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Magnetized electron-positron plasmas, a new mode, stability conditions

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Abstract
In this paper, a set of two-fluids equations based on the quantum magnetic hydrodynamic model (QMHD) for electron-positron plasma were considered and the role of spin in different spin-spin, and spin-field interactions were briefly discussed. Furthermore, effects of density and heterogeneity in the equations were considered. For each of the two electron and positron fluids, the complete theory of spin-1/2 electron-positron quantum plasmas when electrons and positrons move with velocities much smaller than the speed of light was discussed. By considering the two regimes of non-spin and spin plasma separately, new dispersion relations was extracted and analyzed. We also examined the limits of weak and strong magnetic fields and the effect of spin polarization on ripple waves in plasma medium. Results show that the spin-current evolution in a magnetized plasma creates a new dispersion mode. The fast mode generates in the direction perpendicular to the direction of propagation of the waves. The speed of this mode is equal to the speed of the mode in the parallel direction plus an additional term that depends on the characteristics of the system. For high-density plasma, this correction is negligible, but for very low densities and weak magnetic fields, this effect is significant.

Keywords
Particles impurity, Fermi polaron, Electron-positron pair production, Spin-spin interaction, Spin-magnetic field coupling.

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1. Introduction

Magneto-hydrodynamics (MHD) can be considered a suitable formalism for studying magnetized plasma at scales larger than the electronic length $\lambda_e = c/\omega_e$ where ω_e is the electronic plasma frequency [1, 2]. Quantum works become highlighted when the thermal de Broglie wavelength of the plasma particles $\lambda_B (= \hbar/\sqrt{k_B T m})$ is about the average particle distance $\bar{L}(= n_0^{-1/3})$ i.e.,$\lambda_B \gtrsim \bar{L}$. A method for considering the quantum effect is to correct the classical equations. It is natural to see differences between the classical and quantum models, for example, using the quantum hydrodynamic model to see new oscillating modes in a magnetic quantum plasma [3–5]. We discuss the complete theory of spin-1/2 electron-positron quantum plasmas when electrons and positrons move with velocities much smaller than the speed of light and consider the annihilation interaction's contribution in the quantum hydrodynamic equations and in the spectrum of waves in magnetized electron-positron plasmas.
This work organizes as follows. Section 2 uses the two-fluid plasma equations and their oscillation modes concerning quantum effects such as Fermi pressure, Bohm pressure, and spin interactions [6, 7]. We also get a new dispersion mode here. Section 3 investigates the instability of two quantum fluids in magnetized plasma. Unlike the classical case, which has instability for large wavelengths, we can show that there is a minimum cut-off wavelength for instability. Finally, some conclusions are drawn in Section 4.

2. QMHD equations and the effects of electron spin

We first consider a plasma consisting of two fermionic fluids below the Fermi temperature and consider electron-positron as two different species and obtain our equations for the evolution of the spin current. We write the QMHD quantum equations for the two-fluid plasma of electron-positron located in the external magnetic field B.

2.1 Two-fluid plasma equations

We use the method of many-particle quantum hydrodynamics for the two-fluid plasma equations. The main idea of this method is the representation of the many-particle Pauli equation in terms of collective observable variables suitable for the description of quantum plasmas. Let us start with the many-particle Pauli equation,

$$i\hbar\partial_t \psi(r_1, ..., r_N, t) = H\psi(r_1, ..., r_N, t) \tag{1}$$

where $\psi(\mathbf{r}_1, ..., \mathbf{r}_N, t)$ is the many-particle wave function, $\mathbf{r}_i$ is the coordinates of particle with number "i" and N is the total number of particles in the system. Explicit form of the Hamiltonian allows to derive a set of quantum hydrodynamic (QHD) equations. Hamiltonian for electron-positron plasmas of particles moving with velocities $\mathbf{v}$ is [8, 9],

$$H=\sum_{i=1}^{N}[\frac{1}{2m}(\hbar\nabla_i \pm \frac{e}{c}\mathbf{A}_{i,ext})^2 \mp e\Phi_i^{ext} \pm \frac{e\hbar}{2m}\sigma\cdot\mathbf{B}_i^{ext}]-$$
$$\frac{1}{2}\sum_{i,j\neq i}^{N}[\frac{\pi e^2\hbar^2}{m^2c^2}\sigma_i^e\cdot\sigma_j^p]+\sum_{i=1}^{N}\frac{e^2\hbar}{4m^2c^2}\sigma\cdot(\mathbf{E}\times\mathbf{A}_{i,ext})-$$
$$\sum_{i=1}^{N_e}\sum_{j=N_e+1}^{N}\frac{3\pi e^2\hbar^2}{2m^2c^2}\sigma_i^e\cdot\sigma_j^p\delta(\mathbf{r}_i-\mathbf{r}_j) \tag{2}$$

