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Fuzzy Cross Efficiency Measurement for Two-Stage Network DEA

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Abstract. The Data Envelopment Analysis (DEA), as a nonparametric method in operational research, is used to measure the efficiency of a set of homogeneous Decision-Making Units (DMU) with the help of linear programming. Up to now, this method has been extended to be used in various fields. For example, cross-efficiency evaluation, Fuzzy Data Envelopment Analysis (FDEA) and the Network Data Envelopment Analysis (NDEA). The existing models do not work when a decision maker tries to measure the efficiency under all these conditions, and thus there is a need for a unified model that considers all these conditions. In the present study, we present several models to measure the fuzzy cross efficiency of a general two-stage system. The aim of this study is to use the proposed models in the banking industry. In this regard, a case study is conducted to rank the branches of one of Iranian banks. All the proposed models are linear, and some of them are able to calculate all the lower, median and upper bounds of the fuzzy number by solving one model. Finally, we compare the results derived using the proposed models.

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1. Introduction

DEA is a scientific tool that deals with efficiency measurement. The first DEA model was proposed by Charnes, Cooper, and Rhodes [7], which is also known as the CCR model. The CCR model was rapidly expanded by various researchers. The aim of the researchers is to make the proposed model applicable to the real-world problems, reduce its defects, and increase the accuracy and validity of the results derived from the solution of the model. Cross efficiency models were presented to rank efficient DMUs and solve problems arising from self-evaluation and multiple optimum solutions of the traditional models. Network models were developed to measure the efficiency of sub-DMUs in systems with internal processes. Moreover, there are some models that deal with interval, fuzzy, probabilistic,

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and uncertain data.
DEA is widely used in various production and service sectors. The banking industry is one of the important service sectors of any country, so that its performance evaluation is a necessity in today's competitive economic environment. Today, due to the expansion of information technology in society, traditional banking has turned into digital banking. For this reason, banks have adopted the policy of reducing and merging branches. It seems necessary to calculate efficiency in order to identify inefficient branches for integration. The banking procedure is considered by various researcher as a two-stage process with two production and financial intermediation sectors. The present study makes an attempt to measure the fuzzy efficiency of the divisions and rank the units using cross efficiency evaluation. Currently, there is no unified model that takes all these conditions into account, and it is not possible to use multiple models. In this research, several models are proposed to measure the fuzzy cross efficiency in a general two-stage network system. Then the proposed models are implemented on a case study of 105 branches of an Iranian bank. The scope of this article is similar to that of our other research [39] with the difference that in the current research we use the fuzzy data instead of the interval data.
The rest of this study is organized as follows: in Section 2, the literature on DEA, cross efficiency, network efficiency, and fuzzy efficiency are reviewed. In Section 3, several models are proposed to measure the fuzzy cross efficiency in a general two-stage network system. Section 4 is dedicated to discussing the applications of the proposed models in the banking industry and its implementation on a case study. Finally, the conclusions and suggestions for future research are presented in Section 5.

2. Literature review

Economic development of countries has caused many environmental problems [32]. Every organization, institution or company has goals, strategies, and policies within its activity scope. Responding the stakeholder and customer expectations, maintaining the organization survival, having presence in global markets, etc. requires considering the performance management process, which is not possible without performance evaluation. The traditional performance evaluation methods were initially developed based almost entirely on financial indicators. After that, efficiency and effectiveness indicators were used to evaluate the organization's performance, until the notion of DEA was presented to measure efficiency.

The historical background and the beginning of the DEA discussion goes back to Rhodes' doctoral thesis [7]. In collaboration with Cooper and Charnes, he evaluated American national schools, they developed Farrell's method [15], which had two inputs and one output, and proposed the CCR model. Their CCR model was capable of solving problems with multiple inputs and outputs and was based on constant return to scale. Later, Banker, Charnes and Cooper [3] proposed the BCC model, which was based on variable return to scale. Then Charnes et al. [9] presented the multiplicative model and in another research Charnes et al. [8] provided the Slacks-Based Model (SBM). Kaplan and Norton [27] took effective steps and developed the balanced scorecard model.
As a nonparametric linear programming model, DEA measures the efficiency of a number of homogeneous units with multiple inputs and outputs. Homogeneous units have the same inputs and outputs. The first DEA model, known as the fractional CCR model, defines the efficiency of each unit as the ratio of the weighted outputs to weighted inputs. The objective function of this model is to increase the efficiency of DMUs, provided that the maximum value of this fraction does not exceed the interval [0, 1] for each of the units. The weights of inputs and outputs are considered the decision variables of the model.
Let j is a set with n homogeneous units. Each unit is associated with two sets i and r with m inputs and s outputs, respectively. The values of input and output parameters are

represented by x and y, respectively. For each input x_{ij} there is a weight v_i, and for each output y_{rj} there is a weight u_r. The efficiency of DMU_d, $d\epsilon j$ is denoted by E_d. Model (1) is the fractional CCR model of constant return scale, which is transformed into a linear Model (2) by change of variable method proposed by Charnes and Cooper [10]. This model is called the output-oriented model as it increases the outputs by keeping the inputs constant. To measure the efficiency of each unit, the model must be solved individually. Efficient units have an efficiency value of 1 and are located on the frontier of the Production Possibility Set (PPS).

$$
\begin{aligned}
E_d \ &= max \frac{\sum_{r=1}^{s} u_r \ y_{rd}}{\sum_{i=1}^{m} v_i \ \ x_{id}} \\
\frac{\sum_{r=1}^{s} u_r \ y_{rj}}{\sum_{i=1}^{m} v_i \ \ x_{ij}} &\le 1 \quad There\ exists\ j = 1, \dots, n \\
u_r \ &\ge 0 \quad For\ all\ r = 1, \dots, s \\
v_i \ &\ge 0 \quad For\ all\ i = 1, \dots, m.
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
E_d \ &= max \sum_{r=1}^{s} u_r \ y_{rd} \\
&\sum_{i=1}^{m} v_i \ \ x_{id} = 1 \\
\sum_{r=1}^{s} u_r \ y_{rj} - \sum_{i=1}^{m} v_i \ \ x_{ij} &\le 0 \quad There\ exists\ j = 1, \dots, n \\
u_r \ &\ge 0 \quad For\ all\ r = 1, \dots, s \\
v_i \ &\ge 0 \quad For\ all\ i = 1, \dots, m.
\end{aligned} \tag{2}
$$

There are two major problems in traditional models. The first problem is the lack of differentiation between efficient units, as they all have an efficiency value of 1. In this regard, ranking models were developed to solve this problem. Andersen-Petersen's super-efficiency model [2], also known as AP, is one of the famous ranking models. The ranking models have been evaluated by Hosseinzadeh Lotfi et al. [22]. The second problem is the difference in the optimal weights of the units. In other words, the significance of inputs and outputs is different from each unit's point of view. To solve this problem, common weight models were developed, e.g. see Jahanshahloo et al. [23].

2.1. *Cross efficiency*

In traditional models, each DMU tries to increase its efficiency by giving arbitrary weights to inputs and outputs. Accordingly, the measured efficiency is known as self-evaluation. When each unit is evaluated with optimal weights of other units, it is called peer-evaluation. Sexton [36] was the first who gave the idea of forming a cross-efficiency matrix, in which the main diagonal represents the self-evaluation efficiency and the other elements are the efficiency measured using the optimal weights of other units. The cross-efficiency matrix is shown in Table 1, where E_dk denotes the measured efficiency of DMUd with the optimal weights of DMUk, as shown in *Eq. (3)*. Finally, the row average of the matrix represents the DMUd efficiency.

Table 1- Cross efficiency matrix.

		Rated DMU_k			
Rating		1	2	…	n
	1	E_{11}	E_{12}	E_{1k}	E_{1n}

$$\begin{array}{llllll} \text{DMU}_d & 2 & E_{21} & E_{22} & \dots & E_{2n} \\ & \vdots & E_{d1} & \vdots & \ddots & \\ & n & E_{n1} & E_{n2} & \dots & E_{nn} \end{array}$$

$$E_{dk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rd}}{\sum_{i=1}^{m} v_{ik} x_{id}}. \quad (3)$$

Although cross-efficiency solves the major problems of traditional models, it has its own problems, some of which have been pointed out by Song and Liu [38] as follows:

1. The use of average weights has discontinuity [11]. Other central tendency and dispersion measures can be used as well.

2. Average cross efficiency is not Pareto efficiency [42].

3. The self-evaluation efficiency, which is located in the main diagonal of the matrix, plays a significant role in measuring the cross efficiency [41]. While the weight assigned to them is different in DMUs.

4. Average cross efficiency does not reflect the actual performance of all DMUs, as this method simply sums them equally by ignoring the relative significance [40][6].

5. Measuring the row average means that we have assigned the same value to all the elements; i.e. for the self-evaluation efficiency, which is the main diagonal of the matrix, we have assigned a value of 1/n units and for peer evaluation efficiency (n-1)/n units, which does not seem to be a logical choice.

6. Traditional models have multiple optimal solutions for some units. Using each of these solutions will lead to a different outcome in cross efficiency.

7. Cross efficiency has contradictory and unbalanced evaluation, as each unit may determine a different total (or average) value in other units. Different values mean that units with lower efficiency have more effect in the cross-efficiency measurement and vice versa. In other words, if we get the efficiency of the units with the weights of an inefficient unit and add them together, the derived value is greater than the case when we measure the efficiency of the units with the weights of an efficient unit. This means that some units are benevolent and others are not.

Various methods have been proposed to solve the problems related to cross efficiency, one of the most prominent of which is known as the secondary goal. The secondary goal functions are used to solve the non-uniqueness of the optimal weights. The secondary goal acts in the following way: in the first step, Model (1) is solved to measure the DMU efficiency. Then, in the second step, by solving another model, unique weights are obtained to be used for measuring the cross efficiency. The second model deals with the optimization of the secondary goal, provided that the efficiency remains at the level measured in the first step. Two famous models in this field include the benevolent and aggressive models presented by Doyle and Green [12]. Their idea was to measure the weights giving rise to the maximum or minimum sum of efficiencies for all the units except the unit under evaluation, provided that the DMU efficiency remains at the CCR level.
Among other famous approaches, the approach presented by Oral et al. [33] can be mentioned. In the first step, they measured the efficiency of E_{dd} by the use of Model (1). In the second step, they optimized the efficiency of DMU_k in the objective function,

provided that DMU_d remains at the optimal value of E_{dd}. We will use their approach in the present study. Model (4) shows the mentioned approach.

$$
\begin{aligned}
E_{dk} &= max \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \\
\frac{\sum_{r=1}^{s} u_{rk} y_{rd}}{\sum_{i=1}^{m} v_{ik} x_{id}} &= E_{dd} \\
\frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} &\le 1 \quad For\ all\ j = 1, \dots, n \\
u_{rk} &\ge 0 \quad For\ all\ r = 1, \dots, s \\
v_{ik} &\ge 0 \quad For\ all\ i = 1, \dots, m.
\end{aligned} \tag{4}
$$

2.2. *Network DEA*

Most studies conducted on DEA consider the whole system as a black box, i.e. they ignore the internal processes of the system. Decision maker can find it very useful to measure the efficiency of the subsystems in order to identify the weak points of the units. Some studies, e.g. Kao and Hwang [25] and Castelli et al. [6], have shown that ignoring the operation of subsystems can lead to misleading results. For example, the system can be efficient while all the subsystems are inefficient; or all subsystems of a DMU can have greater or equal efficiency when comparing them with each other, while the efficiency of the whole system is much lower. Kao and Liu [26] presented cross efficiency models for two series and parallel network systems. Kao [24] comprehensively reviewed the Network Data Envelopment Analysis (NDEA) in a book with the same title.

One of the applications of DEA is to compare the efficiency of branches in the banking industry. Paradi and Zhu [34] reviewed the literature on DEA in the banking industry. Kassani et al. [28] proposed an integrated approach based on Data Envelopment Analysis (DEA), Clustering algorithms and Polynomial Pattern Classifier for constructing a classifier to identify class of bank branches. A large number of studies consider the banking industry as a two-stage process of production and financial intermediation. In the present study, we implemented our models on a general two-stage system. Figure 1 depicts the structure of a general two-stage system. In this system, the inputs of the first stage, i.e. $x1_i, i = 1, \dots, m_1$, are used to produce the outputs of the first stage. i.e, $y1_r, r = 1, \dots, s_1$, as well as the intermediate products, i.e. $z_g, g = 1, \dots, h$; and the inputs of the second stage, i.e. $x2_i, i = m_{1+1}, \dots, m$, and the intermediate products produce the outputs of the second stage, i.e. $y2_r, r = s_{1+1}, \dots, s$. The efficiencies of the first stage, the second stage, and the whole system are represented by $E1_d$, $E2_d$, and E_d, respectively.

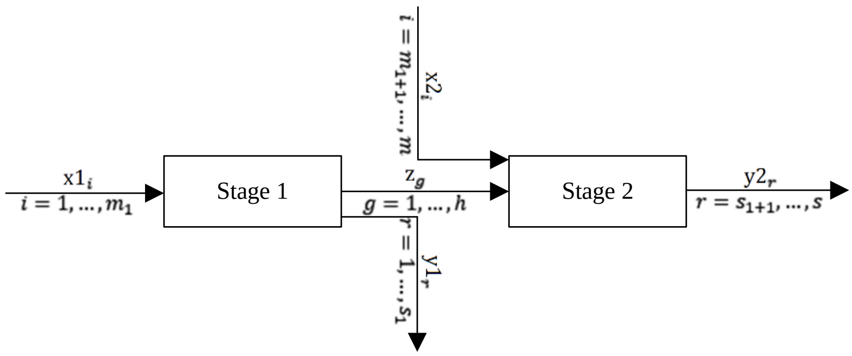

Figure 1- General two-stage system.

