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Solitary and cnoidal solution of ZK equation in multicomponent magnetized dusty plasma with Fermi-Dirac electrons and positrons

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Abstract
The nature of nonlinear ion acoustic waves in multicomponent magnetized dusty plasma have been investigated theoretically. Species of modeled plasma include negative dust particle, ions, electrons and positrons. Both electrons and positrons obey Fermi-Dirac distribution function. Employing reduction perturbation method, the fluid equations was solved to achieve ZK equation. Solitary and cnoidal solution of ZK equation were considered due to number of roots of Sagdeev equation. Results show that both solitonic and cnoidal solutions of ZK equation are under the influence of considered quantumic effect of the modeled plasma. Amplitude of both generated modes in plasma medium varies noticeably with variation of quantum Bohm potential term.

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1. Introduction

There is extremely high number of works on different kinds of linear and nonlinear waves in plasmas includes dusty plasma [1–9]. Since plasma has the largest contribution in the states of matter in the world, it has many different models. Variation of plasma species, their energy and distribution function, being isotropic or anisotropic, may change the scenario. Furthermore, plasma is a multimode medium and there are several modes which can generate and propagate in plasma medium. Here we have come to high density plasma, occur in extreme astrophysical environments, such as white dwarfs, magnetars, or neutron stars and in the core of giant planets [10–15], as well as micro-electronic devices [16] and ultra-cold plasmas [17]. It is believed that the plasmas in the interior of white drafts and in the crust of neutron stars are extremely dense and highly degenerate, (electron number densities $n_{e0} > 10^{20}\text{cm}^{-3}$, temperature is in the range $10^5\text{K} < T < 10^8\text{K}$ and the magnetic field strength reaching very large values ;i.e.,$B \gg 10^9\text{G}$) [18, 19]. For such plasmas, the electron Fermi temperature T_{fe} is much higher than T and the quantum mechanical effects associated with the quantum statistical pressure and due to the quantum tunneling effects, a new force in terms of the gradient of Bohm potential appears in the momentum equation [20]. The plasma frequency is also sufficiently high in such cases due to very large value of the equilibrium density, and the degeneracy parameter $n\lambda_B^3 \geq 1$; i.e., The de Broglie wavelength λ_B of the plasma particles is of the order of the average interparticle distance $n^{-1/3}$, where n is the particle number density. Quantum plasmas are studied mainly by two approaches, viz. quantum kinetic approach and quantum hydrodynamic (QHD) approach. The kinetic approach is needed to discuss the Landau damping [21] of waves in quantum plasmas. The most widely used approach for studying quantum plasmas is QHD approach. Madelung [22] was the first to give the mathematical derivation of QHD model [23]. The dispersion relation of a quantum dusty plasma, based on QHD theory and the propagation of dust-ion acoustic (DIA) shock waves in an unmagnetized collisionless four-component quantum plasma were investigated by some authors [24, 25]. Most of these studies in quantum plasmas have been made by applying the reductive perturbation technique. Recently, M. Hanif and et.al [26] employed a numerical technique to study ion acoustic (IA) shock waves in dense quantum plasmas. Sagdeev's method is applied in order to observe the existence of arbitrary amplitude solitary wave. The inclusion of Bohm potential term in the momentum equation makes the task of finding the closed-form analytical expression of pseudopotential difficult. However, S. Mahmood and et.al. [27] studied IA wave propagation in an unmagnetized quantum plasma by using Sagdeev's pseudopotential approach under quasi-neutrality condition. Later, S. Mahmood [28] employed the same method to study the DIA waves in dense Fermi plasmas. It is worth mentioning that the study of nonlinear periodic waves in plasmas as well as in other dispersive media has become important due to their application in diverse areas of physics such as the nonlinear transport phenomenon. Cnoidal waves based on Jacobian elliptical functions, such as sn, cn, and dn waves, are exact solutions in the form of periodic pulses. Cnoidal waves transform into well-known solitons in the limit of strong spatial localization. Nonlinear periodic wave signals appear beside ion-acoustic soliton and double layer structures in auroral and magnetospheric plasmas [29]. Cnoidal waves have been observed in water, experimentally [30,31]. Moreover, cnoidal waves have been applied as a fundamental basis function to develop a new kind of nonlinear Fourier analysis to explain Adriatic Sea waves [32]. On the other side, Kauschke and Schluter [33] explained single-mode drift wave spectra at the edge of the tokamak plasma on the basis of cnoidal waves [34]. Finally, the purpose of this article is the study of nonlinear propagation of dust ion acoustic compressive solitary waves and cnoidal waves in multi-component plasma. Plasma medium include dust particles, classical cold ions, and quantum magnetized electrons and positrons with Fermi-Dirac distribution function in the presence of external magnetic field. The effects of the electron cyclotron to electron plasma frequency ratio(Ω_c), dust concentration (d), quantum Bohm potential term (H) and the direction cosine of the wave propagation vector with the Cartesian coordinates (l) on the mentioned waves, potential function and electric field are investigated. For this purpose, we choose some of the typical plasma parameters found in astrophysical environments [35–37] which $n_{e0} = 5.9\times10^{28}\text{cm}^{-3}$, $n_{p0} = 5.32\times10^{28}\text{cm}^{-3}$, $n_{i0} = 5.8\times10^{27}\text{cm}^{-3}$ and $B = 10^9$G. The Fermi temperatures of electrons and positrons at such densities are $T_{Fe} = 1.96\times10^8$K and $T_{Fp} = 1.69\times10^8$K.

