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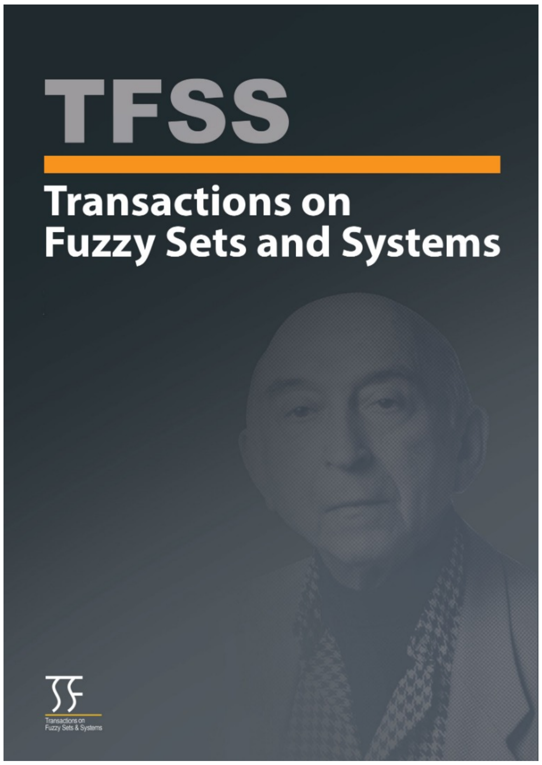

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Confidence Level-Based Hamacher Aggregation Operators for Sustainable Furniture Supplier Selection using p,q-Quasirung Orthopair Fuzzy Sets

Utpal Mandal, **Mijanur Rahaman Seikh***, **Arnab Kumar De**

Abstract. Selecting sustainable furniture suppliers for universities is a complex decision-making problem that must balance environmental, economic, and social factors under conditions of uncertainty. As higher education institutions increasingly adopt green procurement policies, the need for structured, data-driven evaluation methods becomes more pressing. This necessitates robust models capable of handling imprecision and reflecting the trustworthiness of expert opinions. Traditional methods often fall short in handling the vagueness inherent in expert evaluations. To address this, we adopt the recently introduced p,q-quasirung orthopair fuzzy sets (p,q-QOFSs), which provide a more flexible framework for modeling imprecise information. This study proposes novel confidence level-based Hamacher weighted averaging and geometric aggregation operators for p,q-QOFSs to incorporate the reliability of expert judgments into the decision-making process. Using these operators, we develop a robust multi-criteria group decision-making (MCGDM) model for sustainable supplier selection. The model is validated through a real-world case study involving three experts assessing four suppliers against eight sustainability criteria. Comparative analysis with existing methods highlights the superior performance of the proposed approach, while sensitivity analysis confirms its stability and robustness across varying parameter settings. The incorporation of confidence levels not only enhances the credibility of the aggregated evaluations but also allows for more informed and nuanced decision outcomes.

AMS Subject Classification 2020: 90B50; 03E72; 62A86
Keywords and Phrases: Multi-criteria group decision-making, p,q-QOFSs, Hamacher aggregation operators, Confidence levels, Supplier selection.

1 Introduction

Sustainable procurement is critical for universities striving to align their operational activities with environmental, social, and economic sustainability goals. Among various procurement decisions, sustainable furniture supplier selection plays a vital role in ensuring environmentally responsible, cost-effective, and socially ethical choices. Selecting the most appropriate supplier is naturally a multi-criteria decision-making (MCDM) challenge, as it requires assessing various alternatives against multiple sustainability factors that can be diverse and sometimes conflicting. These criteria are typically categorized into three key dimensionsenvironmental, economic, and socialeach contributing to the overall sustainability of the selected supplier. The challenge lies in balancing trade-offs among these factors, such as minimizing environmental impact while maintaining cost-effectiveness and ensuring ethical labor practices.

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Furthermore, the problem extends beyond MCDM into MCGDM since supplier selection is typically carried out by a panel of experts from different domains, such as procurement managers, sustainability officers, and financial analysts. Each expert brings a unique perspective, leading to variations in assessments and preferences. The presence of multiple decision-makers necessitates a structured approach to aggregate diverse viewpoints while considering the varying levels of confidence in their evaluations.

Another significant challenge in sustainable furniture supplier selection is the limitations of real-world data due to uncertainty, vagueness, and incompleteness. Many sustainability factors, such as environmental impact, corporate social responsibility, and fair labor practices, are difficult to quantify with precise numerical values. Additionally, expert evaluations are often subjective, involving linguistic terms such as “moderate durability or “high environmental compliance, which introduce ambiguity into the decision-making process. Moreover, real data may be inconsistent or insufficient, as suppliers may not always provide complete and transparent sustainability reports. To overcome these challenges, methods based on fuzzy sets [1] are essential, as they provide a more effective means of handling uncertainty and vague information.

The p, q-quasirung orthopair fuzzy set (p,q-ROFS) framework [2] is a significant advancement over earlier fuzzy set extensions such as intuitionistic fuzzy sets, Pythagorean fuzzy sets, and q-rung orthopair fuzzy sets. It enhances the ability to represent and process uncertainty, hesitation, and imprecise information in expert evaluations. Defined by two parameters p and q, the p,q-ROFS ensures that the sum of the p^{th} power of the membership degree (MD) added to the q^{th} power of the non-membership degree (NMD) does not exceed one. This parametric flexibility allows for a more nuanced modeling of vague and incomplete data, which is especially beneficial in complex decision-making environments such as sustainable procurement, where expert opinions are often subjective and expressed using linguistic terms. Due to these strengths, the p,q-ROFS framework has been increasingly adopted in various multi-criteria decision-making applications. Rahim et al. [3] developed an integrated decision-making model by combining COPRAS and CRITIC methods under p,q-quasirung orthopar fuzzy environment, then apply this model to green supplier selection. Rahim et al. [4] proposed Dombi aggregation operators (AOs) under the p,q-QOF framework and presented an MCGDM approach utilizing these operators. Later, Rahim et al. [5] proposed Bonferroni mean-based operators formulated within the p,q-ROF structure and applied them for selecting stocks for investment purposes. Ali and Naeem [6] formulated p,q-QOF-based AczelAlsina aggregation methods to address MCDM problems and used them to assess corruption levels. Arya and Pal [7] proposed a decision-making model that incorporates aggregation operators derived from Jaccard and cosine similarity concepts within the p,q-QOF context, applying this model to analyze public healthcare institutions in different Indian states.

Now, in MCGDM, effective information aggregation is crucial for synthesizing diverse opinions and criteria to arrive at a well-informed decision. Aggregation operators (AOs) serve as powerful tools to systematically combine individual preferences or evaluations into a collective assessment. Among various aggregation operators used in fuzzy decision-making, Hamacher AOs are particularly attractive due to their parametric flexibility, which allows them to generalize classical operations such as the algebraic product and sum. Unlike Dombi, Frank, or other Archimedean families that also offer flexible modeling, the Hamacher t-norm (HTN) and t-conorm (HTCN), introduced by Hamacher [8] in 1979, maintain a balanced trade-off between mathematical simplicity, interpretability, and adaptability to complex decision-making environments. Their behavior can be fine-tuned via parameters to model different interaction degrees among decision-makers evaluations. Although Dombi AOs [9, 10], Frank AOs [11], and Archimedean-based approaches [12, 13] have shown strong performance in different fuzzy environments, the application of Hamacher AOs within the p,q-QOFS frameworkespecially while incorporating confidence levelsremains largely unexplored. Numerous studies have incorporated these operators as fundamental tools, exploring their potential to tackle diverse problems and optimize decision-making in different fuzzy environments, for example, Hamacher AOs under intuitionistic fuzzy environment [14], Pythagorean fuzzy Hamacher AOs [15, 16], Fermatean fuzzy environment based Hamacher AOs [17, 18], Hamacher AOs under q-rung orthopair fuzzy environment [19, 20], circular

Pythagorean fuzzy enviornment based Hamacher AOs [21], p,q-QOF environment based Hamacher AOs [22], etc.

In MCGDM, expert judgments often vary in reliability due to differences in domain knowledge, experience, and familiarity with the evaluation criteria. Ignoring these variations can result in biased or suboptimal decisions. Therefore, integrating confidence levels into the aggregation process becomes essential to ensure that more credible and trustworthy evaluations have a greater influence on the final outcome, thereby enhancing the robustness and validity of the decision. Although Hamacher aggregation operators (AOs) under various fuzzy environments have proven effective, they generally overlook the variability in decision-makers confidence levels. These operators typically employ fixed mathematical formulations, which do not account for the credibility or trustworthiness of expert input. As a result, they may be less adaptable in uncertain or imprecise decision environments. Confidence level (CL)-based AOs address this limitation by assigning greater weight to evaluations with higher confidence, thus ensuring that the aggregated outcomes are both accurate and well-supported. Especially in MCGDM scenarios, where subjective assessments and differing levels of certainty are common, incorporating CLs significantly enhances the flexibility, reliability, and overall robustness of the decision-making process. Understanding these benefits, many researchers have created CL-based AOs in different fuzzy environments to make decision-making more accurate and reliable. Qiyaas et al. [23] introduced CL-based bipolar complex fuzzy AOs and discussed their use in decision-making problems. Rahaman and Muhammad [24] suggested improved decision-making by using CL-based complex polytopic fuzzy AOs. Punetha and Komal [25] presented CL-based picture fuzzy hybrid AOs and their role in MCGDM. Raza et al. [26] suggested a MCDM method that uses a complex T-spherical fuzzy Frank prioritized AOs. Chatterjee and Seikh [27] developed a CL-based decision-making method in a q-rung orthopair picture fuzzy setting and used it to assess municipal solid waste management. Mahmood et al. [28] introduced CL-based AOs in intuitionistic fuzzy rough sets for medical diagnosis applications. Lin et al. [29] worked on costprofit analysis with a spherical fuzzy set-based AOs using CL. Seikh and Chatterjee [30] presented CL-based AOs under interval-valued spherical fuzzy environments and also developed the MERCEC-VIKOR approach for selecting sustainable strategies for electric vehicle adoption. Seikh and Chatterjee [31] introduced CL-based AOs within the interval-valued Fermatean fuzzy framework. They also developed a combined SWARA-ARAS model to help identify the most suitable renewable energy sources in India.

From the above discussion, we identify the following research gaps:

- Existing fuzzy set-based approaches struggle to effectively capture hesitation and imprecise expert opinions in sustainable supplier selection. The recently developed p,q-QOFSs offer enhanced flexibility, yet their application in this domain remains underexplored.
- In MCGDM, most aggregation operators treat all expert opinions equally, disregarding variations in reliability. This can lead to biased decision-making, highlighting the need for CL-based aggregation under p,q-QOFSs.
- HTN and HTCN functions provide flexible information fusion, yet their integration with p,q-QOFSs has not been fully investigated. Exploring these operators can enhance decision-making accuracy in uncertain environments.

In light of these research gaps, the study is guided by the following key questions:

- How can uncertainty and subjectivity in sustainable procurement be modelled more accurately?
- How does the incorporation of confidence levels affect the aggregation process and final decision outcomes?
- How effective is the proposed model in comparison to existing aggregation methods?

These questions aim to direct the development of a robust MCGDM model that not only addresses ambiguity and incomplete information but also incorporates decision-makers confidence in a structured and meaningful way.