where N_e, N_p are numbers of electrons and positrons correspondingly, $N=N_e+N_p$, $m=m_e=m_p$, $\Phi_i^e xt$ is the scalar potential of an external electromagnetic, $\mathbf{A}_{(i,ext)}$ is the vector potential of an external electromagnetic, and σ_i is the Pauli matrixes. In Eq. (2), the first term describes the kinetic energy; the second term is the potential energy of charges in the external electric field. The third term is the potential energy of magnetic moments in the external magnetic field. Other terms describe interparticle interactions: the spin-spin, spin-external electromagnetic fields, and annihilation interactions, correspondingly [10–12]. We can write the continuity equation for electrons,

$$\partial_t n_e(\mathbf{r},t)+\nabla\cdot(n_e\mathbf{u}_e)=0 \tag{3}$$

and for positron,

$$\partial_t n_p(\mathbf{r},t)+\nabla\cdot(n_p\mathbf{u}_p)=0 \tag{4}$$

Consider a neutral degenerate plasma and assume we have a two-fluid electron-positron system with number densities n_{ep} and fluid velocities $\mathbf{u}_{ep}$. The scalar pressure of each fluid P_{ep} is

$$P_{ep}\approx\frac{2}{5}n_{ep}E_F=(3\pi^2)^{\frac{2}{3}}\frac{\hbar^2}{5m}n_{ep}^{\frac{5}{3}} \tag{5}$$

where E_F is the Fermi energy. The basic set of equations that we use for the quantum hydrodynamic electron model is

$$mn_e(\partial_t+\mathbf{u}_e\cdot\nabla)\mathbf{u}_e+\nabla p_e-\frac{n_e\hbar^2}{2m}\nabla(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}})=$$
$$-en_e(\mathbf{E}+\mathbf{u}_e\times\mathbf{B})+\frac{\pi e^2\hbar^2}{m^2c^2}n_e\nabla n_e+\frac{\pi e^2\hbar^2}{m^2c^2}n_p\nabla n_e-$$
$$\frac{e\hbar n_e}{2mc}(\mathbf{S}_e\cdot\nabla)\mathfrak{B}_e+\frac{\hbar^2 n_e}{2m}\nabla(\partial_\mu S_\nu^e\partial_\mu S_\nu^e)+2\pi n_e\mu_e^\beta\nabla(n_p\mu_p^\beta) \tag{6}$$

$$d_t\mathbf{S}_e=-\frac{e}{mc}\mathbf{S}\times\mathfrak{B}_e \tag{7}$$

and for positrons is,

$$mn_p(\partial_t+\mathbf{u}_p\cdot\nabla)\mathbf{u}_p+\nabla p_p-\frac{n_p\hbar^2}{2m}\nabla(\frac{\nabla^2\sqrt{n_p}}{\sqrt{n_p}})=$$
$$en_p(\mathbf{E}+\mathbf{u}_p\times\mathbf{B})+\frac{\pi e^2\hbar^2}{m^2c^2}n_p\nabla n_p+\frac{\pi e^2\hbar^2}{m^2c^2}n_e\nabla n_p+$$
$$\frac{\hbar^2 n_p}{2m}\nabla(\partial_\mu S_\nu^p\partial_\mu S_\nu^p)+2\pi n_p\mu_p^\beta\nabla(n_e\mu_e^\beta) \tag{8}$$

and similar to the relation of the evolution in the angular momentum in the magnetic field B, $d_t\mathbf{L}\propto\mathbf{L}\times\mathbf{B}$, one can write,

$$d_t\mathbf{S}_p=+\frac{e}{mc}\mathbf{S}\times\mathfrak{B}_p \tag{9}$$

where p_{ep} is the partial scalar pressure, and the $\mathfrak{B}_{ep}$ is the generalized magnetic field for each species,

$$\mathfrak{B}_e\equiv\mathbf{B}-\frac{\hbar c}{2en_e}\nabla(n_e\nabla,\mathbf{S}) \tag{10}$$

$$\mathfrak{B}_p\equiv\mathbf{B}+\frac{\hbar c}{2en_p}\nabla(n_p\nabla,\mathbf{S}) \tag{11}$$

