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Reconstruction of modified holographic Ricci dark energy in El-Nabulsi fractional action cosmology

Surajit Chattopadhyay^{1*} and Antonio Pasqua²

Abstract

In this work, we discussed the 'modified holographic Ricci dark energy' cosmological model based on El-Nabulsi fractional action cosmology. corresponding related cosmological parameters have been reconstructed. It has been observed that the equation of state parameter behaves like quintessence. Finally, through a test by the squared speed of sound, the proposed reconstruction model is found to be classically unstable.

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Keywords: Fractional action cosmology, Modified holographic Ricci dark energy, Cosmological parameters

Introduction

Based on the concept of fractional calculus of variations mainly the fractional action-like variational approach (FALVA), the fractional action cosmology (FAC) with fractional weight function has been proposed recently [1-7]. FAC is based on the principles and formalism of FALVA applied to cosmology [6,7]. The fractional derivative and fractional integrals are the main tools in fractional calculus, where the order of differentiation or integration is not an integer. The fractional calculus is immensely useful in various branches of mathematics, physics, and engineering [8]. In doing FAC, two different ways are possible [9-11]:

- (1) Replacing the partial derivatives in the Einstein field equations with the corresponding fractional derivatives [11];
- (2) Deriving the field equations and geodesic equations from a more fundamental way, namely, starting with the principle of least action and replacing the usual integral with a fractional integral [9,10].

Mathematical details of El-Nabulsi FAC have been thoroughly discussed in the recent work [4].

The present paper is devoted to the reconstruction of a dark energy (DE) model dubbed as 'modified holographic Ricci dark energy' (MHRDE) in FAC. Prior to the discussion on the methodology, let us have a brief review on DE in general. It is established by cosmological observations (e.g., SNe Ia, CMB) that the present universe is undergoing a phase of accelerated expansion [12,13]. Two approaches have been adopted to explain this accelerated expansion. They are

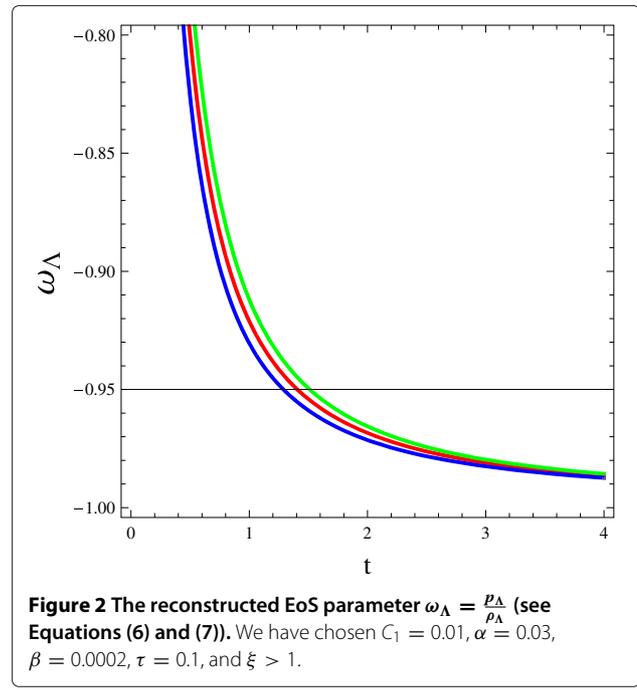
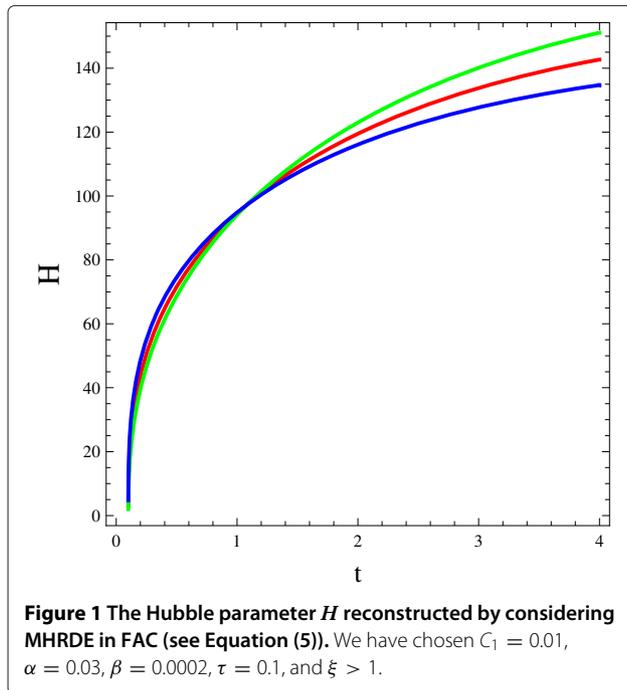
- (1) To introduce DE in the right-hand side of the Einstein equation in the framework of general relativity (recent reviews include [14-18]);
- (2) To introduce 'modified theory of gravity' by modifying the left-hand side of the Einstein equation (recent reviews include [19-21]).