The motivation of the study is given as follows:

- Universities aim to adopt sustainable procurement strategies that align with environmental, social, and economic goals. Selecting a sustainable furniture supplier requires a robust model that effectively handles vague and imprecise data while ensuring fair and informed decisions. This study aims to develop an improved model to support universities in achieving these objectives.
- Expert opinions vary in reliability due to differences in knowledge and expertise. Integrating confidence levels into aggregation operators ensures that more credible evaluations carry greater weight, leading to more justified and trustworthy decisions.
- The HTN and HTCN functions provide parametric flexibility, making them ideal for dealing with uncertainty during decision-making. This study combines these functions with p,q-QOFSs to develop a more flexible aggregation method that can effectively bring together the opinions of experts.

The important contributions to this study are listed below.

- This study introduces Hamacher weighted averaging and geometric AOs under p,q-QOFSs. These operators incorporate confidence levels, ensuring expert reliability influences decision-making outcomes.
- A novel MCGDM model is developed to effectively integrate multiple criteria. It enhances decision accuracy by addressing uncertainty and varying confidence levels in expert evaluations.
- The proposed model is applied to select sustainable furniture suppliers where three decision-makers evaluate four suppliers based on eight criteria. This real-world application demonstrates its practicality and effectiveness in sustainable procurement.
- A comparison with existing methods shows that the proposed approach performs better. Sensitivity analysis confirms its robustness by assessing stability under varying parameter values.

The structure of this study is as follows: Section 2 explains the basic concepts. Section 3 describes the CL-based Hamacher AOs for p,q-QOFSs and their key properties. In Section 4, the MCGDM model is developed using these proposed operators within the p,q-QOFS framework. Section 5 presents a real-world numerical problem on sustainable furniture supplier selection, illustrating the practical applicability of the proposed method. This section also includes a comparative analysis and a sensitivity analysis of the parameters to validate the robustness and effectiveness of the approach. Finally, Section 6 provides the conclusion, along with the study's limitations and suggestions for future research.

2 Preliminaries

This section provides fundamental definitions and preliminary concepts. In the entire paper, the universal set is denoted by D.

Definition 2.1. [2] *A p,q-QOF set I over a universal set D is given by: $I = \{d, \langle \vartheta_I(d), \theta_I(d)\rangle | d \in D\}$, where $\vartheta_I : I \to [0,1], \theta_I : I \to [0,1]$ respectively represents the MD and NMD of an element $d \in D$ such that $(\vartheta_I(d))^p + (\theta_I(d))^q \leq 1$ for $p, q \geq 1$. For simplicity, the set I can be denoted as $I = (\vartheta_I, \theta_I)$, where each ordered pair is known as a p,q-quasirung orthopair fuzzy number (p,q-QOFN).*

Definition 2.2. [2] *The operational rules for the p,q-QOFNs* $I_1 = (\vartheta_{I_1}, \theta_{I_1}), I_2 = (\vartheta_{I_2}, \theta_{I_2}), I = (\vartheta_I, \theta_I)$ *and* $\eta > 0$ *are defined as follows:*

- $I^c = (\theta_I, \vartheta_I)$;
- $I_1 \bigcup I_2 = (max(\vartheta_{I_1}, \vartheta_{I_2}), min(\theta_{I_1}, \theta_{I_2}))$;
- $I_1 \bigcap I_2 = (min(\vartheta_{I_1}, \vartheta_{I_2}), max(\theta_{I_1}, \theta_{I_2}))$;
- $I_1 \bigoplus I_2 = (\sqrt[k]{\vartheta^p_{I_1} + \vartheta^p_{I_2} - \vartheta^p_{I_1}\vartheta^p_{I_2}}, \theta_{I_1}\theta_{I_2})$;
- $I_1 \bigotimes I_2 = (\vartheta_{I_1}\vartheta_{I_2}, \sqrt[k]{\theta^q_{I_1} + \theta^q_{I_2} - \theta^q_{I_1}\theta^q_{I_2}})$;
- $I^\eta = (\vartheta^\eta_I, \sqrt[q]{1 - (1 - \theta^q_I)^\eta})$;
- $\eta I = (\sqrt[k]{1 - (1 - \vartheta^p_I)^\eta}, \theta^\eta_I)$.

Definition 2.3. [2] *The score function* $(\mathcal{S})$ *and the accuracy function* $(\mathcal{A})$ *for the p,q-QOFN* $I = (\vartheta_I, \theta_I)$ *are defined as:*

$$\mathcal{S}(I) = \frac{1 + \vartheta^p_I - \theta^q_I}{2} \tag{1}$$

$$\mathcal{A}(I) = \vartheta^p_I + \theta^q_I \tag{2}$$

where, $0 \leq \mathcal{S}(I) \leq 1$ *and* $0 \leq \mathcal{A}(I) \leq 1$.

Let $I_1 = (\vartheta_{I_1}, \theta_{I_1})$ *and* $I_2 = (\vartheta_{I_2}, \theta_{I_2})$ *be any two p,q- QOFNs, then*

- $I_1 \succ I_2$ *if* $\mathcal{S}(I_1) > \mathcal{S}(I_2)$;
- $I_2 \succ I_1$ *if* $\mathcal{S}(I_1) < \mathcal{S}(I_2)$;
- *If* $\mathcal{S}(I_1) = \mathcal{S}(I_2)$, *then*
  - $I_1 \succ I_2$ *if* $\mathcal{A}(I_1) > \mathcal{A}(I_2)$;
  - $I_2 \succ I_1$ *if* $\mathcal{A}(I_1) < \mathcal{A}(I_2)$;
  - $I_1 \sim I_2$ *if* $\mathcal{A}(I_1) = \mathcal{A}(I_2)$.

Definition 2.4. [8] *For two real numbers a and b, the HTN* (H) *and HTCN* (H^*) *are defined as follows:*

$$H(a,b) = \frac{ab}{\lambda + (1-\lambda)(a + b - ab)} \qquad H^*(a,b) = \frac{a + b - ab - (1-\lambda)ab}{1 - (1-\lambda)ab}$$

Definition 2.5. [22] *The algebraic operational rules using HTN and HTCN for the p,q-QOFNs* $I_1 = (\vartheta_{I_1}, \theta_{I_1}), I_2 = (\vartheta_{I_2}, \theta_{I_2}), I = (\vartheta_I, \theta_I)$, $\lambda > 0$ *and* $\eta > 0$ *are defined by:*

- $I_1 \bigoplus I_2 = \left(\sqrt[p]{\frac{\vartheta^p_{I_1} + \vartheta^p_{I_2} - \vartheta^p_{I_1}\vartheta^p_{I_2} - (1-\lambda)\vartheta^p_{I_1}\vartheta^p_{I_2}}{1 - (1-\lambda)\vartheta^p_{I_1}\vartheta^p_{I_2}}}, \frac{\theta_{I_1}\theta_{I_2}}{\sqrt[q]{1 - (1-\lambda)(\theta^q_{I_1} + \theta^q_{I_2} - \theta^q_{I_1}\theta^q_{I_2})}} \right)$;
- $I_1 \bigotimes I_2 = \left(\frac{\vartheta_{I_1}\vartheta_{I_2}}{\sqrt[p]{1 - (1-\lambda)(\vartheta^p_{I_1} + \vartheta^p_{I_2} - \vartheta^p_{I_1}\vartheta^p_{I_2})}}, \sqrt[q]{\frac{\theta^q_{I_1} + \theta^q_{I_2} - \theta^q_{I_1}\theta^q_{I_2} - (1-\lambda)\theta^q_{I_1}\theta^q_{I_2}}{1 - (1-\lambda)\theta^q_{I_1}\theta^q_{I_2}}} \right)$;

- $\eta I_1 = \left(\sqrt[p]{\frac{(1+(\lambda-1)\vartheta_{I_1}^p)^\eta-(1-\vartheta_{I_1}^p)^\eta}{(1+(\lambda-1)\vartheta_{I_1}^p)^\eta+(\lambda-1)(1-\vartheta_{I_1}^p)^\eta}}, \frac{\sqrt[q]{\lambda}\theta_{I_1}^\eta}{\sqrt[q]{(1+(\lambda-1)(1-\theta_{I_1}^q))^\eta+(\lambda-1)(\theta_{I_1}^q)^\eta}} \right);$
- $I_1^\eta = \left(\frac{\sqrt[p]{\lambda}\vartheta_{I_1}^\eta}{\sqrt[p]{(1+(\lambda-1)(1-\vartheta_{I_1}^p))^\eta+(\lambda-1)(\vartheta_{I_1}^p)^\eta}}, \sqrt[q]{\frac{(1+(\lambda-1)\theta_{I_1}^q)^\eta-(1-\theta_{I_1}^q)^\eta}{(1+(\lambda-1)\theta_{I_1}^q)^\eta+(\lambda-1)(1-\theta_{I_1}^q)^\eta}} \right).$

Definition 2.6. [22] *Assuming $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2, \ldots, n)$ be a collection of p,q-QOFNs, then the p,q-QOF Hamacher weighted averaging aggregation (p,q-QOFHWA) and p,q-QOF Hamacher weighted geometric aggregation (p,q-QOFHWG) operators are defined as the mappings p,q-QOFHWA:$I^n \to I$ and p,q-QOFHWG:$I^n \to I$ and characterized by:*

$$p,q-QOFHWA(I_1, I_2, \ldots, I_n) = \bigoplus_{i=1}^n (w_i I_i) = \left(\sqrt[p]{\frac{\prod_{i=1}^n (1+(\lambda-1)\vartheta_{I_i}^p)^{w_i} - \prod_{i=1}^n (1-\vartheta_{I_i}^p)^{w_i}}{\prod_{i=1}^n (1+(\lambda-1)\vartheta_{I_i}^p)^{w_i} + (\lambda-1)\prod_{i=1}^n (1-\vartheta_{I_i}^p)^{w_i}}}, \frac{\sqrt[q]{\lambda}\prod_{i=1}^n \theta_{I_i}^{w_i}}{\sqrt[q]{\prod_{i=1}^n (1+(\lambda-1)(1-\theta_{I_i}^q))^{w_i} + (\lambda-1)\prod_{i=1}^n (\theta_{I_i}^q)^{w_i}}} \right)$$

and

$$p,q-QOFHWG(I_1, I_2, \ldots, I_n) = \bigotimes_{i=1}^n (w_i I_i) = \left(\frac{\sqrt[p]{\lambda}\prod_{i=1}^n \vartheta_{I_i}^{w_i}}{\sqrt[p]{\prod_{i=1}^n (1+(\lambda-1)(1-\vartheta_{I_i}^p))^{w_i} + (\lambda-1)\prod_{i=1}^n (\vartheta_{I_i}^p)^{w_i}}}, \sqrt[q]{\frac{\prod_{i=1}^n (1+(\lambda-1)\theta_{I_i}^q)^{w_i} - \prod_{i=1}^n (1-\theta_{I_i}^q)^{w_i}}{\prod_{i=1}^n (1+(\lambda-1)\theta_{I_i}^q)^{w_i} + (\lambda-1)\prod_{i=1}^n (1-\theta_{I_i}^q)^{w_i}}} \right).$$

Here, $w = (w_1, w_2, \ldots, w_n)^T$ signifies the weight vector of I_i satisfying the conditions $w_i > 0$ and the constrain $\sum_{i=1}^n w_i = 1$.

Definition 2.7. [32] *The aggregated result of the p,q-QOFNs $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2, \ldots, n)$ using the p,q-QOF weighted averaging (p,q-QOFWA) operator is given as follws:*

$$p,q-QOFWA(I_1, I_2, \ldots, I_n) = \left(\sqrt[p]{1 - \prod_{k=1}^n (1 - \vartheta_{I_k}^p)^{\alpha_k}}, \prod_{k=1}^n (\theta_{I_k})^{\alpha_k} \right). \tag{3}$$

Here, α_k be the weight of I_k satisfying the conditions $\alpha_k > 0$ and $\sum_{k=1}^n \alpha_k = 1$.

