Decentralized Robust Controller for Two-Tank Liquid Flow Process with Recycle

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Abstract–In this paper, we propose the improvement of a modern technique for the synth esis of decentralized robust control for two-input, two-output (MIMO) nonlinear systems. interaction, stability analysis, and controller design for the Two-Tank Liquid Flow Process with recycling have been done. The equations of the system are non-linear and have two inputs and two outputs. The Gershgorin bands check the system's interaction and the system's instability is studied by using a generalized Nyquist diagram. The linear model is used, so that it is necessary to consider uncertainty. In a physical system, it is essential to consider several goals; despite the uncertainty, the system should be stable, desire performance without considering the uncertainty, and minimize the disturbance effect, due to the interaction. In the first step, thepre-compensator is used for decoupling. Then, due to the uncertainty and disturbance, the H_2/H_∞ robust controller is designed to minimize the three conditions of robust stability and nominal performance while minimizing the disturbance effect for each channel. The proposed method, for the synthesis of the decentralized robust nonlinear multivariable control systems, is approved for the two-tank system. The simulation results show the success of the robust decentralized controller with minimum control signal and order.

Keywords: Decentralized Control, H_2/H_∞,liquid level control, MIMO system, Robust stability

1. Introduction

In control engineering and related research areas, liquid-level control is commonly used in industrial automation. In the shipping industry, level control of various containers, such as oil tanks, and freshwater tanks play a vital role. In addition, it is used in various fields such as petrochemical and medical industries, water treatment and power plants. The control of liquid level in tanks and flow between tanks could be an issue within the preparation technologies. The method technologies require liquids to be pumped, put away in tanks, and after that pumped to another tank efficiently. Tank level control systems are often subject to uncertainties such as time-varying state, nonlinearity, and time delay. Many researchers have done a lot of research in this field. In [1], the multivariable controller is centrally designed for the liquid level system with two tanks. In [2], fuzzy logic is used to adjust the PID parameters to control a four-tank system where the liquid level in tanks 1 and 2 is controlled. In [3], a disturbance observer method based on feedback control is presented for a two-tank liquid-level system with external disturbances. In [4], Four control strategies are designed and compared for the conical tank, namely classical PID, Gain Scheduling (GS), Internal Model Control (IMC), and Fuzzy Logic. In [5], A PID controller for the liquid-level system with two single input and single output tanks is designed. In [6], P, PI, PD, and PID controllers for the liquid-level system with two single input and single output tanks without interaction are designed and compared. In [7], a multi-input multi-output model that couples from radial basis function (RBF) neural networks to approximate the coefficients of the autoregressive exogenous (ARX) model is used to describe the dynamics of the fluid system of tanks, then a predictive control strategy is proposed. In [8], based on the robust control, a third-order linear controller is first designed, then using the closed-loop gain shaping algorithm (CGSA), the control law is corrected using the non-linear switching modification technique based on an arcsin function. In [9], a sliding mode controller is designed to control liquid level on a single input single output coupled tank system. In [10], an adaptive fractional order sliding mode controller is designed to control the quadruple tank process. The pump voltage is considered the input and the water level in the two tanks is the output. In [11], a dynamic fractional order sliding mode controller (FOSMC) based on the model is designed to control the liquid level. In [12], the effectiveness of the PSO algorithm in the optimal control of a two-tank system modeled in a piecewise affine manner has been examined. First, the constrained finite time

