_n \frac{kT_c}{q} \tag{4}$$

Thermodynamic efficiency calculation

Resolution of the reaction–diffusion equation

The factorization method [24, 25] is used in this work to solve the reaction–diffusion equation. The equation to solve is Eq. (3).

By setting

$$D = \frac{\partial}{\partial x}, D^2 = \frac{\partial^2}{\partial x^2}, g_1(n,p) = 0, \text{ and } f_1(n,p) = \frac{f_n(n,p)}{D_n},$$

The Eq. (3) can turn to:

$$\left(D^2 + g_1(n,p)D + \frac{f_1(n,p)}{n}\right)n = 0 \tag{5}$$

The Eq. (5) can be factorized as:

$$(D - \psi_{12})(D - \psi_{11})n = 0 \tag{6}$$

where

$$f_1(n,p) = n\psi_{12}\psi_{11} \tag{7}$$

The Eq. (3) can be developed and leads to:

$$n'' - \left(\psi_{11} + \psi_{12} + n\frac{\partial \psi_{11}}{\partial n}\right)n' + \psi_{12}\psi_{11}n = 0 \tag{8}$$

The comparison of Eqs. (3) and (8) leads to:

$$0 = -\left(n\frac{\partial \psi_{11}}{\partial n} + \psi_{11} + \psi_{12}\right) \tag{9}$$

$$\frac{f_n(n,p)}{D_n} = \psi_{12}\psi_{11}n \tag{10}$$

Eq. (1a) leads to:

$$\frac{f_n(n,p)}{D_n} = \frac{1}{K_1 D_n}\left(1 - \frac{(B - X_1)p}{\frac{Y}{n} + X_1(N_D^* - n)}\right) \times K_1\left(\frac{Y}{n} + X_1\left(N_D^* - n\right)\right)n \tag{11}$$

Therefore, referring to Eq. (10), ψ_{ij} could be choosing such as:

$$\psi_{11} = K_1\left(\frac{Y}{n} + X_1\left(N_D^* - n\right)\right) \tag{12a}$$

$$\psi_{12} = \frac{1}{K_1 D_n}\left(1 - \frac{(B - X_1)p}{\frac{Y}{n} + X_1(N_D^* - n)}\right) \tag{12b}$$

where K_1 is an arbitrary constant to be determined.To determine the constants K_1 let us consider Eq. (9). This equation can be developed and leads to,

$$K_1^2 X_1 N_D^* + \frac{1}{D_n} - 2K_1^2 X_1 n - \frac{(B - X_1)p}{D_n\left(\frac{Y}{n} + X_1 N_D^* - X_1 n\right)} = 0 \tag{13}$$

The Eq. (13) admits solution if:

$$\begin{cases} K_1^2 X_1 N_D^* + \dfrac{1}{D_n} = 0, \\ \quad\quad\text{or} \\ -2K_1^2 X_1 n - \dfrac{(B - X_1)p}{D_n\left(\dfrac{Y}{n} + X_1 N_D^* - X_1 n\right)} = 0 \end{cases} \tag{14}$$

The first condition of Eq. (14) leads to:

$$K_1 = i\sqrt{\frac{1}{X_1 N_D^* D_n}}, \tag{15}$$

The Eq. (5) transformed to two possible differential equations of first order such as [25]:

$$n' - \psi_{11}(n,p)n = 0 \tag{16a}$$

$$n' - \psi_{12}(n,p)n = 0 \tag{16b}$$

Let us consider Eq. (16a). The replacement of Eq. (12a) into Eq. (16a) leads to:

$$\int \frac{dn}{Y + X_1(N_D^* - n)n} = \int_{z_0}^{z} K_1 dz \tag{17}$$

The Eq. (17) permits to obtain:

$$\left|\frac{n - \alpha}{n - \beta}\right| = e^{K_1\sqrt{\Delta}(z - z_0)} \tag{18}$$

where

$$\Delta = \left(X_1 N_D^*\right)^2 + 4YX_1, \tag{19a}$$

$$\alpha = \frac{X_1 N_D^* - \sqrt{\Delta}}{2X_1}, \tag{19b}$$

and

$$\beta = \frac{X_1 N_D^* + \sqrt{\Delta}}{2X_1} \tag{19c}$$

We have to consider two cases:

First case

For $\left|\frac{n-\alpha}{n-\beta}\right| = -\frac{n-\alpha}{n-\beta}$, Eq. (18) leads to:

$$n = \frac{\alpha}{1 + e^{K_1\sqrt{\Delta}(x - x_0)}} + \frac{\beta e^{K_1\sqrt{\Delta}(x - x_0)}}{1 + e^{K_1\sqrt{\Delta}(x - x_0)}} \tag{20}$$

By setting $K_1 = iK_2$, Eq. (20) can turn to:

$$n = \frac{\alpha e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}} + \frac{\beta e^{iK_2\sqrt{\Delta}(x-x_0)/2}}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\Delta(x-x_0)/2}} \tag{21}$$

where

$$K_2 = \sqrt{\frac{1}{X_1 N_D^* D_n}} \tag{22}$$

By setting $\gamma = \frac{\alpha e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}$ and $\delta = \frac{\beta e^{iK_2\sqrt{\Delta}(x-x_0)/2}}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}$, one could have:

$$\gamma = \frac{\alpha\left(\frac{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{2}\right) + \alpha e^{-iK_2\sqrt{\Delta}(z-z_0)/2} - \alpha\left(\frac{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{2}\right)}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}} \tag{23a}$$

and

$$\delta = \frac{\beta\left(\frac{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{2}\right) + \beta e^{iK_2\sqrt{\Delta}(x-x_0)/2} - \beta\left(\frac{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}}{2}\right)}{e^{iK_2\sqrt{\Delta}(x-x_0)/2} + e^{-iK_2\sqrt{\Delta}(x-x_0)/2}} \tag{23b}$$

Equations (23a) and (23b) turn to:

$$\gamma = \frac{\alpha}{2}\left(1 - \tan\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right)\right) \tag{24a}$$

and

$$\delta = \frac{\beta}{2}\left(1 + \tan\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right)\right) \tag{24b}$$

Thus, Eq. (20) turns to:

$$n = \frac{\alpha}{2}\left(1 - \tan\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right)\right) + \frac{\beta}{2}\left(1 + \tan\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right)\right) \tag{25}$$

By replacing α and β by their expressions in Eq. (25) one obtains:

$$n = \frac{N_D^*}{2} - \frac{\sqrt{\Delta}}{2X_1}\tan\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right) \tag{26}$$

Second case

For $\left|\frac{n-\alpha}{n-\beta}\right| = \frac{n-\alpha}{n-\beta}$, Eq. (18) leads to:

$$n = \frac{\alpha}{1 - e^{K_1\sqrt{\Delta}(x-x_0)}} - \frac{\beta e^{K_1\sqrt{\Delta}(x-x_0)}}{1 - e^{K_1\sqrt{\Delta}(x-x_0)}} \tag{27}$$

Following a same approach like in the first case, one gets:

$$n = \frac{N_D^*}{2} + \frac{\sqrt{\Delta}}{2X_1}\cot\left(\frac{K_2\sqrt{\Delta}(x-x_0)}{2}\right) \tag{28}$$

Charge current density through the *pn*-junction

According to Ref. [17], the total charge current through a *pn*-junction could be expressed by:

$$j_Q = q\int_{-L_p}^{L_n} \mathrm{div} j_n \mathrm{d}x \tag{29}$$

where

$$L_n = \sqrt{D_n \tau_n} \tag{30a}$$

$$L_p = \sqrt{D_p \tau_p} \tag{30b}$$

$$D_p = \mu_p \frac{kT_c}{q} \tag{30c}$$

From the continuity equation for electrons, one gets:

$$\mathrm{div} j_n = q\left(\frac{\partial n}{\partial t} - f_n(n,p)\right) \tag{31}$$