In Eq. (6), the second and third terms on the left show the contribution of pressure to the motion of the particles and the spinless part of the quantum Bohm potential (When the Schrodinger equation is separated in the polar system, a potential term of $V=-\hbar^2/2m(\nabla^2 r/r)$ is created known as the Boehm potential). The first term on the right represents the Lorentz force; the second term shows the Darwin interaction between electrons (positrons) [11]. The third terms are also the Darwin interaction between electrons and positrons (The Darwin interaction term is due to one particle reacting to the magnetic field generated by the other particle. The Darwin Lagrangian describes the interaction to order V^2/c^2 between two charged particles in a vacuum and is given by $L=L_1+L_2$, where L_1 is the free particle Lagrangian and the L_2 is the interaction Lagrangian, $L_{inter}=L_{Cou}+L_{Dar}$, where the Coulomb interaction L_{Cou} is $-(q_1q_2)/r$, and the Darwin interaction Lagrangian is $L_{Dar}=(q_1q_2)/(2c^2r)(p_1/m_1)$, $(1+\hat{r}\hat{r})$, p_2/m_2, here q_1, q_2, m_1, m_2, v_1, and v_2 are the charges, masses, and the velocities of the particles 1 and 2, respectively, also c is the speed of light, r is the distance between the two particles, and $\hat{r}$ is the unit vector in the direction of r. Darwin's contribution to the particle motion equation is 1 is $d_t\mathbf{p}_{Dar}=(q_1q_2)/(2m_1m_2c^2r^2)$ $\{\mathbf{p}_1(\mathbf{p}_2\cdot\hat{r})+\mathbf{p}_2(\mathbf{p}_1\cdot\hat{r})-\hat{r}[\mathbf{p}_1\cdot(\bar{1}+3\hat{r}\hat{r})\cdot\mathbf{p}_2\}$. For fluid case, we can use alternative $\mathbf{p}_j\equiv\mathbf{n}_j(\hbar/i)\nabla$, and simplify the result.). Other terms describe; the spin interaction with the non-uniform magnetic field, spin interaction with non-uniform spin, and the annihilation interaction (The pair annihilation is the process that occurs when a subatomic particle collides with its respective antiparticle to produce other particles, such as an electron colliding a positron to produce two photons.), non-uniform spin, and the annihilation interaction correspondingly [12–14]. Now, we convert the two-fluid equations to a single fluid.

2.2 Investigation of spin-wave propagation and vibration modes

Let define, $n_e \approx n_p = n$, $2\bar{\mathbf{u}} = \mathbf{u}_e + \mathbf{u}_p$, $\mathbf{S}_p = -\mathbf{S}_e \equiv \mathbf{S}$ and the current density $\mathbf{J} = e(n_p\mathbf{u}_p - n_e\mathbf{u}_e) = en(\mathbf{u}_p - \mathbf{u}_e)$. It can conclude,

$$\mathbf{u}_e = \bar{\mathbf{u}} - \frac{e}{2n}\mathbf{J} \tag{12}$$

$$\mathbf{u}_p = \bar{\mathbf{u}} + \frac{e}{2n}\mathbf{J} \tag{13}$$

According to the definition of the mean velocit $\bar{\mathbf{u}}$ and the total density n, the continuity equation is obtained by adding Eqs. (3), (4).

$$\partial_t n + \nabla\cdot(n\bar{\mathbf{u}}) = 0 \tag{14}$$

and equations of motion for each species become

$$n[\partial_t + (\bar{\mathbf{u}} - \frac{e}{2n}\mathbf{J})\cdot\nabla](\bar{\mathbf{u}} - \frac{e}{2n}\mathbf{J}) + \frac{1}{m}\nabla p_e - \frac{n\hbar^2}{2m^2}\nabla(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}) = -\frac{en}{m}[\mathbf{E} + (\bar{\mathbf{u}} - \frac{e}{2n}\mathbf{J})\times\mathbf{B}] + \frac{en\hbar}{2m^2c}(\mathbf{S}\cdot\nabla)[\mathbf{B} - \frac{\hbar}{2en}\nabla(n\nabla\cdot\mathbf{S})] + \frac{n\hbar^2}{2m^2}\nabla(\partial_\mu S_\nu \partial_\mu S_\nu) + \frac{2\pi e^2\hbar^2}{m^2c^2}n\nabla n + 2\pi n\mu^\beta\nabla(n\mu^\beta) \tag{15}$$

$$n[\partial_t + (\bar{\mathbf{u}} + \frac{e}{2n}\mathbf{J})\cdot\nabla](\bar{\mathbf{u}} + \frac{e}{2n}\mathbf{J}) + \frac{1}{m}\nabla p_p - \frac{n\hbar^2}{2m^2}\nabla(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}) = \frac{en}{m}[\mathbf{E} + (\bar{\mathbf{u}} + \frac{e}{2n}\mathbf{J})\times\mathbf{B}] + \frac{en\hbar}{2m^2c}(\mathbf{S}\cdot\nabla)[\mathbf{B} + \frac{\hbar}{2en}\nabla(n\nabla\cdot\mathbf{S})] + \frac{n\hbar^2}{2m^2}\nabla(\partial_\mu S_\nu \partial_\mu S_\nu) + \frac{2\pi e^2\hbar^2}{m^2c^2}n\nabla n + 2\pi n\mu^\beta\nabla(n\mu^\beta) \tag{16}$$

and, for evolution of spin,

$$d_t\mathbf{S} = \frac{e}{2m}\mathbf{S}\times[\mathbf{B} + \frac{\hbar}{2en}\nabla(n\nabla\cdot\mathbf{S})] \tag{17}$$