2. Model description

2.1 The governing equations for DIAW

We have considered two-dimensional homogeneous collision less four-component quantum plasma, which is containing of inertia less Fermi-Dirac distributed quantum electrons and positrons, classical cold ions and stationary negative dust grains. The ambient magnetic field $\mathbf{B}_0 = B_0\hat{z}$ is assumed to be stationary, pointing along the z-axis. The electrons and positrons are assumed to follow the one-dimensional zero-temperature Fermi gas pressure law $P_j = (m v_{Fj}^2 n_j^{5/3})/(5n_{j0}^{2/3})$, where $v_{Fj} = [2k_B T_{Fj}/m_j]^{1/2}$ is Fermi speed and T_{Fj} is Fermi temperature (here $j = e, p$) [38]. Quasi-neutrality is assumed to be held only at equilibrium (un normalized form), i.e., $n_{i0}+n_{p0} = n_{e0}+Z_{d0}n_{d0}$, where n_{j0} is the equilibrium number density of specie j (here $j = p,e,i,d$ refers to the positrons , electrons , ions and negative dusts respectively), and Z_{d0} is the equilibrium charging state of dust grain. The normalized form of the following fluid equations is used to describe the dynamics of the magnetized four-component dust-plasma model:

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i u_{ix}) + \frac{\partial}{\partial z}(n_i u_{iz}) = 0 \tag{1}$$

$$\frac{\partial \mathbf{u}_i}{\partial t} + (\mathbf{u}_i.\nabla)\mathbf{u}_i = \frac{m_e}{m_i}(-\nabla\phi + \Omega_c(\mathbf{u}_i\times\hat{z})) \tag{2}$$

$$\nabla_{\parallel}\phi - \frac{1}{5n_e}\nabla_{\parallel}n_e^{\frac{5}{3}} + \frac{H^2}{2}\nabla_{\parallel}(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}) = 0 \tag{3}$$

$$-\nabla_{\parallel}\phi - \frac{\sigma_T}{5n_p}\nabla_{\parallel}n_p^{\frac{5}{3}} + \frac{H^2}{2}\nabla_{\parallel}(\frac{\nabla^2\sqrt{n_p}}{\sqrt{n_p}}) = 0 \tag{4}$$

$$\nabla^2\phi = n_e - \beta n_p - \delta n_i + d \tag{5}$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_{jx}) + \frac{\partial}{\partial z}(n_j v_{jz}) = 0, (j = e,p) \tag{6}$$

The second and third terms in equations (3) and (4) show quantum effects which second term is due to Fermi-Dirac distribution and third term is due to quantum diffraction. Integration the electron and positron momentum equations along the z axis with the boundary conditions $n_e = 1$,$n_p = 1$ and $\phi = 0$ at infinity yields

$$\phi - \frac{1}{5}n_e^{\frac{2}{3}} + \frac{H^2}{2}(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}) = 0 \tag{7}$$

$$-\phi - \frac{\sigma_T}{5}n_p^{\frac{2}{3}} + \frac{H^2}{2}(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}) = 0 \tag{8}$$

n_j has been defined as the perturbed part of number density of species j ($j = i,e,p$) which is normalized by its equilibrium value n_{j0}. $\mathbf{u}_i$ is the ion velocity , normalized by v_{Fe}. In addition ϕ is the electrostatic potential which is normalized by $m_e v_{Fe}/e$. In above equations $\sigma_T = T_{Fp}/T_{Fe}$ is the ratio of positron to electron Fermi temperature and $\beta = n_{p0}/n_{e0}$, $\delta = n_{i0}/n_{e0}$, $d = Z_{d0}n_{d0}/n_{e0}$ and $H = \hbar\omega_{pe}m_e^{1/2}/2k_B T_{Fe}$. We assume that the wave propagation is in two dimensions, i.e. $\nabla = (\partial_x, 0, \partial_z)$, and it normalized by v_{Fe}/ω_{pe} where $\omega_{pe} = (4\pi e^2 n_{e0}/m_e)^{1/2}$. The time variable t is normalized by the inverse of ω_{pe}.The quantity Ω_c appeared in equation (2) is tantamount to ω_c/ω_{pe} where $\omega_c(= eB_0/m_e c)$ is electron cyclotron frequency.

2.2 Derivation of Zkharov-Kuznetsov (ZK) equation

To elucidate the dynamics of weakly nonlinear dust ion acoustic waves, we adopt the reductive perturbation technique (RPT) to achieve the Zkharov-Kuznetsov (ZK) equation in

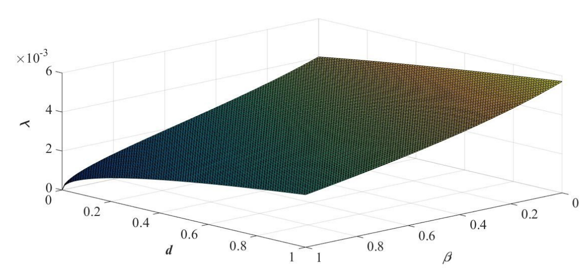

Figure 1. Phase velocity versus d and β.