3 Confidence Level-Based Hamacher AOs for p,q-QOFSs

In this section, we develop the Hamacher AOs within the p,q-QOF environment, incorporating the confidence level.

Definition 3.1. *The aggregated value of the p,q-QOFNs $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2 \ldots, n)$ using the confidence level p,q-quasirung orthopair fuzzy Hamacher weighted averaging (Cp,q-QOFHWA) operator is given by*

$$Cp,q-QOFHWA(I_1, I_2, \ldots, I_n) = \bigoplus_{i=1}^n (w_i l_i) I_i$$

where $l_i (0 \leq l_i \leq 1)$ be the confidence level I_i, and w_i's are given in Definition 2.6.

Definition 3.2. *The aggregated value of the p,q-QOFNs $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2 \ldots, n)$ using the confidence level p,q-quasirung orthopair fuzzy Hamacher weighted geometric (Cp,q-QOFHWG) operator operator is given by*

$$Cp,q-QOFHWG(I_1, I_2, \ldots, I_n) = \bigotimes_{i=1}^n I_i^{(w_i l_i)}$$

where l_i, w_i are described in Definition 3.1.

Theorem 3.3. *If $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2, \ldots, n)$ be a collection of p,q-QOFNs, where each I_i has a CL l_i. Then, the combined value of these p,q-QOFNs using the Cp,q-QOFHWA operator can be expressed as:*

$$Cp,q-QOFHWA(I_1, I_2, \ldots, I_n) = \left(\sqrt[p]{\frac{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} - \prod_{i=1}^{n}(1-\vartheta_{I_i}^{p})^{l_i w_i}}{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(1-\vartheta_{I_i}^{p})^{l_i w_i}}}, \frac{\sqrt[q]{\lambda}\prod_{i=1}^{n}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{n}(1+(\lambda-1)(1-\theta_{I_i}^{q}))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(\theta_{I_i}^{q})^{l_i w_i}}} \right).$$

Proof. This result can be simply shown by using mathematical induction.
The theorem is true for the case $n = 1$.
Now, let us assume that the theorem is also true for $n = k$, where k is any positive integer. Based on this assumption, we proceed as follows:

$$Cp,q-QOFHWA(I_1, I_2, \ldots, I_k) =$$
$$\left(\sqrt[p]{\frac{\prod_{i=1}^{k}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} - \prod_{i=1}^{k}(1-\vartheta_{I_i}^{p})^{l_i w_i}}{\prod_{i=1}^{k}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k}(1-\vartheta_{I_i}^{p})^{l_i w_i}}}, \frac{\sqrt[q]{\lambda}\prod_{i=1}^{k}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{k}(1+(\lambda-1)(1-\theta_{I_i}^{q}))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k}(\theta_{I_i}^{q})^{l_i w_i}}} \right).$$

Now, for $n = k + 1$,

$$\begin{aligned}
Cp,q\text{-}QOFHWA(I_1, I_2, \ldots, I_k, I_{k+1}) &= \bigoplus_{i=1}^{k+1}(w_i l_i)I_i \\
&= \bigoplus_{i=1}^{k}(w_i l_i)I_i \bigoplus (w_{k+1}l_{k+1})I_{k+1} \\
&= \left(\sqrt[p]{\frac{\prod_{i=1}^{k}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} - \prod_{i=1}^{k}(1-\vartheta_{I_i}^{p})^{l_i w_i}}{\prod_{i=1}^{k}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k}(1-\vartheta_{I_i}^{p})^{l_i w_i}}}, \right. \\
&\qquad \left. \frac{\sqrt[q]{\lambda}\prod_{i=1}^{k}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{k}(1+(\lambda-1)(1-\theta_{I_i}^{q}))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k}(\theta_{I_i}^{q})^{l_i w_i}}} \right) \\
&\bigoplus \left(\sqrt[p]{\frac{(1+(\lambda-1)\vartheta_{I_{k+1}}^{p})^{l_{k+1}w_{k+1}} - (1-\vartheta_{I_{k+1}}^{p})^{l_{k+1}w_{k+1}}}{(1+(\lambda-1)\vartheta_{I_{k+1}}^{p})^{l_{k+1}w_{k+1}} + (\lambda-1)(1-\vartheta_{I_{k+1}}^{p})^{l_{k+1}w_{k+1}}}}, \right. \\
&\qquad \left. \frac{\sqrt[q]{\lambda}\theta_{I_{k+1}}^{l_{k+1}w_{k+1}}}{\sqrt[q]{(1+(\lambda-1)(1-\theta_{I_{k+1}}^{q}))^{l_{k+1}w_{k+1}} + (\lambda-1)(\theta_{I_{k+1}}^{q})^{l_{k+1}w_{k+1}}}} \right) \\
&= \left(\sqrt[p]{\frac{\prod_{i=1}^{k+1}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} - \prod_{i=1}^{k+1}(1-\vartheta_{I_i}^{p})^{l_i w_i}}{\prod_{i=1}^{k+1}(1+(\lambda-1)\vartheta_{I_i}^{p})^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k+1}(1-\vartheta_{I_i}^{p})^{l_i w_i}}}, \right. \\
&\qquad \left. \frac{\sqrt[q]{\lambda}\prod_{i=1}^{k+1}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{k+1}(1+(\lambda-1)(1-\theta_{I_i}^{q}))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{k+1}(\theta_{I_i}^{q})^{l_i w_i}}} \right).
\end{aligned}$$

Thus, the theorem is valid for any natural numbers. □

Theorem 3.4. *If $I_i = (\vartheta_{I_i}, \theta_{I_i})(i = 1, 2, \ldots, n)$ be a collection of p,q-QOFNs, where each I_i has a CL l_i. Then, the combined value of these p,q-QOFNs using the Cp,q-QOFHWG operator can be expressed as:*

$$Cp,q-QOFHWG(I_1, I_2, \ldots, I_n) = \left(\frac{\sqrt[p]{\lambda}\prod_{i=1}^{n}\vartheta_{I_i}^{l_i w_i}}{\sqrt[p]{\prod_{i=1}^{n}(1+(\lambda-1)(1-\vartheta_{I_i}^{p}))^{l_i w_i}+(\lambda-1)\prod_{i=1}^{n}(\vartheta_{I_i}^{p})^{l_i w_i}}}, \sqrt[q]{\frac{\prod_{i=1}^{n}(1+(\lambda-1)\theta_{I_i}^{q})^{l_i w_i}-\prod_{i=1}^{n}(1-\theta_{I_i}^{q})^{l_i w_i}}{\prod_{i=1}^{n}(1+(\lambda-1)\theta_{I_i}^{q})^{l_i w_i}+(\lambda-1)\prod_{i=1}^{n}(1-\theta_{I_i}^{q})^{l_i w_i}}} \right).$$

Proof. The proof is carried out using the same method as used in the proof of Theorem 3.3. □

Next, we will explain some important and useful properties of the proposed Cp,q-QOFHWA and Cp,q-QOFHWG operators in simple terms.

Property 1. *If all the p,q-QOFNs $I_i = (\vartheta_{I_i}, \theta_{I_i}), (i = 1, 2, \ldots, n)$ are equal to $I = (\vartheta_I, \theta_I)$ and $l_1 = l_2 = \ldots = l_n = l$ then $Cp,q-QOFHWA(I_1, I_2, \ldots, I_n) = lI$ and $Cp,q-QOFHWG(I_1, I_2, \ldots, I_n) = I^l$.*

Proof. Let each $I_i = I = (\vartheta_I, \theta_I)$ for all i, and $l_i = l$, w_i be weights such that $\sum_{i=1}^{n} w_i = 1$.

Therefore, we have

$$Cp,q\text{-}QOFHWA(I_1, I_2 \ldots, I_n) = \left(\sqrt[p]{\frac{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^{p})^{lw_i}-\prod_{i=1}^{n}(1-\vartheta_{I_i}^{p})^{lw_i}}{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^{p})^{lw_i}+(\lambda-1)\prod_{i=1}^{n}(1-\vartheta_{I_i}^{p})^{lw_i}}}, \frac{\sqrt[q]{\lambda}\prod_{i=1}^{n}\theta_{I_i}^{lw_i}}{\sqrt[q]{\prod_{i=1}^{n}(1+(\lambda-1)(1-\theta_{I_i}^{q}))^{lw_i}+(\lambda-1)\prod_{i=1}^{n}(\theta_{I_i}^{q})^{lw_i}}} \right).[\because l_i = l, \forall i]$$

Since $\vartheta_{I_i} = \vartheta_I$ for all i, define: $A = 1 + (\lambda-1)\vartheta_I^p, \quad B = 1 - \vartheta_I^p$.

Then:

$$\prod_{i=1}^{n} A^{lw_i} = A^{l\sum w_i} = A^l, \quad \prod_{i=1}^{n} B^{lw_i} = B^l.$$

So the membership component becomes:

$$\sqrt[p]{\frac{A^l - B^l}{A^l + (\lambda-1)B^l}} = \sqrt[p]{\frac{(1+(\lambda-1)\vartheta_I^p)^l - (1-\vartheta_I^p)^l}{(1+(\lambda-1)\vartheta_I^p)^l + (\lambda-1)(1-\vartheta_I^p)^l}}.$$

Again, since $\theta_{I_i} = \theta_I$ for all i, define:

$$C = \theta_I^q, \quad D = 1 + (\lambda-1)(1-C) = 1 + (\lambda-1)(1-\theta_I^q).$$

Then:

$$\prod_{i=1}^{n}\theta_{I_i}^{lw_i} = \theta_I^l, \quad \prod_{i=1}^{n} D^{lw_i} = D^l, \quad \prod_{i=1}^{n} C^{lw_i} = C^l.$$

Hence, the non-membership component becomes:

$$\frac{\sqrt[q]{\lambda}\cdot\theta_I^l}{\sqrt[q]{1+(\lambda-1)(1-\theta_I^q)^l+(\lambda-1)(\theta_I^q)^l}}.$$

Finally, from Definition 2.5, we obtain $Cp,q-QOFHWA(I_1, I_2, \ldots, I_n) = lI$.

A similar approach can be used to prove the remaining part. □

Property 2. *Consider a set of p,q-QOFNs, represented as:* $\{I_i = (\vartheta_{I_i}, \theta_{I_i}) : (i = 1, 2, \ldots, n)\}$. *Let us define:* $I^- = \min_i\{I_i\}$ *and* $I^+ = \max_i\{I_i\}$. *Then,*

$$l^- I^- \leq Cp,q - QOFHWA(I_1, I_2, \ldots, I_n) \leq l^+ I^+$$

$$l^- I^- \leq Cp,q - QOFHWG(I_1, I_2, \ldots, I_n) \leq l^+ I^+$$

where $min\{I_i\} = (min(\vartheta_{I_i}), max(\theta_{I_i}))$, $max\{I_i\} = (max(\vartheta_{I_i}), min(\theta_{I_i})), \forall i$. *Also* l^- *and* l^+ *are the confidence levels of* I^- *and* I^+ *respectively.*

Proof. Let $Cp,q - QOFHWA(I_1, I_2, \ldots, I_n) = (\mu^*, \nu^*)$;

$$\mu^* = \sqrt[p]{\frac{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^p)^{l_i w_i} - \prod_{i=1}^{n}(1-\vartheta_{I_i}^p)^{l_i w_i}}{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^p)^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(1-\vartheta_{I_i}^p)^{l_i w_i}}},$$

$$\nu^* = \frac{\sqrt[q]{\lambda}\prod_{i=1}^{n}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{n}(1+(\lambda-1)(1-\theta_{I_i}^q))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(\theta_{I_i}^q)^{l_i w_i}}}.$$

Let us denote: $a_i = \vartheta_{I_i}^p, \quad b_i = \theta_{I_i}^q$. Since $\vartheta_{I_i} \in [0,1]$, we have $a_i \in [0,1]$. The numerator and denominator of μ^* are monotonic in a_i, and similarly, the components of ν^* are monotonic in b_i.