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optimal control (CFTOC) problem is considered and then the PSO algorithm is used to reduce the computational complexity of the control law and adjust the physical parameters of the system to improve performance at the same time [12]. In [13], a proficient computational approach is proposed to optimize and control a coupled tanks framework. In this way, the optimal solution of the model utilized can be overhauled iteratively. On this premise, system optimization and parameter estimation are integrated. In [14], created the two tank interacting framework numerical demonstration and simulated with PID controller and Fuzzy controller. In [15], they displayed a strategy to control the liquid level in a second tank of a coupled-tank plant through variable control of a water pump within the, to begin with, the tank. The ideal controller parameters of this plant are calculated utilizing radial basis function neural network demonstration. This paper is organized as follows; first, the Modeling of two-tank liquid level system with Recycle is briefly described. Having introduced the Stability analysis, interaction and decentralized liquid level control system 3, in section 4, the H_2/H_∞ robust controller problem is discussed, and finally, the simulation results and conclusion are presented. In this study, the liquid level system with two inputs and two Outputs and disturbance is considered. The mentioned system is non-linear, unstable, and has interaction. Considering that the system is not diagonal dominant, first, a decoupling matrix is determined and then two controllers are designed separately. Due to the uncertainty and disturbance, the controller is robust and the goal is to minimize the three conditions of robust stability, nominal performance, and minimize the effect of disturbance simultaneously.

2. Modeling of two-tank liquid level system with Recycle

Consider the liquid level system of Figure 1.

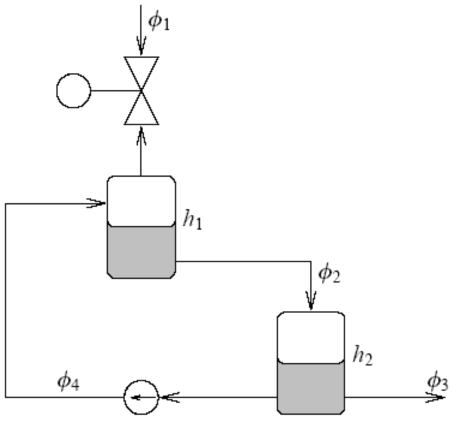

Fig. 1. Two-tank liquid flow process with recycle [16]

Input flow 1 and return flow 4 act as manipulable input variables to the system and output flow 3 acts as a disturbance.The purpose of liquid level control is to keep h_1 and h_2 between the acceptable limits by the measured value which is used as feedback. Deriving the dynamic model of the liquid level control process of two tanks has been done with the following assumptions: the liquid is incompressible; the pressure in both tanks is atmospheric. Flow φ_2 depends on the static pressure determined by the liquid level h1.Flow φ_4 is the instantaneous result of the controlled return pump. Flow$\varphi_3$is determined by a control valve or pumps outside the process system and is independent of the variables considered in the model. The equations of each tank are (1) and (2).

$$A_1\rho\dot{h}_1(t) = \rho[\varphi_1(t) + \varphi_4(t) - \varphi_2(t)](1)$$

$$A_2\rho\dot{h}_2(t) = \rho[\varphi_2(t) - \varphi_4(t) - \varphi_3(t)](2)$$

Where, ρ is the volume density of the liquid in kg/m^3, A_1 and A_2 are the cross-sectional area of each tank in m^2.Flow φ_2 is related to the height of tank 1 by (3).K depends on the control valve and is constant. At the equilibrium points$(h_{k0} - \varphi_{i0})$, the equations are linearized.

$$\varphi_2(t) = K\sqrt{h_1(t)}(3)$$

$$\varphi_i(t) = \varphi_{i0} + \tilde{\varphi}_i(t) \;\; i = 1{:}4$$

$$h_k(t) = h_{k0} + \tilde{h}_k(t) \quad k = 1{:}2 \quad (4)$$

Equation (3) is linearized as (5) and equations (1 and 2) are linearized as (6).

$$\tilde{\varphi}_2(t) = \frac{K}{2\sqrt{h_{10}}}\dot{\tilde{h}}_1(t) = \frac{\varphi_{20}}{2h_{10}}\dot{\tilde{h}}_1(t)(5)$$

$$\begin{bmatrix}\dot{\tilde{h}}_1(t)\\ \dot{\tilde{h}}_2(t)\end{bmatrix} = \begin{bmatrix}-\frac{\varphi_{20}}{2A_1h_{10}} & 0\\ \frac{\varphi_{20}}{2A_1h_{10}} & 0\end{bmatrix}\begin{bmatrix}\tilde{h}_1(t)\\ \tilde{h}_2(t)\end{bmatrix} + \begin{bmatrix}\frac{1}{A_1} & \frac{1}{A_1}\\ 0 & -\frac{1}{A_2}\end{bmatrix}\begin{bmatrix}\tilde{\varphi}_1(t)\\ \tilde{\varphi}_4(t)\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{A_2}\end{bmatrix}\tilde{\varphi}_3(t)(6)$$