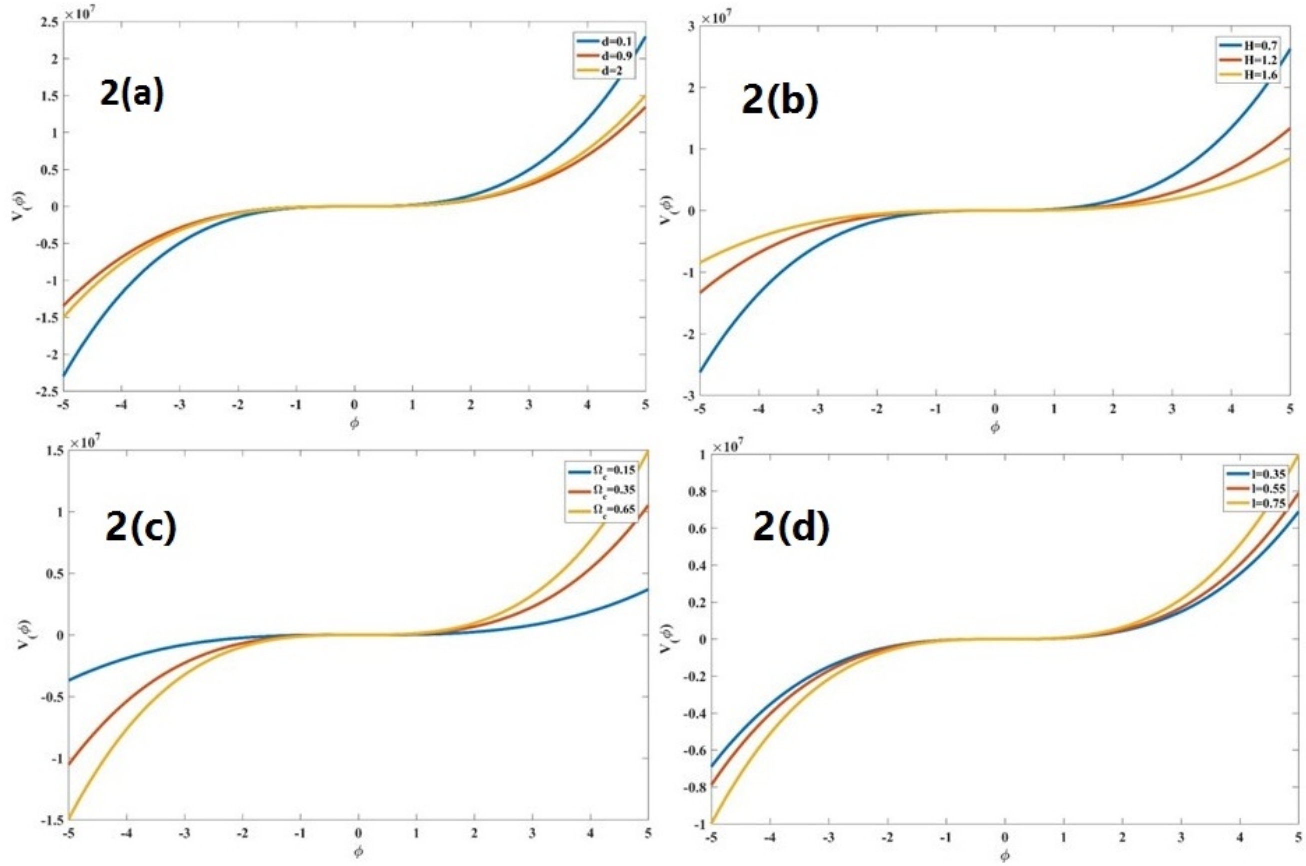

Figure 2. (a) The variation of $V(\phi)$ versus ϕ at different values of d for $H = 1.2, \Omega_c = 0.5, l = 0.9, \sigma_T = 0.86$. (b) The variation of versus $V(\phi)$ at ϕ different values of H for $d = 0.6, \Omega_c = 0.5, l = 0.9, \sigma_T = 0.86$. (c) The variation of $V(\phi)$ versus ϕ at different values of l for $d = 0.6, \Omega_c = 0.5, l = 0.9, \sigma_T = 0.86$. (d) The variation of $V(\phi)$ versus ϕ at different values of Ω_c for $d = 0.6, \Omega_c = 0.5, l = 0.9, \sigma_T = 0.86$.

a four-component magnetized dusty plasma model. The stretched coordinates are defined as

$$T = \varepsilon^{\frac{3}{2}} t, Z = \varepsilon^{\frac{1}{2}}(lz - \lambda t), X = \varepsilon^{\frac{1}{2}}(mx), (0 < \varepsilon \ll 1) \quad (9)$$

l and m are direction cosines of the wave vector k along the directions z and x, respectively, which are defined as $l^2 + m^2 = 1$. $\varepsilon$is a small expansion parameter measuring the weakness of the dispersion as well as λ is the phase velocity of the solitary wavefront. The value of λ will later be determined by compatibility requirements. The perturbed quantities (dependent variables $\varphi, n_j, \mathbf{u}_i$) can be expanded in power series of ε as:

$$(n_j, \varphi, u_{iz}) = (1, 0, 0) + \varepsilon(n_j^{(1)}, \varphi^{(1)}, u_{iz}^{(1)}) + \varepsilon^2(n_j^{(2)}, \varphi^{(2)}, u_{iz}^{(2)}) + \ldots,$$

$$(u_{ix}, u_{iy}) = (0, 0) + \varepsilon^{\frac{3}{2}}(u_{ix}^{(1)}, u_{iy}^{(1)}) + \varepsilon^2(u_{ix}^{(2)}, u_{iy}^{(2)}) + \varepsilon^{\frac{5}{2}}(u_{ix}^{(3)}, u_{iy}^{(3)}) + \ldots, \quad (10)$$

We can now substitute equations (9) and (10) in equations (1) and extract a collection of equations in various powers of ε. By keeping the lowest-order ε terms, we achieve the following equations:

$$n_e^{(1)} - \beta n_p^{(1)} - \delta n_i^{(1)} = 0 \quad (11)$$

$$\phi^{(1)} - \frac{1}{3} n_e^{(1)} = 0 \quad (12)$$

$$-\phi^{(1)} - \frac{1}{3} \sigma_T n_p^{(1)} = 0 \quad (13)$$

The next higher order equations in ε are given by

$$-\lambda \frac{\partial}{\partial Z} n_i^{(1)} + l \frac{\partial}{\partial Z} u_{iz}^{(1)} = 0 \quad (14)$$

$$-\frac{m_e}{m_i}(m \frac{\partial}{\partial X} \phi^{(1)}) + \frac{m_e}{m_i} \Omega_c u_{iy}^{(1)} = 0 \quad (15)$$

$$\lambda \frac{\partial}{\partial Z} u_{iz}^{(1)} = \frac{m_e}{m_i}(l \frac{\partial}{\partial Z} \phi^{(1)}) \quad (16)$$