(i) Bounds on μ^**:*

- **Lower Bound:** Set $a_i = \min_j \vartheta_{I_j}^p =: a^-$ for all i. Then:

$$\mu^* \geq \sqrt[p]{\frac{(1+(\lambda-1)a^-)^{\sum l_i w_i} - (1-a^-)^{\sum l_i w_i}}{(1+(\lambda-1)a^-)^{\sum l_i w_i} + (\lambda-1)(1-a^-)^{\sum l_i w_i}}} \geq l^- \cdot \min_i \vartheta_{I_i}$$

- **Upper Bound:** Set $a_i = \max_j \vartheta_{I_j}^p =: a^+$. Then:

$$\mu^* \leq \sqrt[p]{\frac{(1+(\lambda-1)a^+)^{\sum l_i w_i} - (1-a^+)^{\sum l_i w_i}}{(1+(\lambda-1)a^+)^{\sum l_i w_i} + (\lambda-1)(1-a^+)^{\sum l_i w_i}}} \leq l^+ \cdot \max_i \vartheta_{I_i}$$

(ii) Bounds on ν^**:*

- **Upper Bound:** Since non-membership decreases preference, ν^* is maximized when $\theta_{I_i} = \max_j \theta_{I_j} =: \theta^-$. Then:

$$\nu^* \leq \frac{\sqrt[q]{\lambda}(\theta^-)^{\sum l_i w_i}}{\sqrt[q]{(1+(\lambda-1)(1-(\theta^-)^q))^{\sum l_i w_i} + (\lambda-1)(\theta^-)^{q\sum l_i w_i}}} \leq l^- \cdot \max_i \theta_{I_i}$$

- **Lower Bound:** When $\theta_{I_i} = \min_j \theta_{I_j} =: \theta^+$, we get:

$$\nu^* \geq \frac{\sqrt[q]{\lambda}(\theta^+)^{\sum l_i w_i}}{\sqrt[q]{(1+(\lambda-1)(1-(\theta^+)^q))^{\sum l_i w_i} + (\lambda-1)(\theta^+)^{q\sum l_i w_i}}} \geq l^+ \cdot \min_i \theta_{I_i}$$

Hence, $l^- I^- \leq (\mu^*, \nu^*) \leq l^+ I^+$.
The other part of the proof follows similarly. □

Property 3. *Let $\{I_i = (\vartheta_{I_i}, \theta_{I_i}) | i = 1(1)n\}$ and $\{I_i^* = (\vartheta_{I_i}^*, \theta_{I_i}^*) | i = 1(1)n\}$ be the sets of p,q-QOFNs. If for each i, we have $I_i \preceq I_i^*$ i.e. if $\vartheta_{I_i} \le \vartheta_{I_i}^*$ and $\theta_{I_i} \ge \theta_{I_i}^*$, then the following holds:*

$$Cp,q\text{-}QOFHWA(I_1, I_2, \ldots, I_n) \le C_{p,q}\text{-}QOFHWA(I_1^*, I_2^*, \ldots, I_n^*),$$
$$Cp,q\text{-}QOFHWG(I_1, I_2, \ldots, I_n) \le C_{p,q}\text{-}QOFHWG(I_1^*, I_2^*, \ldots, I_n^*).$$

Proof. Let us denote: Cp,q-QOFHWA$(I_1, I_2, \ldots, I_n) = (\Theta, \Delta)$, where

$$\Theta = \sqrt[p]{\frac{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^p)^{l_i w_i} - \prod_{i=1}^{n}(1-\vartheta_{I_i}^p)^{l_i w_i}}{\prod_{i=1}^{n}(1+(\lambda-1)\vartheta_{I_i}^p)^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(1-\vartheta_{I_i}^p)^{l_i w_i}}},$$

$$\Delta = \frac{\sqrt[q]{\lambda}\prod_{i=1}^{n}\theta_{I_i}^{l_i w_i}}{\sqrt[q]{\prod_{i=1}^{n}(1+(\lambda-1)(1-\theta_{I_i}^q))^{l_i w_i} + (\lambda-1)\prod_{i=1}^{n}(\theta_{I_i}^q)^{l_i w_i}}}.$$

Similarly, define the aggregated value based on the p,q-QOFNs $(\vartheta_{I_i}^*, \theta_{I_i}^*)$ as

$$Cp,q\text{-QOFHWA}(I_1^*, I_2^*, \ldots, I_n^*) = (\Theta^*, \Delta^*).$$

Here,

- The function $(1 + (\lambda - 1)\vartheta_{I_i}^p)$ increases with ϑ_{I_i}.
- The function $(1 - \vartheta_{I_i}^p)$ decreases with ϑ_{I_i}.
- Therefore, the numerator increases with increasing ϑ_{I_i}.
- The denominator also increases, but the rate is slower due to the $(\lambda - 1)$ term.

Hence, Θ increases with ϑ_{I_i} and since $\vartheta_{I_i} \le \vartheta_{I_i}^*$, it follows that $\Theta \le \Theta^*$.
Again,

- The numerator $\prod \theta_{I_i}^{l_i w_i}$ increases with θ_{I_i}.
- In the denominator:
  - $(1 + (\lambda - 1)(1 - \theta_{I_i}^q))$ increases as θ_{I_i} decreases.
  - $(\theta_{I_i}^q)$ decreases as θ_{I_i} decreases.
- Hence, the denominator increases when θ_{I_i} decreases.

Therefore, Δ increases with θ_{I_i}, and since $\theta_{I_i} \ge \theta_{I_i}^*$, we conclude $\Delta \le \Delta^*$.
Since both components satisfy $\Theta \le \Theta^*, \quad \Delta \le \Delta^*$, we obtain:

$$Cp,q\text{-QOFHWA}(I_1, \ldots, I_n) \le Cp,q\text{-QOFHWA}(I_1^*, I_2^*, \ldots, I_n^*).$$

The proof of the remaining part proceeds in the same way. □

4 A MCGDM Process using p,q-Quasi Rung Hamacher Aggregation Operators with Confidence Level

Here, we construct an MCGDM framework utilizing the proposed Cp,q-QOFHWA and Cp,q-QOFHWG AOs.

Consider the set $\{K_1, K_2, \ldots, K_m\}$ representing m alternatives and the set $\{R_1, R_2, \ldots, R_n\}$ representing n criteria. The weights corresponding to these criteria are denoted as $\alpha_1, \alpha_2, \ldots, \alpha_n$, which fulfill the conditions $\sum_{b=1}^{n} \alpha_b = 1$ and $0 \leq \alpha_b \leq 1$ for each n. Let us consider a group of l experts, $E_1, E_2, \ldots, E_l$, who evaluate a set of alternatives based on multiple criteria. Using the p,q-QOFN, each expert assesses each alternative and provides their evaluations in the form of p,q-QOFNs as, $\gamma^c_{ab} = (\vartheta^c_{I_{ab}}, \theta^c_{I_{ab}})$, where $a = 1, 2, \ldots, m$; $b = 1, 2, \ldots, n$; $c = 1, 2, \ldots, t$, and $(\vartheta^c_{I_{ab}})^p + (\theta^c_{I_{ab}})^q \leq 1$. The experts also specify the extent of their familiarity with the given alternatives by assigning confidence levels l^c_{ab}, where $0 \leq l^c_{ab} \leq 1$, thereby incorporating the notion of confidence into the evaluation process. Also assume that experts weights are $\beta_1, \beta_2, \ldots, \beta_l$, where $0 \leq \beta_c \leq 1$ and $\sum_{c=1}^{l} \beta_c = 1$.

The proposed MCGDM framework, which uses Cp,q-QOFHWA and Cp,q-QOFHWG aggregation operators, follows these steps.

Step 1. Identify the problem, select the experts along with their confidence levels, and define the alternatives and criteria associated with the given problem.

Step 2. The weight of each decision expert is computed using Equation (4). Let β_c denote the weight of the c^{th} expert, then

$$\beta_c = \frac{y_c}{\sum_{c=1}^{t} y_c}, \tag{4}$$

where y_c represents the number of years of experience of the c^{th} expert.

Step 3. Rank the criteria based on their importance and use the RS method following Equation (5) to evaluate the weights of criteria.

$$\alpha_b = \frac{n - rank + 1}{\sum_{b=1}^{n} (n - rank + 1)}, \ b = 1(1)n \tag{5}$$

where n denotes the number of criteria.

Step 4. Gather the decision matrices expressed in linguistic variables (LVs) together with their level of confidence. Then, Transform the LVs assigned by the decision experts (DEs) into p,q-QOFNs using the predefined linguistic scale presented in Table 1. This scale is constructed based on the semantics and relative strength of each linguistic term, where each LV is mapped to a p,q-QOFN of the form (μ, ν), where μ and ν denote the membership and non-membership degrees respectively, satisfying the condition $\mu^p + \nu^q \leq 1$ for the chosen values of p and q. This results in the p,q-QOF decision matrix with their level of confidence $\widetilde{M}_r = \langle \tilde{\gamma}^c_{ab}, l^c_{ab} \rangle_{m \times n} = \langle (\tilde{\vartheta}^c_{I_{ab}}, \tilde{\theta}^c_{I_{ab}}), l^c_{ab} \rangle_{m \times n}$ for $c = 1, 2, \ldots, t$, where $a = 1, 2, \ldots, m$ and $b = 1, 2, \ldots, n$.

Step 5. Normalize the p,q-QOF decision matrix by using Equation (6). Let

$$M_r = \langle \gamma^c_{ab}, l^c_{ab} \rangle_{m \times n} = \langle (\vartheta^c_{I_{ab}}, \theta^c_{I_{ab}}), l^c_{ab} \rangle_{m \times n}$$

where

$$\gamma^c_{ab} = \begin{cases} (\tilde{\vartheta}^c_{I_{ab}}, \tilde{\theta}^c_{I_{ab}}), & \text{if } R_b \text{ is benifit attribute;} \\ (\tilde{\theta}^c_{I_{ab}}, \tilde{\vartheta}^c_{I_{ab}}), & \text{if } R_b \text{ is cost attribute.} \end{cases} \tag{6}$$

Table 1: DEs' confidence level for each attribute

LVs	p,q-QOFNs
Very bad (VB)	(0.20, 0.98)
Bad (B)	(0.30,0.80)
Medium (M)	(0.45,0.55)
Good (G)	(0.81,0.42)
Very good (VG)	(0.98,0.15)

Step 6. Determine the aggregated decision matrix by applying the CLs and weights of the decision experts, utilizing the Cp,q-QOFHWA (or Cp,q-QOFHWG) AO given in Definition 3.1. Consider the aggregated decision matrix $\overline{M} = (\overline{\gamma}_{cd})_{m\times n}$, where each element is given by $\overline{\gamma}_{ab} = (\vartheta_{I_{ab}}, \theta_{I_{ab}})$ and calculated using Equation (7).

$$\overline{\gamma}_{ab} = (\vartheta_{I_{ab}}, \theta_{I_{ab}}) = \left(\sqrt[p]{\frac{\prod_{c=1}^{t}(1+(\lambda-1)\vartheta^p_{I_{ab}})^{l^c_{ab}\beta_c} - \prod_{c=1}^{t}(1-\vartheta^p_{I_{ab}})^{l^c_{ab}\beta_c}}{\prod_{c=1}^{t}(1+(\lambda-1)\vartheta^p_{I_{ab}})^{l^c_{ab}\beta_c} + (\lambda-1)\prod_{i=1}^{n}(1-\vartheta^p_{I_{ab}})^{l^c_{ab}\beta_c}}}, \frac{\sqrt[q]{\lambda}\prod_{c=1}^{t}\theta_{I_{ab}}^{l^c_{ab}\beta_c}}{\sqrt[q]{\prod_{c=1}^{t}(1+(\lambda-1)(1-\theta^q_{I_{ab}}))^{l^c_{ab}\beta_c} + (\lambda-1)\prod_{c=1}^{t}(\theta^q_{I_{ab}})^{l^c_{ab}\beta_c}}} \right) \tag{7}$$

Step 7. Compute the combined value for each alternative by using the p,q-QOFWA operator as defined in Definition 2.7. Then, aggregate the overall value of each alternative, denoted as ρ_c, by Equation (8).

$$\rho_a = \left(\sqrt[p]{1 - \prod_{b=1}^{n}(1-\vartheta^p_{I_{ab}})^{\alpha_b}}, \prod_{b=1}^{n}(\theta_{I_{ab}})^{\alpha_b} \right). \tag{8}$$

Step 8. Determine the score value $\Phi(\rho_a)(a = 1(1)m)$ utilizing Equation (1) for each combined value ρ_c.