Holographic reconstruction of DE models is not new. Significant studies are available in the area of reconstruction of various DE candidates based on holographic dark energy (HDE) [22-27] originated from the 'holographic principle' [28]. A few works on the holographic reconstruction of different DE candidates are reviewed below. Setare [29] studied the cosmological application of holographic dark energy density in the Brans-Dicke framework and suggested a correspondence between the holographic dark energy scenario in flat universe and the phantom dark energy model in the

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framework of Brans-Dicke theory with potential. In another work, Setare [30] reconstructed the potential and the dynamics of the scalar field which describes the Chaplygin cosmology. Reference [31] considered a correspondence between the holographic dark energy density and tachyon energy density in Friedmann-Robertson-Walker (FRW) universe. The present work is centered around the MHRDE proposed in [32]. Mathematical background of the MHRDE and its reconstruction through FAC would be discussed in the subsequent section.

The paper is organized as follows: In section ‘Reconstruction of MHRDE in FAC’, we give a brief overview of the MHRDE and then explain the reconstruction procedure through FAC. In section ‘Discussion’, we present a discussion on the results, and in the last section, we stated the concluding remarks.

Reconstruction of MHRDE in FAC

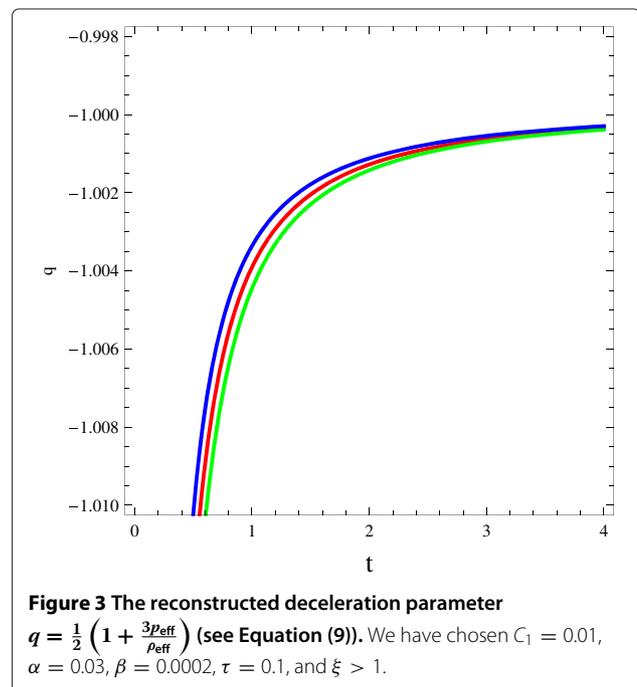
Spatially flat, homogeneous and isotropic universe in FRW line element is given by