Assuming the numerical values A_1=A_2=1, $h_{1.0}$=1,

$\varphi_{10} = \varphi_{40} = 1 \;\; \varphi_{20} = 2\varphi_{10}$,Equation (7) is obtained in the Laplace domain. The effect of inputs φ_1 and φ_2 on outputs 1 and 2 is as (8).

$$\begin{bmatrix}H_1(s)\\ H_2(s)\end{bmatrix} = \begin{bmatrix}\frac{1}{s+1} & \frac{1}{s+1}\\ \frac{1}{s(s+1)} & -\frac{1}{s+1}\end{bmatrix}\begin{bmatrix}\emptyset_1(s)\\ \emptyset_4(s)\end{bmatrix} + \begin{bmatrix}0\\ -\frac{1}{s}\end{bmatrix}\emptyset_3(s)\ (7)$$

$$P(s)=\begin{bmatrix}\frac{1}{s+1} & \frac{1}{s+1}\\ \frac{1}{s(s+1)} & -\frac{1}{s+1}\end{bmatrix}(8)$$

3. Stability analysis, interaction and decentralized liquid level control system

Figure. 2 shows a two-input and two-output liquid-level system with two tanks. In multivariable systems, it is important to determine the input-output couples and the amount of interaction [17]. To check the stability, we draw the generalized Nyquist diagram. In SISO systems, we used to draw a Nyquist diagram for one gain and comment on the rest of the gains because the k (gain) equation was linear. In multivariable systems, in general, the relationship is not linear; as a result, the transfer function $|\lambda_i I - G(s)| = 0$, λ_i‘ is obtained from the eigenvalues.λ_iGraphs are called characteristic loci. Then they draw these graphs in a diagram.