Step 9. If a particular alternative achieves the highest score among all options, it is considered the most preferred choice. Thus, alternative K_a deemed the best if $\Phi(\rho_a) = \max_{1\leq s\leq m}\{\Phi(\rho_s)\}$. However, if no unique alternative holds the highest score, proceed to the next step.

Step 10. In cases where multiple alternatives attain the same score, Equation (2) should be used to calculate the accuracy function Ψ for these alternatives. Then:

- If several alternatives share the highest score, the final selection is based on the one with the maximum $\Psi-$functional value.
- If multiple alternatives also have identical functional values, any of them can be regarded as an acceptable choice.

Figure 1 illustrates the step of the proposed MCGDM approach.

5 Sustainable Furniture Supplier Selection for a University: a MCGDM Problem

Sustainable supplier selection plays a vital role in the procurement process of universities, particularly when sourcing furniture, as it significantly influences environmental stewardship, economic prudence, and social

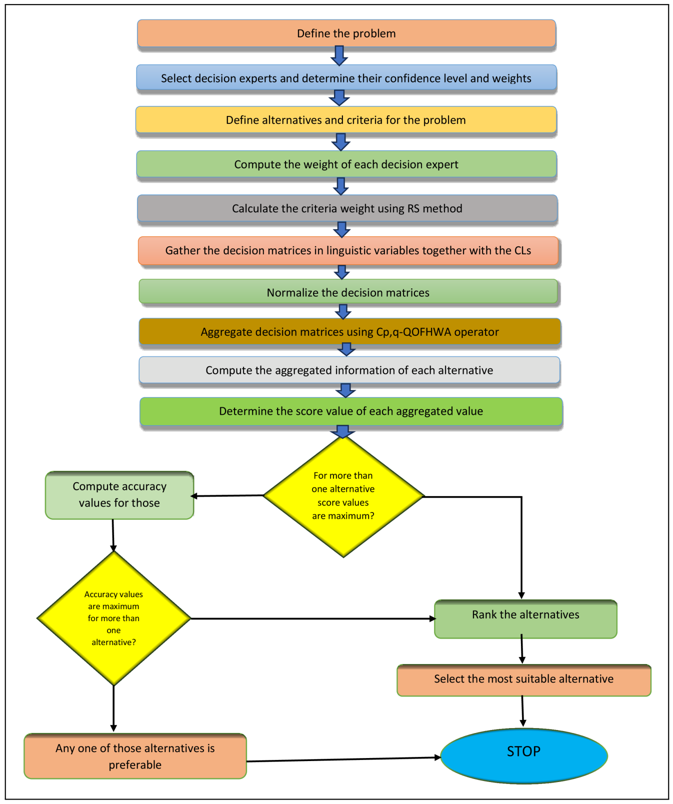

Figure 1: Framework of the proposed model

responsibility within the academic community. Universities are increasingly expected to look beyond traditional parameters such as cost and durability, and to integrate sustainability considerations into their supplier evaluation and selection processes. This approach supports long-term institutional goals, strengthens the university's commitment to sustainability, and aligns with global environmental and social development objectives.

To make well-informed procurement decisions, universities must assess furniture suppliers across a range of sustainability criteria. These typically encompass environmental impact, economic feasibility, and social responsibilityeach essential for ensuring a sustainable campus infrastructure. However, selecting the most suitable supplier is a complex task, given the presence of conflicting priorities, data uncertainties, and subjective expert assessments. To address these challenges, researchers and decision-makers have proposed various sophisticated MCDM methods to support transparent and balanced supplier selection.

5.1 Review of Decision-Making Models for Supplier Selection

As supplier selection becomes increasingly complex, researchers have introduced a variety of MCDM frameworks that incorporate fuzzy logic and hybrid techniques. These approaches are designed to manage uncertainty in data and support balanced evaluations across environmental, economic, and social sustainability aspects.

For instance, Tronnebati et al. [33] developed a comprehensive decision-making framework that integrates AHP, TOPSIS, and WASPAS techniques within a fuzzy environment. They effectively utilized this approach to address the problem of selecting green suppliers in Moroccos automotive sector. Kara et al. [34] developed a MCDM model based on Dempster-Shafer theory to select green supplier in the automotive industry.

Pinar [35] utilized q-rung orthopair fuzzy MCDM model for supplier selection in E-commerce. Banerjee et al. [36] explored the impact of social media on supplier selection in online B2B markets using a MCDM approach. Wu and Gu [37] developed an integrated decision making framework by combining Generalized Autoregressive Conditional Heteroscedasticity model with the fuzzy TOPSIS model for hydrogen energy supplier selection. Magableh [38] developed an integrated MCDM model by integrating fuzzy entropy method, fuzzy multi-objective optimization based on ratio analysis (MOORA), fuzzy complex proportional assessment (COPRAS) methods and applied this model in selecting rice supplier selection strategies. Kamran [39] proposed an integrated MCDM model by combining AHP and TOPSIS, which was utilized for evaluating and selecting suppliers in the oil and gas industry. Abdulla and Baryannis [40] proposed a hybrid supplier selection framework that combines interpretable data-driven AI techniques with MCDM approaches and then validated the framework through two real-world case studies supporting supplier selection decisions in oil, gas, and aerospace manufacturing companies. Baki et al. [41] enhanced the VIKOR method by incorporating interval-valued intuitionistic fuzzy sets and demonstrated its effectiveness through an application to supplier selection within digital supply chains of e-commerce platforms. Wang et al. [42] proposed an integrated MCDM framework by combining CRITIC and MARCOS methods within the Pythagorean fuzzy environment to identify the most suitable sustainable food supplier, aiming to support the development of resilient cities. Pamucar et al. [43] proposed a MCGDM approach based on the Preference Selection Index and Combined Compromise Solution methods under the Fermatean Fuzzy environment for green supplier selection is proposed.

Although these existing models have contributed greatly to improving supplier evaluation techniques, many of them fall short in considering the confidence levels associated with expert opinionsan important factor in assessing the reliability of those assessments. In real-world decision-making, it is essential to employ not only advanced analytical tools but also well-defined problem frameworks that accommodate expert knowledge, real-life limitations, and varying levels of certainty in expert evaluations. To address this gap, the present study enhances prior methodologies by incorporating confidence levels within the decision-making framework. By combining expert insights with realistic supplier data, this approach offers a more systematic and dependable method for selecting sustainable suppliers.

5.2 Problem Statement

A university aims to procure sustainable furniture for its newly constructed academic block. The selection process requires evaluating multiple suppliers based on sustainability and other key factors. Since the decision impacts various stakeholders, the university forms a committee of three decision experts (DEs) with distinct roles and responsibilities. The role and expertise of the experts is given in Table 2.

Table 2: Details of experts involved in the decision-making process

Expert	Designation	Area of expertise	Years of experience
E1	Procurement Officer	Supplier reliability, contract evaluation, compliance with procurement policies	12 years
E2	Environmental Sustainability Expert	Environmental impact, material sustainability, regulatory certifications	10 years
E3	Finance Manager	Cost-effectiveness analysis, budget management, financial feasibility assessment	15 years

The objective is to identify the most appropriate furniture supplier by taking into account economic, environmental, and social sustainability factors.

The alternatives to this problem are four potential furniture suppliers, denoted as $K1, K2, K3$, and $K4$. Each supplier offers a range of sustainable furniture options and has been shortlisted based on preliminary screening for compliance with the universitys procurement guidelines. These suppliers differ in their sustainability practices, product quality, cost structures, and corporate social responsibility initiatives. The selection process requires a comparative evaluation of these suppliers based on multiple criteria to determine the best fit for the universitys sustainability goals.

The suppliers are evaluated based on eight criteria categorized into three sustainability dimensions:

1. **Economic criteria:** Financial feasibility plays a key role in sustainable procurement, ensuring that the selected supplier offers cost-effective yet high-quality products. By considering aspects such as durability, pricing, and supply chain efficiency, universities can make informed decisions that maximize value for money. A supplier who provides long-term economic benefits through durable and reliable furniture helps institutions maintain financial stability while promoting sustainability.

    - Cost-effectiveness (R1): The total cost of procuring furniture, including initial purchase price, transportation, maintenance, and disposal, is assessed. Suppliers offering high-quality products at competitive prices without compromising sustainability receive priority. Cost-effectiveness is analyzed over the products life cycle, ensuring long-term financial benefits for the university. Choosing an optimal supplier helps in budget optimization and financial sustainability.
    - Product durability (R2): This criterion evaluates the lifespan and resilience of the furniture, considering factors like material strength, wear resistance, and maintenance requirements. Durable furniture reduces the need for frequent replacements, lowering costs and environmental impact. Suppliers providing warranties and conducting product quality testing are preferred. Higher durability also contributes to overall economic efficiency and sustainability.
    - Supply chain reliability (R3): Suppliers are analyzed based on their ability to provide consistent, timely, and efficient delivery of products. Factors like lead time, inventory management, responsiveness to demand fluctuations, and transportation efficiency are considered. A reliable supplier ensures an uninterrupted supply, reducing delays in university projects. This criterion also examines risk management strategies for handling disruptions in the supply chain.

2. **Environmental criteria:** Environmental sustainability is a crucial aspect of supplier selection, focusing on reducing negative environmental effects while promoting eco-friendly practices. Selecting suppliers with strong environmental commitments helps institutions align with global sustainability goals and reduce their overall carbon footprint.

    - Eco-friendly materials (R4): This criterion assesses the supplier's commitment to using environmentally friendly materials, including options like recycled wood, bamboo, and biodegradable elements. It ensures that raw materials are sourced responsibly, minimizing deforestation and environmental degradation. The preference is given to suppliers who hold certifications like FSC (Forest Stewardship Council) or similar eco-labels. Using eco-friendly materials reduces carbon footprint and promotes circular economy practices.
    - Energy efficiency (R5): Suppliers are assessed based on their energy consumption in manufacturing, transportation, and operational activities. Suppliers that utilize renewable energy sources, like solar or wind power, and adopt energy-efficient technologies are awarded higher ratings. Energy-efficient production methods help in reducing greenhouse gas emissions and operational costs. This criterion aligns with global sustainability goals for reducing fossil fuel dependency.

