Kuchin and Maksimenko *Journal of Theoretical and Applied Physics* 2013, **7**:47
http://www.jtaphys.com/content/7/1/47

Journal of Theoretical and Applied Physics
a SpringerOpen Journal

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Characteristics of charged pions in the quark model with potential which is the sum of the Coulomb and oscillator potential

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Abstract

In this paper, we calculate the polarizability of the charged pions in the nonrelativistic quark potential model.

Keywords: Schrödinger equation; Characteristics of charged pions; Polarizability

Introduction

The theoretical description of the present experimental data on the interaction of photons with hadrons at high energies and large momentum transfer is carried out, mainly in the framework of perturbation theory of quantum chromodynamics. However, the structural degrees of freedom, which appear in low energy, cannot be reduced to simple ideas about the interaction of electromagnetic fields with hadrons. The response of the quark degrees of freedom for the action of the electromagnetic field can be determined phenomenologically on the basis of self-consistent description of the polarizabilities, root mean square radius of the charge, and other electromagnetic properties of hadrons. The unique sensitivity of these values to theoretical models puts them among the most important characteristics, with which you can set the features of the structure of hadrons, exhibited at low energies. Therefore, the study of the electromagnetic characteristics of related systems in the framework of potential models is becoming one of the most effective methods for studying the characteristics of the quark-quark interaction.

At present, there are quite a number of theoretical studies providing the electric polarizabilities of charged hadrons, including mesons. Among them, we can mention calculations that make use of effective Lagrangians [1,2] and current algebra [3]. Nucleon and meson polarizabilities were also computed in the context of the nonrelativistic quark model [4-12] and other potential models [13-17].

In this article, we look at the mass spectrum, mean square radius, leptonic decay constant, and electric polarizability of charged pions in the quark model with potential, which is the sum of the Coulomb and oscillator potential. This potential is used, for example, in [18,19] to describe the mass spectrum of quarkonia. In addition, in [20-27], they conducted research into this building, which contains the linear part. The good agreement between the results of this work with the experimental data is of interest for the use of this potential for the description of the component systems, consisting of not only heavy but also of light quarks.

In this paper, we calculate the characteristics of the pions in the quark model with the potential, which is the sum of the Coulomb and oscillator potential. The results are in good agreement with the experimental data.

Solution of the Schrodinger equation

To find the wave function of the relative motion of a quark and an antiquark, we solve the Schrodinger equation:

$$\Delta\Psi + 2\mu[\mathrm{E}-U(r)]\Psi = 0, \tag{1}$$

where μ is the reduced mass, $U(r)$ is the quark-antiquark interaction potential, and r is the relative coordinate.

As the potential $U(r)$ is spherically symmetric, the variables in the Schrodinger Equation 1 are separable [28],

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and the equation can be reduced to the equation for radial wave functions $R_{nl}(r)$:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{nl(r)}}{dr}\right)+\left[2\mu(E-U(r))-\frac{l(l+1)}{r^2}\right]R_{nl}(r)=0. \tag{2}$$

Introducing 'reduced' radial wave functions $\chi_{nl}(r) = rR_{nl}(r)$ normalized by the condition, $\int_0^\infty |\chi_{nl}(r)|^2 dr = 1$, we rewrite Equation 2 as

$$\chi''_{nl}(r)+\left[2\mu(E-U(r))-\frac{l(l+1)}{r^2}\right]\chi_{nl}(r)=0, \tag{3}$$

where $\chi''_{nl}(r)=\frac{d^2}{dr^2}=\frac{d^2}{dr^2}\chi_{nl}(r)$.

The interaction potential between a quark and antiquark can be written as

$$U(r)=ar^2-\frac{b}{r}+c, \tag{4}$$

where a and b are nonnegative constants and r is the interquark distance. This potential has two parts: the first is ar^2 accounts for quark confinement at large distances, while the second part $-\frac{b}{r'}$ which corresponds to the potential induced by one gluon exchange between the quark and antiquark that dominated at short distances.

Substituting the quark-antiquark interaction potential $U(r)$ in Equation 3, we obtain

$$\chi''_{nl}(r)+2\mu\left[\left(E-ar^2+\frac{b}{r}-c\right)-\frac{l(l+1)}{2\mu r^2}\right]\chi_{nl}(r)=0. \tag{5}$$

The approximate solution of this equation was obtained in [29] using the method Nikiforov-Uvarov [30]. Here are the expressions obtained in [29] for the eigenvalues and eigenfunctions:

$$E_{nl}=\frac{6a}{\delta^2}-\frac{2\mu\left(b+\frac{8a}{\delta^3}\right)}{\left[(2n+)\pm\sqrt{1+4l(l+1)+\frac{24\mu a}{\delta^4}}\right]^2}+c. \tag{6}$$

$$R_{nl}(r)=N_{nl}r^{-\frac{B}{\sqrt{2A}}-1}e^{\sqrt{2A}r}\left(-r^2\frac{d}{dr}\right)^n\left(r^{-2n+\frac{2B}{\sqrt{2A}}}e^{-2\sqrt{2A}r}\right), \tag{7}$$

where $A=\mu\left(-E+c+\frac{6a}{\delta^2}\right), B=\mu\left(b+\frac{8a}{\delta^3}\right)$, and $\delta\equiv\frac{1}{r_0}, r_0$– the characteristic radius of the meson, n = 0, 1, 2, ...