Akbarian et al. [1] presented a network-based data envelope analysis model in a dynamic Balanced Score Card (BSC). In their paper, an integrated framework of the BSC and DEA models is proposed for measuring the efficiency during the time and along with strategies based on the time delay of the lag Key Performance Indicators (KPIs) of the BSC model. In *chapter 11* of Kao's book [24], he has discussed various methods of measuring efficiency in a general two-stage structure. In the current study, we use the mentioned linear models. The simplest case is the use of the arithmetic average of the efficiencies of two stages as the efficiency of the whole system. In this case, the efficiency of the first stage, the second stage, and the whole system can be measured using Model (5) and Model (6) and Eq. (7), respectively.

$$
\begin{aligned}
E1_d &= max \sum_{r=1}^{s_1} u_r \; y1_{rd} + \sum_{g=1}^{h} w_g \; z_{gd} \\
&\sum_{i=1}^{m_1} v_i \; x1_{id} = 1 \\
&\sum_{r=1}^{s_1} u_r \; y1_{rj} + \sum_{g=1}^{h} w_g \; z_{gj} - \sum_{i=1}^{m_1} v_i \; x1_{ij} \leq 0 \quad For\ all\ j = 1, \dots, n \\
&u_r \geq 0 \quad For\ all\ r = 1, \dots, s_1 \\
&v_i \geq 0 \quad For\ all\ i = 1, \dots, m_1 \\
&w_g \geq 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned}
\tag{5}
$$

$$
\begin{aligned}
E2_d &= max \sum_{r=s_1+1}^{s} u_r \; y2_{rd} \\
&\sum_{i=m_{1+1}}^{m} v_i \; x2_{id} + \sum_{g=1}^{h} w_g \; z_{gd} = 1 \\
&\sum_{r=s_{1+1}}^{s} u_r \; y2_{rj} - \sum_{i=m_{1+1}}^{m} v_i \; x2_{ij} - \sum_{g=1}^{h} w_g \; z_{gj} \leq 0 \quad For\ all\ j = 1, \dots, n \\
&u_r \geq 0 \quad For\ all\ r = s_{1+1}, \dots, s \\
&v_i \geq 0 \quad For\ all\ i = m_{1+1}, \dots, m \\
&w_g \geq 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned}
\tag{6}
$$

$$
E_d = \frac{1}{2}\left(E1_d + E2_d\right). \tag{7}
$$

Liang et al. [30] proposed a game approach, in which two stages are regarded as two players who try to increase the efficiency of the whole system. This game can be played in two ways: with or without cooperation. For the case of without cooperation, which is a linear model, one stage is the leader and the other is the follower. In this way, first, the efficiency of the leader is measured using Model (5) or Model (6), and then the efficiency of the follower is measured by adding Constraints (8) or Constraints (9), which guarantees that the efficiency of the leader remains at the previously measured value. Finally, the efficiency of the whole system is measured by averaging using Eq. (7).

$$\sum_{r=1}^{s_1} u_r\ y1_{rd} + \sum_{g=1}^{h} w_g\ z_{gd} - E1_{dd} \sum_{i=1}^{m_1} v_i\ x1_{id} = 0$$
$$\sum_{r=1}^{s_1} u_r\ y1_{rj} + \sum_{g=1}^{h} w_g\ z_{gj} - \sum_{i=1}^{m_1} v_i\ x1_{ij} \le 0 \quad For\ all\ j = 1, \dots, n. \tag{8}$$

$$\sum_{r=s_{1+1}}^{s} u_r\ y2_{rj} - E2_{dd}\left(\sum_{i=m_{1+1}}^{m} v_i\ x2_{ij} + \sum_{g=1}^{h} w_g\ z_{gj}\right) = 0$$
$$\sum_{r=s_{1+1}}^{s} u_r\ y2_{rj} - \sum_{i=m_{1+1}}^{m} v_i\ x2_{ij} - \sum_{g=1}^{h} w_g\ z_{gj} \le 0 \quad For\ all\ j = 1, \dots, n. \tag{9}$$

There is another approach that aggregates the efficiencies. Unlike the previous approaches, this approach first measures the overall efficiency, followed by measuring the efficiency of the stages. This approach measures the overall efficiency by calculating the weighted average of the efficiency of the stages. The weight of each stage is calculated as the ratio of that stage input to the whole system inputs. After simplification and linearization, the efficiency of the whole system is measured using Model (10).

$$\begin{aligned}
E_d &= max \sum_{r=1}^{s} u_r\ y_{rd} + \sum_{g=1}^{h} w_g\ z_{gd} \\
&\sum_{i=1}^{m} v_i\ x_{id} + \sum_{g=1}^{h} w_g\ z_{gd} = 1 \\
&\sum_{r=1}^{s_1} u_r\ y1_{rj} + \sum_{g=1}^{h} w_g\ z_{gj} - \sum_{i=1}^{m_1} v_i\ x1_{ij} \le 0 \quad For\ all\ j = 1, \dots, n \\
&\sum_{r=s_{1+1}}^{s} u_r\ y2_{rj} - \sum_{i=m_{1+1}}^{m} v_i\ x2_{ij} - \sum_{g=1}^{h} w_g\ z_{gj} \le 0 \quad For\ all\ j = 1, \dots, n \\
&u_r \ge 0 \quad For\ all\ r = 1, \dots, s \\
&v_i \ge 0 \quad For\ all\ i = 1, \dots, m \\
&w_g \ge 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned} \tag{10}$$

2.3. *Fuzzy DEA*

In the real world, humans understand and use many concepts in a fuzzy manner (i.e. imprecise, vague and ambiguous). For example, although words and concepts such as hot, cold, long, short, old, young, and the like do not refer to a specific and precise number, the human mind understands them all with surprising speed and flexibility and use them to make decisions and conclusions. Fuzzy logic first appeared in the scene of novel computing after the formulation of the theory of fuzzy sets by Zadeh [44]. In fuzzy logic, each element of an uncertain set is assigned a membership value from the interval [0, 1]. A membership value of zero for an element of a set means that that member does not exist in the set, and a membership value of one for an element indicates that the member is completely included in the set.

First, Lertworasirikul [29] and then Hatami-Marbini et al. [20] in their review articles have divided and described fuzzy data envelopment analytical solution methods in 4 main groups, i.e. tolerance approach, α-level based approach, ranking approach, and possibility

approach, among which the α-level based approach has received more attention. In the the tolerance approach, Sengupta [35] presented the first DEA model by defining the tolerance level in the objective function and constraints. The main idea of the α-level based approach is to transform the fuzzy DEA model into a pair of parametric programming in order to find the lower and upper bounds of the α-level efficiency score of the membership function. For the first time, Girod [17] used Carlsson and Korhonen's method [5] to model the fuzzy BCC in his doctoral thesis in order to measure the radial efficiency.
The main idea of the ranking approach is to find the fuzzy efficiency score of DMUs using fuzzy linear programming, which requires the ranking of fuzzy sets. Guo and Tanaka [18] first proposed a fuzzy CCR model that transforms the fuzzy constraints (including fuzzy equalities and inequalities) into crisp constraints by determining a probability level and using the comparison rule for the fuzzy numbers. In fuzzy linear model, fuzzy coefficients can be considered as fuzzy variables and constraints can be considered as fuzzy events. Hence, the possibility of fuzzy events (i.e. fuzzy constraints) can be determined using the possibility theory. Guo et al. [19] presented FDEA models based on possibility and necessity measures.

Before fuzzification of a problem, various aspects of fuzzy problem assumptions should be specified. Although many types of fuzzy numbers with different names and characteristics have been presented and used so far, an important principle in applying fuzzy theory is its computational efficiency. The triangular and trapezoidal fuzzy numbers can be mentioned as examples of commonly used fuzzy numbers with high computational efficiency. Therefore, we use the Triangular Fuzzy Number (TFN) in the present study. TFN can be displayed in two ways. In the first case, it is displayed as F=(l,m,u) with three real numbers. The upper bound *u* and the lower bound *l* are respectively the maximum and minimum values that the fuzzy number F can take and they have the lowest degree of membership. The value of m has the highest membership degree of the fuzzy number. Another type of representation of fuzzy numbers is LR, which for TFN is represented by F=(m,α,β). Here, n is core and α and β are left and right spreads of F, respectively. The membership degree of a TFN can be calculated using *Eq. (11)* and *Eq. (12)* and is displayed in the coordinate system as shown in Figure 2.

$$\mu_f(x) = \begin{cases} \dfrac{x-l}{m-l} & l < x < m, \\ \dfrac{u-x}{u-m} & m < x < u, \\ 0 & else. \end{cases} \quad (11)$$

$$\mu_f(x) = \begin{cases} L\left(\dfrac{m-x}{\alpha}\right) & x \le m\,, \alpha > 0, \\ R\left(\dfrac{x-m}{\beta}\right) & x \ge m\,, \beta > 0. \end{cases} \quad (12)$$

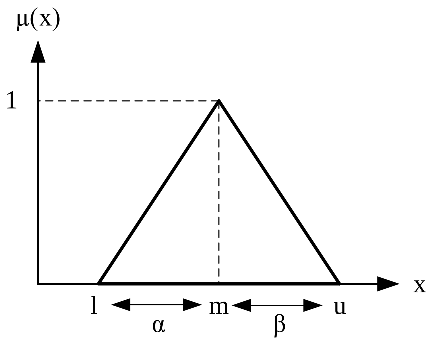

Figure 2. Triangular fuzzy number.

In some problems, only some parts are considered to be fuzzy. For example, only the numbers on the right hand side or the objective function coefficients are considered as fuzzy. However, there are some linear programming problems where all variables and parameters are assumed to be fuzzy. Such problems are known as Fully Fuzzy Linear Programming (FFLP). Model (13) represents an FFLP problem.

$$
\begin{gathered}
Maximize\ (or\ Minimize)\left(\tilde{C}^T \otimes \tilde{X}\right) \\
subject\ to: \\
\tilde{A} \otimes \tilde{X} \leq, =, \geq \tilde{b} \\
\tilde{X}\ is\ a\ non-negative\ fuzzy\ number \\
\tilde{C}^T = \left[\tilde{c}_j\right]_{1\times n}, \tilde{X} = \left[\tilde{x}_j\right]_{n\times 1}, \tilde{A} = \left[\tilde{a}_{ij}\right]_{m\times n}, \tilde{b} = \left[\tilde{b}_i\right]_{m\times 1}, \tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \epsilon F(R).
\end{gathered}
\tag{13}
$$

There are different methods for solving FFLP problems, which are given in the review studies conducted by Ebrahimnejad and Verdegay [13] and Ghanbari et al. [16]. In this research, two fuzzy solution approaches are used, which are explained in the next sections. Hatami Marbini et al. [21] used LR triangular fuzzy numbers in their model and presented Model. The advantage of their model is in reducing the calculations, so that the lower, median, and upper bounds of the optimal value can be calculated by solving only one *model (14)*.

$$
\begin{aligned}
max\ \tilde{E}_d\ &= \sum_{r=1}^{s}[u_r^m\left(y_{rd}^m + \frac{1}{4}y_{rd}^\beta - \frac{1}{4}y_{rd}^\alpha\right) + u_r^\beta(\frac{1}{4}y_{rd}^m) - u_r^\alpha(\frac{1}{4}y_{rd}^m)] \\
s.t{:}\ &\sum_{i=1}^{m}[v_i^m\left(x_{id}^m + \frac{1}{4}x_{id}^\beta - \frac{1}{4}x_{id}^\alpha\right) + v_i^\beta(\frac{1}{4}x_{id}^m) - v_i^\alpha(\frac{1}{4}x_{id}^m)] = 1 \\
&\sum_{r=1}^{s}[u_r^m\left(y_{rj}^m + \frac{1}{4}y_{rj}^\beta - \frac{1}{4}y_{rj}^\alpha\right) + u_r^\beta(\frac{1}{4}y_{rj}^m) - u_r^\alpha(\frac{1}{4}y_{rj}^m)] \\
&\quad \leq \sum_{i=1}^{m}[v_i^m\left(x_{ij}^m + \frac{1}{4}x_{ij}^\beta - \frac{1}{4}x_{ij}^\alpha\right) + v_i^\beta(\frac{1}{4}x_{ij}^m) - v_i^\alpha(\frac{1}{4}x_{ij}^m)] \quad \forall j \\
&\quad = 1, \dots, n\ \ , j \\
&u_r^m - u_r^\alpha \geq 0\ \ For\ all\ r = 1, \dots, s \\
&u_r^m - \frac{1}{4}u_r^\alpha + \frac{1}{4}u_r^\beta \geq 0\ \ For\ all\ r = 1, \dots, s \\
&v_i^m - v_i^\alpha \geq 0\ \ For\ all\ i = 1, \dots, m \\
&v_i^m - \frac{1}{4}v_i^\alpha + \frac{1}{4}v_i^\beta \geq 0\ \ For\ all\ i = 1, \dots, m.
\end{aligned}
\tag{14}
$$