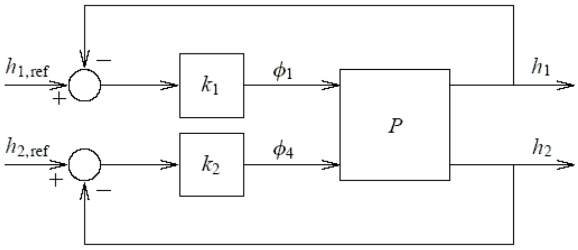

Fig. 2. Two loop feedback control for liquid level system

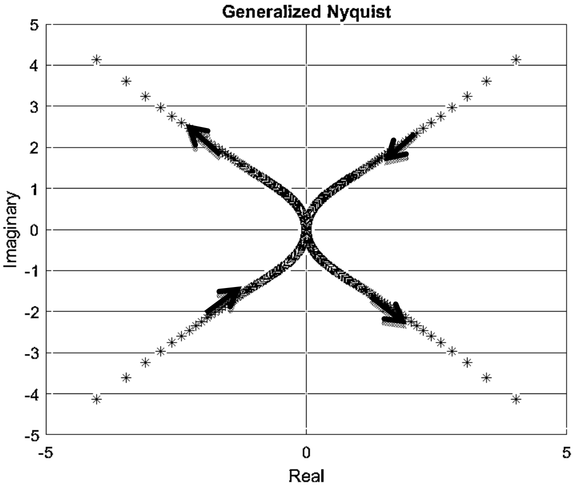

Fig. 3. Generalized Nyquist Diagram for liquid level system

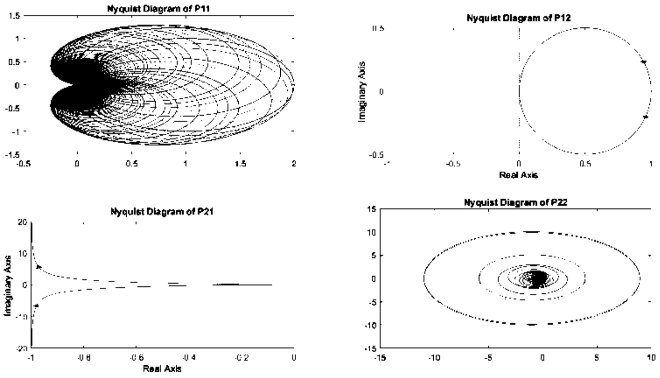

Fig. 4. Gershgorin bands and Nyquist arrays for liquid level system

These graphs are well-behaved and intersect each other at some points. This set of locations is called the generalized Nyquist diagram [18]. According to Figure. 3 and that the poles of the system are 0 and -1, as a result, the system is unstable.Using Gershgorin bands, we want to comment on the diagonal dominance or not, if at any frequency, we draw a circle with center P_{ii}(jw) and radius $\sum_{j=1}^{m} P_{ij}\,(j\omega)\ \ j \neq i$,these circles are called Gershgorin circles and the bands that are created from the formation of this circle at different frequencies are called Gershgorin bands [19]. A system is a diagonal dominance in that none of these bands include the coordinate origin. Figure. 4 shows the Gershgorin bands and Nyquist arrays for the liquid level system. According to Figure4, the system is not diagonal dominance system and the interaction of the system is high.In a MIMO system there's a phenomenon called interference. This phenomenon causes all of the inputs to impact all of the outputs. Mayne recommended that, sometime recently beginning to plan the individual loop compensators, a cross-coupling arrange of compensation ought to be presented. This organize ought to comprise of either a steady gain matrix or a sequence of elementary operations and its reason is to redistribute the “difficulty of control” among the loops. Mayne proposes three compensations or decoupling matrix [20].One way that's compelling in these systems is the decentralized control method in which the system interference is reduced through the decoupling matrix, and after that the control of the system is comparable to a SISO model [21]. We use a pre-compensator to eliminate the interaction. For the liquid level system with two tanks, we introduce the pre-compensator matrix, this matrix is able to convert the mentioned system into a decoupled system (9).

$$K^{pre} = \begin{bmatrix} 1 & -1 \\ \frac{1}{s} & 1 \end{bmatrix} \qquad p(s)K^{pre} = \begin{bmatrix} \frac{1}{s} & 0 \\ 0 & -\frac{1}{s} \end{bmatrix} \tag{9}$$

Now we are dealing with two SISO systems and we can design a diagonal controller. Due to the existence of disturbance and uncertainty and linearization of the system, a robust controller is used [22-23].

4. H_2/H_∞ robust controller

Un-modeled dynamics, non-linearity of systems and the presence of disturbances cause that in most cases, it is not possible to reach the desired response with the theory of linear control systems, for this purpose, several goals are considered in controlling a system [24]. Robust stability, the system is stable despite the uncertainty. Nominal performance is zero without considering the system error uncertainty. Minimizing the effect of disturbance, disturbance can have an adverse effect on the transient response. So, one of the control goals is to minimize the disturbance effect, or noise rejection. Consider the uncertainty with cumulative deviation of Figure. 5 ($\Delta_f = 0$). K controller is designed in such a way that it has three nominal performance, robust stability and noise reduction conditions [25].

Assuming Δ=0, the nominal performance condition is$\|F(s)S\|_\infty \leq 1$, where $S=(I+GK)^{-1}$ and F(s) is the weight function. Assuming Δ≠0and $\bar{\sigma}(\Delta(s)) \leq \beta$ shown in Figure.6, the condition of robust stability is (12)[26];γ (s) is the weight function.