Method of estimating electric polarizabilities

In this section, we shall give a general method of estimating the static electric polarizability of a bound system [14] that includes deriving the lower and upper boundaries for this quantity.

Consider the equation

$$\hat{H}|\Phi\rangle=E|\Phi\rangle \tag{8}$$

with a Hamiltonian consisting of a sum of two operators:

$$\hat{H}=\hat{H}_0+\Delta\hat{H}, \tag{9}$$

where $\hat{H}_0$ is a Hamiltonian of an 'unperturbed' system while $\Delta\hat{H}$ is a small additional term (a perturbation operator). We also assume that Equation (8) has the form

$$\hat{H}_0|\psi_n\rangle=\varepsilon_n|\psi_n\rangle, n=0,1,2,... \tag{10}$$

provided there are no perturbations.

According to the stationary perturbation theory, the energy value additional to that of the ground state ε_0 will be looked for in the form of a series:

$$E=\varepsilon_0+\Delta\varepsilon^{(1)}+\Delta\varepsilon^{(2)}+\cdots \tag{11}$$

Respectively, the wave function will also be represented as a series in a small parameter, which is part of $\Delta\hat{H}$:

$$|\Phi\rangle=|\Psi\rangle+|\Delta\Psi\rangle+\cdots \tag{12}$$

When $\varepsilon_0 \le \varepsilon_1 \ldots \le \varepsilon_n$, we find that the value of additional energy $\Delta\varepsilon^{(2)}$ belongs to the interval [14]:

$$\frac{F}{\varepsilon_0-\varepsilon_1}\le\Delta\varepsilon^{(2)}\le\frac{\left(G^2-F\right)^2}{F\varepsilon_0-D'} \tag{13}$$

where the following notation is introduced: $D=\Psi_0\left|\Delta\hat{H}\hat{H}_0\Delta\hat{H}\right|\Psi_0,\ F=\langle\Psi_0\left|\Delta\hat{H}^2\right|\Psi_0\rangle,$

$$G=\Psi_0|\Delta\hat{H}|\Psi_0. \tag{14}$$

Therefore, in order to find the interval boundaries (13), it is necessary to determine the wave function of the ground state Ψ_0 as well as the energies of the ground and first radially excited states. Unlike the case when it is needed to find an exact value of $\Delta\varepsilon^{(2)}$, it is not required to completely solve an unperturbed problem in the case at hand.

Correction $\Delta\varepsilon^{(2)}$ to the ground-state energy of a bound system, when the role of perturbation is played by the external stationary field with strength E, is related to the electric polarizability of system a_0 as follows [4]:

$$\Delta\varepsilon^{(2)}=-\frac{a_0}{2}E^2. \tag{15}$$

Note that, when the ground state $|\langle\Psi_0$ is spherically symmetric, the value of $\Delta\varepsilon^{(1)}$ is zero, i.e.,

$$\Delta\varepsilon^{(1)} = G = 0. \tag{16}$$

Using Equations 13 and 16, we find that the value of static electric polarizability a_0 lies within the interval

$$\frac{2F^2/E^2}{D - F\varepsilon_0} \le a_0 \le \frac{2F/E^2}{\varepsilon_1 - \varepsilon_0}. \tag{17}$$

Pion static polarizabilities

In order to fix the model parameters of interquark potential and quark masses, we shall use experimental data on lepton decay constants, masses, and electromagnetic radius of charged pions. For experimental values [31,32], we have

$$M^{1S}_{\exp} = 139.57018 \pm 0.00035 \text{ MeV},$$

$$M^{2S}_{\exp} = 1300 \pm 100 \text{ MeV},$$

$$f_{\pi^\pm} = 130.70 \pm 0.10 \pm 0.36 \text{ MeV},$$

$$r^2_{\pi\pm\exp} = (0.420 \pm 0.014) \text{ fm}^2.$$

These data lead us to the following parameter values:

$m_u = m_d$ = 0.176 GeV; a = 0.00766 GeV^3; b = 0.3; δ = 0.142 GeV; c = −0.982 GeV.