By replacing Eq. (1a) into Eq. (31) and Eq. (31) into Eq. (29), and considering the fact that $\frac{\partial n}{\partial t} = 0$ (steady state condition), then Eq. (29) turns to:

$$j_Q = -q^2 \int_{-L_p}^{L_n} Y\mathrm{d}x - q^2 X_1 N_D^* \int_{-L_p}^{L_n} n\mathrm{d}x + q^2 X_1 \int_{-L_p}^{L_n} n^2\mathrm{d}x + q^2(B - X_1) \int_{-L_p}^{L_n} pn\mathrm{d}x \quad (32)$$

For simplicity let us set:

$$\theta_1 = q^2 \int_{-L_p}^{L_n} Y\mathrm{d}x, \quad (33a)$$

$$\theta_2 = q^2 X_1 N_D^* \int_{-L_p}^{L_n} n\mathrm{d}x, \quad (33b)$$

$$\theta_3 = q^2 X_1 \int_{-L_p}^{L_n} n^2\mathrm{d}x, \quad (33c)$$

$$\theta_4 = q^2(B - X_1) \int_{-L_p}^{L_n} pn\mathrm{d}x \quad (33d)$$

The calculation of θ_1, θ_2, θ_3 and θ_4 leads to:

$$\theta_1 = q^2 Y(L_n + L_p) \quad (34a)$$

$$\theta_2 = q^2 X_1 N_D^* \left(\frac{N_D^*}{2}(L_n + L_p) + \frac{1}{K_2 X_1} \log\left|\cos\left(\frac{K_2\sqrt{\Delta}}{2}(L_n - x_0)\right)\right| - \frac{1}{K_2 X_1} \log\left|\cos\left(\frac{K_2\sqrt{\Delta}}{2}(-L_p - x_0)\right)\right| \right) \quad (34b)$$

$$\theta_3 = q^2 X_1 \left(\left(\frac{N_D^{*2}}{4} - \frac{\Delta}{4X_1^2}\right)(L_n + L_p) + \frac{N_D^*}{K_2 X_1} \log\left|\cos\left(\frac{K_2\sqrt{\Delta}}{2}(L_n - x_0)\right)\right| - \frac{N_D^*}{K_2 X_1} \log\left|\cos\left(\frac{K_2\sqrt{\Delta}}{2}(-L_p - x_0)\right)\right| + \frac{\sqrt{\Delta}}{2K_2 X_1^2} \tan\left(\frac{K_2\sqrt{\Delta}}{2}(L_n - x_0)\right) - \frac{\sqrt{\Delta}}{2K_2 X_1^2} \tan\left(\frac{K_2\sqrt{\Delta}}{2}(-L_p - x_0)\right) \right) \quad (34c)$$

$$\theta_4 = q^2 n_i^2 (B - X_1)(L_n + L_p)\exp\left(\frac{qV}{kT_c}\right) \quad (34d)$$

Thus, Eq. (32) could be rewritten as:

$$j_Q = -\theta_1 - \theta_2 + \theta_3 + q^2 n_i^2 (B - X_1)(L_n + L_p)\exp\left(\frac{qV}{kT_c}\right) \quad (35)$$

The short-circuit current j_{sc} (when $V = 0$) is defined as:

$$j_{sc} = -\theta_1 - \theta_2 + \theta_3 + q^2 n_i^2 (B - X_1)(L_n + L_p) \quad (36)$$

Equation (35) could be rewritten as:

$$j_Q = j_{sc} + q^2 n_i^2 (B - X_1)(L_n + L_p)\left(\exp\left(\frac{qV}{kT_c}\right) - 1\right) \quad (37)$$

By considering the dark where $j_{sc} = 0$ and for large negative voltages (where $\exp\left(\frac{qV}{kT_c}\right) \ll 1$), one gets the reverse saturation current j_s as:

$$j_s = q^2 n_i^2 (B - X_1)(L_n + L_p) \quad (38)$$

From the relation

$$j_Q = j_{sc} + j_s\left(\exp\left(\frac{qV}{kT_c}\right) - 1\right), \quad (39)$$

the open circuit voltage (when $j_Q = 0$) is deducted as:

$$V_{oc} = \frac{kT_c}{q} \ln\left(1 - \frac{j_{sc}}{j_s}\right) \quad (40)$$

According to Ref. [17], the thermodynamic efficiency is given by:

$$\eta_{\text{thermodynamic}} = \frac{qV_{oc}}{E_G + 3kT_c} \quad (41)$$

According to Ref. [1], the photo-generation rate of hole-electron pairs Y is defined by:

$$Y = Sf_\omega t_s Q_s \quad (42)$$

where

$$Q_s = \left[2\pi(kT_s)^3/(h^3c^2)\right] \int_{x_g}^{\infty} \frac{x^2\mathrm{d}x}{e^x - 1} \quad (43)$$

$$x_g = \frac{E_G}{kT_s} \quad (44)$$

.