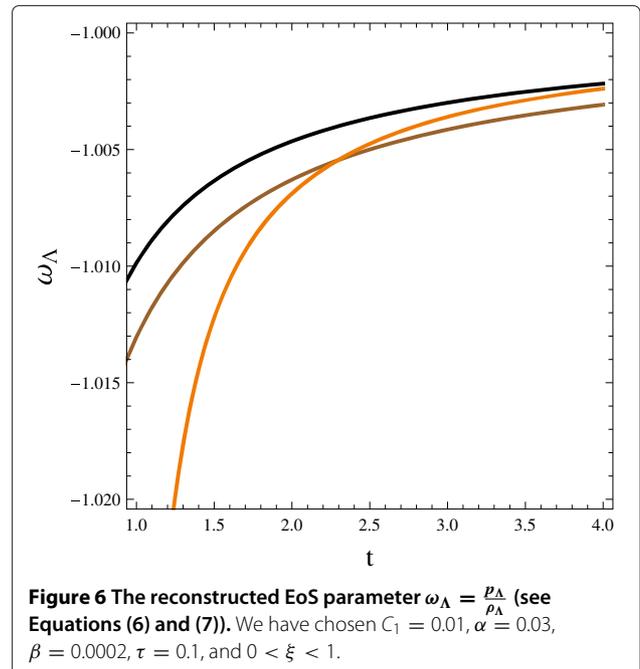
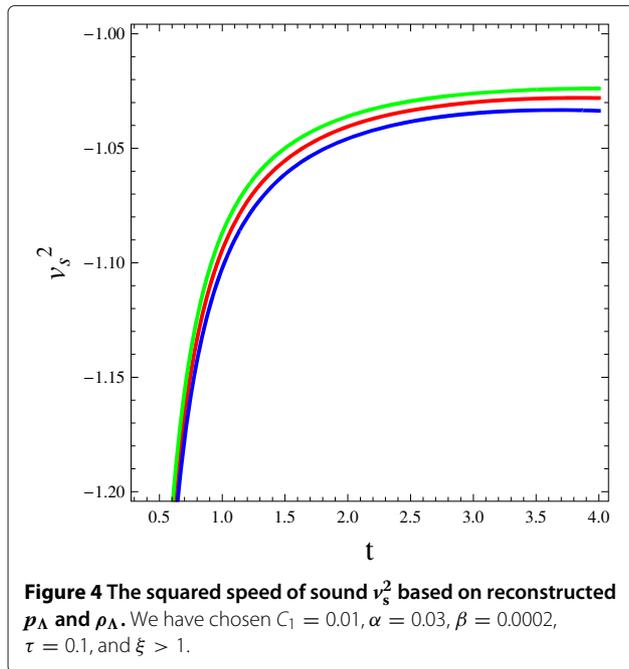
$$ds^2 = dt^2 - a^2(t) [dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)], \quad (1)$$

where t represents the cosmic time, r is the radial component, and (θ, φ) are the two angular coordinates. In this work, we consider a recently proposed holographic cosmological model with IR cut-off given by the modified Ricci radius so that L^{-2} is a combination of H^2 and \dot{H} (with H and \dot{H} being, respectively, the Hubble parameter and its first derivative with respect to the cosmic time t) [32-34]. After that, the energy density ρ_Λ of the MHRDE model becomes [32]:

$$\rho_\Lambda = \frac{2}{\alpha - \beta} \left(\dot{H} + \frac{3\alpha}{2} H^2 \right), \quad (2)$$

where α and β are free constants. The theoretical background of the MHRDE is thoroughly discussed in [32]; thus, we are skipping any detailed discussion on this dark energy density.