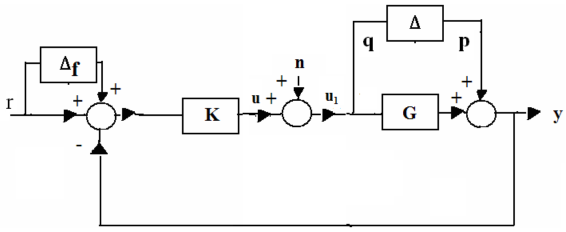

Fig. 5. Closed-Loop System with additive uncertainty

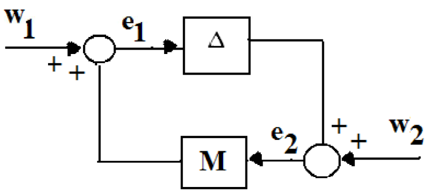

Fig. 6. M-Δ Model

$$\mathbf{q} = (I + KG)^{-1} K \mathbf{p} \quad \Rightarrow \quad \left\| (I + KG)^{-1} K \right\|_\infty < \beta^{-1} \tag{10}$$

$$\| \quad (s)M(s)\|_\infty < 1, \quad M(s) = (I + KG)^{-1}K(11)$$

Reducing the effect of noise with condition$\|R(s)T_{nu1}\|_2 < 1$.R(s) is the weight function.If the white noise input is unity, the variance of u1 will be reduced; it can be written as equation (12).

$$\|T_{nu1}\|_2 \leq \|T_{nu1}\|_\infty \leq 1 \tag{12}$$

From parseval equation and objective 3:

$$\|Y\|_2^2 = \int_0^\infty Y^T(t)Y(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} Y^*(j\omega)Y(j\omega)d\omega \text{ , } Y(j\omega) = G(j\omega)U(j\omega)$$

$$\Rightarrow \|Y\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} U^*(j\omega)G^*(j\omega)G(j\omega)U(j\omega)d\omega$$

$$\leq \frac{1}{2\pi}\sup\bar{\sigma}(G^*(j\omega)G(j\omega))\int_{-\infty}^{\infty} U^*(j\omega)U(j\omega)d\omega$$

$$= \|G(s)\|_\infty^2 \|U(s)\|_2^2 \Rightarrow \sup\frac{\|Y\|_2^2}{\|U\|_2^2} = \|G(s)\|_\infty^2 \quad \rightarrow \quad \frac{\|U_1\|_2}{\|n\|_2} \leq \left\|T_{nU_1}\right\|_\infty < 1$$

(13)

So there are three conditions$\|F(s)S\|_\infty < 1, \| \quad (s)M\|_\infty < 1, \|R(s)T_{nu1}\|_\infty < 1$for design. It is enough to solve problem$\left\| \begin{matrix} F(s)S(KG) \\ (s)M(KG) \\ R(s)T_{nu1}(KG) \end{matrix} \right\|_\infty < 1$. A huge class of systems with uncertainty can be treated as LFT (Linear Factorial Transform).LFT model has appeared in Figure 7 [24] in which, W: the disturbance signals to the system which won't be a function of states of the system, Z: the variable that will be controlled, P: the nominal open loop system, Y: the system's measurable output. The proposed problem can be drawn as Figure. 8. It is important to determine the three weight matrices F(s),γ(s) and R(S) in Figure 8. For F and R ,γ we use inverse sensitivity function.

