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Optimal Shape Design for 2D Radiative Enclosures Using NURBS

Farbod Fakhrabadi [a,*], Farshad Kowsary [b]

[a] *Department of Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *School of Mechanical Engineering, Faculty of Engineering, University of Tehran,Tehran, Iran*

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Abstract

This article presents an optimal shape design methodology for 2D diffuse-walled radiant enclosures. In this study, the shape of the enclosure is parameterized by means of non-uniform rational B-spline (NURBS) surfaces, and their control points represent the design variables. The enclosure geometry is discretized by choosing the parameters of NURBS surfaces as generalized curvilinear coordinates, and the radiosity equation is solved using the infinitesimal-area analysis technique developed by Daun and Hollands [1]. The simplified conjugate-gradient method (SCGM) is used as the optimization method to obtain the optimal shape and adjust the design variables intelligently. The methodology is demonstrated by optimizing the shape profile of a cavity with the objective of enhancing the apparent emittance.

Keywords: Optimal shape design, Radiative enclosures, NURBS

1. Introduction

Optimal shape design for heat transfer problems is of great importance, since using an optimal design reduces the consumption of energy, matter and time. The aim of optimal shape design for a heat transfer system is to improve the performance of the system or to meet some specific heat transfer requirements such as specified heat flux or temperature distribution.

Extensive work has been done in shape design problems, such as fin profile optimization [2-4], shape design for heat conduction problems [5,6], shape design of a cylinder with heat transfer [7], shape design of millimeter-scale air channels [8], geometric optimization of radiative enclosures [9], shape optimization of convective periodic channels [10], shape optimization of a heat exchanger [11] and optimization of steady fluid-thermal systems [12].

In general, optimal shape design problems require a great amount of computation time and memory space. This paper is aimed at describing a robust and efficient method for shape optimization of radiative enclosures by reducing the computation time and improving the accuracy and the quality of the optimal design.

In the discussion that follows, a parametric representation of the enclosure geometry is presented. A computational method for solving the radiosity equation is then discussed. Subsequently, the simplified conjugate-gradient method (SCGM) is described as the optimization method. Finally the methodology is demonstrated by optimizing the shape profile of a cavity with the objective of enhancing the apparent emittance.

*Corresponding author Email address: Farbod.Fakhrabadi@iau.ac.ir

2. Parametric Representation of the Enclosure Geometry

The first step in optimal shape design for radiative enclosures is to specify the enclosure geometry. The shape profile of the enclosure could be either represented parametrically or built by using a point-by-point approach [5,6]. Parametric representation of the shape profile reduces the overall number of design variables and consequently the computation time. However, the point-by-point approach gives a wider range of shape alternatives.

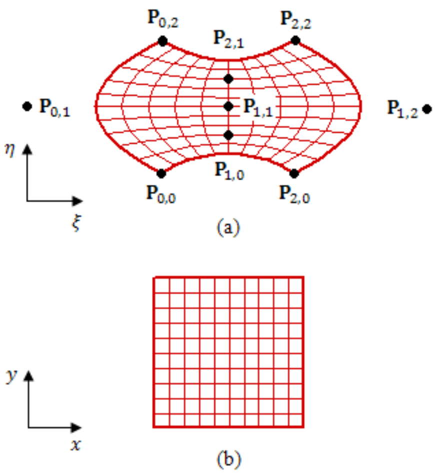

Fig 1. Transformation of the physical domain (a) into the computational domain (b).

In the present study, the shape of the enclosure is parameterized by means of non-uniform rational B-spline

(NURBS) surfaces, and their control points represent the design variables. These parametric surfaces allow free-form representation with total geometry control over the surface. The number of control points, and hence the number of degrees of freedom (DOFs) of the shape profile could be increased, if a finer description of the shape and more flexibility in shape design are required.

A non-uniform rational B-spline (NURBS) surface is defined as

$$\mathbf{S}(\mathbf{u}) = \mathbf{S}(u,v) = \frac{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)w_{i,j}\mathbf{P}_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m} N_{i,p}(u)N_{j,q}(v)} \quad 0 \le u,v \le 1 \tag{1}$$

where the $\mathbf{P}_{i,j}$ are the control points that form a bidirectional control net. The n and m are the number of control points in the ξ and η directions, respectively. The $w_{i,j}$ are the weights. The $N_{i,p}(u)$ and $N_{j,q}(v)$ are the non-rational B-spline basis functions of degree p and degree q, respectively, defined on the non-decreasing knot vectors

$$U = \left\{\underbrace{0,\dots,0}_{p+1}, u_{p+1},\dots,u_{r-p-1}, \underbrace{1,\dots,1}_{p+1}\right\} \tag{2}$$

$$V = \left\{\underbrace{0,\dots,0}_{q+1}, v_{q+1},\dots,v_{s-q-1}, \underbrace{1,\dots,1}_{q+1}\right\} \tag{3}$$

where $r = n + p + 1$ and $s = m + q + 1$.