Adding Eqs. (15), (16), and considering $p_e + p_p \approx p_F = [(2(3\pi^2)^{2/3}\hbar^2)/(5m)]n^{5/3} \equiv \eta_0 n^{5/3}$, and using total spin currents density, we obtain the equation of evolution for $\bar{\mathbf{u}}$,

$$\partial_t\bar{\mathbf{u}} = -(\bar{\mathbf{u}}\cdot\nabla)\bar{\mathbf{u}} - \frac{e^2}{4n^2}(\mathbf{J}\cdot\nabla)\mathbf{J} - \frac{\eta_0}{2m}\nabla n^{\frac{2}{3}} + \frac{e^2}{2nm}\mathbf{J}\times\mathbf{B} + \frac{e\hbar}{2m^2c}(\mathbf{S}\cdot\nabla)\mathbf{B} + \frac{\hbar^2}{2m^2}\nabla\times\nabla\times\mathbf{S} - \frac{\hbar^2}{2m^2}\nabla(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}) + \frac{2\pi e^2\hbar^2}{nm^2c^2}n\nabla n \tag{18}$$

By calculating E of Eq. (15) and then taking the curl on both sides of the result, all terms resulting from the gradient become zero. So, the evolution of the magnetic field $\nabla\times\mathbf{E} = -\partial_t\mathbf{B}$ becomes,

$$\partial_t\mathbf{B} = \nabla\times(\bar{\mathbf{u}}\times\mathbf{B}) + \frac{m}{e}\partial_t(\nabla\times\bar{\mathbf{u}}) + \frac{e\hbar}{8m^2}\nabla\mathbf{S}\times\nabla\mathbf{B} \tag{19}$$

In the first term in the right hand, $(\bar{\mathbf{u}}\times\mathbf{B})$ is Hall term, and the second term represents a battery effect due to electron inertia. The first two terms are the classic ones of MHD, the last term is a spin electromotive force.

Let us consider the equilibrium state where $\mathbf{B} = B\hat{z} + \delta\mathbf{B}$, $\langle\mathbf{u}\rangle = 0$, $\mathbf{k} = k(0, \sin\varphi, \cos\varphi)$, where φ is the angle between wave vector and magnetic field.

For parallel propagation, $\mathbf{k}\|\mathbf{B}$, the linear perturbations around the equilibrium state, $\mathbf{k} = k\hat{z}$, $\delta\mathbf{B} = \delta B_x\hat{x} + \delta B_y\hat{y}$. Using Eqs. (15-19), we can write,

$$\begin{bmatrix} C_4 & -i\omega/k & 0 & 0 & -C_5 & 0 & 0 & 0 \\ -i\omega/k & -C_4 & 0 & C_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & \omega/k & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & C_1 & -C_3 & -C_2 & \cos\theta & 0 \\ 0 & 0 & 0 & \cos\theta & 0 & 0 & \omega/k & 0 \\ 0 & 0 & 0 & 0 & \cos\theta & \omega/k & 0 & 0 \\ 0 & 0 & 0 & C_3 & C_1 & \cos\theta & C_2 & \sin\theta \\ 0 & 0 & -\eta & 0 & C_6 & 0 & 0 & \omega/k \end{bmatrix} \times \begin{bmatrix} S_x \\ S_y \\ n \\ \delta B_y \\ \delta B_x \\ u_n \\ u_y \\ u_z \end{bmatrix} = 0 \tag{20}$$

where,

$C_1 \equiv \frac{\omega}{k}[1 + \frac{mc^2k^2}{8\pi n^{1/3}e^2}]$,

$C_2 \equiv i\sqrt{\frac{mc^2}{8\pi n^{1/3}e^2}}\omega$,

$C_3 \equiv i\sqrt{\frac{mc^2}{8\pi n^{1/3}e^2}}k\cos\theta$,

$C_4 \equiv \sqrt{\frac{8\pi n^{1/3}e^2}{mc^2}}[1 + \frac{\hbar cn^{2/3}}{2eB_0}k^2]$,

$C_5 \equiv \sqrt{\frac{8\pi n^{1/3}e^2}{mc^2}}$,

$C_6 = [1 + k\frac{2\pi\hbar en^{5/6}}{cmB_0}]\sin\theta$,

$\eta \equiv \frac{\pi\hbar^2 n^{5/3}}{2mB^2}(1 + 4\pi^2k^2)$.