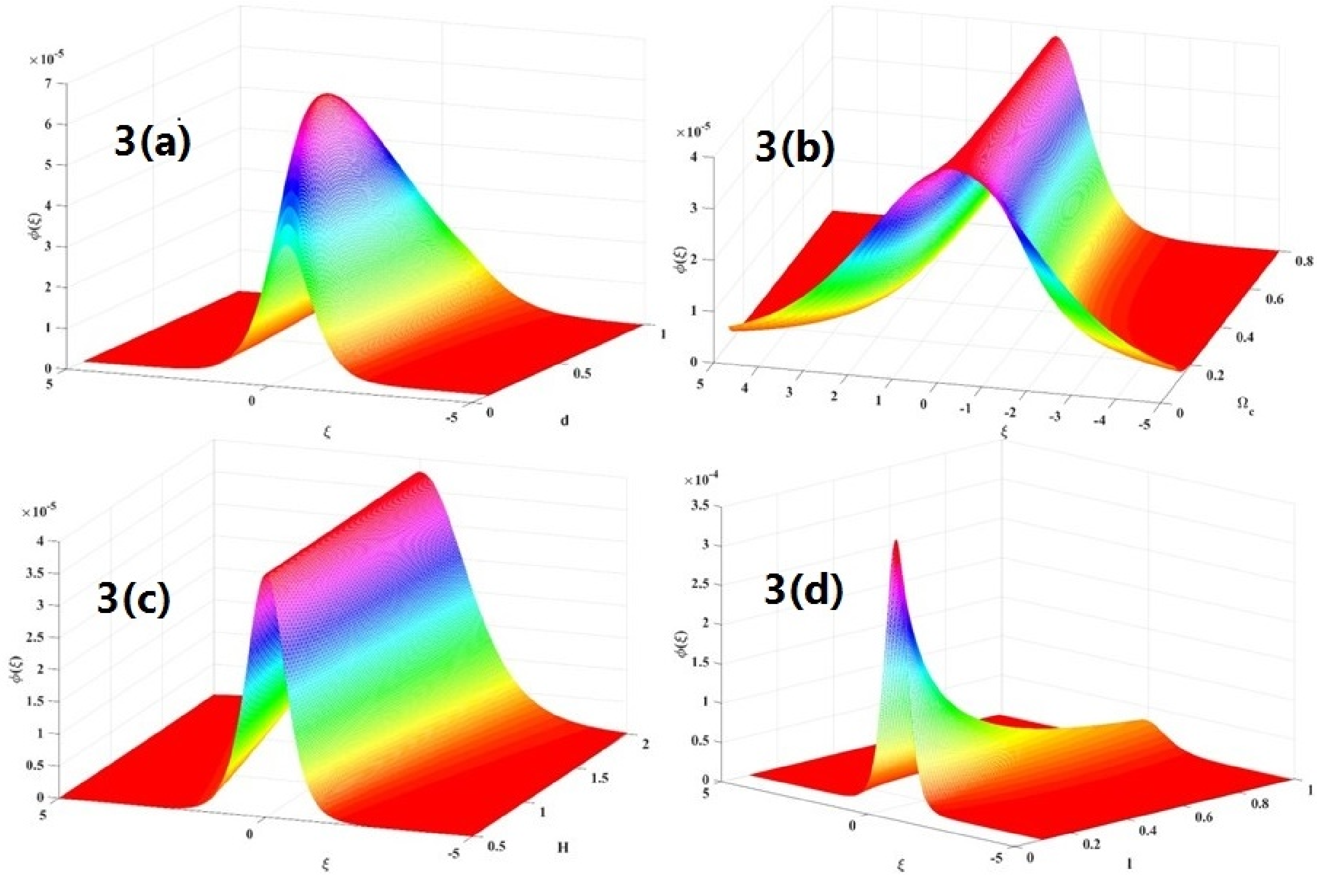

Figure 3. (a) 3D profile $\phi(\xi)$ versus $\xi$and d,where $H = 1.2$,$\Omega_c = 0.5$,$l = 0.9$,$\sigma_T = 0.86$. (b) 3D profile $\phi(\xi)$ versus $\xi$and Ω_c,where $H = 1.2$,$d = 0.76$,$l = 0.9$,$\sigma_T = 0.86$. (c) 3D profile $\phi(\xi)$ versus $\xi$and l,where $H = 1.2$,$d = 0.76$,$\Omega_c = 0.5$,$l = 0.9$,$\sigma_T = 0.86$. (d) 3D profile $\phi(\xi)$ versus ξ and H,where $l = 0.9$,$d = 0.76$,$\Omega_c = 0.5$,$\sigma_T = 0.86$.