Singh and Yadav [37] presented models for solving problems of Fully Fuzzy Data Envelopment Analysis (FFDEA) using α-cut. They implemented their model on TFN of type $\tilde{F}=(f^l,f^m,f^u)$. Their ranking method works as follows: it first calculates the minimum and maximum efficiency values of the units by solving two models for different α-cuts; and then it ranks the units using a simple calculation algorithm. Only the lower and upper bounds of the efficiency were calculated, because the median value m is the same for all αs. *Model (15)* and *Model (16)* were respectively used to calculate the lower and upper bounds of the efficiency for different αs. The lower bound of efficiency occurs when the DMU is in the worst condition (the lowest output with the highest input) and the other units are in the best condition (the highest output with the lowest input) and vice versa. The constraints for the extremes of the model ensure that the coefficients of the median m are greater than those of the lower bound l, and the upper bound u is greater than the median.

$$\begin{gathered}
E_d^l = max\sum_{r=1}^{s}\alpha u_{rd}^m y_{rd}^m + (1-\alpha)u_{rd}^l y_{rd}^l \\
\sum_{i=1}^{m}\alpha v_{id}^m x_{id}^m + (1-\alpha)v_{id}^m x_{id}^m = 1 \\
\sum_{r=1}^{s}\alpha u_{rd}^m y_{rd}^m + (1-\alpha)u_{rd}^m y_{rd}^m - \sum_{i=1}^{m}\alpha v_{id}^m x_{id}^m + (1-\alpha)v_{id}^u x_{id}^u \le 0 \\
\sum_{r=1}^{s}\alpha u_{rd}^m y_{rj}^m + (1-\alpha)u_{rd}^u y_{rj}^u - \sum_{i=1}^{m}\alpha v_{id}^m x_{ij}^m + (1-\alpha)v_{id}^l x_{ij}^l \le 0 \quad For\ all\ j = 1,\dots,n\ , j\ne d \\
u_{rd}^l - u_{rd}^m \le 0 \quad For\ all\ r=1,\dots,s \\
u_{rd}^m - u_{rd}^u \le 0 \quad For\ all\ r=1,\dots,s \\
v_{id}^l - v_{id}^m \le 0 \quad For\ all\ i=1,\dots,m \\
v_{id}^m - v_{id}^u \le 0 \quad For\ all\ i=1,\dots,m \\
\alpha\epsilon[0,1].
\end{gathered} \tag{14}$$

$$\begin{gathered}
E_d^u = max\sum_{r=1}^{s}\alpha u_{rd}^m y_{rd}^m + (1-\alpha)u_{rd}^u y_{rd}^u \\
\sum_{i=1}^{m}\alpha v_{id}^m \tilde{x}_{id}^m + (1-\alpha)v_{id}^l x_{id}^l = 1 \\
\sum_{r=1}^{s}\alpha u_{rd}^m y_{rd}^m + (1-\alpha)u_{rd}^u y_{rd}^u - \sum_{i=1}^{m}\alpha v_{id}^m x_{id}^m + (1-\alpha)v_{id}^l x_{id}^l \le 0 \\
\sum_{r=1}^{s}\alpha u_{rd}^m y_{rj}^m + (1-\alpha)u_{rd}^l y_{rj}^l - \sum_{i=1}^{m}\alpha v_{id}^m x_{ij}^m + (1-\alpha)v_{id}^u x_{ij}^u \le 0 \quad Forall\ j = 1,\dots,n\ , j\ne d \\
u_{rd}^l - u_{rd}^m \le 0 \quad For\ all\ r=1,\dots,s \\
u_{rd}^m - u_{rd}^u \le 0 \quad For\ all\ r=1,\dots,s \\
v_{id}^l - \tilde{v}_{id}^m \le 0 \quad For\ all\ i=1,\dots,m \\
v_{id}^m - v_{id}^u \le 0 \quad For\ all\ i=1,\dots,m \\
\alpha\epsilon[0,1].
\end{gathered} \tag{15}$$

There are various models that aggregate cross efficiency, uncertainty, and network systems. In another study of ours [39], we presented models to solve the interval cross efficiency of a general two-stage network system. Kao and Liu [26] implemented the cross efficiency method on two series and parallel network systems. Yu and Hou [43] presented a model for interval data by aggregating the cross efficiency and super efficiency methods. Liu and Lee [31] presented a model for ranking based on cross efficiency with fuzzy data. Fan et al. [14] proposed a method to evaluate cross efficiency in an environment of Hesitant Fuzzy Sets (HFS).

3. Solution method

In this section, we present several models for measuring cross efficiency of a fuzzy network. In the cross-efficiency section, we use the approach presented by Oral et al. [33] considering the advantages of using secondary goals in the model. In the network efficiency section, considering the high compatibility of the general two-stage network structure with the banking process, we use the linear approaches presented in *Chapter 11* of Kao's book [24], which include the average approach, the leader and follower approach, and the aggregation approach. The problem data were collected in 12 months of a one-year period. We made TFN based on the minimum, average, and maximum values of the observations, assuming that all parts of the problem are fuzzy. To solve fully fuzzy problems with TFN, we use the two approaches of Hatami-Marbini et al. [21] due to the simplicity in calculations and the approach of Singh and Yadav [37] due to the commonness of the α-cut approach.

In the modeling process, it is complicated and impossible to present the model in one working step. Therefore, we completed and integrated the models step by step, but for the sake of brevity, we present only the final model. In the first step, we combined the cross efficiency and fuzzy mode, and in the second step, we combined the network efficiency and fuzzy mode. Finally, from their combination, we presented the network fuzzy cross efficiency. Combination of different approaches give rise to six models, which are described below.

3.1. *Network Fuzzy Cross Efficiency: Average Approach and α-Cut Approach*

Suppose the efficiency of the whole system is measured by averaging the efficiencies of the stages. In the first step, the CCR efficiency of each stage is measured separately, and in the second step, their cross efficiency is measured. The solution method of Singh and Yadav [37] is as follows: first, the median of TFN is measured, and then the left and right bounds are calculated separately. Thus, for each stage and each step, we need to solve three models, i.e. a total of twelve models are solved. To calculate the lower bound of the fuzzy efficiency of the first step, Stage 1 and Stage 2 using α-cut are presented in Model (17) and Model (18), respectively. In the second step, the cross efficiency of Stage 1 and Stage 2 can be calculated by Model (19) and Model (20), respectively. It is enough to present only the lower bound, as the upper bound can be calculated by changing the index l to u and vice versa, and the median is obtained by setting α=1 in the proposed models.

$$
\begin{aligned}
E1_{dd}^{l} = max \sum_{r=1}^{s_1} \alpha u_{rd}^{m} y1_{rd}^{m} + (1-\alpha) u_{rd}^{l} y1_{rd}^{l} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{l} z_{gd}^{l} \\
\sum_{i=1}^{m_1} \alpha v_{id}^{m} x1_{id}^{m} + (1-\alpha) v_{id}^{u} x1_{id}^{u} = 1 \\
\sum_{r=1}^{s_1} \alpha u_{rd}^{m} y1_{rj}^{m} + (1-\alpha) u_{rd}^{u} y1_{rj}^{u} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gj}^{m} + (1-\alpha) w_{gd}^{u} z_{gj}^{u} \\
- \sum_{i=1}^{m_1} \alpha v_{id}^{m} x1_{ij}^{m} + (1-\alpha) v_{id}^{l} x1_{ij}^{l} \le 0 \quad For\ all\ j = 1, \dots, n, j \neq d \\
\sum_{r=1}^{s_1} \alpha u_{rd}^{m} y1_{rd}^{m} + (1-\alpha) u_{rd}^{l} y1_{rd}^{l} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{l} z_{gd}^{l} \\
- \sum_{i=1}^{m_1} \alpha v_{id}^{m} x1_{id}^{m} + (1-\alpha) v_{id}^{u} x1_{id}^{u} \le 0 \\
u_{rd}^{l} - u_{rd}^{m} \le 0 \quad For\ all\ r = 1, \dots, s_1 \\
u_{rd}^{m} - u_{rd}^{u} \le 0 \quad For\ all\ r = 1, \dots, s_1 \\
v_{id}^{l} - v_{id}^{m} \le 0 \quad For\ all\ i = 1, \dots, m_1 \\
v_{id}^{m} - v_{id}^{u} \le 0 \quad For\ all\ i = 1, \dots, m_1 \\
w_{gd}^{l} - w_{gd}^{m} \le 0 \quad For\ all\ g = 1, \dots, h \\
w_{gd}^{m} - w_{gd}^{u} \le 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned}
\tag{16}
$$

$$E2_{dd}^{l} = max \sum_{r=s_1+1}^{s} \alpha u_{rd}^{m} y2_{rd}^{m} + (1-\alpha) u_{rd}^{l} y2_{rd}^{l}$$

$$\sum_{i=m_1+1}^{m} \alpha v_{id}^{m} x2_{id}^{m} + (1-\alpha) v_{id}^{u} x2_{id}^{u} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{u} z_{gd}^{u} = 1$$

$$\sum_{r=s_1+1}^{s} \alpha u_{rd}^{m} y2_{rj}^{m} + (1-\alpha) u_{rd}^{u} y2_{rj}^{u} - \sum_{i=m_1+1}^{m} \alpha v_{id}^{m} x2_{ij}^{m} + (1-\alpha) v_{id}^{l} x2_{ij}^{l} - \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gj}^{m} + (1-\alpha) w_{gd}^{l} z_{gj}^{l} \leq 0 \quad For\ all\ j = 1, \dots, n, j \neq d$$

$$\sum_{r=s_1+1}^{s} \alpha u_{rd}^{m} y2_{rd}^{m} + (1-\alpha) u_{rd}^{l} y2_{rd}^{l} - \sum_{i=m_1+1}^{m} \alpha v_{id}^{m} x2_{id}^{m} + (1-\alpha) v_{id}^{u} x2_{id}^{u} - \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{u} z_{gd}^{u} \leq 0 \tag{17}$$

$$u_{rd}^{l} - u_{rd}^{m} \leq 0 \quad For\ all\ r = s_1 + 1, \dots, s$$

$$u_{rd}^{m} - u_{rd}^{u} \leq 0 \quad For\ all\ r = s_1 + 1, \dots, s$$

$$v_{id}^{l} - v_{id}^{m} \leq 0 \quad For\ all\ i = m_1 + 1, \dots, m$$

$$v_{id}^{m} - v_{id}^{u} \leq 0 \quad For\ all\ i = m_1 + 1, \dots, m$$

$$w_{gd}^{l} - w_{gd}^{m} \leq 0 \quad For\ all\ g = 1, \dots, h$$

$$w_{gd}^{m} - w_{gd}^{u} \leq 0 \quad For\ all\ g = 1, \dots, h$$

$$\alpha \epsilon [0,1].$$

$$E1_{dk}^{l} = max\sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rk}^{m} + (1-\alpha) u_{rk}^{l} y1_{rk}^{l} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{l} z_{gk}^{l}$$

$$\sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{ik}^{m} + (1-\alpha) v_{ik}^{u} x1_{ik}^{u} = 1$$

$$\sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rj}^{m} + (1-\alpha) u_{rk}^{u} y1_{rj}^{u} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gj}^{m} + (1-\alpha) w_{gk}^{u} z_{gj}^{u} - \sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{ij}^{m} + (1-\alpha) v_{ik}^{l} x1_{ij}^{l} \le 0 \quad For\ all\ j = 1, \dots, n\ , j \neq d, k$$

$$\sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rk}^{m} + (1-\alpha) u_{rk}^{l} y1_{rk}^{l} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{l} z_{gk}^{l} - \sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{ik}^{m} + (1-\alpha) v_{ik}^{u} x1_{ik}^{u} \le 0 \tag{18}$$

$$E1_{dd}^{l} \times \sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{id}^{m} + (1-\alpha) v_{ik}^{u} x1_{id}^{u} = \sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rd}^{m} + (1-\alpha) u_{rk}^{l} y1_{rd}^{l} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gd}^{m} + (1-\alpha) w_{gk}^{l} z_{gd}^{l}$$

$$u_{rd}^{l} - u_{rd}^{m} \le 0 \quad For\ all\ r = 1, \dots, s_1$$

$$u_{rd}^{m} - u_{rd}^{u} \le 0 \quad For\ all\ r = 1, \dots, s_1$$

$$v_{id}^{l} - v_{id}^{m} \le 0 \quad For\ all\ i = 1, \dots, m_1$$

$$v_{id}^{m} - v_{id}^{u} \le 0 \quad For\ all\ i = 1, \dots, m_1$$

$$w_{gd}^{l} - w_{gd}^{m} \le 0 \quad For\ all\ g = 1, \dots, h$$

$$w_{gd}^{m} - w_{gd}^{u} \le 0 \quad For\ all\ g = 1, \dots, h$$

$$\alpha \epsilon [0,1].$$

$$
\begin{aligned}
&E2_{dk}^{l} = max \sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rk}^{m} + (1-\alpha) u_{rk}^{l} y2_{rk}^{l} \\
&\sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{ik}^{m} + (1-\alpha) v_{ik}^{u} x2_{ik}^{u} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{u} z_{gk}^{u} = 1 \\
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rj}^{m} + (1-\alpha) u_{rk}^{u} y2_{rj}^{u} - \sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{ij}^{m} + (1-\alpha) v_{ik}^{l} x2_{ij}^{l} - \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gj}^{m} + (1-\alpha) w_{gk}^{l} z_{gj}^{l} \le 0 \quad For\ all\ j = 1, \dots, n, j \neq d, k \\
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rk}^{m} + (1-\alpha) u_{rk}^{l} y2_{rk}^{l} - \sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{ik}^{m} + (1-\alpha) v_{ik}^{u} x2_{ik}^{u} - \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{u} z_{gk}^{u} \le 0 \\
&E2_{dd}^{l} \times \left(\sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{id}^{m} + (1-\alpha) v_{ik}^{u} x2_{id}^{u} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gd}^{m} + (1-\alpha) w_{gk}^{u} z_{gd}^{(u)} \right) = \sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rd}^{m} + (1-\alpha) u_{rk}^{l} y2_{rd}^{l} \\
&u_{rd}^{l} - u_{rd}^{m} \le 0 \quad For\ all\ r = s_1+1, \dots, s \\
&u_{rd}^{m} - u_{rd}^{u} \le 0 \quad For\ all\ r = s_1+1, \dots, s \\
&v_{id}^{l} - v_{id}^{m} \le 0 \quad For\ all\ i = m_1+1, \dots, m \\
&v_{id}^{m} - v_{id}^{u} \le 0 \quad For\ all\ i = m_1+1, \dots, m \\
&w_{gd}^{l} - w_{gd}^{m} \le 0 \quad For\ all\ g = 1, \dots, h \\
&w_{gd}^{m} - w_{gd}^{u} \le 0 \quad For\ all\ g = 1, \dots, h \\
&\alpha \epsilon [0,1].
\end{aligned} \tag{19}
$$