- Waste management (R6): The ability of the supplier to minimize waste generation and implement recycling strategies is evaluated. This includes practices such as repurposing manufacturing by-products, reducing packaging waste, and ensuring responsible disposal of materials. Suppliers with circular economy initiatives and zero-waste policies are preferred. Proper waste management reduces environmental pollution and promotes resource efficiency.

3. **Social criteria:** Sustainable procurement goes beyond environmental and economic concerns; it also involves evaluating suppliers ethical and social responsibilities. Fair labor practices, workplace safety, and community contributions reflect a companys commitment to positive social impact. Universities that prioritize socially responsible suppliers not only uphold ethical business practices but also foster a culture of sustainability and fairness within their ecosystem.

   - Labor conditions and ethical practices (R7): The suppliers adherence to fair labor standards, worker rights, and ethical business practices is assessed. This includes compliance with labor laws, prevention of child labor, and ensuring safe working conditions. Suppliers that maintain transparency in their workforce policies and provide fair wages are prioritized. Ethical sourcing enhances the universitys reputation and promotes social responsibility.
   - Corporate social responsibility (CSR) initiatives (R8): Suppliers are evaluated based on their commitment to social and community welfare programs. This includes involvement in charitable activities, environmental conservation projects, and contributions to local communities. Suppliers with CSR policies aligned with sustainability principles are preferred. Strong CSR engagement reflects the suppliers dedication to ethical and responsible business practices.

The criteria are classified into benefit and cost categories based on their impact on decision-making. Benefit criteria are those where higher values are desirable, directly contributing to the sustainability, efficiency, and overall quality of the supplier selection. Product durability (R2) is a benefit criterion because higher durability ensures longer-lasting products, reducing replacement frequency. Supply chain reliability (R3) is also a benefit criterion, as reliable suppliers ensure consistent, timely delivery. Eco-friendly materials (R4) are preferred for their positive environmental impact, while energy efficiency (R5) reduces energy consumption and emissions. Waste management (R6) is beneficial because better waste practices promote resource efficiency. Labor conditions and ethical practices (R7) and corporate social responsibility (CSR) initiatives (R8) are benefit criteria as they reflect suppliers commitment to social responsibility and ethical conduct. On the other hand, cost-effectiveness (R1) is a cost criterion because lower values are preferable, directly impacting financial sustainability. Lower total procurement costs, including purchase, maintenance, and disposal, are desirable for achieving budget optimization. This classification ensures a balanced evaluation of suppliers, considering both financial and sustainability aspects.

The hierarchy of the selection of sustainable furniture suppliers is presented in Figure 2.

5.3 Solution Procedure

Step 1. In this decision-making problem, four alternatives $(K1, K2, K3, K4)$ are evaluated based on eight criteria $(R1, R2, \dots, R8)$. The evaluation is carried out by three decision experts $(E1, E2, E3)$, possessing 18, 15, and 12 years of experience, respectively.

Step 2. The weights of the decision experts based on their years of experience are calculated by using Equation (4) as follows: $\beta_1 = \frac{18}{45} = 0.400, \beta_2 = \frac{15}{45} = 0.333, \beta_3 = \frac{12}{45} = 0.267$.

Step 3. Based on the detailed descriptions provided for the criteria R1 to R8, the criteria can be ranked based on their importance as follows: $1, 2, 5, 3, 4, 6, 7, 8$. Now, by using Equation (5), we obtain the criteria weights of the eight criteria as $0.222, 0.194, 0.111, 0.167, 0.139, 0.083, 0.056$, and 0.028.

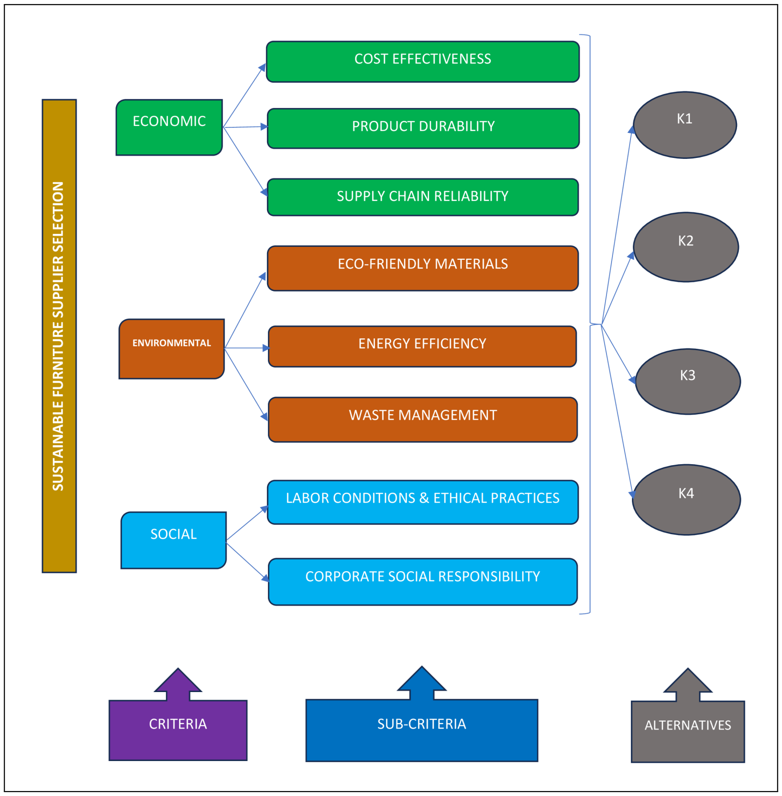

Figure 2: Hierarchy for sustainable furniture supplier selection

Step 4. The decision matrices, expressed in linguistic variables is presented in Table 3. The p,q-QOF decision matrix (as per Table 1) with their level of confidence is presented in Table 4.

Step 5. The normalized decision matrices, computed by using Equation (6), are presented in Table 5.

Step 6. The individual decision matrices provided by the three DEs are aggregated using Equation (7), where the parameters are set as $p = 5$, $q = 4$, and $\lambda = 5$. The resulting aggregated decision matrix is presented in Table 6.

Step 7. Next, the combined values for each alternative with respect to each criterion are determined by applying Equation (8). The aggregated results of each alternative are presented in Table 7.

Step 8. Following this, the score values of the aggregated results are computed by using Equation (1), and the alternatives are ranked accordingly. The score values and final rankings are presented in Table 7.

Step 9. Based on Table 7, the ranking order of the five suppliers is determined as $K1 \succ K4 \succ K3 \succ K2$. Therefore, $K1$ is identified as the best supplier using the proposed method.

5.4 Comparison Analysis

This section presents a comparative analysis of the results generated by the proposed method alongside those obtained from several existing techniques.

Table 3: The decision matrices given in LVs.

Decision maker	Alternative	Sub-criteria							
		R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8
E1	$K1$	M	B	G	VG	M	B	VB	G
	$K2$	VG	G	B	G	B	G	M	B
	$K3$	G	M	VB	M	G	VG	B	M
	$K4$	B	VG	VG	B	VB	M	G	VB
E2	$K1$	VB	G	VB	B	G	VG	VB	B
	$K2$	G	B	M	M	M	G	VG	M
	$K3$	M	VB	VG	G	B	M	G	G
	$K4$	B	M	G	VG	VB	B	M	VG
E3	$K1$	VB	B	M	G	VG	VB	B	M
	$K2$	G	VG	VB	B	M	G	VG	VB
	$K3$	B	M	G	VG	VB	B	M	G
	$K4$	VG	VB	B	M	G	VG	VB	B

Table 4: Decision matrix using p,q-QOFNs with confidence levels

DEs	**Alternatives**	**Sub criteria**							
		$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$
E1	$K1$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K2$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$
	$K3$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K4$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.2,0.98),0.88\rangle$
E2	$K1$	$\langle(0.20,0.98),0.88\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$
	$K2$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K3$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K4$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.98,0.15),0.80\rangle$
E3	$K1$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K2$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$
	$K3$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K4$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$

Table 5: Normalized decision matrices

DEs	**Alternatives**	**Sub criteria**							
		$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$
E1	$K1$	$\langle(0.55,0.45),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K2$	$\langle(0.15,0.98),0.80\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.30,0.80),0.90\rangle$
	$K3$	$\langle(0.42,0.81),0.75\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K4$	$\langle(0.80,0.30),0.90\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.2,0.98),0.88\rangle$
E2	$K1$	$\langle(0.98,0.20),0.88\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$
	$K2$	$\langle(0.42,0.81),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K3$	$\langle(0.55,0.45),0.85\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K4$	$\langle(0.80,0.30),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.98,0.15),0.80\rangle$
E3	$K1$	$\langle(0.98,0.20),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$
	$K2$	$\langle(0.42,0.81),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.20,0.98),0.88\rangle$
	$K3$	$\langle(0.80,0.30),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$
	$K4$	$\langle(0.15,0.98),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$	$\langle(0.45,0.55),0.85\rangle$	$\langle(0.81,0.42),0.75\rangle$	$\langle(0.98,0.15),0.80\rangle$	$\langle(0.2,0.98),0.88\rangle$	$\langle(0.30,0.80),0.90\rangle$

Table 6: Aggregated decision matrix

	R1	R2	R3	R4	R5	R6	R7	R8
K1	(0.902,0.349)	(0.593,0.717)	(0.621,0.741)	(0.837,0.458)	(0.792,0.472)	(0.768,0.596)	(0.24,0.953)	(0.622,0.672)
K2	(0.358,0.923)	(0.798,0.506)	(0.36,0.812)	(0.624,0.672)	(0.4,0.702)	(0.757,0.567)	(0.88,0.343)	(0.36,0.812)
K3	(0.611,0.632)	(0.402,0.766)	(0.809,0.558)	(0.792,0.472)	(0.615,0.792)	(0.805,0.48)	(0.599,0.669)	(0.68,0.574)
K4	(0.726,0.502)	(0.805,0.529)	(0.845,0.454)	(0.771,0.5)	(0.563,0.878)	(0.735,0.524)	(0.623,0.731)	(0.768,0.635)

Table 7: Ranking outcomes

Alternative	Aggregated value	Score value	Rank
$K1$	(0.800, 0.536)	0.6226	1
$K2$	(0.684, 0.669)	0.4747	4
$K3$	(0.702, 0.622)	0.5104	3
$K4$	(0.758, 0.559)	0.5763	2

Table 8: Comparison with the current approach of averaging aggregation method.

Methods	Score Values				Ranking
	$\mathcal{S}(K1)$	$\mathcal{S}(K2)$	$\mathcal{S}(K3)$	$\mathcal{S}(K4)$	
p,q-CQOFWA [44]	0.6920	0.5235	0.5781	0.6579	$K1 \succ K4 \succ K3 \succ K2$
p,q-CQOFWG [44]	0.5124	0.4458	0.4301	0.3587	$K1 \succ K2 \succ K3 \succ K4$
CFWA$_q$ [45]	0.6778	0.5151	0.5650	0.6400	$K1 \succ K4 \succ K3 \succ K2$
CFWG$_q$ [45]	0.5124	0.4458	0.4301	0.3587	$K1 \succ K2 \succ K3 \succ K4$
CLp,q-QOFDWA [46]	0.8765	0.7151	0.7928	0.8715	$K1 \succ K4 \succ K3 \succ K2$
CLp,q-QOFDWG [46]	0.4004	0.3356	0.3037	0.1784	$K1 \succ K2 \succ K3 \succ K4$
Cp,q-QOFHWA (Proposed)	0.6226	0.4547	0.5104	0.5763	$K1 \succ K4 \succ K3 \succ K2$

5.4.1 Comparison with several existing operator based methods

To ensure a consistent evaluation, various aggregation operator-based approaches are applied to the same numerical example detailed in Subsection 5.2. The alternatives $K1$ through $K4$ are assessed based on their score values $\mathcal{S}(K_i)$, which are subsequently used to establish their final ranking.