The masses of mesons, lepton decay constant, and mean square radius are determined by the following formulas:

$$M_{nl} = 2m + E_{nl}, \tag{18}$$

$$f_p^2 = \frac{3|R(0)|^2}{\pi M_p}\left[1 - \frac{3b}{2\pi}\right], \tag{19}$$

$$\left\langle r^2 \right\rangle = \left\langle \sum e_i (\boldsymbol{r}_i - \boldsymbol{R}_{c.m})^2 \right\rangle. \tag{20}$$

Calculations by formula (17)-obtained values of parameters give the following interval for the static polarizability:

$$a_0^{\pi^\pm} = (0.21 \pm 0.18) \times 10^{-4} fm^3.$$

To estimate polarizability, a nonrelativistic operator of the electric dipole interaction was used:

$$\boldsymbol{DE} = \frac{1}{2}(e_1 - e_2)(\boldsymbol{rE}),$$

where e_i are quark charge operators acting upon that part of the wave function which depends on the unitary spin and, for $\pi^\pm$ – mesons, have the following form [4]:

$$\psi^{\pi^+}(\xi) = \left(\frac{1}{\sqrt{2}}\right)\left[\left|\bar{d}\uparrow u\downarrow\right\rangle - \left|\bar{d}\downarrow u\uparrow\right]\right., \tag{21}$$

$$\psi^{\pi^-}(\xi) = \left(\frac{1}{\sqrt{2}}\right)[|\bar{u}\uparrow d\downarrow\rangle - |\bar{u}\downarrow d\uparrow], \tag{22}$$

where $\bar{u}$ *and* $\bar{d}$ are antiquarks.

The relation

$$\left\langle \pi^\pm \left| (e_1 - e_2)^2 \right| \pi^\pm \right\rangle = \frac{e^2}{9} \tag{23}$$

was also used in the calculations.

Compton electric polarizability of π meson

As was, for example, shown in [4], generalized electric polarizability $\bar{a}$ can be represented as a sum of two parts:

$$\bar{a} = a_0 + \Delta a. \tag{24}$$

The quantity a_0 is called static polarizability and is related with the induced electric dipole moment in the approximation of its pointness; i.e., a deformed composite system is described as a pointlike dipole.

The term Δa takes into account the structure of a composite system and is expressed in the leading approximation through the rms radius of that same system. The quantity Δa is relativistic in nature and can be explained as describing a transition from Thomson scattering on pointlike particles to that on composite ones with an electromagnetic radius [33]. For a spinless system, this term is written as follows:

$$\Delta a = \frac{a\langle r^2\rangle}{3M}, \tag{25}$$

where r is a pion electromagnetic radius, a is a fine structure constant, and M for the nonrelativistic case is the sum of the masses of the constituent particles.

Calculating Δa within the framework of the given model with pointlike quarks, we find that, in the approach proposed, the term related with a pion electromagnetic radius has the following value:

$$\Delta a^{\pi^\pm} = 5.81 \times 10^{-4} \text{ fm}^3.$$

Thus, we obtain the next value Compton polarizabilities of charged pions in this model:

$$\bar{a}_{\pi^\pm} = (6.02 \pm 0.18) \times 10^{-4} \text{ fm}^3.$$

We compare the results of our calculations with the experimental values obtained in [34,35]. In [34], the following value of Compton polarizability is $\bar{a}_{\pi^\pm} = (6.8 \pm 1.4 \pm 1.2) \times 10^{-4} \text{ fm}^3$, and in Equation 35, it is $\bar{a}_{\pi^\pm} = (6.5 \pm 1.1) \times 10 \text{ fm}^3$. Thus, the results of our calculations are in good agreement with the experimental values.

Conclusions

In this paper, in nonrelativistic quark model with potential, which is the sum of the Coulomb and oscillator potential, we evaluate the characteristics of the charged pions. It is in good agreement with the experimental data. The resulting value of electric polarizability of charged pions in the model is slightly higher than the results of calculations of this value in chiral theories [36-39] but is in good agreement with the results obtained in [2,40,41]. However, the numerical value of r_0 has turned quite large, which indicates that the description of a system such as those of pions account for relativistic corrections or application of relativistic quark models.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

SMK and NVM carried out all the calculations, the analysis, designed the study, and drafted the manuscript together. Both authors read and approved the final manuscript.

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Received: 16 April 2013 Accepted: 2 September 2013
Published: 12 September 2013

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doi:10.1186/2251-7235-7-47
Cite this article as: Kuchin and Maksimenko: **Characteristics of charged pions in the quark model with potential which is the sum of the Coulomb and oscillator potential.** *Journal of Theoretical and Applied Physics* 2013 **7**:47.