3.2. *Network Fuzzy Cross Efficiency: Non-Cooperative Game Approach and α-Cut Approach*

In this approach, measuring the efficiency of the first step of CCR requires to measure first the efficiency of the leader stage and then the efficiency of the follower stage, provided that the efficiency of the leader stage remains at the measured value. Measuring the efficiency of the leader is the same as the average method. When the first stage is the leader, the lower bound of the second stage can be measured by adding the Constraint (21) to Model (18). When the second stage is the leader, the efficiency of the first one can be measured by adding the Constraint (22) to Model (17). In the second step, when the first stage is the leader, the lower bound of the cross efficiency of the second stage can be measured by adding the Constraint (23) to Model (20). And when the second stage is the leader, the cross efficiency of the first stage can be measured by adding the Constraint (24) to Model (19). The upper bound of the efficiency can be measured by changing the index l to u and vice versa, and the median can be measured by setting α=1 in the proposed models.

$$\begin{aligned}&\sum_{r=1}^{s_1}\alpha u_{rd}^m y1_{rj}^m+(1-\alpha)u_{rd}^u y1_{rj}^u+\sum_{g=1}^{h}\alpha w_{gd}^m z_{gj}^m+(1-\alpha)w_{gd}^u z_{gj}^u\\&\quad-\sum_{i=1}^{m_1}\alpha v_{id}^m x1_{ij}^m+(1-\alpha)v_{id}^l x1_{ij}^l\le 0\quad For\ all\ j=1,\dots,n\ ,j\neq d\\&\sum_{r=1}^{s_1}\alpha u_{rd}^m y1_{rd}^m+(1-\alpha)u_{rd}^l y1_{rd}^l+\sum_{g=1}^{h}\alpha w_{gd}^m z_{gd}^m+(1-\alpha)w_{gd}^l z_{gd}^l\\&\quad-\sum_{i=1}^{m_1}\alpha v_{id}^m x1_{id}^m+(1-\alpha)v_{id}^u x1_{id}^u\le 0\\&E1_{dd}^l\times\sum_{i=1}^{m_1}\alpha v_{id}^m x1_{id}^m+(1-\alpha)v_{id}^u x1_{id}^u=\sum_{r=1}^{s_1}\alpha u_{rd}^m y1_{rd}^m+(1-\alpha)u_{rd}^l y1_{rd}^l\\&\quad+\sum_{g=1}^{h}\alpha w_{gd}^m z_{gd}^m+(1-\alpha)w_{gd}^l z_{gd}^l.\end{aligned}\tag{20}$$

$$\begin{aligned}&\sum_{r=s_1+1}^{s}\alpha u_{rd}^m y2_{rj}^m+(1-\alpha)u_{rd}^u y2_{rj}^u\\&\quad-\sum_{i=m_1+1}^{m}\alpha v_{id}^m x2_{ij}^m+(1-\alpha)v_{id}^l x2_{ij}^l\\&\quad-\sum_{g=1}^{h}\alpha w_{gd}^m z_{gj}^m+(1-\alpha)w_{gd}^l z_{gj}^l\le 0\quad For\ all\ j=1,\dots,n,j\neq d\\&\sum_{r=s_1+1}^{s}\alpha u_{rd}^m y2_{rd}^m+(1-\alpha)u_{rd}^l y2_{rd}^l\\&\quad-\sum_{i=m_1+1}^{m}\alpha v_{id}^m x2_{id}^m+(1-\alpha)v_{id}^u x2_{id}^u\\&\quad-\sum_{g=1}^{h}\alpha w_{gd}^m z_{gd}^m+(1-\alpha)w_{gd}^u z_{gd}^u\le 0\\&E2_{dd}^l\times\left(\sum_{i=m_1+1}^{m}\alpha v_{id}^m x2_{id}^m+(1-\alpha)v_{id}^u x2_{id}^u+\sum_{g=1}^{h}\alpha w_{gd}^m z_{gd}^m+(1-\alpha)w_{gd}^u z_{gd}^u\right)\\&\quad=\sum_{r=s_1+1}^{s}\alpha u_{rd}^m y2_{rd}^m+(1-\alpha)u_{rd}^l y2_{rd}^l.\end{aligned}\tag{21}$$

$$\begin{aligned}
&\sum_{r=1}^{s_1} \alpha u_{rk}^m y1_{rj}^m + (1-\alpha) u_{rk}^u y1_{rj}^u + \sum_{g=1}^{h} \alpha w_{gk}^m z_{gj}^m + (1-\alpha) w_{gk}^u z_{gj}^u \\
&\quad - \sum_{i=1}^{m_1} \alpha v_{ik}^m x1_{ij}^m + (1-\alpha) v_{ik}^l x1_{ij}^l \le 0 \quad For\ all\ j = 1, \dots, n\ , j \neq k \\
&\sum_{r=1}^{s_1} \alpha u_{rk}^m y1_{rk}^m + (1-\alpha) u_{rk}^l y1_{rk}^l + \sum_{g=1}^{h} \alpha w_{gk}^m z_{gk}^m + (1-\alpha) w_{gk}^l z_{gk}^l \\
&\quad - \sum_{i=1}^{m_1} \alpha v_{ik}^m x1_{ik}^m + (1-\alpha) v_{ik}^u x1_{ik}^u \le 0 \\
&E1_{dk}^l \times \sum_{i=1}^{m_1} \alpha v_{ik}^m x1_{ik}^m + (1-\alpha) v_{ik}^u x1_{ik}^u = \sum_{r=1}^{s_1} \alpha u_{rk}^m y1_{rk}^m + (1-\alpha) u_{rk}^l y1_{rk}^l \\
&\quad + \sum_{g=1}^{h} \alpha w_{gk}^m z_{gk}^m + (1-\alpha) w_{gk}^l z_{gk}^l .
\end{aligned} \tag{22}$$

$$\begin{aligned}
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^m y2_{rj}^m + (1-\alpha) u_{rk}^u y2_{rj}^u \\
&\quad - \sum_{i=m_1+1}^{m} \alpha v_{ik}^m x2_{ij}^m + (1-\alpha) v_{ik}^l x2_{ij}^l \\
&\quad - \sum_{g=1}^{h} \alpha w_{gk}^m z_{gj}^m + (1-\alpha) w_{gk}^l z_{gj}^l \le 0 \quad For\ all\ j = 1, \dots, n, j \neq k \\
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^m y2_{rk}^m + (1-\alpha) u_{rk}^l y2_{rk}^l \\
&\quad - \sum_{i=m_1+1}^{m} \alpha v_{ik}^m x2_{ik}^m + (1-\alpha) v_{ik}^u x2_{ik}^u \\
&\quad - \sum_{g=1}^{h} \alpha w_{gk}^m z_{gk}^m + (1-\alpha) w_{gk}^u z_{gk}^u \le 0 \\
&E2_{dk}^l \times \left(\sum_{i=m_1+1}^{m} \alpha v_{ik}^m x2_{ik}^m + (1-\alpha) v_{ik}^u x2_{ik}^u + \sum_{g=1}^{h} \alpha w_{gk}^m z_{gk}^m + (1-\alpha) w_{gk}^u z_{gk}^u \right) \\
&\quad = \sum_{r=s_1+1}^{s} \alpha u_{rk}^m y2_{rk}^m + (1-\alpha) u_{rk}^l y2_{rk}^l .
\end{aligned} \tag{23}$$

3.3. *Network fuzzy cross efficiency: Aggregation approach and α-cut Approach*

In the aggregation approach, were first measure the efficiency of the whole system and then we measure the efficiencies of the stages. The lower bound of the efficiency of the first stage of CCR and that of the cross efficiency of the second stage are measured using Models (25) and Model (26), respectively. The upper bound of the efficiency can be measured by changing the index l to u and vice versa, and the median can be measured by setting α=1 in the proposed models.

$$
\begin{aligned}
&E_{dd}^{l} = max \sum_{r=1}^{s} \alpha u_{rd}^{m} y_{rd}^{m} + (1-\alpha) u_{rd}^{l} y_{rd}^{l} \\
&\sum_{i=1}^{m} \alpha v_{id}^{m} x_{id}^{m} + (1-\alpha) v_{id}^{u} x_{id}^{u} = 1 \\
&\sum_{r=s_1+1}^{s} \alpha u_{rd}^{m} y2_{rj}^{m} + (1-\alpha) u_{rd}^{u} y2_{rj}^{u} \\
&- \sum_{i=m_1+1}^{m} \alpha v_{id}^{m} x2_{ij}^{m} + (1-\alpha) v_{id}^{l} x2_{ij}^{l} \\
&- \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gj}^{m} + (1-\alpha) w_{gd}^{l} z_{gj}^{l} \leq 0 \quad For\ all\ j = 1, \dots, n, j \neq d \\
&\sum_{r=1}^{s_1} \alpha u_{rd}^{m} y1_{rj}^{m} + (1-\alpha) u_{rd}^{u} y1_{rj}^{u} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gj}^{m} + (1-\alpha) w_{gd}^{u} z_{gj}^{u} \\
&- \sum_{i=1}^{m_1} \alpha v_{id}^{m} x1_{ij}^{m} + (1-\alpha) v_{id}^{l} x1_{ij}^{l} \leq 0 \quad For\ all\ j = 1, \dots, n, j \neq d \\
&\sum_{r=s_1+1}^{s} \alpha u_{rd}^{m} y2_{rd}^{m} + (1-\alpha) u_{rd}^{l} y2_{rd}^{l} \\
&- \sum_{i=m_1+1}^{m} \alpha v_{id}^{m} x2_{id}^{m} + (1-\alpha) v_{id}^{u} x2_{id}^{u} \\
&- \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{u} z_{gd}^{u} \leq 0 \\
&\sum_{r=1}^{s_1} \alpha u_{rd}^{m} y1_{rd}^{m} + (1-\alpha) u_{rd}^{l} y1_{rd}^{l} + \sum_{g=1}^{h} \alpha w_{gd}^{m} z_{gd}^{m} + (1-\alpha) w_{gd}^{l} z_{gd}^{l} \\
&- \sum_{i=1}^{m_1} \alpha v_{id}^{m} x1_{id}^{m} + (1-\alpha) v_{id}^{u} x1_{id}^{u} \leq 0 \\
&u_{rd}^{l} - u_{rd}^{m} \leq 0 \quad For\ all\ r = 1, \dots, s \\
&u_{rd}^{m} - u_{rd}^{u} \leq 0 \quad For\ all\ r = 1, \dots, s \\
&v_{id}^{l} - v_{id}^{m} \leq 0 \quad For\ all\ i = 1, \dots, m \\
&v_{id}^{m} - v_{id}^{u} \leq 0 \quad For\ all\ i = 1, \dots, m \\
&w_{gd}^{l} - w_{gd}^{m} \leq 0 \quad For\ all\ g = 1, \dots, h \\
&w_{gd}^{m} - w_{gd}^{u} \leq 0 \quad For\ all\ g = 1, \dots, h \\
&\alpha \epsilon [0,1].
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
&E_{dk}^{l} = max\sum_{r=1}^{s} \alpha u_{rk}^{m} y_{rk}^{m} + (1-\alpha) u_{rk}^{l} y_{rk}^{l} \\
&\sum_{i=1}^{m} \alpha v_{ik}^{m} x_{ik}^{m} + (1-\alpha) v_{ik}^{u} x_{ik}^{u} = 1 \\
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rj}^{m} + (1-\alpha) u_{rk}^{u} y2_{rj}^{u} \\
&- \sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{ij}^{m} + (1-\alpha) v_{ik}^{l} x2_{ij}^{l} \\
&- \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gj}^{m} + (1-\alpha) w_{gk}^{l} z_{gj}^{l} \leq 0 \quad For\ all\ j = 1, \dots, n, j \neq d, k \\
&\sum_{r=s_1+1}^{s} \alpha u_{rk}^{m} y2_{rk}^{m} + (1-\alpha) u_{rk}^{l} y2_{rk}^{l} \\
&- \sum_{i=m_1+1}^{m} \alpha v_{ik}^{m} x2_{ik}^{m} + (1-\alpha) v_{ik}^{u} x2_{ik}^{u} \\
&- \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{u} z_{gk}^{u} \leq 0 \\
&\sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rj}^{m} + (1-\alpha) u_{rk}^{u} y1_{rj}^{u} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gj}^{m} + (1-\alpha) w_{gk}^{u} z_{gj}^{u} \\
&- \sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{ij}^{m} + (1-\alpha) v_{ik}^{l} x1_{ij}^{l} \leq 0 \quad For\ all\ j = 1, \dots, n, j \neq d, k \\
&\sum_{r=1}^{s_1} \alpha u_{rk}^{m} y1_{rk}^{m} + (1-\alpha) u_{rk}^{l} y1_{rk}^{l} + \sum_{g=1}^{h} \alpha w_{gk}^{m} z_{gk}^{m} + (1-\alpha) w_{gk}^{l} z_{gk}^{l} \\
&- \sum_{i=1}^{m_1} \alpha v_{ik}^{m} x1_{ik}^{m} + (1-\alpha) v_{ik}^{u} x1_{ik}^{u} \leq 0 \\
&E_{dd}^{l} \times \sum_{i=1}^{m} \alpha v_{ik}^{m} x_{id}^{m} + (1-\alpha) v_{ik}^{u} x_{id}^{u} = \sum_{r=1}^{s} \alpha u_{rk}^{m} y_{rd}^{m} + (1-\alpha) u_{rk}^{l} y_{rd}^{l} \\
&u_{rd}^{l} - u_{rd}^{m} \leq 0 \quad For\ all\ r = 1, \dots, s \\
&u_{rd}^{m} - u_{rd}^{u} \leq 0 \quad For\ all\ r = 1, \dots, s \\
&v_{id}^{l} - v_{id}^{m} \leq 0 \quad For\ all\ i = 1, \dots, m \\
&v_{id}^{m} - v_{id}^{u} \leq 0 \quad For\ all\ i = 1, \dots, m \\
&w_{gd}^{l} - w_{gd}^{m} \leq 0 \quad For\ all\ g = 1, \dots, h \\
&w_{gd}^{m} - w_{gd}^{u} \leq 0 \quad For\ all\ g = 1, \dots, h \\
&\alpha \epsilon [0,1].
\end{aligned} \tag{25}
$$