The methods considered for comparison include:

- CL-based p,q-QOF weighted averaging/geometric (p,q-CQOFWA/p,q-CQOFWG) AOs [44], assuming $p = 5$ and $q = 4$.
- Confidence q-rung orthopair fuzzy weighted averaging/geometric (CFWA$_q$, CFWG$_q$) AOs [45], with $q = 4$.
- CL-based p,q-QOF Dombi weighted averaging/geometric (CLp,q-QOFDWA/CLp,q-QOFDWG) AOs [46], using $p = 5$ and $q = 4$.

The comparative results are presented in Table 8 and Figure 3.

Based on the score values presented in Table 8 and ranking outcomes exhibited in Figure 3 for the various alternatives, the following key observations can be made:

- p,q-CQOFWA and p,q-CQOFWG [44]: These methods demonstrate distinct score value patterns for the alternatives. The p,q-CQOFWA method produces relatively higher scores for all alternatives, with K1 having the highest score (0.6920) and K2 the lowest (0.5235). Conversely, the p,q-CQOFWG method shows significantly lower score values for all alternatives, with K1 also having the highest (0.5124) but K4 the lowest (0.3587). This difference highlights the contrasting nature of weighted averaging and weighted geometric aggregation.
- CFWA$_q$ and CFWG$_q$ [45]: These methods exhibit a similar trend to the p,q-CQOFWA and p,q-CQOFWG methods. The CFWA$_q$ method generates relatively higher score values, with K1 (0.6778)

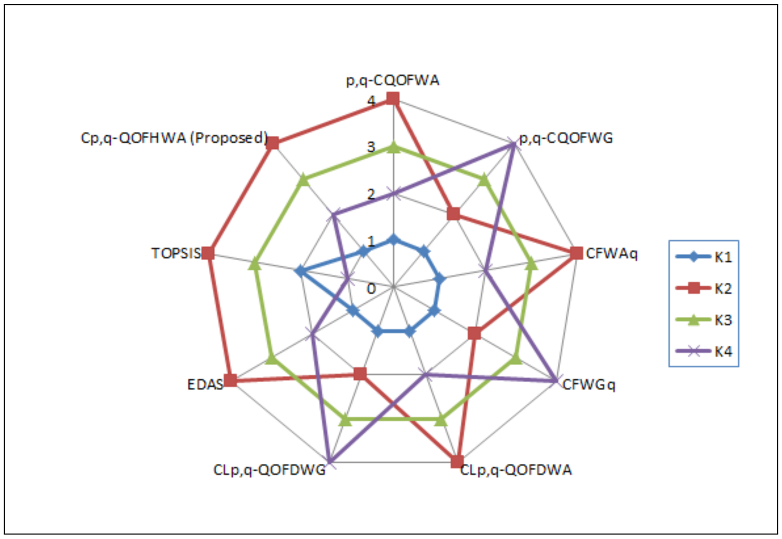

Figure 3: Ranking outcomes using different methods

achieving the highest value and K2 (0.5151) the lowest. The CFWG$_q$ method again shows lower values, with K1 (0.5124) being the highest and K4 (0.3587) the lowest. This consistency suggests that the use of weighted geometric aggregation reduces the score values compared to weighted averaging.

- CLp,q-QOFDWA and CLp,q-QOFDWG [46]: These methods provide another layer of comparison. The CLp,q-QOFDWA method achieves significantly higher scores for all alternatives, with K1 (0.8765) having the highest score and K2 (0.7151) the lowest. The CLp,q-QOFDWG method, however, shows much lower values, with K1 (0.4004) being the highest and K4 (0.1784) the lowest. The notable difference between these two methods highlights the strong impact of the type of aggregation (weighted averaging vs. weighted geometric).
- Proposed Cp,q-QOFHWA method: The proposed Cp,q-QOFHWA method demonstrates a balanced score range. It produces a high score for K1 (0.6226) and relatively moderate values for the other alternatives, maintaining consistency without extreme values. This balanced performance reflects the method's effectiveness in providing reliable aggregation results without excessive sensitivity.

Table 8 reveals several key observations:

- For p,q-CQOFWA, CFWA$_q$, CLp,q-QOFDWA, and the proposed Cp,q-QOFHWA methods, the ranking sequence is identical: $K1 \succ K4 \succ K3 \succ K2$. This consistency suggests a strong preference for K1 across these averaging-based methods.
- In contrast, p,q-CQOFWG, CFWG$_q$, and CLp,q-QOFDWG follow a different ranking pattern: $K1 \succ K2 \succ K3 \succ K4$. This change is consistent for the geometric-based methods, demonstrating their unique approach to aggregation, where lower score values are generally preferred due to the nature of geometric averaging.
- The proposed Cp,q-QOFHWA method aligns with the majority of the averaging-based methods, ensuring stability in ranking outcomes. This stability further reinforces the robustness of the proposed method in maintaining reliable decision-making results.

These findings highlight the effectiveness and reliability of the proposed Cp,q-QOFHWA method as a robust averaging aggregation approach compared to existing methods in the literature.

5.4.2 Comparison with the EDAS method

In this section, the remaining steps of the EDAS method [47] are applied to the aggregated decision matrix presented in Table 6 of Subsection 5.3.

- First, use Equation (9) to calculate the average solution $A = (A_1, A_2, \ldots, A_n)$ of all the alternatives under each attribute.

$$A_b = \left(\sqrt[p]{1 - \prod_{a=1}^{m} (1 - \vartheta_{I_{ab}}^{p})^{1/m}}, \prod_{a=1}^{m} (\theta_{I_{ab}})^{1/m} \right). \tag{9}$$

Taking $p = 5$ and m(number of alternatives) $= 4$ we obtain the average solution as

A= (0.691,0.623),(0.579,0.695),(0.618,0.710),(0.636,0.585),(0.543,0.756),(0.623,0.613),(0.619,0.692),(0.528,0.745).

- Next, we compute the positive distance from average solution matrix $A^+ = (A^+_{ab})_{m\times n}$ and the negative distance from average solution matrix $A^- = (A^-_{ab})_{m\times n}$ where

$$A^+_{ab} = \frac{\max\{0, \mathcal{S}(\bar{\gamma}_{ab}) - \mathcal{S}(A_b)\}}{\mathcal{S}(A_b)} \text{ and } A^-_{ab} = \frac{\max\{0, \mathcal{S}(A_b)\} - \mathcal{S}(\bar{\gamma}_{ab})}{\mathcal{S}(A_b)}.$$

$$A^+ = \begin{bmatrix} 0.3747 & 0 & 0 & 0.1076 & 0.4577 & 0 & 0 & 0 \\ 0 & 0.2195 & 0 & 0 & 0 & 0.0134 & 0.4494 & 0 \\ 0 & 0 & 0.0900 & 0.0642 & 0 & 0.0720 & 0 & 0.2128 \\ 0 & 0.1414 & 0.3438 & 0 & 0 & 0 & 0 & 0.0385 \end{bmatrix}$$

$$A^- = \begin{bmatrix} 0 & 0.1979 & 0.3476 & 0 & 0 & 0.1201 & 0.8702 & 0.0008 \\ 0.8353 & 0 & 0.4711 & 0.2592 & 0.0419 & 0 & 0 & 0.3967 \\ 0.2118 & 0.3919 & 0 & 0 & 0.3691 & 0 & 0.1400 & 0 \\ 0.0093 & 0 & 0 & 0.0130 & 0.6195 & 0.0141 & 0.3403 & 0 \end{bmatrix}$$

- Calculate the positive weighted distance PWD_a and the negative weighted distance NWD_a for each alternative $a = 1, 2, \ldots, m$. Here,

$$PWD_a = \sum_{b=1}^{n} \alpha_b A^+_{ab} \text{ and } NWD_a = \sum_{b=1}^{n} \alpha_b A^-_{ab}$$

where α_b be the weight of the bth criteria.

The values of PWD and NWD are exhibited in Table 9.

- Next, we normalize the PWD_a and NWD_a for each alternative $a = 1, 2, \ldots, m$ by Equation (10).

$$NPWD_a = \frac{PWD_a}{\max\{PWD_1, PWD_2, \ldots, PWD_m\}}, NNWD_a = \frac{NWD_a}{\max\{NWD_1, NWD_2, \ldots, NWD_m\}}. \tag{10}$$

The normalized values of PWD_a and NWD_a are exhibited in Table 9.

- Finally, we derive the integrative appraisal score IAS_a for each alternative $a = 1, 2, \ldots, m$, by using Equation (11).

$$IAS_a = \frac{NPWD_a + 1 - NNWD_a}{2}. \tag{11}$$

The integrative appraisal score for each alternative and the ranking outcomes are presented in Table 9.

Table 9: Calculated values of PWD, NWD, NPWD, NNWD, and IAS for each alternative

Alternative	PWD	NWD	NPWD	NNWD	IAS	Ranking
K1	0.1648	0.1357	0.9786	0.4555	0.7616	1
K2	0.0689	0.2979	0.4091	1.0000	0.2046	4
K3	0.0326	0.1822	0.1936	0.6116	0.2910	3
K4	0.0667	0.1106	0.3961	0.3713	0.5124	2

- Therefore, from Table 9 we obtain the ranking outcomes of the four supplier using EDAS method [47] as $K1 \succ K4 \succ K3 \succ K2$. Thus, the best supplier is $K1$.

To provide a better visualization, the results of the ranking obtained using the EDAS method and the proposed method are illustrated in Figure 3. The ranking results obtained using the proposed method are consistent with those derived using the EDAS method [47], yielding the identical preference order: $K1 \succ K4 \succ K3 \succ K2$. This agreement demonstrates the reliability and validity of the proposed approach, confirming that it produces results in alignment with an established and widely accepted method. Such consistency reinforces the robustness of the proposed model in capturing the relative performance of the alternatives.

5.4.3 Comparison with the TOPSIS method

In this section, the remaining steps of the TOPSIS method [2] are applied to the aggregated decision matrix presented in Table 6 of Subsection 5.3.

The positive ideal solution (P^+) and negative ideal solution (P^-) are computed as:
$P^+ = \langle(0.835, 0.385), (0.631, 0.623), (0.723, 0.522), (0.700, 0.523), (0.671, 0.541), (0.559, 0.581), (0.508, 0.826),(0.556, 0.735)\rangle$
and
$P^- = \langle(0.341, 0.577), (0.384, 0.771), (0.342, 0.835), (0.514, 0.556), (0.385, 0.927), (0.556, 0.682),(0.237, 0.971), (0.342, 0.741)\rangle$.

Then we computed the Hamming distance from the positive ideal solution (DPIS) and negative ideal solution (DNIS) for each alternative. The distances are shown in Table 10. Then we calculate the closeness coefficient (CC) of each alternative. We exhibit the values of CC in Table 10.

Table 10: Calculated values of DPIS, DNIS, and CC for each alternative

Alternative	DPIS	DNIS	CC	Ranking
K1	2.5030	3.2777	0.5670	2
K2	4.7148	5.1658	0.5228	4
K3	3.2721	3.6201	0.5450	3
K4	2.3895	3.5319	0.5964	1

Therefore, from Table 10 we obtain the ranking outcomes of the four supplier using TOPSIS method [2] as $K4 \succ K1 \succ K3 \succ K2$. Thus, the best supplier is $K4$.

The ranking outcomes using the proposed method are: $K1 \succ K4 \succ K3 \succ K2$ whereas the rankings obtained from the TOPSIS method are: $K4 \succ K1 \succ K3 \succ K2$. To provide a better visualization, the results of the ranking obtained using the TOPSIS method and the proposed method are illustrated in Figure 3.