Table 1 Typical materials parameters corresponding to α-si near room temperature for the g–r process of band-trap impact ionization [18]

Parameters	Value
X_1	$3 \times 10^{-5}\exp(-2 \times 10^4/E_o)$ Cm^3 S^{-1}
B	10^{-10} Cm^3 S^{-1}
N_D^*	2×10^{15} Cm^{-3}
N_t	3×10^{15} Cm^{-3}
μ_n/μ_p	1
μ_n	2 Cm^2/Vs

Discussion

All the needed parameters for simulation are presented in Table 1.

Figure 2 presents the thermodynamic efficiency of the studied silicon solar cell modeled as an increase function for the external applied electric field in the range of 0–1586 V/Cm. An efficiency of 67% is reached for $E_o = 1586$ V/Cm. From $E_o > 1586$ V/Cm an efficiency fluctuation (increasing and decreasing) is noted. This fluctuation could be due to current instabilities which emerge from solar cells (which are semiconductors) when the applied electric field is increasing [26]. Figure 3 has been plotted for $0 < E_o < 10^6$ V/Cm and shows that the solar cell could reach a high efficiency for strong electric field as it is the case for $E_o = 8 \times 10^5$ V/Cm where an efficiency of 72.7% has been reached. In this case, the problem is that, for the same value of electric field, there are different values of efficiency because of fluctuations. Therefore, it could be difficult to determinate exactly the high efficiency of the solar cell subjected to impact ionization (induced by an external applied electric field) in the margin of which efficiency fluctuates and is unstable. The results obtained shows that the band-trap impact ionization of charge carriers induced by an applied electric field could be an interesting solution to reach a high efficiency of the photovoltaic solar cells. However, it is very important to know and avoid applying electric field belonging to the range which induces efficiency instabilities. Theoretically a high efficiency of solar cell could be reached even for applied electric fields of average intensity (67% has been reached at $E_o = 1586$ V/Cm).

Conclusion

In this paper, the effect of an external applied electric field on the thermodynamic efficiency of a silicon photovoltaic solar cell has been studied. Theoretically, it has been shown that an auxiliary applied electric field could be a very promising solution to reach a high efficiency