$$-\lambda \frac{\partial}{\partial Z} u_{iz}^{(1)} = \frac{m_e}{m_i} \Omega_c u_{iy}^{(2)} \tag{17}$$

$$-\lambda \frac{\partial}{\partial Z} u_{iy}^{(1)} = -\frac{m_e}{m_i} \Omega_c u_{ix}^{(2)} \tag{18}$$

$$(m^2 \frac{\partial^2}{\partial X^2 + l^2 \frac{\partial^2}{\partial Z^2}})\phi^{(1)} = n_e^{(2)} - n_i^{(2)} + \beta(n_e^{(2)} - n_i^{(2)}) - dn_i^{(2)} \tag{19}$$

$$\phi^{(2)} - \frac{2}{15} n_e^{(2)} + \frac{H^2}{2}(m^2 \frac{\partial^2}{\partial X^2} + l^2 \frac{\partial^2}{\partial Z^2})(\frac{1}{2} n_e^{(1)}) = 0 \tag{20}$$

$$-\phi^{(2)} - \frac{2\sigma_T}{15} n_p^{(2)} + \frac{H^2}{2}(m^2 \frac{\partial^2}{\partial X^2} + l^2 \frac{\partial^2}{\partial Z^2})(\frac{1}{2} n_p^{(1)}) = 0 \tag{21}$$

$$-\lambda \frac{\partial}{\partial Z} n_i^{(2)} + \frac{\partial}{\partial T} n_i^{(1)} + m \frac{\partial}{\partial X} u_{ix}^{(2)} + l \frac{\partial}{\partial Z} u_{iz}^{(2)} + l \frac{\partial}{\partial Z}(n_i^{(1)} u_{iz}^{(1)}) = 0 \tag{22}$$

$$-\lambda \frac{\partial}{\partial Z} u_{iz}^{(2)} + \frac{\partial}{\partial T} u_{iz}^{(1)} + u_{iz}^{(1)} l \frac{\partial}{\partial Z} u_{iz}^{(1)} = -\frac{m_e}{m_i} l \frac{\partial}{\partial Z} \phi^{(2)} \tag{23}$$

$$l \frac{\partial}{\partial Z} \phi^{(2)} - \frac{l}{3} \frac{\partial}{\partial Z} n_e^{(2)} + \frac{l}{3} n_e^{(1)} \frac{\partial}{\partial Z} n_e^{(1)}$$

$$+\frac{H^2}{4} l \frac{\partial}{\partial Z}(m^2 \frac{\partial^2}{\partial X^2} + l^2 \frac{\partial^2}{\partial Z^2}) n_e^{(1)} = 0 \tag{24}$$

$$-l \frac{\partial}{\partial Z} \phi^{(2)} - \frac{l\sigma_T}{3} \frac{\partial}{\partial Z} n_p^{(2)} + \frac{l\sigma_T}{3} n_p^{(1)} \frac{\partial}{\partial Z} n_p^{(1)}$$

$$+\frac{H^2}{4}l\frac{\partial}{\partial Z}(m^2\frac{\partial^2}{\partial X^2}+l^2\frac{\partial^2}{\partial Z^2})n_p^{(1)}=0 \tag{25}$$

From the above relations, we get

$$\lambda^2=\frac{(1+d-\beta)(m_e/m_i)l^2}{3(1+\beta/\sigma_T)} \tag{26}$$

Figure 1 shows the phase velocity graph versus d and β.As it is seen, with the increase of d, phase speed decreases.
From equations (11) -(26), after some simplification, the ZK equation for DIA waves in the mentioned four-component plasma in the presence of external magnetic field, obtained as follows:

$$\frac{\partial\phi}{\partial T}+A\phi\frac{\partial\phi}{\partial Z}+B\frac{\partial^3\phi}{\partial Z^3}+C\frac{\partial}{\partial Z}\frac{\partial^2\phi}{\partial X^2}=0 \tag{27}$$

where the nonlinearity coefficient A, dispersive coefficient B and higher order coefficient C are defined as:

$$A=\frac{\frac{9}{4}(1+\frac{1}{\sigma_T})[\frac{\lambda}{\delta}(2+\frac{\beta}{\sigma_T})-\frac{m_el^2}{3m_i\lambda}]+(\frac{m_e}{m_i})^2(\frac{3l^4}{4\lambda^3})}{\frac{3m_el^2}{m_i\lambda^2}-\frac{3}{\delta}(1+\frac{\beta}{\sigma_T})}$$

$$B=\frac{\lambda m^2(\frac{1}{\delta}+\frac{m_i}{m_e\Omega_c^2})+\frac{9H^2m^2}{4}[(1-\frac{1}{\sigma_T})(\frac{\beta\lambda}{\delta\sigma_T}-\frac{m_el^2}{3m_i\lambda})]}{\frac{3m_el^2}{m_i\lambda^2}-\frac{3}{\delta}(1+\frac{\beta}{\sigma_T})}$$

$$C=\frac{\frac{l^2\lambda}{\delta}[1+\frac{9H^2\beta}{4\sigma_T}(1-\frac{1}{\sigma_T})]-\frac{3H^2l^4m_e}{4\lambda m_i}}{\frac{3m_el^2}{m_i\lambda^2}-\frac{3}{\delta}(1+\frac{\beta}{\sigma_T})} \tag{28}$$

3. Results and discussion

In order to investigate the localized electrostatic excitations, a pulse type solitary wave solution of the ZK equations considered which is instantly made through utilizing the famous tangent hyperbolic ($tanh$) method [39,40]. Leaving the localized solution behind and taking the mentioned method into account, it is demanded to consider the variable transformation $\xi=X+Z-UT$, which U is the velocity of nonlinear structure moving with the frame:

$$-U\frac{\partial\phi}{\partial\xi}+A\phi\frac{\partial\phi}{\partial\xi}+(B+C)\frac{\partial^3\phi}{\partial\xi^3}=0 \tag{29}$$