A close examination reveals that both methods agree on the relative positioning of the alternatives $K3$ and $K2$, consistently ranking $K3$ higher than $K2$. However, a notable difference lies in the ordering of the top two alternatives. The proposed method ranks $K1$ as the best alternative, followed by $K4$, while the TOPSIS method [2] ranks $K4$ higher than $K1$.

One key distinction that may contribute to this variation is that the TOPSIS method [2], does not incorporate criteria weights in the decision-making process, because the rankings were derived from the aggregated decision matrix rather than a weighted aggregated decision matrix. As a result, all criteria are implicitly treated with equal importance. However, it is possible to extend TOPSIS by employing weighted distances, in which the criteria weights may come into play.

In contrast, the proposed method takes into account the relative importance of criteria, often through expert-defined weights or confidence levels, leading to a more realistic and nuanced evaluation of the alternatives. This capability enhances the discriminatory power of the proposed method and enables it to better reflect the decision-makers preferences and the practical significance of each criterion. Therefore, the proposed method can be considered more robust and adaptable in complex multi-criteria decision-making environments.

5.5 Sensitivity Analysis

The sensitivity analysis of the parameter η plays a crucial role in understanding the stability and robustness of the decision-making model. The parameter η is a crucial component in the Dombi aggregation operators, influencing the aggregation process. For solving the numerical problem, we initially set $\eta = 1$ arbitrarily. To conduct the sensitivity analysis, we systematically vary the value of η from 1 to 100, solving the numerical problem for each λ and analyzing its effect on the ranking outcomes.

The table presented in Table 11 provides a comprehensive sensitivity analysis of the parameter λ across 23 distinct values ranging from $\lambda = 1$ to $\lambda = 100$. The analysis is primarily focused on the performance scores $\mathcal{S}(K1)$, $\mathcal{S}(K2)$, $\mathcal{S}(K3)$, and $\mathcal{S}(K4)$, with the ranking of the alternatives clearly depicted for each value of λ. This comparative analysis aims to elucidate the impact of varying λ on the performance of the four alternatives (K1, K2, K3, and K4) and their relative rankings.

From Table 11 we can observe that, as λ increases from 1 to 100, the performance scores of all alternatives decrease gradually. This decline is more noticeable for $K2$ and $K3$ compared to $K1$ and $K4$. $K1$ exhibits the slowest rate of performance decline, which supports its position as the most robust alternative. The sensitivity of each alternatives performance to the variation of λ can be assessed as follows: $K1$ has the highest resistance to change, indicating stability. $K4$ shows moderate sensitivity, maintaining a strong second position, while $K3$ and $K2$ display higher sensitivity, with their performance scores decreasing at a faster rate.

Overall, across all values of λ, Alternative $K1$ consistently exhibits the highest performance score among the four alternatives, maintaining the top rank. This indicates the robustness of $K1$ as the most preferred alternative regardless of the variation in the parameter λ. Alternative $K4$ is consistently ranked second, demonstrating relatively stable performance values, albeit lower than $K1$ but superior to $K3$ and $K2$. Alternatives $K3$ and $K2$ consistently occupy the third and fourth ranks, respectively, indicating a relatively lower and stable performance throughout the range of λ values.

The ranking pattern remains constant across all values of λ as $K1 \succ K4 \succ K3 \succ K2$. This stable ranking implies that changes in λ do not alter the relative performance order of the alternatives. Such consistency in ranking highlights the robustness of the ranking method used, ensuring that the impact of λ is primarily on the magnitude of the performance scores rather than their relative order.

The consistent ranking of the alternatives, despite varying λ, indicates that the decision-maker can confidently rely on the dominance of $K1$ and $K4$ over $K3$ and $K2$. In scenarios where the decision environment is highly uncertain, selecting $K1$ or $K4$ as preferred choices is justified. The analysis further demonstrates the importance of evaluating sensitivity to ensure that alternative rankings are not misleadingly influenced by parameter variations.

To visually illustrate the trend of sensitivity analysis, Figure 4 presents a graphical representation of the variation in scores $\mathcal{S}(Ki)$ for different values of λ.

Table 11: Sensitivity of the parameter λ

λ	$\mathcal{S}(K1)$	$\mathcal{S}(K2)$	$\mathcal{S}(K3)$	$\mathcal{S}(K4)$	**Rank**
1	0.6937	0.5260	0.5777	0.6660	$K1 \succ K4 \succ K3 \succ K2$
2	0.6630	0.5040	0.5488	0.6237	$K1 \succ K4 \succ K3 \succ K2$
3	0.6454	0.4909	0.5313	0.6018	$K1 \succ K4 \succ K3 \succ K2$
4	0.6324	0.4810	0.5200	0.5872	$K1 \succ K4 \succ K3 \succ K2$
5	0.6226	0.4747	0.5104	0.5763	$K1 \succ K4 \succ K3 \succ K2$
6	0.6143	0.4679	0.5032	0.5673	$K1 \succ K4 \succ K3 \succ K2$
7	0.6075	0.4635	0.4973	0.5600	$K1 \succ K4 \succ K3 \succ K2$
8	0.6015	0.4596	0.4920	0.5534	$K1 \succ K4 \succ K3 \succ K2$
9	0.5971	0.4559	0.4877	0.5479	$K1 \succ K4 \succ K3 \succ K2$
10	0.5928	0.4521	0.4836	0.5433	$K1 \succ K4 \succ K3 \succ K2$
15	0.5771	0.4405	0.4629	0.5263	$K1 \succ K4 \succ K3 \succ K2$
20	0.5669	0.4312	0.4587	0.5147	$K1 \succ K4 \succ K3 \succ K2$
25	0.5589	0.4252	0.4515	0.5065	$K1 \succ K4 \succ K3 \succ K2$
30	0.5534	0.4202	0.4459	0.5001	$K1 \succ K4 \succ K3 \succ K2$
35	0.5479	0.4159	0.4413	0.4948	$K1 \succ K4 \succ K3 \succ K2$
40	0.5440	0.4130	0.4373	0.4906	$K1 \succ K4 \succ K3 \succ K2$
45	0.5404	0.4095	0.4337	0.4869	$K1 \succ K4 \succ K3 \succ K2$
50	0.5376	0.4066	0.4311	0.4838	$K1 \succ K4 \succ K3 \succ K2$
60	0.5325	0.4017	0.4256	0.4780	$K1 \succ K4 \succ K3 \succ K2$
70	0.5288	0.3976	0.4211	0.4737	$K1 \succ K4 \succ K3 \succ K2$
80	0.5255	0.3941	0.4183	0.4704	$K1 \succ K4 \succ K3 \succ K2$
90	0.5218	0.3914	0.4148	0.4670	$K1 \succ K4 \succ K3 \succ K2$
100	0.5192	0.3890	0.4123	0.4642	$K1 \succ K4 \succ K3 \succ K2$

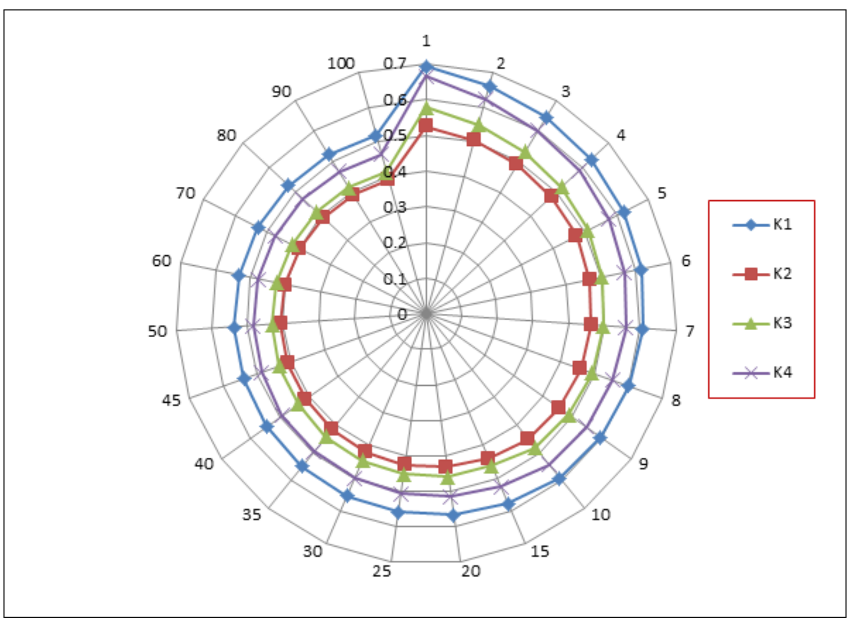

Figure 4: Sensitivity analysis of the parameter λ

6 Conclusion, Limitations and Future Scope

This study introduced a comprehensive MCGDM framework that integrates confidence level-driven Hamacher aggregation operators within the p,q-QOFS environment. To facilitate effective aggregation under uncertainty, two novel operatorsCp,q-QOFHWA and Cp,q-QOFHWGwere developed, enabling the incorporation of varying degrees of expert confidence into the decision-making process. These operators were then utilized to construct a structured methodology tailored for complex MCGDM problems. The proposed framework was applied to a real-world numerical problem concerning the selection of a sustainable furniture supplier for a university, thereby demonstrating its practical utility and robustness. A comparative analysis with existing approaches further confirmed the advantages of the proposed model in delivering accurate and interpretable outcomes under uncertain and subjective conditions. Additionally, sensitivity analysis was conducted for different parameter values to test the stability of the decision outcomes under variations in operator parameters.

The key findings of this study highlight several important contributions. Firstly, the incorporation of confidence levels into the aggregation process enhances the reliability of group decisions by ensuring that expert evaluations with higher credibility have a greater influence on the final outcome. Secondly, the use of Hamacher t-norm and t-conorm functions within the p,q-QOFS framework provides enhanced flexibility for modeling complex interactions between membership and non-membership degrees. Lastly, the empirical results reveal that the proposed model offers superior decision-making performance compared to traditional methods, particularly in handling vague, hesitant, and inconsistent information often encountered in real-world decision environments.

Despite its strengths, the study is not without limitations. The current framework employs only Hamacher-based aggregation operators, which may not be optimal in all decision contexts, and the reliance on predefined crisp weights for criteria, experts, and confidence levels may oversimplify the inherent uncertainty in these parameters. Additionally, the model considers only a single expert per criterion, which limits the richness and diversity of decision-making process. The comparison conducted was restricted to a few existing MCGDM approaches, suggesting the need for a broader benchmarking to validate the generalizability and competitiveness of the proposed model across different scenarios. Also, the sensitivity analysis was restricted to a few specific parameter values, which, while useful, may not capture the full range of potential outcomes.

Building on these observations, several directions for future research emerge. One promising extension involves developing confidence level-based aggregation operators using alternative t-norm and t-conorm families such as Frank, Yager, or Dombi, to assess the relative performance of different mathematical formulations. Another valuable enhancement would be the integration of subjective and objective weight-determination methods, such as SWARA, AHP, or entropy, to derive more representative weights for experts and criteria. Future studies could also incorporate multiple experts per criterion to better capture diverse perspectives and improve consensus reliability. Furthermore, validating the proposed model against a wider array of MCDM methods, including COPRAS, VIKOR, and PROMETHEE, would provide a more comprehensive evaluation of its effectiveness. The frameworks adaptability also lends itself well to applications in broader decision-making domains such as biomedical waste management, site selection for software units, and recruitment processes. Finally, expanding the model to work with more complex fuzzy environmentssuch as interval-valued extensionswould enhance its ability to capture multi-dimensional uncertainty and further broaden its applicability.

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