The spin effect appears only in the four elements

$$\begin{bmatrix} C_4 & -i\omega/k \\ -i\omega/k & -C_4 \end{bmatrix}$$

and does not appear in the rest of them. The frequency of this spin mode is equal to ω_{spin}, so,

$$\omega_{spin} = \sqrt{\frac{8\pi n^{1/3}e^2}{mc^2}}[1 + \frac{\hbar cn^{2/3}}{2eB_0}k^2]k \tag{21}$$

We considered spin effects for only the spin- magnetic field coupling, as it is linear in $\hbar$. The other spin interactions are of the second order in $\hbar$ and have weaker effects.
Use the properties of determinants, we have [15, 16],

$$\det(8\times 8)=\det(6\times 6)\times\det\begin{vmatrix} C_4 & -i\omega/k \\ -i\omega/k & -C_4\end{vmatrix}=0 \quad (22)$$

Solving $\det(6\times 6=0)$, normal modes without spin can obtain.

$$\begin{bmatrix} \omega/k & 0 & 0 & 0 & 0 & -1 \\ 0 & C_1 & -C_3 & -C_2 & \cos\theta & 0 \\ 0 & \cos\theta & 0 & 0 & \omega/k & 0 \\ 0 & 0 & \cos\theta & \omega/k & 0 & 0 \\ 0 & C_3 & C_1 & \cos\theta & C_2 & \sin\theta \\ -\eta & 0 & C_6 & 0 & 0 & \omega/k \end{bmatrix}$$

$$\times\begin{bmatrix} n \\ \delta B_y \\ \delta B_x \\ u_n \\ u_y \\ u_z \end{bmatrix}=0 \quad (23)$$

In the Eqs. (23), (u_z, n) correspond to the fast mode, but $(u_y, \delta B_y, u_n, \delta B_x)$ correspond to the Alfven-slow mode. For parallel propagation $(\theta = 0)$, one of the solusions is

$\begin{bmatrix} \omega/k & -1 \\ -\eta & \omega/k\end{bmatrix}\times\begin{bmatrix} n \\ u_z\end{bmatrix}=0$ or,

$$\omega^2=\frac{\hbar^2 n^{2/3}}{8m^2}\frac{\omega_p^2}{\omega_c^2}(k^2+4\pi^2k^4) \quad (24)$$

At very long wavelengths, the value of k is small and ω can be approximated as $\omega\approx(\hbar n^{1/3})/(2\sqrt{2}m)(\omega_p/\omega_c)k$, which does not show any dispersion. At low wavelengths, the value of k is high and ω can be approximated as $\omega\approx(\hbar n^{1/3})/(\sqrt{2}m)(\omega_p/\omega_c)\pi k^2$.
The others have modes with the characteristic equation,

$$\begin{vmatrix} C_1 & -C_3 & -C_2 & 1 \\ 1 & 0 & 0 & \omega/k \\ 0 & 1 & \omega/k & 0 \\ C_3 & C_1 & 1 & C_2\end{vmatrix}=0$$

where

$C_1\equiv\omega/k[1+\frac{mc^2k^2}{8\pi n^{1/3}e^2}]$,

$C_2\equiv i\sqrt{\frac{mc^2}{8\pi n^{1/3}e^2}}\omega$,

$C_3\equiv i\sqrt{\frac{mc^2}{8\pi n^{1/3}e^2}}k$.

After calculating the determinant and placing the values and simplification, we get,

$$\frac{\omega^2}{k^2}=\frac{4\omega_p^4}{4\omega_p^4+2c^2k^2n^{2/3}\omega_p^2-c^4k^4n^{4/3}} \quad (25)$$

where $\omega_{pe}=(=\sqrt{\frac{4\pi ne^2}{m}})$ is the plasma frequency. At very long wavelengths, the value of k is small and can be approximated as

$\omega^2/k^2\approx(2\omega_p^2)/(2\omega_p^2+c^2k^2n^{2/3})$,

and for very short wavelengths, it can be approximated,

$\omega^2/k^2\approx(4\omega_p^2)/(2c^2k^2n^{2/3}\omega_p^2-c^4k^4n^{4/3})$.