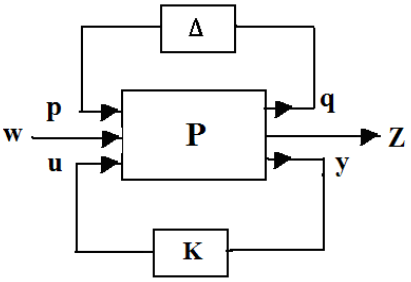

Fig. 7.LFT Model

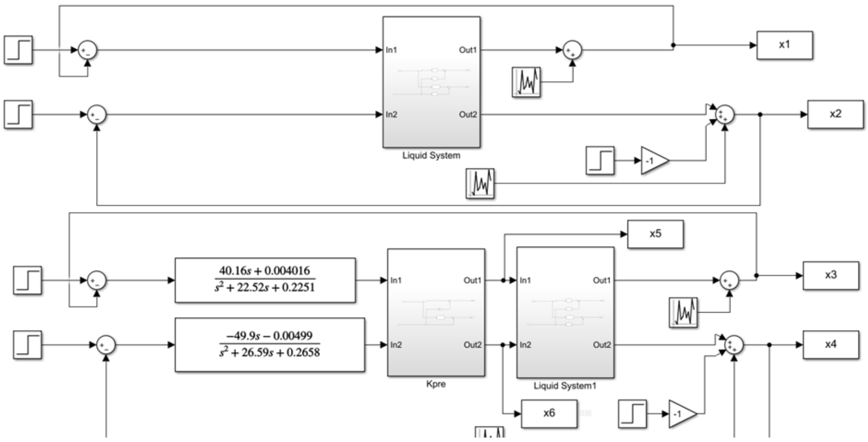

Fig. 9. The simulation of the liquid level system with two tanks and feedback in two states with/without controller

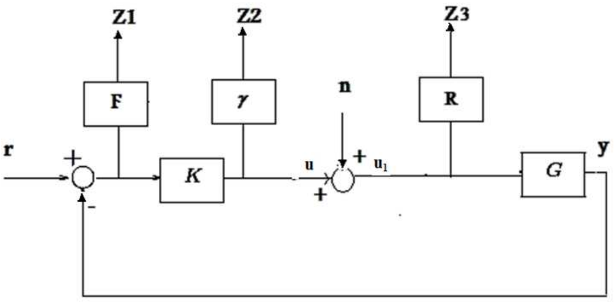

Fig.8. Robust Stability+Nominal Performance+ disturbance rejection

5. Simulation results

The simulation is done in MATLAB software. Figure. 9 shows the simulation of the liquid level system with two tanks and feedback in two states with/without the decentralized control structure is broadly utilized in numerous industrial multivariable processes. In this approach, the control structure plan and in specific input–output pairing may be an imperative arrangement within the plan strategy. There are a few effective strategies to choose the suitable input–output pair in linear multivariable plants. Before designing the controller, we apply the first input with the unit step value and the second input with zero value to the system. The first output is as Figure. 10 and the dual output is as Figure. 11.First, the system has strong interaction because the second output reaches -11 when the second input is zero; second, the system is unstable. Figures. 12 and 13 show the first and second outputs for the first input with a zero value and the second input with a unit step value. According to Figures.10-13, input 1→output 1 and input 2→output 2 were selected. The most straightforward approach to a multivariable design is to disregard its multivariable nature. A SISO controller is designed for one pair of input and output variables. When this design has been effectively completed another SISO controller is designed for a second pair of variables. To design the controller, first according to (9) the pre-compensator for decoupling is used and then the H_2/H_∞ robust controller is designed in a decentralized manner. The response of the first output step is shown in Figure. 14 and the response of the second output step is shown in Figure. 15.

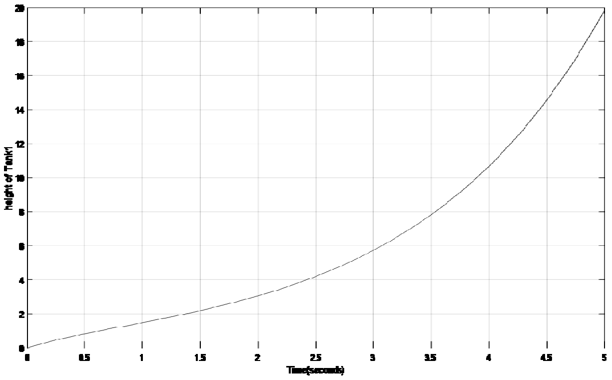

Fig.10. Step Response of closed-Loop System Without Controller, Liquid Level Tank1, u1=1, u2=0