3.4. *Network fuzzy cross efficiency: Average approach and Hatami-Marbini's [39] approach*

In the average approach, to measure the cross efficiency, in the first step, separate CCR efficiencies of the both stages and in the second step, their cross-efficiencies are measured.

Hatami Marbini's et al. [39] solution method works as follows: by solving a model, the values of the upper, left, and right bounds of TFN are determined. They used LR triangular fuzzy numbers in their model. We denote the median of the TFN by m, the left spread by α, and the right spread by β. Thus, we need to solve one model for each step and each stage, and a total of four models must be solved. To measure the fuzzy efficiency of the first step of Stage 1 and Stage 2, Model (27) and Model (28) are presented respectively using Hatami Marbini's et al. [36] approach. In the second step, the cross efficiencies of Stage 1 and Stage 2 can be measured by the use of Model (29) and Model (30), respectively.

$$
\begin{aligned}
E1_{dd} = max \sum_{r=1}^{s_1} &\left[u_{rd}^{m}\left(y1_{rd}^{m} + \frac{1}{4}y1_{rd}^{\beta} - \frac{1}{4}y1_{rd}^{\alpha}\right) + \frac{1}{4}u_{rd}^{\beta}y1_{rd}^{m} - \frac{1}{4}u_{rd}^{\alpha}y1_{rd}^{m}\right] \\
&+ \sum_{g=1}^{h}\left[w_{gd}^{m}\left(z_{rd}^{m} + \frac{1}{4}z_{gd}^{\beta} - \frac{1}{4}z_{gd}^{\alpha}\right) + \frac{1}{4}w_{gd}^{\beta}z_{gd}^{m} - \frac{1}{4}w_{gd}^{\alpha}z_{gd}^{m}\right] \\
\sum_{i=1}^{m_1}&\left[v_{id}^{m}\left(x1_{id}^{m} + \frac{1}{4}x1_{id}^{\beta} - \frac{1}{4}x1_{id}^{\alpha}\right) + \frac{1}{4}v_{id}^{\beta}x1_{id}^{m} - \frac{1}{4}v_{id}^{\alpha}x1_{id}^{m}\right] = 1 \\
\sum_{r=1}^{s_1}&\left[u_{rd}^{m}\left(y1_{rj}^{m} + \frac{1}{4}y1_{rj}^{\beta} - \frac{1}{4}y1_{rj}^{\alpha}\right) + \frac{1}{4}u_{rd}^{\beta}y1_{rj}^{m} - \frac{1}{4}u_{rd}^{\alpha}y1_{rj}^{m}\right] \\
&+ \sum_{g=1}^{h}\left[w_{gd}^{m}\left(z_{rj}^{m} + \frac{1}{4}z_{gj}^{\beta} - \frac{1}{4}z_{gj}^{\alpha}\right) + \frac{1}{4}w_{gd}^{\beta}z_{gj}^{m} - \frac{1}{4}w_{gd}^{\alpha}z_{gj}^{m}\right] \\
&- \sum_{i=1}^{m_1}\left[v_{id}^{m}\left(x1_{ij}^{m} + \frac{1}{4}x1_{ij}^{\beta} - \frac{1}{4}x1_{ij}^{\alpha}\right) + \frac{1}{4}v_{id}^{\beta}x1_{ij}^{m} - \frac{1}{4}v_{id}^{\alpha}x1_{ij}^{m}\right] \\
&\leq 0 \quad For\ all\ j = 1, \dots, n \\
&u_{rd}^{m} - u_{rd}^{\alpha} \geq 0 \quad For\ all\ r = 1, \dots, s_1 \\
&u_{rd}^{m} - \frac{1}{4}u_{rd}^{\alpha} + \frac{1}{4}u_{rd}^{\beta} \geq 0 \quad For\ all\ r = 1, \dots, s_1 \\
&v_{id}^{m} - v_{id}^{\alpha} \geq 0 \quad For\ all\ i = 1, \dots, m_1 \\
&v_{id}^{m} - \frac{1}{4}v_{id}^{\alpha} + \frac{1}{4}v_{id}^{\beta} \geq 0 \quad For\ all\ i = 1, \dots, m_1 \\
&w_{gd}^{m} - w_{gd}^{\alpha} \geq 0 \quad For\ all\ g = 1, \dots, h \\
&w_{gd}^{m} - \frac{1}{4}w_{gd}^{\alpha} + \frac{1}{4}w_{gd}^{\beta} \geq 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned} \tag{26}
$$

$$E2_{dd} = max \sum_{r=s_1+1}^{s} \left[u_{rd}^{m}\left(y2_{rd}^{m} + \frac{1}{4}y2_{rd}^{\beta} - \frac{1}{4}y2_{rd}^{\alpha}\right) + \frac{1}{4}u_{rd}^{\beta}y2_{rd}^{m} - \frac{1}{4}u_{rd}^{\alpha}y2_{rd}^{m}\right]$$

$$\sum_{i=m_1+1}^{m} \left[v_{id}^{m}\left(x2_{id}^{m} + \frac{1}{4}x2_{id}^{\beta} - \frac{1}{4}x2_{id}^{\alpha}\right) + \frac{1}{4}v_{id}^{\beta}x2_{id}^{m} - \frac{1}{4}v_{id}^{\alpha}x2_{id}^{m}\right]$$

$$+ \sum_{g=1}^{h} \left[w_{gd}^{m}\left(z_{rd}^{m} + \frac{1}{4}z_{gd}^{\beta} - \frac{1}{4}z_{gd}^{\alpha}\right) + \frac{1}{4}w_{gd}^{\beta}z_{gd}^{m} - \frac{1}{4}w_{gd}^{\alpha}z_{gd}^{m}\right]$$

$$= 1$$

$$\sum_{r=s_1+1}^{s} \left[u_{rd}^{m}\left(y2_{rj}^{m} + \frac{1}{4}y2_{rj}^{\beta} - \frac{1}{4}y2_{rj}^{\alpha}\right) + \frac{1}{4}u_{rd}^{\beta}y2_{rj}^{m} - \frac{1}{4}u_{rd}^{\alpha}y2_{rj}^{m}\right]$$

$$- \sum_{i=m_1+1}^{m} \left[v_{id}^{m}\left(x2_{ij}^{m} + \frac{1}{4}x2_{ij}^{\beta} - \frac{1}{4}x2_{ij}^{\alpha}\right) + \frac{1}{4}v_{id}^{\beta}x2_{ij}^{m} - \frac{1}{4}v_{id}^{\alpha}x2_{ij}^{m}\right] \tag{27}$$

$$- \sum_{g=1}^{h} \left[w_{gd}^{m}\left(z_{rj}^{m} + \frac{1}{4}z_{gj}^{\beta} - \frac{1}{4}z_{gj}^{\alpha}\right) + \frac{1}{4}w_{gd}^{\beta}z_{gj}^{m} - \frac{1}{4}w_{gd}^{\alpha}z_{gj}^{m}\right]$$

$$\leq 0 \quad For\ all\ j = 1, \dots, n$$

$$u_{rd}^{m} - u_{rd}^{\alpha} \geq 0 \qquad For\ all\ r = s_1 + 1, \dots, s$$

$$u_{rd}^{m} - \frac{1}{4}u_{rd}^{\alpha} + \frac{1}{4}u_{rd}^{\beta} \geq 0 \qquad For\ all\ r = s_1 + 1, \dots, s$$

$$v_{id}^{m} - v_{id}^{\alpha} \geq 0 \qquad For\ all\ i = m_1 + 1, \dots, m$$

$$v_{id}^{m} - \frac{1}{4}v_{id}^{\alpha} + \frac{1}{4}v_{id}^{\beta} \geq 0 \qquad For\ all\ i = m_1 + 1, \dots, m$$

$$w_{gd}^{m} - w_{gd}^{\alpha} \geq 0 \qquad For\ all\ g = 1, \dots, h$$

$$w_{gd}^{m} - \frac{1}{4}w_{gd}^{\alpha} + \frac{1}{4}w_{gd}^{\beta} \geq 0 \qquad For\ all\ g = 1, \dots, h.$$

$$E1_{dk} = max\sum_{r=1}^{s_1}\left[u_{rk}^m\left(y1_{rk}^m+\frac{1}{4}y1_{rk}^\beta-\frac{1}{4}y1_{rk}^\alpha\right)+\frac{1}{4}u_{rk}^\beta y1_{rk}^m-\frac{1}{4}u_{rk}^\alpha y1_{rk}^m\right]$$
$$+\sum_{g=1}^{h}\left[w_{gk}^m\left(z_{rk}^m+\frac{1}{4}z_{gk}^\beta-\frac{1}{4}z_{gk}^\alpha\right)+\frac{1}{4}w_{gd}^\beta z_{gk}^m-\frac{1}{4}w_{gk}^\alpha z_{gk}^m\right]$$
$$\sum_{i=1}^{m_1}\left[v_{ik}^m\left(x1_{ik}^m+\frac{1}{4}x1_{ik}^\beta-\frac{1}{4}x1_{ik}^\alpha\right)+\frac{1}{4}v_{ik}^\beta x1_{ik}^m-\frac{1}{4}v_{ik}^\alpha x1_{ik}^m\right]=1$$
$$\sum_{r=1}^{s_1}\left[u_{rk}^m\left(y1_{rj}^m+\frac{1}{4}y1_{rj}^\beta-\frac{1}{4}y1_{rj}^\alpha\right)+\frac{1}{4}u_{rk}^\beta y1_{rj}^m-\frac{1}{4}u_{rk}^\alpha y1_{rj}^m\right]$$
$$+\sum_{g=1}^{h}\left[w_{gk}^m\left(z_{rj}^m+\frac{1}{4}z_{gj}^\beta-\frac{1}{4}z_{gj}^\alpha\right)+\frac{1}{4}w_{gk}^\beta z_{gj}^m-\frac{1}{4}w_{gk}^\alpha z_{gj}^m\right]$$
$$-\sum_{i=1}^{m_1}\left[v_{ik}^m\left(x1_{ij}^m+\frac{1}{4}x1_{ij}^\beta-\frac{1}{4}x1_{ij}^\alpha\right)+\frac{1}{4}v_{ik}^\beta x1_{ij}^m-\frac{1}{4}v_{ik}^\alpha x1_{ij}^m\right]$$
$$\leq 0 \quad For\ all\ j=1,\dots,n\ ,j\neq d$$
$$E1_{dd}\times\sum_{i=1}^{m_1}\left[v_{ik}^m\left(x1_{id}^m+\frac{1}{4}x1_{id}^\beta-\frac{1}{4}x1_{id}^\alpha\right)+\frac{1}{4}v_{ik}^\beta x1_{id}^m-\frac{1}{4}v_{ik}^\alpha x1_{id}^m\right] \tag{28}$$
$$=\left(\sum_{r=1}^{s_1}\left[u_{rk}^m\left(y1_{rd}^m+\frac{1}{4}y1_{rd}^\beta-\frac{1}{4}y1_{rd}^\alpha\right)+\frac{1}{4}u_{rk}^\beta y1_{rd}^m-\frac{1}{4}u_{rk}^\alpha y1_{rd}^m\right]\right.$$
$$\left.+\sum_{g=1}^{h}\left[w_{gk}^m\left(z_{rd}^m+\frac{1}{4}z_{gd}^\beta-\frac{1}{4}z_{gd}^\alpha\right)+\frac{1}{4}w_{gk}^\beta z_{gd}^m-\frac{1}{4}w_{gk}^\alpha z_{gd}^m\right]\right)$$