This relationship indicates that for very short wavelengths, there is a cut-off frequency.
The answer ω_+^2 is always positive, and the system oscillations are stable. The answer ω_-^2 can be negative, and system oscillations become unstable. For perpendicular propagation, $\mathbf{k}\perp\mathbf{B}$, $\mathbf{k}=k\hat{y}$, $\delta\mathbf{B}=\delta B_y\hat{y}-\delta B_z\hat{z}$, $\theta=\pi/2$, we can write,

$C_1\equiv\frac{\omega}{k}[1+\frac{mc^2k^2}{8\pi n^{1/3}e^2}]$,

$C_2\equiv i\sqrt{\frac{mc^2}{8\pi n^{1/3}e^2}}\omega$,

$C_3\equiv 0$,

$C_4\equiv\sqrt{\frac{8\pi n^{1/3}e^2}{mc^2}}[1+\frac{\hbar cn^{2/3}}{2eB_0}k^2]$,

$C_5=\sqrt{\frac{8\pi n^{1/3}e^2}{mc^2}}$,

$C_6=[1+k\frac{2\pi\hbar en^{5/6}}{cmB_0}]$.

$$\begin{vmatrix} \omega/k & 0 & 0 & -1 \\ 0 & 0 & \omega/k & 0 \\ 0 & C_1 & C_2 & 1 \\ -\eta & C_6 & 0 & \omega/k\end{vmatrix}=0 \quad (26)$$

after calculating the determinants and placing the values and simplification, we get,

$$\frac{\omega^2}{k^2}[1+\frac{c^2k^2}{2\omega_p^2}]=c^2[1+\frac{\hbar\omega_p^2}{2cm\omega_c^2}k -\frac{3\pi^2\omega_p^2\hbar^2}{8m^2c^2\omega_c^2}k^2-\frac{\pi^2\hbar^3\omega_p^2}{4m^3c^3\omega_c^2}k^3] \quad (27)$$

$$\omega^2=\frac{c^2k^2}{(1+\frac{c^2k^2}{2\omega_p^2})}[1+\frac{\hbar\omega_p^2}{2cm\omega_c^2}k -\frac{3\pi^2\omega_p^2\hbar^2}{8m^2c^2\omega_c^2}k^2+\frac{\pi^2\hbar^3\omega_p^2}{4m^3c^3\omega_c^2}k^3] \quad (28)$$

At very long wavelengths, the value of k is small and can be approximated $\omega^2\approx c^2(k^2+(\hbar\omega_p^2)/(2cm\omega_c^2)k^3)$, and for very short wavelengths, the value of k is high and can be approximated

$$\omega^2\approx\frac{c^2k^2}{\frac{c^2k^2}{2\omega_p^2}}(\frac{\pi^2\hbar^3\omega_p^2}{4m^3c^3\omega_c^2}k^3)\approx\frac{\pi^2\hbar^3\omega_p^4}{2m^3c^3\omega_c^2}k^3$$

In the absence of spin effects, for parallel propagation, in the fast magnetosonic domain, the frequency is determined by the Fermi pressure, Bohm forces (quantum effects), and in the Alfven-slow domain, is determined by the Hall effect (non quantum effects). Indeed, for the high wavenumbers, the MHD Alfven mode separates into whistler and ion-cyclotron modes but for perpendicular propagation, there is only fast mode.

3. Study of instability

When a disturbance occurs in the environment, it affects many physical quantities. The relationship between dispersion waves as they propagate through matter and these quantities is an interesting subject to study. The behavior of a quantity can be very different at different frequencies, and even for a range of frequencies, it wastes less wave energy and, conversely, absorbs wave energy strongly over a range of frequencies and does not allow it to pass. The relationship between this quantity and vibrational modes is called the stability and instability of those modes against this quantity. The stability and instability of those modes depends on the type of material and the type of quantity. Historically, in the discussion of multi-fluid plasma, one of the first quantities to be studied is the dielectric function.

This section discusses the stability and instability of turbulence-induced modes in electron-positron plasma. An external magnetic field can have a stabilizing effect. This section aims to investigate the instability of two quantum fluids in magnetized dense plasma in a specific transverse configuration. In the following, the application conditions of the model and its specific parameters are discussed. Unlike the classical case, which has instability for large wavelengths, Fermi pressure effects have a minimum cut-off wavelength for instability; Therefore, a comparison of the strengths of each of these effects is considered. We assume the plasma as a non-relativistic Fermi gas and obtain a new transverse mode in the two-current magnetic quantum plasma.