The i-th B-spline basis function of degree p, denoted by $N_{i,p}(\xi)$, is defined recursively by the Cox-De Boor formula as

$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u < u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$
$$N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i}N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) \tag{4}$$

The shape of NURBS surfaces could be locally changed by moving the control points or modifying the weights. These surfaces have several unique properties that are effective and well suited for shape optimization. The surface interpolates the four corner control points, i.e. $\mathbf{S}(0,0) = \mathbf{P}_{0,0}$, $\mathbf{S}(1,0) = \mathbf{P}_{n,0}$, $\mathbf{S}(0,1) = \mathbf{P}_{0,m}$, and $\mathbf{S}(1,1) = \mathbf{P}_{n,m}$. Furthermore, the control points approximate the surface and the surface is contained in the convex hull of its control points. This property is very useful, especially in defining the geometric constraints. An example of a NURBS surface with its defining control points is depicted in Fig. 1. A complete description of NURBS surfaces can be found in [13].

3. Solution of the Radiasity Equation

The second step in optimal shape design for radiative enclosures is to solve the radiosity equation in the specified geometry. To this end, the enclosure should be discretized first. As in this study the enclosure geometry is represented parametrically, through NURBS surfaces, it can be discretized by choosing the parameters of NURBS surfaces as generalized curvilinear coordinates. This method reduces the CPU time needed for grid generation significantly. The use of generalized curvilinear coordinates transforms an irregular region in the physical domain into a rectangular region in the computational domain (Fig. 1).

The radiosity equation in generalized coordinates can be expressed as [1]

$$q_o(\mathbf{u}) = B(\mathbf{u}) + G(\mathbf{u})\int_0^1\int_0^1 q_o(u')K(\mathbf{u},\mathbf{u}')\,du'dv' \tag{5}$$

where, if $T(\mathbf{u})$ is specified,

$$B(\mathbf{u}) = \varepsilon(\mathbf{u})\sigma T^4(\mathbf{u}), \quad G(\mathbf{u}) = 1 - \varepsilon(\mathbf{u}) \tag{6}$$

or, if $q(\mathbf{u})$ is specified,

$$B(\mathbf{u}) = q(\mathbf{u}), \quad G(\mathbf{u}) = 1 \tag{7}$$

The kernel, $K(\mathbf{u},\mathbf{u}')$, is equal to the shape factor between the differential area element at $\mathbf{u}$ and the differential area element at $\mathbf{u}'$, divided by $du'dv'$.

In the present study, an iterative solver is used to obtain the radiosity distribution. When the radiosity distribution has been found, the unknown heat flux or temperature distribution can be determined by the following relations [14].

$$q(\mathbf{u}) = \frac{\varepsilon(\mathbf{u})}{1-\varepsilon(\mathbf{u})}[\sigma T^4(\mathbf{u}) - q_o(\mathbf{u})], \tag{8}$$

$$\sigma T^4(\mathbf{u}) = \frac{1-\varepsilon(\mathbf{u})}{\varepsilon(\mathbf{u})}q(\mathbf{u}) + q_o(\mathbf{u}) \tag{9}$$

4. Optimization Method

The last step in optimal shape design for radiative enclosures is to use an optimization method to adjust the design variables intelligently. The two most commonly used optimization methods for shape design problems are the genetic algorithms and the gradient-based optimization algorithms. The genetic algorithms are robust, and can be used for multi- objective problems. However, the main drawback to them is the reduced convergence rate. The gradient-based optimization algorithms are computationally efficient. But, the drawback of these methods is their tendency to get trapped in local optima. To remedy this problem, multiple optimizations should be performed, each starting from different values of design variables.