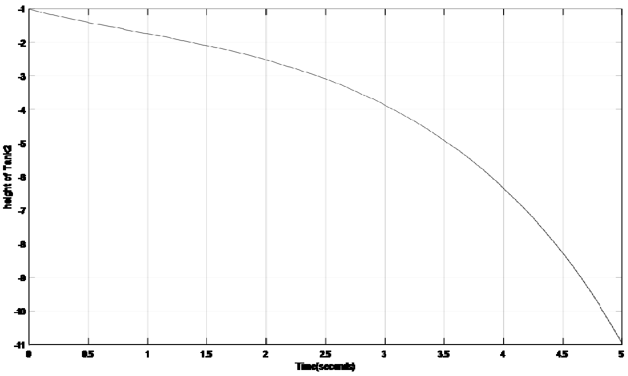

Fig .11. Step Response of closed-Loop System Without Controller, Liquid Level Tank2, u1=1, u2=0

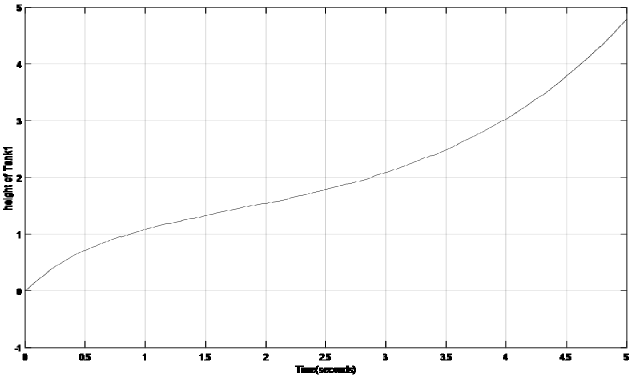

Fig. 12. Step Response of closed-Loop System without Controller, Liquid Level Tank1, u1=0, u2=1

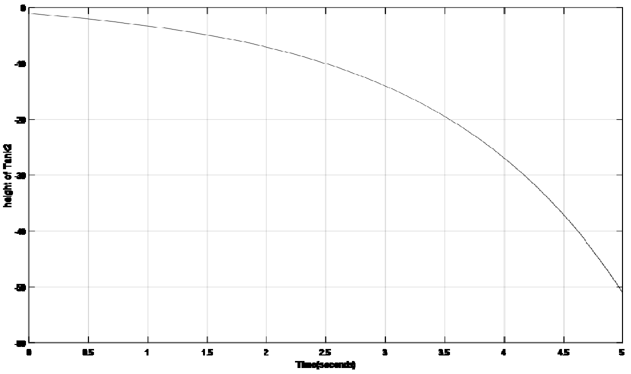

Fig. 13. Step Response of closed-Loop System without Controller, Liquid Level Tank2, u1=0, u2=1

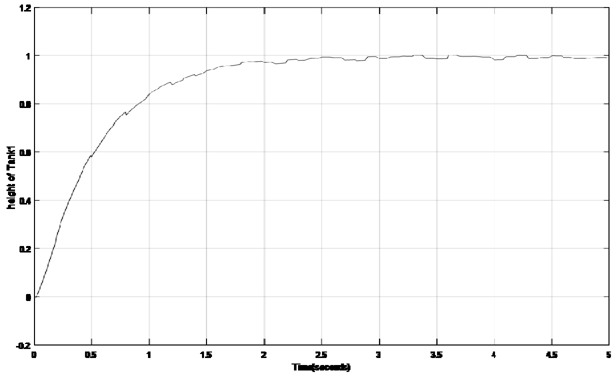

Fig. 14. Step Response of closed-Loop System with Robust Controller, Liquid Level Tank1, u1=1, u2=1