3.1 Dielectric function

The dielectric function of plasma expresse as follows, [17, 18]

$$\varepsilon(\mathbf{k},\omega)=1+\frac{4\pi^2e^2}{k^2}(\chi_e+\chi_p) \tag{29}$$

where χ is the susceptibility of the particles; If the equation $\varepsilon(\mathbf{k},\omega)=0$ as a function of ω has roots in the upper half of the complex plane, the plasma is unstable. In classical plasma, the susceptibility is given by [19, 20],

$$\chi^c_{ep}(\mathbf{k},\omega)=\frac{e^2n_{ep}}{m}\int d^3V[\frac{k^\beta\partial_\beta f_{ep}}{\omega-k^\beta u_\beta}] \tag{30}$$

where f_i is the normalization distribution function; for degenerate electrons, the susceptibility χ_e of the electrons and protons is obtained by the Lindhard equation,

$$\chi^Q_e(\mathbf{k},\omega)=\frac{3n_e}{mv_F^2}L(u,\beta) \tag{31}$$

and

$$\chi^Q_p(\mathbf{k},\omega)=\frac{3n_p}{mv_F^2}L(u,\beta) \tag{32}$$

where v_F is Fermi velocity and k_F is Fermi wave vector, $\beta\equiv k/2k_F$, $u=\omega/kv_F$, and L is Lindhard function; suppose that two currents of electrons and positrons are moving in opposite directions in a one-dimensional space. The longitudinal dielectric function of this one-dimensional plasma for the electron and positron currents spin is written as [21–24],

$$\varepsilon=1+\frac{4\pi e^2}{k^2}[\chi^Q_e(\mathbf{k},\omega+k^\beta u_{e\beta})+\chi^Q_P(\mathbf{k},\omega-k^\beta u_{p\beta})] \tag{33}$$

where u_e and u_p are the drift velocity of the electrons and positrons, respectively $L(u,\beta)$ is Linard function,

$$L_{re}=\frac{1}{2}+\frac{1}{8z}[1-(\beta-u)^2]\log(\frac{|\beta-u+1|}{|\beta+u+1|})$$
$$+\frac{1}{8\beta}[1-(\beta+u)^2]\log(\frac{|\beta+u+1|}{|\beta-u+1|}) \tag{34}$$

Using Eqs. (33,34), expansion of the logarithmic terms, linearization of the result, and a little mathematical work, we obtain,

$$\varepsilon(\mathbf{k},\omega)=1-\frac{1}{2}\frac{\omega_p^2}{(\omega-ku_e)^2-\frac{\hbar^2k^4}{4m^2}}-\frac{1}{2}\frac{\omega_p^2}{(\omega+ku_p)^2-\frac{\hbar^2k^4}{4m^2}} \tag{35}$$

The condition for the stability of the answer is $\varepsilon(\mathbf{k},\omega)\geq 0$. Assuming $\mathbf{u}_e\approx-\mathbf{u}_p\equiv u\hat{z}$, the equation $\varepsilon(\mathbf{k},\omega)=0$ has two answers to ω^2,

$$\omega^2_\pm=\frac{1}{2}\omega_p^2+k^2u^2+\frac{\hbar^2k^4}{4m^2}\pm\frac{1}{2}\omega_p^2[1+\frac{8k^2u^2}{\omega_p^2}+\frac{4\hbar^2k^6u^2}{m^2\omega_p^4}]^{\frac{1}{2}} \tag{36}$$

The first solution (36) describes a sound-like wave existing in magnetized dielectrics due to different equilibrium distribution of particles. The quantum pressure produces a nonlinear contribution $\sim k^4$ that may be important at large k. The quantum correction associated to the Bohm potential is taken into account. The answer ω^2_+ is always positive, and its oscillations are stable. The answer ω^2_- can be negative $\omega^2_-<0$, and its oscillations become unstable. By defining dimensionless variable $\Re\equiv ku/\omega_p$, Eq. (36) can rewrite as follow,

$$\omega^2_-=\frac{1}{2}\omega_p^2+k^2u^2+\frac{\hbar^2k^4}{4m^2}-\frac{1}{2}\omega_p^2[1+8\Re^2+\frac{4\hbar^2\omega_p^2\Re^6}{m^2u^4}]^{\frac{1}{2}} \tag{37}$$

then by approximating the radical term in Eq. (36) and assuming $\Re\ll 1$ (long wavelengths or small k) and also $(4\hbar^2\omega_p^2\Re^6)/(m^2u^4)\ll 1$, we obtain,

$$\omega^2_-=-k^2u^2+\frac{\hbar^2k^4}{4m^2}(1-4\Re^2) \tag{38}$$

The condition of instability is that $\omega_-^2 < 0$, which is equivalent to (for $\Re \ll 1$) $[(\hbar^2 k^2)/(4m^2)](1-4\Re^2) < u^2$. In this case, the amplitude of the dispersion wave is proportional to $\exp[-\gamma t]$, where γ is the instability parameter and is given by,

$$\gamma = ku[1 - \frac{\hbar^2 k^2}{4m^2 u^2}(1-4\Re^2)]^{\frac{1}{2}} \quad (39)$$

now, suppose again,

$$\mathbf{u}_e = \mathbf{u}_p = u\hat{z}, \qquad \mathbf{B} = B\hat{z}, \qquad \mathbf{E} = 0 \quad (40)$$