Integrating equation (29) with regard to ξ and ignoring the integration constant, the form of an ordinary differential equation can be derived as follows

$$\frac{d^2\phi}{d\xi^2}=r_1\phi+r_2\phi^2 \tag{30}$$

where

$$r_1=\frac{U}{B+C},r_2=-\frac{A}{2(B+C)} \tag{31}$$

At this point, the above equation can be rewritten as the following dynamical system of travelling wave equations varied by plasma parameters

$$\frac{d\phi}{d\xi}=y,\frac{dy}{d\xi}=(r_1+r_2\phi)\phi \tag{32}$$

The last equation defines a planar Hamiltonian system with the following Hamiltonian function

$$H(\phi,y)=\frac{y^2}{2}-(\frac{r_1}{2}+\frac{r_2}{3}\phi)\phi^2=g \tag{33}$$

In the above relation, the term $-(\frac{r_1}{2}+\frac{r_2}{3}\phi)\phi^2$ is the electric potential ($V(\phi)$).The potential function $V(\phi)$ is plotted as a function of ϕ for different values of d, H, l and Ω_c in figures 2(a)-2(d). It is clear that the potential has one pit and a hump. The potential becomes more wide and flat as H and d increases but Ω_c and l decreases. When $r_1r_2\neq 0$, then there exist two equilibrium points at $(0,0)$ and $(\phi_1,0)$, with

$$\phi_1=-\frac{r_1}{r_2}=\frac{2U}{A} \tag{34}$$

If we consider that $M(\phi_i,0)$ is the coefficient matrix of the linearized system of equation (32) at an equilibrium point, then one can obtain the following determinant

$$J=\det M(\phi_i,0)=-r_1-2r_2\phi_i \tag{35}$$

By the theory of planar dynamical systems, we know that an equilibrium point $(\phi_1,0)$ of the planar dynamical system is a saddle point when $J<0$ and the equilibrium point $(\phi_1,0)$ of the planar dynamical system is a center when $J>0$ [39–42]. At this point, applying the planar dynamical system in equation (32) and the Hamiltonian function in equation (33) with $g=0$, we obtain two types of solitary wave solution and periodic travelling wave solution of equation (30) depending on the different physical situation:
When the condition (nonlinear coefficient) $A>0$ is satisfied, corresponding to the homoclinic orbit at the equilibrium point $(0,0)$, the system (equation (29)) has a dust ion acoustic compressive solitary wave solution:

$$\phi(\xi)=-\frac{3r_1}{2r_2}\sec h^2(\frac{1}{2^{3/2}}\sqrt{r_1}\xi) \tag{36}$$

By plotting 3D profile $\phi(\xi)$ versus ξ (equation 36), and through changing d, l, H and Ω_c the behavior of dust ion acoustic compressive solitary wave was investigated in figures 3(a)-3(d). It is shown in figure 3(a) that with the increase of d (from about 0.1 to 0.5), wave amplitude increases and the width becomes narrower. Then, in the range of d (0.5 to 1), the amplitude decreases and the width becomes broader.