In this study, the simplified conjugate gradient method (SCGM), proposed by Cheng and Chang [16], is used as the optimization method. The SCGM is capable of dealing with various forms of the objective functions, and thus it is a well suited method for shape optimization. The iterative procedure of SCGM for finding the optimum design variables $\vec{\phi}$ and hence the optimal shape can be stated as follows:

(1) Define an objective function $f(\vec{\phi})$ that the minimum point of it corresponds with the optimal shape.
(2) Make an initial guess for $\vec{\phi}$ (as initial point). Set iteration number as $i = 1$.
(3) Solve the radiosity equation and find the heat flux or objective function $f(\vec{\phi})$ associated with the latest values of design variables.
(4) Compute the gradient of the objective function, $\vec{\nabla} f_i$, at the point $\vec{\phi}_i$, by means of the direct numerical sensitivity analysis [15].
(5) Compute the conjugate gradient coefficients β_i, and the search directions $\vec{S}_i$ as

$$\beta_i = \frac{\vec{\nabla} f_i^T \, \vec{\nabla} f_i}{\vec{\nabla} f_{i-1}^T \, \vec{\nabla} f_{i-1}}, \tag{10}$$

$$\vec{S}_i = -\vec{\nabla} f_i + \beta_i \vec{S}_{i-1} \tag{11}$$

(6) Assign an appropriate fixed value to the step sizes λ_i^*, and update the design variables as

$$\vec{\phi}_{i+1} = \vec{\phi}_i + \lambda_i^* \vec{S}_i \tag{12}$$

(7) Test the new point $\vec{\phi}_{i+1}$ for optimality. If $\vec{\phi}_{i+1}$ is optimal, terminate the iteration process. Otherwise, set the new iteration number $i = i + 1$, and go to step (3).

5. Implementation

To demonstrate the performance of the methodology presented in this paper, the shape profile of a cavity is optimized with the objective of enhancing the apparent emittance. The cavity geometry is shown in Fig. 2. For simplicity, the shape profile of the cavity is assumed 2-D, and it is presented parametrically, through a B-spline surface of degree five in the u direction and degree two in the v direction. B-spline surfaces are a special subclass of NURBS surfaces with $w_{i,j} = 1$ and the uniform knots distribution. The wall emissivity is set equal to 0.5. The temperature and the emissivity of the surrounding have been assumed constant and equal to 0 K and 1, respectively.

As shown in Fig. 2, the coordinates of selected control points represent the design variables $\vec{\phi}$. The following constraints have been imposed on the design variables $\vec{\phi}$ to restrict the cavity dimensions and to force the enclosure to remain unobstructed.

$$\begin{aligned} &0.0 \le \phi_1 \le 0.07, \\ &0.01 \le \phi_2 \le \phi_3, \\ &0.01 \le \phi_3 \le 0.05 \end{aligned} \tag{13}$$

Now, in order to find vector of unknowns $\vec{\phi}$, an objective function $f(\vec{\phi})$ is defined as

$$f(\vec{\phi}) = \frac{1}{C\varepsilon_a + \frac{1}{A}} \tag{14}$$

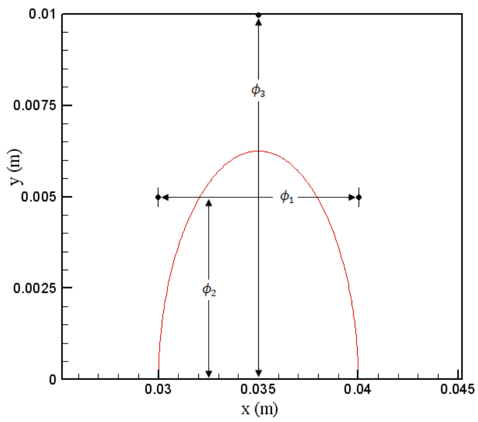

Fig 2. Shape profile of the cavity

where A is the area of the cavity surface, C is a constant that its value depends on the requirement of the design purpose and ε_a is the apparent emittance, given by

$$\varepsilon_a = \frac{Q_o}{A_o \sigma T_c^4} \tag{15}$$

where Q_o is the total heat transfer rate leaving the opening of the cavity, A_o is the cavity opening area and T_c is the cavity temperature.

The minimum point of function f corresponds to the solution $\vec{\phi}$ of the problem. As explained previously, the computational method of the minimization procedure consists of two main modules; the direct problem solver and the search modules. The heat flux distribution along the wall of the optimal cavity profile is shown in Fig. 3. Considering both the accuracy and the computational cost, the calculations were performed on a 200×1 grid system. Finer grids have been tested without finding any significant changes in the results.