Moreover, a small amplitude is applied to the system where $\mathbf{k} = k\hat{x}$ is the wave vector. In addition, we assume that the disturbance of the electric field is along the magnetic $\delta\mathbf{B} = \delta B\hat{y}$. Since the current velocities are in the direction of field B, they have no role in transverse modes. By linearizing the quantities in Eq. (30) and Fourier analyzing into space and time, we obtain the following dispersion relation,

$$\omega^2 - c^2k^2 = \omega_p^2[1 + \frac{u^2k^2}{\omega^2 - \frac{3}{5}v_F^2 - \omega_c^2 - \frac{\hbar^2 k^4}{4m^2}}] \quad (41)$$

This equation can quickly solve in terms of ω. In Eq. (34), factors such as ω_p and ω_c can significantly stabilize unstable modes. In low-density plasmas, the role of the magnetic field in stabilizing these modes is significant. In dense plasma, the two effects of Fermi pressure and Bohm pressure have a stabilizing effect. The answer to Eq. (34) is given by,

$$\frac{\omega_\pm^2}{\omega_P^2} = \frac{1}{2}[1 + \frac{c^2k^2}{\omega_p^2}(1 + \frac{3}{5}v_F^2) + \frac{\omega_c^2}{\omega_p^2} + \frac{1}{4}\frac{\hbar^2 k^4}{m^2\omega_p^2}]$$
$$\pm\frac{1}{2}\{[1 + \frac{c^2k^2}{\omega_p^2}(1 - \frac{3}{5}V_F^2) - \frac{\omega_c^2}{\omega_p^2} - \frac{1}{4}\frac{\hbar^2 k^4}{m^2\omega_p^2}]^2 + \frac{4u^2k^2}{\omega_p^2}\}^{\frac{1}{2}} \quad (42)$$

It is possible to omit terms appropriate to $\hbar^2$ due to their small size compared to other terms. Like Eq. (29), the answer ω_+^2 is always positive, and its oscillations are stable. Answer ω_-^2 can become unstable, provided this relation is established,

$$\frac{u^2k^2\omega_p^2}{\omega_p^2 + c^2k^2} > \omega_c^2 + \frac{3}{5}k^2 v_F^2 \quad (43)$$

In the quantum regime $\omega_c \ll kv_F$, the instability condition (36) becomes simpler,

$$u^2 > \frac{3}{5}\frac{\omega_p + c^2k^2}{\omega_p^2}V_F^2 or k^2 < \frac{\omega_p^2}{c^2}(\frac{5}{3}\frac{u^2}{V_F^2} - 1) \quad (44)$$

According to this condition and using $\lambda = 2\pi/k$, we obtain a cut-off wave number

$$k_{cut}^2 = \frac{\omega_p^2}{c^2}(\frac{5}{3}\frac{u^2}{V_F^2} - 1)$$

$$k_{cut} = \frac{\omega_p}{c}(\frac{5}{3}\frac{u^2}{V_F^2} - 1)^{\frac{1}{2}}$$

then

$$\lambda_{cut} = \frac{2\pi c}{\omega_p}[\frac{5u^2}{3v_F^2} - 1]^{-\frac{1}{2}} \quad (45)$$

as a result, $\lambda < \lambda_{cut}$ is definitely stable.

4. Conclusion

In this paper, we write a set of two-fluid electron-positron plasma equations based on the hydrodynamic model and briefly discuss its various terms and the role of spin in different spin-spin, spin-field, and spin interactions. We discussed their density and the effect of heterogeneity in the equations. These equations can determine what dispersion relations to expect if turbulence occurs in equilibrium plasma. Here, we take a path for discussion that differs from the typical path taken in other articles and obtain new dispersion equations. We also examined the limits of weak and strong magnetic fields and the effect of spin polarization on ripple waves in plasma. Again, assuming that the external magnetic field is weak, we found a quasi-sound wave due to differences in electron-positron states' distribution in the two-fluid electron-positron model. We found that the spin-current evolution in a magnetized plasma creates a new dispersion mode. We also showed that we have only the fast mode in the direction perpendicular to the direction of propagation of the waves. The speed of this mode is equal to the speed of the mode in the parallel direction plus an additional term that depends on the system's characteristics. For high-density plasma, this correction is negligible, but for very low densities and weak magnetic fields, this effect is significant. Then, we discussed the stability and instability of oscillating plasma modes, including electron-positron as two stream plasma, and determined these vibration modes' range of stability and instability. Finally, we calculated the cut-off wavelength.

Conflict of interest statement:
The authors declare that they have no conflict of interest.

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