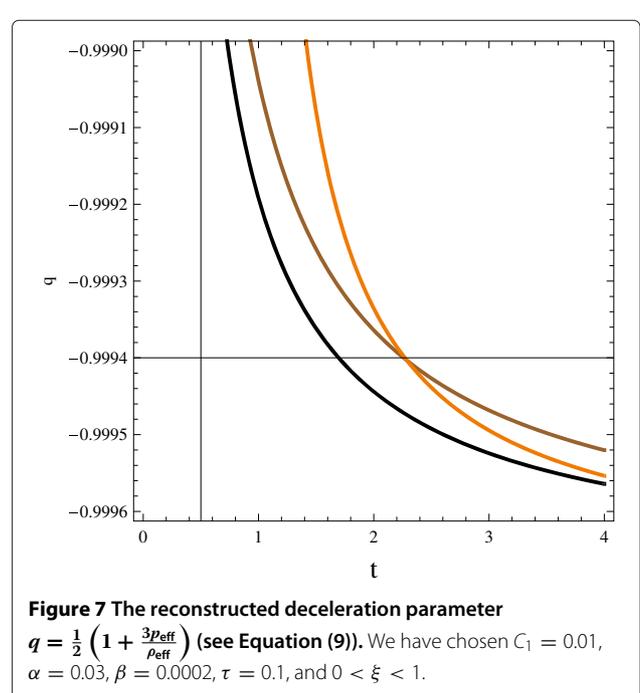
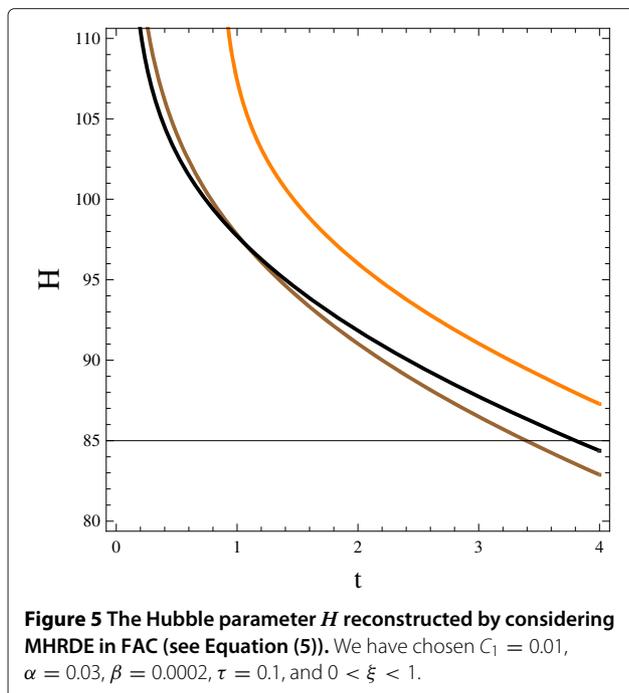
In FAC, the field equations take the following form [1,2]:

$$H^2 + \frac{2(\xi - 1)}{T_1}H + \frac{k}{a^2} = \frac{8\pi G}{3}\rho, \quad (3)$$

$$\dot{H} - \frac{(\xi - 1)}{T_1}H - \frac{k}{a^2} = -4\pi G(\rho + p), \quad (4)$$

where $T_1 = t - \tau$. Here, τ is the intrinsic time, and t is the observer time. As we are considering a spatially flat FRW universe, we have the curvature $k = 0$.

In the present work, we assume that the universe is filled with MHRDE. Then, we replace ρ of the Equation (3) by the MHRDE density as given in Equation (2). Clearly, this generated a differential equation of first order



on H with t as the independent variable. Solving the differential equation, we express Hubble parameter H as a function of t as follows:

$$H = \left(\frac{3\beta(t-\tau)}{2+6\alpha(\xi-1)-6\beta(\xi-1)} + C_1(t-\tau)^{-3(\alpha-\beta)(\xi-1)} \right)^{-1}. \tag{5}$$

Consequently, from Equations (3) and (5), we have the reconstructed density of MHRDE as:

$$\rho_\Lambda = \frac{3\alpha - \frac{3\beta(t-\tau)}{1+3\alpha(\xi-1)-3\beta(\xi-1)} + 6(\alpha-\beta)(\xi-1)C_1(t-\tau)^{-3(\alpha-\beta)(\xi-1)}}{3(\alpha-\beta) \left[\frac{3\beta(t-\tau)}{2+6\alpha(\xi-1)-6\beta(\xi-1)} + C_1(t-\tau)^{-3(\alpha-\beta)(\xi-1)} \right]^2}. \tag{6}$$

Using the reconstructed ρ_Λ in Equation (4), we get the reconstructed pressure of MHRDE as

$$p_\Lambda = -4[1+3(\xi-1)-3\beta(\xi-1)] \times \frac{\{6C_1(\alpha-\beta)[1+3(\xi-1)-3\beta(\xi-1)](\xi-1) + [1+3(\xi-1)-3\beta\xi](t-\tau)^{1+3(\alpha-\beta)(\xi-1)}\}}{\{C_1[2+6\alpha(\xi-1)-6\beta(\xi-1)] + 3\beta(t-\tau)^{1+3(\alpha-\beta)(\xi-1)}\}^2}. \tag{7}$$