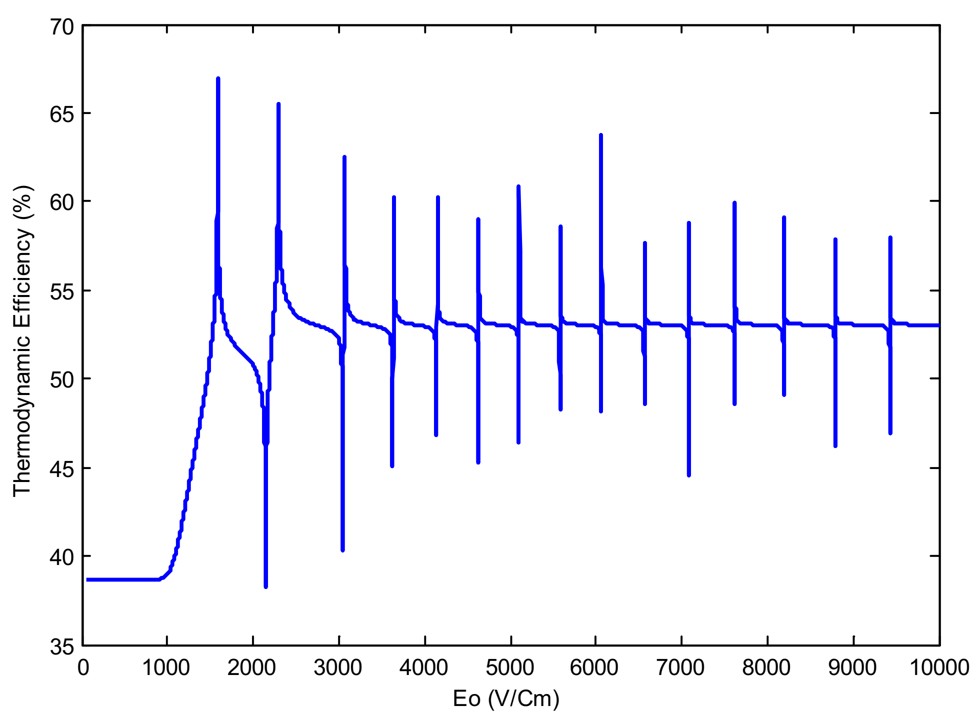

Fig. 2 Thermodynamic efficiency versus external applied electric field for $0 < Eo < 10^4$ V/Cm. $T_c = 300$ K, $T_s = 6000$ K, $S = 1$ Cm^2, $f_\omega = 2.18 \times 10^{-5}$, $t_s = 1$, $\tau_n = \tau_p = 10^{-6}$ s, $x_0 = 0$

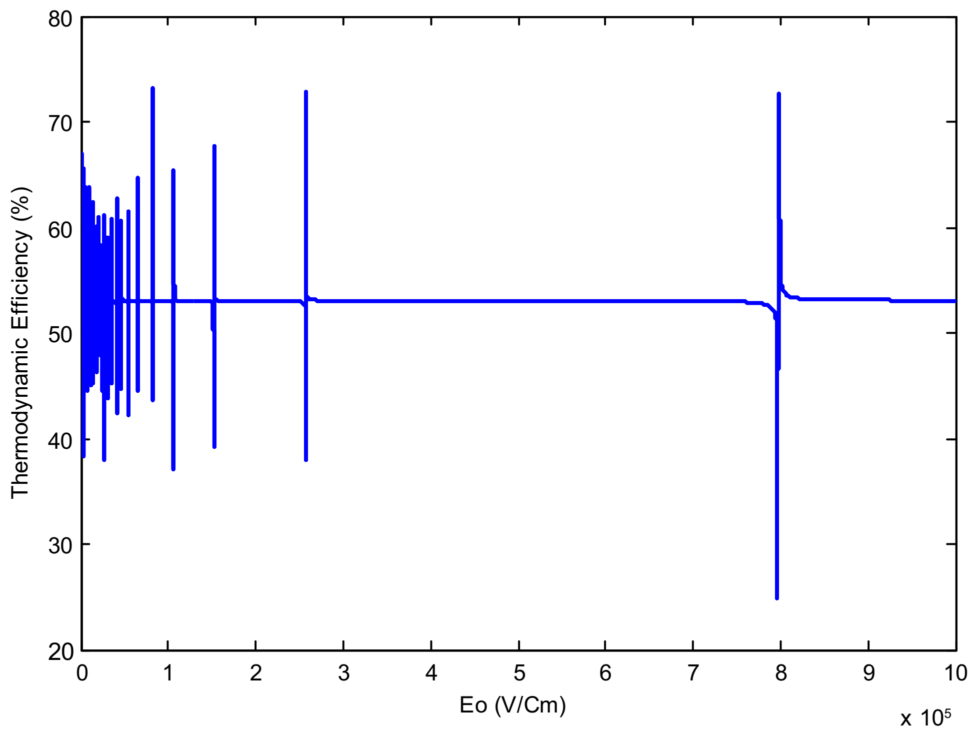

Fig. 3 Thermodynamic efficiency versus external applied electric field for $0 < Eo < 10^6$ V/Cm. $T_c = 300$ K, $T_s = 6000$ K, $S = 1$ Cm^2, $f_\omega = 2.18 \times 10^{-5}$, $t_s = 1$, $\tau_n = \tau_p = 10^{-6}$ s, $x_0 = 0$

of the solar cells. However, it is not always the stronger electric field which is necessary to induce the higher efficiency. There are efficiency instabilities for strong applied electric field to solar cells.

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