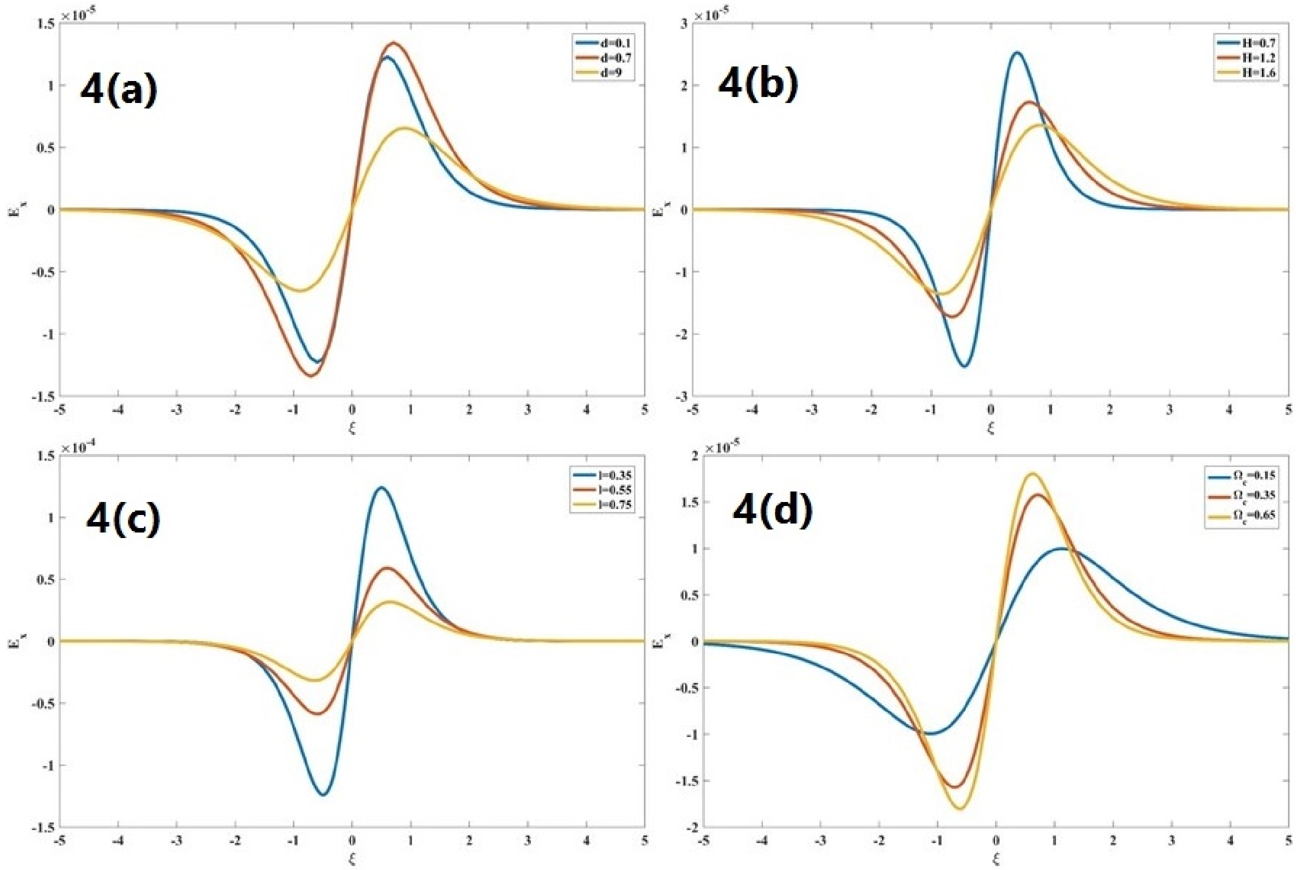

Figure 4. (a)The variation of E_x versus ξ at different values of d for $l = 0.9, H = 1.2, \Omega_c = 0.5, \sigma_T = 0.86$.(b)The variation of E_x versus ξ at different values of H for $l = 0.9, d = 0.6, \Omega_c = 0.5, \sigma_T = 0.86$.(c)The variation of E_x versus ξ at different values of l for $H = 1.2, d = 0.6, \Omega_c = 0.5, \sigma_T = 0.86$.(d)The variation of E_x versus ξ at different values of Ω_c for $H = 1.2, d = 0.6, l = 0.9, \sigma_T = 0.86$.

Figure 3(b) shows with the increase of Ω_c, wave amplitude is constant but the wave width decreases. It is shown in figure 3(c) that with the increase of l, wave width and amplitude decreases. With the increase of H as illustrated in figure 3(d), the amplitude is constant but wave becomes more flat (its width increases).
The perturbed electric field is attained as $\mathbf{E} = -\nabla\phi$ then, it is expressed as

$$E = \begin{pmatrix} E_x \\ E_z \end{pmatrix} = \frac{2\phi_m}{W} seh^2(\frac{\xi}{W}) \tanh(\frac{\xi}{W}) \begin{pmatrix} m \\ l \end{pmatrix} \qquad (37)$$

where $\phi_m = -3r_1/2r_2 = 3U/Al$ and $W = 2\sqrt{(B+C)/U}$ are the amplitude and the width of ion acoustic solitary wave, respectively. It is clear that the width W is determined by the dispersive coefficients B and C while the amplitude ϕ_m is dependent on the nonlinearity coefficient A. Figures 4(a) – 4(d) illustrates the behavior of the electric field E_x. In the figures, the variation of the electric field $\mathbf{E}$ is shown as a function of ξ for various values of d, H, l and Ω_c. It is seen in figures 4(a) and 4(b) that by increasing d and H, the amplitude of electric field is decreased and its width is increased. By increasing l in 4(c), the amplitude of electric field is decreased. In figure 4(d), the amplitude of the electric field increases with increasing the value of Ω_c, while the width of the electric field decreases.
Otherwise, equation (29) has the periodic travelling wave solution in terms of Jacobian elliptic functions [43, 44].The travelling wave system, equation (32), has a family of periodic orbits about the equilibrium point $(\phi_1, 0)$ as mentioned before, furthermore at $\phi_1 = 2U/A$ we obtain

$$-\frac{A}{U}\phi_n^3 + 3\phi_n^2 - \frac{4U^2}{A^2} = 0 \qquad (38)$$

where n= 1, 2 and 3. The three real zeros of equation (38) are ϕ_1, ϕ_2 and ϕ_3. It should be mentioned here that the conditions for the existence of a periodic travelling wave solution of equation (27) require that $\phi_1 > \phi_2 > \phi_3$. Accordingly, the periodic wave solution of equation (27) is given by

$$\phi(\xi) = \phi_3 + (\phi_2 - \phi_3) Sn^2[I\xi, k] \qquad (39)$$

where Sn is the Jacobian elliptic function. For the nonlinear coefficient $A < 0$, the amplitude of the periodic travelling