Thus, the reconstructed equation of state (EoS) parameter is

$$\omega_\Lambda = \frac{-6C_1(\alpha-\beta)[1+3\alpha(\xi-1)-3\beta(\xi-1)](\xi-1) + [-1-3\alpha(\xi-1)+3\beta\xi](t-\tau)^{1+3(\alpha-\beta)(\xi-1)}}{(2C_1)[1+3\alpha(\xi-1)-3\beta(\xi-1)](\xi-1) + [1+3\alpha(\xi-1)](t-\tau)^{1+3(\alpha-\beta)(\xi-1)}} \tag{8}$$

Next, we study the deceleration parameter given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2}. \tag{9}$$

Using the H reconstructed in Equation (5), we get the deceleration parameter as a function of t using Equation (9):

$$q = -1 + \frac{3\beta}{2+6\alpha(\xi-1)-6\beta(\xi-1)} - 3C_1(\alpha-\beta)(\xi-1)(t-\tau)^{-1-3(\alpha-\beta)(\xi-1)}. \tag{10}$$

Discussion

In this section, we are going to present what is apparent from the figures plotted based on the equations derived above. The equations relevant to the individual figures are mentioned in the figure captions. In the Figures 1, 2, 3, and 4 the red, green, and blue lines correspond to $\xi = 5$, $\xi = 5.5$, and $\xi = 4.5$, respectively. In Figure 1, we have plotted the reconstructed Hubble parameter H against the cosmic time t . We observed that the reconstructed H is exhibiting an increasing pattern with the evolution of the universe. Moreover, we observed that at later stages of the universe (after $t \approx 2$), the rate of increase in H is increasing with ξ . However, in the early stage, the value of ξ does not have that influence on the behavior of H . After reconstructing the energy density and pressure of the MHRDE in FAC, we have reconstructed the EoS parameter ω_Λ . The reconstructed EoS parameter is

plotted in Figure 2. A quintessence-like behavior is apparent, i.e., $\omega_\Lambda \geq -1$. With the evolution of the universe, ω_Λ is approaching towards -1 . However, it never crossed the phantom boundary. The deceleration parameter, as derived in Equation (9) has been plotted in Figure 3, where an accelerated universe is observable due to its negative sign. However, a decay in the acceleration is observable from its upward approach in the negative side of q . Lastly,

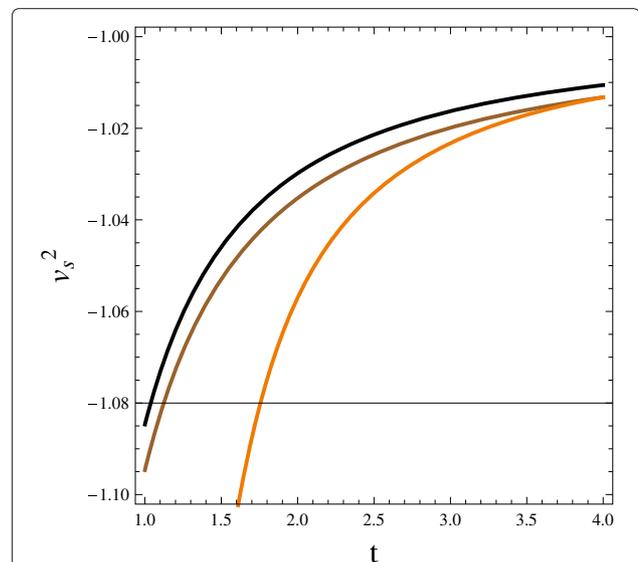


Figure 8 The squared speed of sound v_s^2 based on reconstructed p_Λ and ρ_Λ . We have chosen $C_1 = 0.01$, $\alpha = 0.03$, $\beta = 0.0002$, $\tau = 0.1$, and $0 < \xi < 1$.