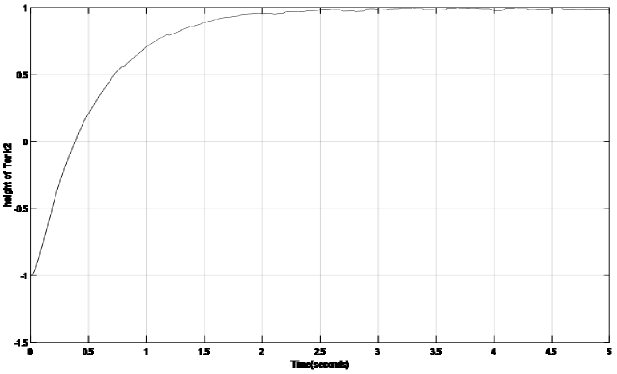

Fig. 15. Step Response of closed-Loop System with Robust Controller, Liquid Level Tank2, u1=1, u2=1

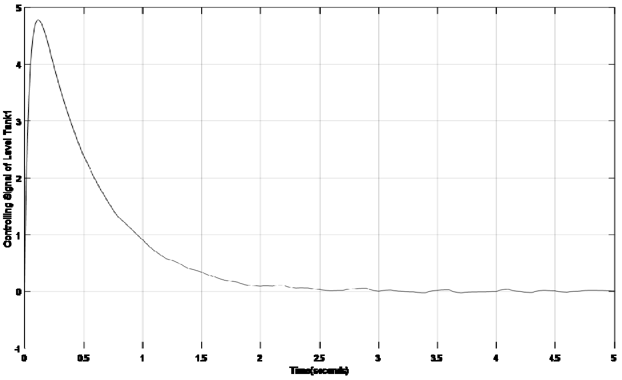

Fig.16. Controlling Signal of Level Tank1, u1=1, u2=1

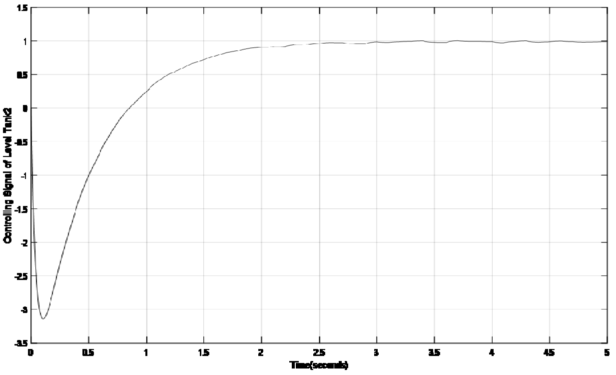

Fig. 17. Controlling Signal of Level Tank2, u1=1, u2=1

Figures. 16 and 17 show the control signal of two tanks. Although the disturbance is considered for both tanks and the uncertainty for the second tank, two independent H_2/H_∞ robust controllers have been designed with the minimum order (second order). Also, the maximum range of the control signal for the first tank is 4.78, and for the second tank is 3.15.

6. Conclusion

In this paper, stability analysis has been done using a generalized Nyquist diagram and interaction by Gershgorin bands for the liquid level system with two tanks and feedback. Considering that the system has two inputs and two outputs and its interaction is high and unstable. The most straightforward approach to a multivariable design is to disregard its multivariable nature. A SISO controller is designed for one pair of input and output variables. In the first step, input-output pairing is selected and the pre-compensator is designed to eliminate the decoupling system and interaction. Several goals are considered in controlling a system. 1. robust stability, the system stable despite the uncertainty. 2. Nominal performance, without considering the uncertainty, error of system is zero. 3. minimizing the effect of the disturbance, and disturbance can hurt the transient response. So, one of the control targetsis to minimize the disturbance effect or noise rejection. A SISO controller is designed for one pair of input and output variables. When this design has been effectively completed another SISO controller is designed for a second pair of variables. Then, due to the disturbance in the two tanks and the uncertainty in the second tank, to achieve these goals, a robust H_2/H_∞ controller is designed. The proposed strategy, for the synthesis of the decentralized robust nonlinear multivariable control systems, is affirmed for the two-tank framework.The simulation results show two independent H_2/H_∞ robust controllers are designed with the minimum order (second order) and the maximum range of the controlling signal for the first tank is 4.78 and for the second tank is 3.15.For the future work, it is suggested that in addition to robust stability, robust performance should also be considered.

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