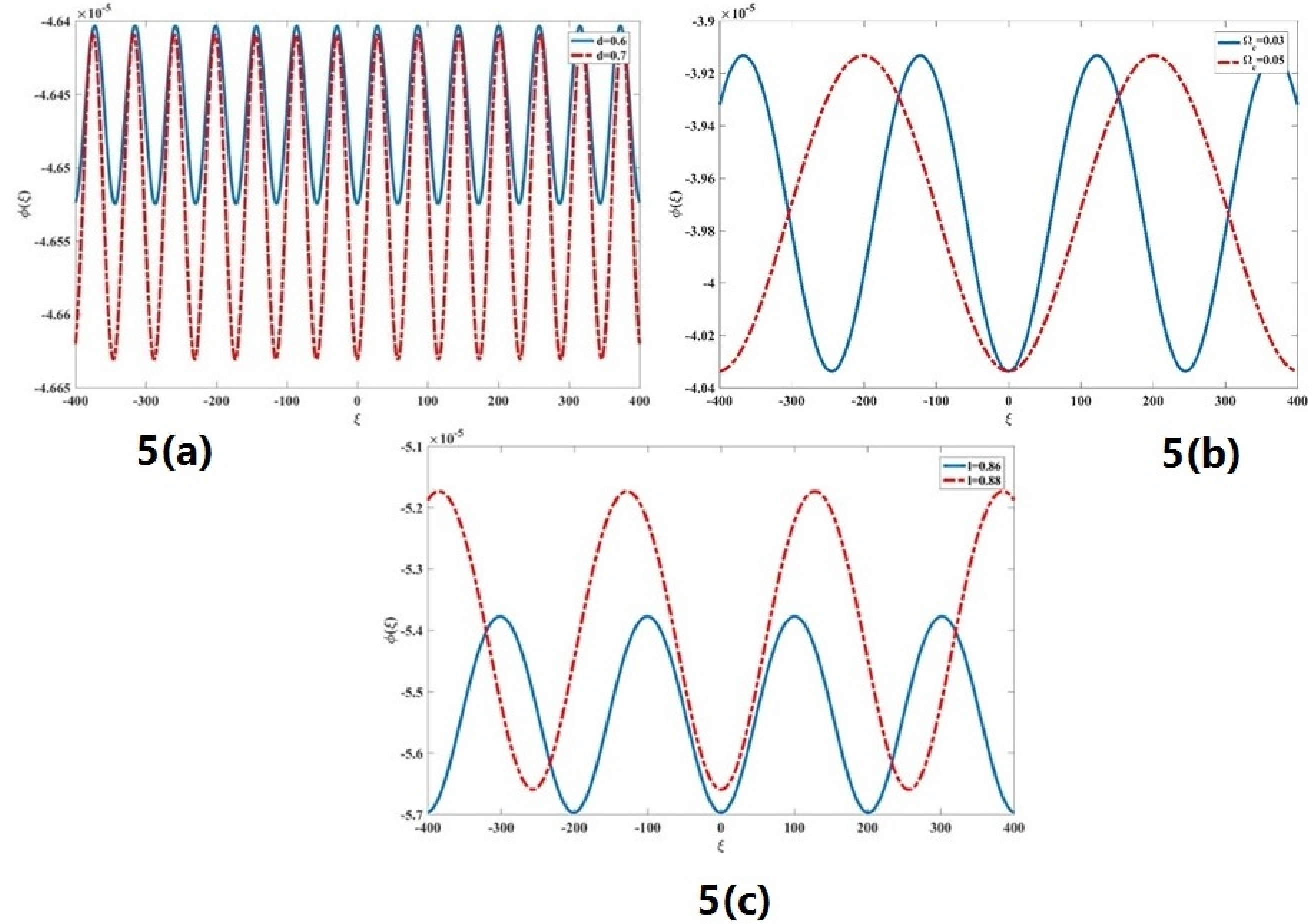

Figure 5. (a)Variation of periodic travelling wave $\phi(\xi)$ versus ξ at different values of d for $H = 0.9, \Omega_c = 0.01, l = 0.8, U = 1.5$. (b)Variation of periodic travelling wave $\phi(\xi)$ versus ξ at different values of Ω_c for $H = 1.2, d = 0.6, l = 0.9, U = 1.5$. (c)Variation of periodic travelling wave $\phi(\xi)$ versus ξ at different values of l for $H = 1.2, \Omega_c = 0.01, d = 0.6, U = 1.5$.

wave is given by $\phi_2 - \phi_3 > 0$. The elliptic parameter k is

$$k = \sqrt{\frac{\phi_2 - \phi_3}{\phi_1 - \phi_3}} \tag{40}$$

Refers physically to the nonlinearity with the linear limit corresponds to $k \to 0$ and the extreme nonlinear limit corresponds to $k \to 1$. The parameter I is given by

$$I = \sqrt{\frac{U(\phi_1 - \phi_3)}{12(B+C)}} \tag{41}$$

In this part, the impacts of d, l and Ω_c will be investigated on the basic features of periodic travelling waves. In figures 5(a)-5(c), the results are displayed numerically. In figure 5(a), the effect of d is investigated on the profile of the periodic travelling waves against the space coordinate ξ. It is definite that by increasing d, the amplitude of periodic travelling wave is increased and its width is decreased. In figures 5(b) and 5(d), the variation of the profile of the periodic travelling waves with l and Ω_c is examined. The impact of Ω_c on the periodic travelling waves is exhibited in figure 5(b). It is seen that the width of the periodic travelling waves increases as Ω_c increases, on the other hand, Ω_c has no effect on the amplitude of the periodic travelling waves. In figure 5(c), the variation of the profile of the periodic travelling waves with l is examined. It is obvious that both of the amplitude and the width increase as l increases. Physically, one can predict that the amplitude and the width of nonlinear periodic travelling wave become extremely large as it approaches the direction perpendicular to the magnetic field, Therefore the nonlinear periodic travelling wave disappears.

4. Conclusion

In this work the nonlinear propagation of dust ion acoustic compressive solitary and dust ion acoustic travelling waves in two-dimensional magneto plasma in the presence of the external magnetic field was studied. Plasma system containing quantum electron and positron with Fermi-Dirac distribution,

classical cold ion and negative dust grains. As the plasma particles obey Fermi–Dirac distribution, the pressure term in the momentum equation is described by the Fermi pressure law, which includes the quantum statistical effects. Using the standard reductive perturbation method, the ZK equation was attained. The bifurcation theory of planar dynamical systems used to describe the nonlinear propagation of solitary and periodic travelling wave. We have investigated the effects of the electron cyclotron to electron plasma frequency ratio(Ω_c), dust concentration (d), quantum Bohm potential term (H) and the direction cosine of the wave propagation vector with the Cartesian coordinates (l) on the nonlinear propagation of dust ion acoustic compressive solitary and ion acoustic travelling waves, on the potential function and electric field .The numerical results show these parameters have strong effect on the propagation of mentioned waves. As the graphs show, the electric potential becomes wider by increasing d and H and decreasing l and Ω_c. The behavior of dust ion acoustic compressive solitary wave shows that increasing H and Ω_c has no effect on the wave amplitude, but increasing l decreases the wave amplitude. As Ω_c and l increase, the wave width decreases, but when H increases, the width increases. The variation of E_x versus ξ shows that increasing l, d and H decreases the electric field amplitude and increases its width, while increasing Ω_c increases the amplitude and decreases the electric field width. The profile of the periodic travelling waves against the space coordinate ξ indicates that increasing d and l increases the amplitude, but the Ω_c change has no effect on the wave amplitude. Increasing l and Ω_c increases the width and increasing d decreases the width.
These results could be helpful to understand the formation and propagation of dust ion acoustic solitary and travelling waves in dense magnetized dusty quantum $e-p-i$ plasmas which may be found in extreme astrophysical environments, such as white dwarfs, magnetars, or neutron stars and in the core of giant planets.

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Conflict of interest statement:
The authors declare that they have no conflict of interest.

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