we consider an important quantity to check the stability of HDE $f(G)$ model, namely the squared speed of sound v_s^2 , defined as:

$$v_s^2 = \frac{\dot{p}_\Lambda}{\dot{\rho}_\Lambda}. \quad (11)$$

The sign of v_s^2 is important for the stability of a background evolution. A negative value implies a classical instability of a given perturbation in general relativity [35,36]. Recently, Sharif and Jawad [37] have shown that interacting new HDE is characterized by negative v_s^2 . In Figure 4, we have plotted the v_s^2 based on the reconstructed p_Λ and ρ_Λ . Its negative sign throughout the evolution of the universe indicates that the reconstructed MHRDE model is classically unstable. In Figures 5, 6, 7, and 8, we have investigated the cases for $0 < \xi < 1$. In these figures, the brown, black, and orange lines correspond to $\xi = 0.35$, $\xi = 0.5$, and $\xi = 0.55$, respectively. The behavior of the cosmological parameters are found to be influenced by the values of ξ . Contrary to what happened for $\xi > 1$, we found in Figure 5 that the reconstructed Hubble parameter is exhibiting a decreasing pattern (see Figure 5). In Figure 6, we found the contrast in behavior in the EoS parameter. Here, $\omega_\Lambda < -1$, i.e., the EoS parameter is behaving like phantom for $0 < \xi < 1$, whereas it behaves like quintessence for $\xi > -1$. Like Figure 3, Figure 7 exhibits an accelerated universe. However, unlike Figure 3, Figure 7 exhibits an increase in the acceleration with passage of cosmic time. However, instability of the model is apparent in Figure 8. Thus, irrespective of the value of ξ , the model under consideration is classically unstable.

Concluding remarks

In this work, we considered a recently proposed holographic cosmological model with IR cut-off given by the modified Ricci radius dubbed as 'modified holographic Ricci dark energy' [32]. The dark energy candidate has been reconstructed in fractional action cosmology proposed by [1]. Primarily, solving an ordinary differential equation, the Hubble parameter has been reconstructed, and this has led to the reconstruction of the density and pressure of the said dark energy candidate. The deceleration parameter and the equation of state parameter have also been derived. With suitable choice of free constants, we have plotted the derived cosmological parameters. Some influence of ξ present in the modified field equations pertaining to fractional action cosmology has been observed:

For $\xi > 1$, (1) the Hubble parameter has exhibited an increasing pattern with the evolution of the universe. (2) The equation of state parameter has behaved like quintessence. (3) The deceleration parameter derived from the reconstructed density, and pressure has stayed

at negative level throughout. This has indicated that the universe is accelerated in time, and the acceleration has a decreasing pattern with passage of cosmic time. (4) The stability of the proposed reconstructed modified holographic Ricci dark energy has been investigated by means of the squared speed of sound, and its negative sign has proved the model to be classically unstable.

For $0 < \xi < 1$, (i) the Hubble parameter has exhibited a decreasing pattern with the evolution of the universe. (2) We have observed that the equation of state parameter has behaved like phantom. (3) We have seen that the deceleration parameter derived from the reconstructed density and pressure has stayed at negative level throughout, and this has indicated an accelerated expansion of the universe and the acceleration has an increasing pattern with passage of cosmic time. (4) The negative sign of the squared speed of sound has proved the model to be classically unstable.

It is important in the future to reconstruct holographic theory by means of FALVA not obligatorily based on Riemann-Liouville fractional integral operators which give accordingly Equations (2) and (3), but by means of different more generalized integral operators like Erdelyi-Kober operators, extended exponentially fractional integral, etc. References [38-40] should be mentioned in this context.

Competing interests

The authors declare that they have no competing interests.

Authors' contribution

The work is an output of the collaborative effort of SC and AP. Both of the authors have contributed equally to the calculation, the plotting, and the preparation of the draft. Both authors read and approved the final manuscript.

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SC completed his MSc in Mathematics from Jadavpur University, Calcutta (India) in 1999 and his PhD (Science) in Mathematics from Bengal Engineering and Science University, Shibpur (India) in 2010. He is a Visiting Associate of the Inter-University Centre for Astronomy and Astrophysics (IUCAA), Pune (India) since August 2011. He is working as an Assistant Professor of Mathematics at Pailan College of Management and Technology, Kolkata, India since 2006. His areas of research interest include dark energy models and modified gravity theories. The author has 80 research papers published in various peer-reviewed journals.

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