$$u_{rk}^m-u_{rk}^\alpha\geq 0 \quad For\ all\ r=1,\dots,s_1$$
$$u_{rk}^m-\frac{1}{4}u_{rk}^\alpha+\frac{1}{4}u_{rk}^\beta\geq 0 \quad For\ all\ r=1,\dots,s_1$$
$$v_{ik}^m-v_{ik}^\alpha\geq 0 \quad For\ all\ i=1,\dots,m_1$$
$$v_{ik}^m-\frac{1}{4}v_{ik}^\alpha+\frac{1}{4}v_{ik}^\beta\geq 0 \quad For\ all\ i=1,\dots,m_1$$
$$w_{gk}^m-w_{gk}^\alpha\geq 0 \quad For\ all\ g=1,\dots,h$$
$$w_{gk}^m-\frac{1}{4}w_{gk}^\alpha+\frac{1}{4}w_{gk}^\beta\geq 0 \quad For\ all\ g=1,\dots,h.$$

$$
\begin{aligned}
E2_{dk} = max \sum_{r=s_1+1}^{s} & \left[u_{rk}^{m}\left(y2_{rk}^{m}+\frac{1}{4}y2_{rk}^{\beta}-\frac{1}{4}y2_{rk}^{\alpha}\right)+\frac{1}{4}u_{rk}^{\beta}y2_{rk}^{m}-\frac{1}{4}u_{rd}^{\alpha}y2_{rk}^{m}\right] \\
\sum_{i=m_1+1}^{m} & \left[v\left(x2_{ik}^{m}+\frac{1}{4}x2_{ik}^{\beta}-\frac{1}{4}x2_{ik}^{\alpha}\right)+\frac{1}{4}v_{ik}^{\beta}x2_{ik}^{m}-\frac{1}{4}v_{ik}^{\alpha}x2_{ik}^{m}\right] \\
& +\sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rk}^{m}+\frac{1}{4}z_{gk}^{\beta}-\frac{1}{4}z_{gk}^{\alpha}\right)+\frac{1}{4}w_{gk}^{\beta}z_{gk}^{m}-\frac{1}{4}w_{gk}^{\alpha}z_{gk}^{m}\right]=1 \\
\sum_{r=s_1+1}^{s} & \left[u_{rk}^{m}\left(y2_{rj}^{m}+\frac{1}{4}y2_{rj}^{\beta}-\frac{1}{4}y2_{rj}^{\alpha}\right)+\frac{1}{4}u_{rk}^{\beta}y2_{rj}^{m}-\frac{1}{4}u_{rk}^{\alpha}y2_{rj}^{m}\right] \\
& -\sum_{i=m_1+1}^{m}\left[v_{ik}^{m}\left(x2_{ij}^{m}+\frac{1}{4}x2_{ij}^{\beta}-\frac{1}{4}x2_{ij}^{\alpha}\right)+\frac{1}{4}v_{ik}^{\beta}x2_{ij}^{m} \right. \\
& \left. -\frac{1}{4}v_{ik}^{\alpha}x2_{ij}^{m}\right] \\
& -\sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rj}^{m}+\frac{1}{4}z_{gj}^{\beta}-\frac{1}{4}z_{gj}^{\alpha}\right)+\frac{1}{4}w_{gk}^{\beta}z_{gj}^{m}-\frac{1}{4}w_{gk}^{\alpha}z_{gj}^{m}\right] \\
& \leq 0 \quad For\ all\ j=1,\dots,n\ ,j\neq d \\
E2_{dd} \times & \left(\sum_{i=m_1+1}^{m}\left[v_{ik}^{m}\left(x2_{id}^{m}+\frac{1}{4}x2_{id}^{\beta}-\frac{1}{4}x2_{id}^{\alpha}\right)+\frac{1}{4}v_{ik}^{\beta}x2_{id}^{m}-\frac{1}{4}v_{ik}^{\alpha}x2_{id}^{m}\right]\right. \\
& \left. +\sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rd}^{m}+\frac{1}{4}z_{gd}^{\beta}-\frac{1}{4}z_{gd}^{\alpha}\right)+\frac{1}{4}w_{gk}^{\beta}z_{gd}^{m}-\frac{1}{4}w_{gk}^{\alpha}z_{gd}^{m}\right]\right) \\
& =\sum_{r=s_1+1}^{s}\left[u_{rk}^{m}\left(y2_{rd}^{m}+\frac{1}{4}y2_{rd}^{\beta}-\frac{1}{4}y2_{rd}^{\alpha}\right)+\frac{1}{4}u_{rk}^{\beta}y2_{rd}^{m}\right. \\
& \left. -\frac{1}{4}u_{rk}^{\alpha}y2_{rd}^{m}\right] \\
& u_{rk}^{m}-u_{rk}^{\alpha}\geq 0 \quad For\ all\ r=s_1+1,\dots,s \\
& u_{rk}^{m}-\frac{1}{4}u_{rk}^{\alpha}+\frac{1}{4}u_{rk}^{\beta}\geq 0 \quad For\ all\ r=s_1+1,\dots,s \\
& v_{ik}^{m}-v_{ik}^{\alpha}\geq 0 \quad For\ all\ i=m_1+1,\dots,m \\
& v_{ik}^{m}-\frac{1}{4}v_{ik}^{\alpha}+\frac{1}{4}v_{ik}^{\beta}\geq 0 \quad For\ all\ i=m_1+1,\dots,m \\
& w_{gk}^{m}-w_{gk}^{\alpha}\geq 0 \quad For\ all\ g=1,\dots,h \\
& w_{gk}^{m}-\frac{1}{4}w_{gk}^{\alpha}+\frac{1}{4}w_{gk}^{\beta}\geq 0 \quad For\ all\ g=1,\dots,h.
\end{aligned}
\tag{29}
$$

3.5. *Network fuzzy cross efficiency: Non-cooperative game approach and Hatami-Marbini's [39] approach*

In the non-cooperative game approach, to measure the efficiency of the first stage of CCR, the efficiency of the leader stage must be first calculated, and then the efficiency of the follower stage is measured, provided that the efficiency of the leader stage remains at the measured value. Measuring the efficiency of the leader is the same as the average approach. When the first stage is the leader, the efficiency of the second stage can be measured by adding the Constraint (31) to Model (28). When the second stage is the leader, the efficiency of the first one can be measured by adding the Constraint (32) to Model (27). In the second step, when the first stage is the leader the cross-efficiency of the second stage can be measured by adding the Constraint (33) to Model (30). On the other hand, when the second stage is the leader, the efficiency of the first one can be measured by adding the Constraint (34) to Model (29).

$$\sum_{r=1}^{s_1}\left[u_{rd}^{m}\left(y1_{rj}^{m}+\frac{1}{4}y1_{rj}^{\beta}-\frac{1}{4}y1_{rj}^{\alpha}\right)+\frac{1}{4}u_{rd}^{\beta}y1_{rj}^{m}-\frac{1}{4}u_{rd}^{\alpha}y1_{rj}^{m}\right]$$
$$+\sum_{g=1}^{h}\left[w_{gd}^{m}\left(z_{rj}^{m}+\frac{1}{4}z_{gj}^{\beta}-\frac{1}{4}z_{gj}^{\alpha}\right)+\frac{1}{4}w_{gd}^{\beta}z_{gj}^{m}-\frac{1}{4}w_{gd}^{\alpha}z_{gj}^{m}\right]$$
$$-\sum_{i=1}^{m_1}\left[v_{id}^{m}\left(x1_{ij}^{m}+\frac{1}{4}x1_{ij}^{\beta}-\frac{1}{4}x1_{ij}^{\alpha}\right)+\frac{1}{4}v_{id}^{\beta}x1_{ij}^{m}-\frac{1}{4}v_{id}^{\alpha}x1_{ij}^{m}\right]$$
$$\leq 0 \quad For\ all\ j$$
$$E1_{dd}\times\sum_{i=1}^{m_1}\left[v_{id}^{m}\left(x1_{id}^{m}+\frac{1}{4}x1_{id}^{\beta}-\frac{1}{4}x1_{id}^{\alpha}\right)+\frac{1}{4}v_{id}^{\beta}x1_{id}^{m}-\frac{1}{4}v_{id}^{\alpha}x1_{id}^{m}\right] \tag{30}$$
$$=\left(\sum_{r=1}^{s_1}\left[u_{rd}^{m}\left(y1_{rd}^{m}+\frac{1}{4}y1_{rd}^{\beta}-\frac{1}{4}y1_{rd}^{\alpha}\right)+\frac{1}{4}u_{rd}^{\beta}y1_{rd}^{m}\right.\right.$$
$$\left.-\frac{1}{4}u_{rd}^{\alpha}y1_{rd}^{m}\right]$$
$$\left.+\sum_{g=1}^{h}\left[w_{gd}^{m}\left(z_{rd}^{m}+\frac{1}{4}z_{gd}^{\beta}-\frac{1}{4}z_{gd}^{\alpha}\right)+\frac{1}{4}w_{gd}^{\beta}z_{gd}^{m}-\frac{1}{4}w_{gd}^{\alpha}z_{gd}^{m}\right]\right).$$

$$\sum_{r=s_1+1}^{s}\left[u_{rd}^{m}\left(y2_{rj}^{m}+\frac{1}{4}y2_{rj}^{\beta}-\frac{1}{4}y2_{rj}^{\alpha}\right)+\frac{1}{4}u_{rd}^{\beta}y2_{rj}^{m}-\frac{1}{4}u_{rd}^{\alpha}y2_{rj}^{m}\right]$$
$$-\sum_{i=m_1+1}^{m}\left[v_{id}^{m}\left(x2_{ij}^{m}+\frac{1}{4}x2_{ij}^{\beta}-\frac{1}{4}x2_{ij}^{\alpha}\right)+\frac{1}{4}v_{id}^{\beta}x2_{ij}^{m}\right.$$
$$\left.-\frac{1}{4}v_{id}^{\alpha}x2_{ij}^{m}\right]$$
$$-\sum_{g=1}^{h}\left[w_{gd}^{m}\left(z_{rj}^{m}+\frac{1}{4}z_{gj}^{\beta}-\frac{1}{4}z_{gj}^{\alpha}\right)+\frac{1}{4}w_{gd}^{\beta}\tilde{z}_{gj}^{m}-\frac{1}{4}w_{gd}^{\alpha}z_{gj}^{m}\right]$$
$$\leq 0 \quad For\ all\ j=1,\dots,n \tag{31}$$
$$E2_{dd}\times\left(\sum_{i=m_1+1}^{m}\left[v_{ik}^{m}\left(x2_{id}^{m}+\frac{1}{4}x2_{id}^{\beta}-\frac{1}{4}x2_{id}^{\alpha}\right)+\frac{1}{4}v_{ik}^{\beta}x2_{id}^{m}-\frac{1}{4}v_{ik}^{\alpha}x2_{id}^{m}\right]\right.$$
$$\left.+\sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rd}^{m}+\frac{1}{4}z_{gd}^{\beta}-\frac{1}{4}z_{gd}^{\alpha}\right)+\frac{1}{4}w_{gk}^{\beta}z_{gd}^{m}-\frac{1}{4}w_{gk}^{\alpha}z_{gd}^{m}\right]\right)$$
$$=\sum_{r=s_1+1}^{s}\left[u_{rk}^{m}\left(y2_{rd}^{m}+\frac{1}{4}y2_{rd}^{\beta}-\frac{1}{4}y2_{rd}^{\alpha}\right)+\frac{1}{4}u_{rk}^{\beta}y2_{rd}^{m}\right.$$
$$\left.-\frac{1}{4}u_{rk}^{\alpha}y2_{rd}^{m}\right].$$

$$\sum_{r=1}^{s_1}\left[u_{rd}^m\left(y1_{rj}^m+\frac{1}{4}y1_{rj}^\beta-\frac{1}{4}y1_{rj}^\alpha\right)+\frac{1}{4}u_{rd}^\beta y1_{rj}^m-\frac{1}{4}u_{rd}^\alpha y1_{rj}^m\right]$$
$$+\sum_{g=1}^{h}\left[w_{gd}^m\left(z_{rj}^m+\frac{1}{4}z_{gj}^\beta-\frac{1}{4}z_{gj}^\alpha\right)+\frac{1}{4}w_{gd}^\beta z_{gj}^m-\frac{1}{4}w_{gd}^\alpha z_{gj}^m\right]$$
$$-\sum_{i=1}^{m_1}\left[v_{id}^m\left(x1_{ij}^m+\frac{1}{4}x1_{ij}^\beta-\frac{1}{4}x1_{ij}^\alpha\right)+\frac{1}{4}v_{id}^\beta x1_{ij}^m-\frac{1}{4}v_{id}^\alpha x1_{ij}^m\right]$$
$$\leq 0 \quad For\ all\ j$$
$$E1_{dk}\times\sum_{i=1}^{m_1}\left[v_{ik}^m\left(x1_{ik}^m+\frac{1}{4}x1_{ik}^\beta-\frac{1}{4}x1_{ik}^\alpha\right)+\frac{1}{4}v_{ik}^\beta x1_{ik}^m-\frac{1}{4}v_{ik}^\alpha x1_{ik}^m\right] \tag{32}$$
$$=\left(\sum_{r=1}^{s_1}\left[u_{rk}^m\left(y1_{rk}^m+\frac{1}{4}y1_{rk}^\beta-\frac{1}{4}y1_{rk}^\alpha\right)+\frac{1}{4}u_{rk}^\beta y1_{rk}^m-\frac{1}{4}u_{rk}^\alpha y1_{rk}^m\right]\right.$$
$$\left.+\sum_{g=1}^{h}\left[w_{gk}^m\left(z_{rk}^m+\frac{1}{4}z_{gk}^\beta-\frac{1}{4}z_{gk}^\alpha\right)+\frac{1}{4}w_{gk}^\beta z_{gk}^m-\frac{1}{4}w_{gk}^\alpha z_{gk}^m\right]\right).$$