A personal computer with a Pentium IV 3.2GHz processor has been used to perform the calculations. The CPU time required for the shape optimization problem is approximately 10 seconds. Fig. 4 shows the dependence of optimal shape on the value of C. The reduction history of the objective function f is shown in Fig. 5. The convergence criterion is set at $\|\nabla f\| \le 10^{-10}$.

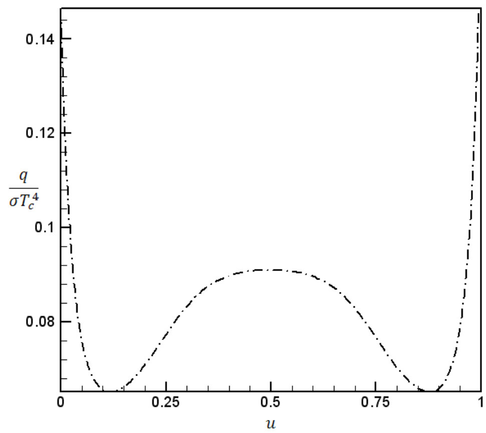

Fig 3. Heat flux distribution in the optimal cavity profile (C=1000)

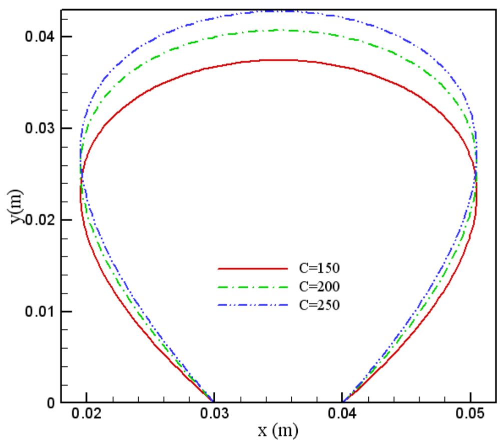

Fig 4. Dependence of optimal shape on the value of C

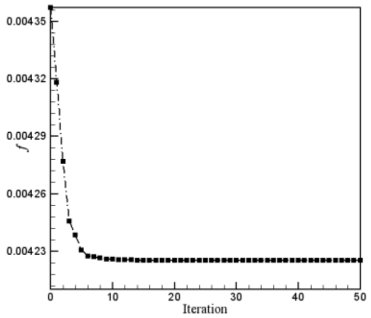

Fig 5. Reduction history of the objective function per each cycle of the SCGM

6. Conclusion

In this paper a method is presented for shape optimization of radiative enclosures. The shape of the enclosure is represented parametrically, through non-uniform rational B-spline (NURBS) surfaces, and their control points represent the design variables. These parametric surfaces allow free-form representation with total geometry control over the surface. Moreover, parametric representation of the shape profile reduces the overall number of design variables and consequently the computation time. The simplified conjugate-gradient method (SCGM) is used as the optimization method to obtain the optimal shape and adjust the design variables intelligently. The SCGM is capable of dealing with various forms of the objective functions, and thus it is a well suited method for shape optimization. The performance of the proposed method is demonstrated by optimizing the shape profile of a cavity with the objective of enhancing the apparent emittance.

Nomenclature

A	=	area of the cavity surface, m^2
A_o	=	cavity opening area, m^2
$B(\mathbf{u})$	=	generalized emissive power term, W/m^2
C	=	weighting factor
f	=	objective function
$G(\mathbf{u})$	=	generalized reflectivity term
$K(\mathbf{u},\mathbf{u}')$	=	kernel function
m	=	number of NURBS surface control points in the η direction
N	=	NURBS basis function
n	=	number of NURBS surface control points in the ξ direction
$\mathbf{P}$	=	NURBS surface control net
p	=	NURBS degree in the u direction
Q_o	=	heat transfer rate of the cavity, W
q_o	=	radiosity, W/m^2
q	=	heat flux, W/m^2
q	=	NURBS degree in the v direction
$\mathbf{S}(\xi,\eta)$	=	NURBS surface
$\vec{S}$	=	search direction
T	=	temperature, K
T_c	=	temperature of the cavity, K
$\mathbf{u}$	=	two-component vector equivalent to (u,v)
(u,v)	=	computational coordinates
(x,y)	=	physical coordinates, m
Greek symbols		
β	=	conjugate gradient coefficient
ε	=	emissivity
ε_a	=	apparent emissivity
λ^*	=	optimal step size
ϕ	=	design variable
∇	=	gradient
Superscripts		
$\rightarrow$	=	vector
T	=	transpose symbol
Subscripts		
i	=	iteration number
i,j	=	control point indices

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