$$\sum_{r=s_1+1}^{s}\left[u_{rd}^m\left(y2_{rj}^m+\frac{1}{4}y2_{rj}^\beta-\frac{1}{4}y2_{rj}^\alpha\right)+\frac{1}{4}u_{rd}^\beta y2_{rj}^m-\frac{1}{4}u_{rd}^\alpha y2_{rj}^m\right]$$
$$-\sum_{i=m_1+1}^{m}\left[v_{id}^m\left(x2_{ij}^m+\frac{1}{4}x2_{ij}^\beta-\frac{1}{4}x2_{ij}^\alpha\right)+\frac{1}{4}v_{id}^\beta x2_{ij}^m-\frac{1}{4}v_{id}^\alpha x2_{ij}^m\right]$$
$$-\sum_{g=1}^{h}\left[w_{gd}^m\left(z_{rj}^m+\frac{1}{4}z_{gj}^\beta-\frac{1}{4}z_{gj}^\alpha\right)+\frac{1}{4}w_{gd}^\beta \tilde{z}_{gj}^m-\frac{1}{4}w_{gd}^\alpha z_{gj}^m\right]$$
$$\leq 0 \quad For\ all\ j=1,\dots,n \tag{33}$$
$$E2_{dk}\times\left(\sum_{i=m_1+1}^{m}\left[v_{id}^m\left(x2_{id}^m+\frac{1}{4}x2_{id}^\beta-\frac{1}{4}x2_{id}^\alpha\right)+\frac{1}{4}v_{id}^\beta x2_{id}^m-\frac{1}{4}v_{id}^\alpha x2_{id}^m\right]\right.$$
$$\left.+\sum_{g=1}^{h}\left[w_{gd}^m\left(z_{rd}^m+\frac{1}{4}z_{gd}^\beta-\frac{1}{4}z_{gd}^\alpha\right)+\frac{1}{4}w_{gd}^\beta z_{gd}^m-\frac{1}{4}w_{gd}^\alpha z_{gd}^m\right]\right)$$
$$=\sum_{r=s_1+1}^{s}\left[u_{rk}^m\left(y2_{rk}^m+\frac{1}{4}y2_{rk}^\beta-\frac{1}{4}y2_{rk}^\alpha\right)+\frac{1}{4}u_{rk}^\beta y2_{rk}^m-\frac{1}{4}u_{rd}^\alpha y2_{rk}^m\right].$$

3.6. *Network fuzzy cross efficiency: Aggregation approach and Hatami-Marbini's [39] approach*

In the aggregation approach, first the efficiency of the whole system and then the efficiencies of the stages are measured. The efficiency of the first stage of CCR of the aggregation approach is measured using Hatami Marbini's et al. [21] approach based on *Model (35)*. The cross efficiency of the second stage is measured by the use of *Model (36)*.

$$
\begin{aligned}
E_{dd} = max \sum_{r=1}^{s} \Big[u_{rd}^{m} \Big(y_{rd}^{m} + \frac{1}{4} y_{rd}^{\beta} - \frac{1}{4} y_{rd}^{\alpha} \Big) + \frac{1}{4} u_{rd}^{\beta} y_{rd}^{m} - \frac{1}{4} u_{rd}^{\alpha} y_{rd}^{m} \Big] \\
\sum_{i=1}^{m} \Big[v_{id}^{m} \Big(x_{id}^{m} + \frac{1}{4} x_{id}^{\beta} - \frac{1}{4} x_{id}^{\alpha} \Big) + \frac{1}{4} v_{id}^{\beta} x_{id}^{m} - \frac{1}{4} v_{id}^{\alpha} x_{id}^{m} \Big] = 1 \\
\sum_{r=1}^{s_1} \Big[u_{rd}^{m} \Big(y1_{rj}^{m} + \frac{1}{4} y1_{rj}^{\beta} - \frac{1}{4} y1_{rj}^{\alpha} \Big) + \frac{1}{4} u_{rd}^{\beta} y1_{rj}^{m} - \frac{1}{4} u_{rd}^{\alpha} y1_{rj}^{m} \Big] \\
+ \sum_{g=1}^{h} \Big[w_{gd}^{m} \Big(z_{rj}^{m} + \frac{1}{4} z_{gj}^{\beta} - \frac{1}{4} z_{gj}^{\alpha} \Big) + \frac{1}{4} w_{gd}^{\beta} z_{gj}^{m} - \frac{1}{4} w_{gd}^{\alpha} z_{gj}^{m} \Big] \\
- \sum_{i=1}^{m_1} \Big[v_{id}^{m} \Big(x1_{ij}^{m} + \frac{1}{4} x1_{ij}^{\beta} - \frac{1}{4} x1_{ij}^{\alpha} \Big) + \frac{1}{4} v_{id}^{\beta} x1_{ij}^{m} - \frac{1}{4} v_{id}^{\alpha} x1_{ij}^{m} \Big] \\
\leq 0 \quad \forall j = 1, \dots, n \\
\sum_{r=s_1+1}^{s} \Big[u_{rd}^{m} \Big(y2_{rj}^{m} + \frac{1}{4} y2_{rj}^{\beta} - \frac{1}{4} y2_{rj}^{\alpha} \Big) + \frac{1}{4} u_{rd}^{\beta} y2_{rj}^{m} - \frac{1}{4} u_{rd}^{\alpha} y2_{rj}^{m} \Big] \\
- \sum_{i=m_1+1}^{m} \Big[v_{id}^{m} \Big(x2_{ij}^{m} + \frac{1}{4} x2_{ij}^{\beta} - \frac{1}{4} x2_{ij}^{\alpha} \Big) + \frac{1}{4} v_{id}^{\beta} x2_{ij}^{m} \\
- \frac{1}{4} v_{id}^{\alpha} x2_{ij}^{m} \Big] \\
- \sum_{g=1}^{h} \Big[w_{gd}^{m} \Big(z_{rj}^{m} + \frac{1}{4} z_{gj}^{\beta} - \frac{1}{4} z_{gj}^{\alpha} \Big) + \frac{1}{4} w_{gd}^{\beta} \tilde{z}_{gj}^{m} - \frac{1}{4} w_{gd}^{\alpha} z_{gj}^{m} \Big] \\
\leq 0 \quad \forall j = 1, \dots, n \\
u_{rd}^{m} - u_{rd}^{\alpha} \geq 0 \quad For\ all\ r = 1, \dots, s \\
u_{rd}^{m} - \frac{1}{4} u_{rd}^{\alpha} + \frac{1}{4} u_{rd}^{\beta} \geq 0 \quad \forall r = 1, \dots, s \\
v_{id}^{m} - v_{id}^{\alpha} \geq 0 \quad For\ all\ i = 1, \dots, m \\
v_{id}^{m} - \frac{1}{4} v_{id}^{\alpha} + \frac{1}{4} v_{id}^{\beta} \geq 0 \quad \forall i = 1, \dots, m \\
w_{gd}^{m} - w_{gd}^{\alpha} \geq 0 \quad For\ all\ g = 1, \dots, h \\
w_{gd}^{m} - \frac{1}{4} w_{gd}^{\alpha} + \frac{1}{4} w_{gd}^{\beta} \geq 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
E_{dk} = max \sum_{r=1}^{s} \left[u_{rk}^{m}\left(y_{rk}^{m} + \frac{1}{4}y_{rk}^{\beta} - \frac{1}{4}y_{rk}^{\alpha}\right) + \frac{1}{4}u_{rk}^{\beta}y_{rk}^{m} - \frac{1}{4}u_{rk}^{\alpha}y_{rk}^{m}\right] \\
\sum_{i=1}^{m}\left[v_{ik}^{m}\left(x_{ik}^{m} + \frac{1}{4}x_{ik}^{\beta} - \frac{1}{4}x_{ik}^{\alpha}\right) + \frac{1}{4}v_{ik}^{\beta}x_{ik}^{m} - \frac{1}{4}v_{ik}^{\alpha}x_{ik}^{m}\right] = 1 \\
\sum_{r=1}^{s_1}\left[u_{rk}^{m}\left(y1_{rj}^{m} + \frac{1}{4}y1_{rj}^{\beta} - \frac{1}{4}y1_{rj}^{\alpha}\right) + \frac{1}{4}u_{rk}^{\beta}y1_{rj}^{m} - \frac{1}{4}u_{rk}^{\alpha}y1_{rj}^{m}\right] \\
+ \sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rj}^{m} + \frac{1}{4}z_{gj}^{\beta} - \frac{1}{4}z_{gj}^{\alpha}\right) + \frac{1}{4}w_{gk}^{\beta}z_{gj}^{m} - \frac{1}{4}w_{gk}^{\alpha}z_{gj}^{m}\right] \\
- \sum_{i=1}^{m_1}\left[v_{ik}^{m}\left(x1_{ij}^{m} + \frac{1}{4}x1_{ij}^{\beta} - \frac{1}{4}x1_{ij}^{\alpha}\right) + \frac{1}{4}v_{ik}^{\beta}x1_{ij}^{m} - \frac{1}{4}v_{ik}^{\alpha}x1_{ij}^{m}\right] \\
\leq 0 \quad For\ all\ j = 1, \dots, n \\
\sum_{r=s_1+1}^{s}\left[u_{rk}^{m}\left(y2_{rj}^{m} + \frac{1}{4}y2_{rj}^{\beta} - \frac{1}{4}y2_{rj}^{\alpha}\right) + \frac{1}{4}u_{rk}^{\beta}y2_{rj}^{m} - \frac{1}{4}u_{rk}^{\alpha}y2_{rj}^{m}\right] \\
- \sum_{i=m_1+1}^{m}\left[v_{ik}^{m}\left(x2_{ij}^{m} + \frac{1}{4}x2_{ij}^{\beta} - \frac{1}{4}x2_{ij}^{\alpha}\right) + \frac{1}{4}v_{ik}^{\beta}x2_{ij}^{m} \right.\\
\left. - \frac{1}{4}v_{ik}^{\alpha}x2_{ij}^{m}\right] \\
- \sum_{g=1}^{h}\left[w_{gk}^{m}\left(z_{rj}^{m} + \frac{1}{4}z_{gj}^{\beta} - \frac{1}{4}z_{gj}^{\alpha}\right) + \frac{1}{4}w_{gk}^{\beta}z_{gj}^{m} - \frac{1}{4}w_{gk}^{\alpha}z_{gj}^{m}\right] \\
\leq 0 \quad For\ all\ j = 1, \dots, n \\
E_{dd} \times \sum_{i=1}^{m}\left[v_{ik}^{m}\left(x_{id}^{m} + \frac{1}{4}x_{id}^{\beta} - \frac{1}{4}x_{id}^{\alpha}\right) + \frac{1}{4}v_{ik}^{\beta}x_{id}^{m} - \frac{1}{4}v_{ik}^{\alpha}x_{id}^{m}\right] \\
= \sum_{r=1}^{s}\left[u_{rk}^{m}\left(y_{rd}^{m} + \frac{1}{4}y_{rd}^{\beta} - \frac{1}{4}y_{rd}^{\alpha}\right) + \frac{1}{4}u_{rk}^{\beta}y_{rd}^{m} - \frac{1}{4}u_{rk}^{\alpha}y_{rd}^{m}\right] \\
u_{rk}^{m} - u_{rk}^{\alpha} \geq 0 \quad For\ all\ r = 1, \dots, s \\
u_{rk}^{m} - \frac{1}{4}u_{rk}^{\alpha} + \frac{1}{4}u_{rk}^{\beta} \geq 0 \quad For\ all\ r = 1, \dots, s \\
v_{ik}^{m} - v_{ik}^{\alpha} \geq 0 \quad For\ all\ i = 1, \dots, m \\
v_{ik}^{m} - \frac{1}{4}v_{ik}^{\alpha} + \frac{1}{4}v_{ik}^{\beta} \geq 0 \quad For\ all\ i = 1, \dots, m \\
w_{gk}^{m} - w_{gk}^{\alpha} \geq 0 \quad For\ all\ g = 1, \dots, h \\
w_{gk}^{m} - \frac{1}{4}w_{gk}^{\alpha} + \frac{1}{4}w_{gk}^{\beta} \geq 0 \quad For\ all\ g = 1, \dots, h.
\end{aligned}
\tag{35}
$$

4. Application example

The aim of this research is to rank the branches of Iranian banks using DEA. As mentioned by some researchers, the banking process is a two-stage process with two parts of production and financial intermediation. In the production stage, financial institutions absorb capital to allocate loans. Financial institutions are thought of as primarily producing services for account holders. They perform transactions and process documents for customers, including loan applications, credit reports, checks or other payment instruments, and insurance policy or claim forms [4].

In the financial intermediation stage, financial institutions are thought of as primarily

intermediating funds between savers and investors. The financial resources gained through the first-stage are granted to investors in the form of loans with higher interest rates than the interest given to depositors. Since the personnel, the Automated Teller Machine (ATM), the Point of Sale (POS) and the Internet Payment Gateway (IPG), and internet banking demand their own costs and service revenues are earned through them, they are considered as the first-stage inputs. Because letter of guarantee, and inexpensive deposits are profitable for the bank, they are considered as first-stage outputs. The deferred loan of the bank, defined as the product of financial intermediation processes, is considered as the second-stage input. The amount of loans and net interest incomes are considered as the second-stage outputs. Expensive deposits are a source of loss to the bank because they receive interest from the bank. On the other hand, these deposits are a good source for loan payments, from which banks gain profits.
Therefore, in some researches, they are considered as the input and, in some others, as the output. An advantage of network systems is parameter definition for both input and output nature. We have considered the expensive deposits as the first-stage output and the second-stage input. Overall, the banking procedure can be illustrated as a general two-stage process with middle inputs and outputs as in Figure 3.This process has been described in our other study [39]. In the present study, we used the same process, with the difference that due to the lack of access to the data of the interest and fee incomes part, this part was removed from the application example.

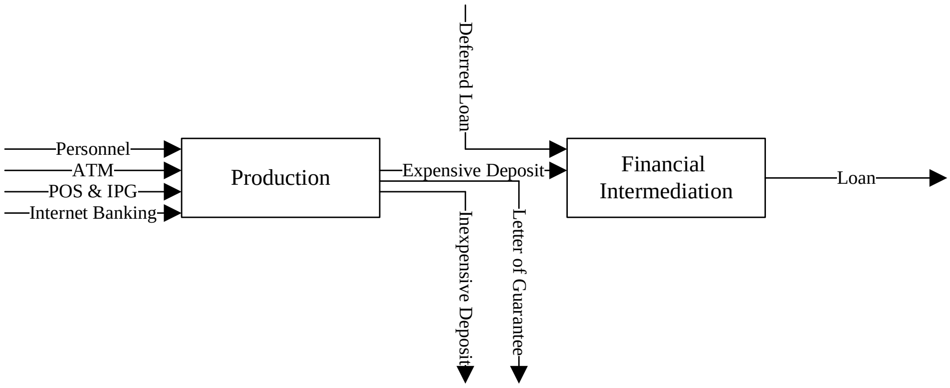

Figure 3. The network system of banking procedure.

In this study, we evaluated the input and output values of 105 branches of one of the banks in Tehran in a one-year period. The minimum, average, and maximum values were considered as TFNs for evaluation. The data of the problem are listed in *Table 4* presented in Appendix. After solving the problem with the models presented in the previous section, the obtained values of the fuzzy efficiency of the stages and the whole system using the α-cut approach for α=0.5 as well as using Hatami-Marbini's approach [21] are presented in the *Table 2* and *Table 3*. The fuzzy ranking values of the stages and the whole system using the α-cut approach for α=0.5 and Hatami-Marbini's approach [21] are listed *Table 5* and *Table 5* in Appendix, respectively.

The results show the closeness of the total efficiency values and the big difference between the ranks derived from different approaches, so that the decision maker finds it difficult to make a final decision. One solution is to use the average rankings derived using different approaches as the final ranking for decision making. Another solution is to choose one of the approaches according to the organization's strategy. If the organization's strategy is the growth of the organization in all departments with the view of decentralization, it seems logical to use the average approach. If the decision maker finds a part of the process more important (e.g. profitability by increasing services or profitability by increasing loens), he/she should use the leader and follower approaches. If the goal is to create confidence levels different from the measured ones, the α-cut approach is recommended, and if the goal is to reduce the calculations, Hatami Marbini's approach [21] should be used. Although measuring the efficiency of the stages is possible in the aggregation approach, after measuring the efficiency of the stages, there are many stages with zero efficiency. If the derived efficiency variance is used as a dispersion measure to compare the separability of different approaches, Hatami Marbini's approach [21], among the fuzzy approaches, has a higher separability than the α-cut approach, and the aggregation approach, among the network approaches, has lower separability than the other approaches.

Table 1. Cross-fuzzy efficiency of the stages and the whole system using α-cut approach. for α=0.5.

Branch	Average			Stage 1 is the leader			Stage 2 is the leader			Aggregation
	Stage 1	Stage 2	System	Stage 1	Stage 2	System	Stage 1	Stage 2	System	System
1	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.96,0.41,1)
2	(0.16,0.2,1)	(0.12,0.16,1)	(0.14,0.18,1)	(0.16,0.2,1)	(0.12,0.16,1)	(0.14,0.18,1)	(0.16,0.2,1)	(0.12,0.16,1)	(0.14,0.18,1)	(0.96,0.15,1)
3	(0.27,0.33,0.89)	(0.11,0.16,1)	(0.19,0.24,0.95)	(0.27,0.33,0.89)	(0.11,0.16,1)	(0.19,0.24,0.95)	(0.27,0.33,0.89)	(0.11,0.16,1)	(0.19,0.24,0.95)	(0.96,0.29,1)
4	(0.31,0.39,0.98)	(0.11,0.16,1)	(0.21,0.27,0.99)	(0.31,0.39,0.98)	(0.11,0.16,1)	(0.21,0.27,0.99)	(0.31,0.39,0.98)	(0.11,0.16,1)	(0.21,0.27,0.99)	(0.96,0.31,1)
5	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.97)	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.97)	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.97)	(0.96,0.27,1)
6	(0.3,0.38,1)	(0.12,0.16,0.94)	(0.21,0.27,0.97)	(0.3,0.38,1)	(0.12,0.16,0.94)	(0.21,0.27,0.97)	(0.3,0.38,1)	(0.12,0.16,0.94)	(0.21,0.27,0.97)	(0.96,0.33,1)
7	(0.42,0.49,1)	(0.06,0.09,0.98)	(0.24,0.29,0.99)	(0.42,0.49,1)	(0.06,0.09,0.98)	(0.24,0.29,0.99)	(0.42,0.49,1)	(0.06,0.09,0.98)	(0.24,0.29,0.99)	(0.95,0.45,1)
8	(0.27,0.33,1)	(0.11,0.06,0.99)	(0.19,0.2,0.99)	(0.27,0.33,1)	(0.11,0.06,0.99)	(0.19,0.2,0.99)	(0.27,0.33,1)	(0.11,0.06,0.99)	(0.19,0.2,0.99)	(0.93,0.27,1)
9	(0.31,0.39,1)	(0.11,0.16,0.83)	(0.21,0.27,0.91)	(0.31,0.39,1)	(0.11,0.16,0.83)	(0.21,0.27,0.91)	(0.31,0.39,1)	(0.11,0.16,0.83)	(0.21,0.27,0.91)	(0.93,0.31,1)
10	(0.3,0.36,1)	(0.11,0.16,0.99)	(0.21,0.26,1)	(0.3,0.36,1)	(0.11,0.16,0.99)	(0.21,0.26,1)	(0.3,0.36,1)	(0.11,0.16,0.99)	(0.21,0.26,1)	(0.93,0.29,1)
11	(0.32,0.44,0.98)	(0.06,0.09,0.96)	(0.19,0.27,0.97)	(0.32,0.44,0.98)	(0.06,0.09,0.96)	(0.19,0.27,0.97)	(0.32,0.44,0.98)	(0.06,0.09,0.96)	(0.19,0.27,0.97)	(0.4,0.36,1)
12	(0.51,0.58,0.88)	(0.04,0.04,0.96)	(0.27,0.31,0.92)	(0.51,0.58,0.88)	(0.04,0.04,0.96)	(0.27,0.31,0.92)	(0.51,0.58,0.88)	(0.04,0.04,0.96)	(0.27,0.31,0.92)	(0.57,0.51,0.96)
13	(0.25,0.33,1)	(0.04,0.04,0.99)	(0.14,0.18,0.99)	(0.25,0.33,1)	(0.04,0.04,0.99)	(0.14,0.18,0.99)	(0.25,0.33,1)	(0.04,0.04,0.99)	(0.14,0.18,0.99)	(0.59,0.04,1)
14	(0.26,0.33,0.94)	(0.03,0.06,1)	(0.15,0.2,0.97)	(0.26,0.33,0.94)	(0.04,0.06,1)	(0.15,0.2,0.97)	(0.26,0.33,0.94)	(0.04,0.06,1)	(0.15,0.2,0.97)	(0.59,0.29,0.99)
15	(0.32,0.36,0.99)	(0.11,0.16,0.96)	(0.21,0.26,0.98)	(0.32,0.36,0.99)	(0.11,0.16,0.96)	(0.22,0.26,0.98)	(0.32,0.36,0.99)	(0.11,0.16,0.96)	(0.22,0.26,0.98)	(0.59,0.27,1)
16	(0.32,0.39,0.88)	(0.06,0.09,1)	(0.19,0.24,0.94)	(0.32,0.39,0.88)	(0.06,0.09,1)	(0.19,0.24,0.94)	(0.32,0.39,0.88)	(0.06,0.09,1)	(0.19,0.24,0.94)	(0.59,0.4,1)
17	(0.33,0.47,0.95)	(0.06,0.09,0.92)	(0.19,0.28,0.93)	(0.33,0.47,0.95)	(0.06,0.09,0.92)	(0.2,0.28,0.93)	(0.33,0.47,0.95)	(0.06,0.09,0.92)	(0.19,0.28,0.93)	(0.43,0.4,0.99)
18	(0.42,0.37,1)	(0.11,0.16,0.97)	(0.27,0.26,0.98)	(0.42,0.37,1)	(0.11,0.16,0.97)	(0.27,0.26,0.98)	(0.42,0.37,1)	(0.11,0.16,0.97)	(0.27,0.26,0.98)	(0.29,0.45,1)
19	(0.21,0.3,1)	(0.11,0.16,0.87)	(0.16,0.23,0.94)	(0.21,0.3,1)	(0.11,0.16,0.87)	(0.16,0.23,0.94)	(0.21,0.3,1)	(0.11,0.16,0.87)	(0.16,0.23,0.94)	(0.29,0.33,1)
20	(0.29,0.38,1)	(0.11,0.16,0.95)	(0.2,0.27,0.98)	(0.29,0.38,1)	(0.11,0.16,0.95)	(0.2,0.27,0.98)	(0.29,0.38,1)	(0.11,0.16,0.95)	(0.2,0.27,0.98)	(0.29,0.33,1)
21	(0.3,0.36,0.99)	(0.11,0.16,0.98)	(0.21,0.26,0.99)	(0.3,0.36,0.99)	(0.11,0.16,0.98)	(0.21,0.26,0.99)	(0.3,0.36,0.99)	(0.11,0.16,0.98)	(0.21,0.26,0.99)	(0.29,0.29,1)
22	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.29,0.45,1)
23	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.96)	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.96)	(0.27,0.33,1)	(0.11,0.16,0.93)	(0.19,0.24,0.96)	(0.29,0.29,1)
24	(0.3,0.36,1)	(0.12,0.09,1)	(0.21,0.23,1)	(0.3,0.36,1)	(0.11,0.09,1)	(0.21,0.23,1)	(0.3,0.36,1)	(0.12,0.09,1)	(0.21,0.23,1)	(0.29,0.29,1)
25	(0.27,0.33,1)	(0.11,0.16,1)	(0.19,0.24,1)	(0.27,0.33,1)	(0.11,0.16,1)	(0.19,0.24,1)	(0.27,0.33,1)	(0.11,0.16,1)	(0.19,0.24,1)	(0.29,0.29,1)
26	(0.28,0.36,0.99)	(0.12,0.16,0.92)	(0.2,0.26,0.95)	(0.28,0.36,0.99)	(0.11,0.16,0.92)	(0.2,0.26,0.95)	(0.28,0.36,0.99)	(0.12,0.16,0.92)	(0.2,0.26,0.95)	(0.29,0.34,0.99)
27	(0.31,0.39,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.31,0.39,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.31,0.39,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.67,0.34,1)
28	(0.32,0.38,1)	(0.06,0.09,0.99)	(0.19,0.24,1)	(0.32,0.38,1)	(0.06,0.09,0.99)	(0.19,0.24,1)	(0.32,0.38,1)	(0.06,0.09,0.99)	(0.19,0.24,1)	(0.87,0.31,1)
29	(0.42,0.49,0.97)	(0.12,0.09,0.95)	(0.27,0.29,0.96)	(0.42,0.49,0.97)	(0.12,0.09,0.95)	(0.27,0.29,0.96)	(0.42,0.49,0.97)	(0.12,0.09,0.95)	(0.27,0.29,0.96)	(0.9,0.45,1)
30	(0.33,0.47,0.98)	(0.12,0.16,0.91)	(0.22,0.31,0.95)	(0.33,0.47,0.98)	(0.13,0.16,0.91)	(0.23,0.31,0.95)	(0.33,0.47,0.98)	(0.12,0.16,0.91)	(0.22,0.31,0.95)	(0.93,0.34,0.99)
31	(0.42,0.39,1)	(0.11,0.16,0.83)	(0.27,0.27,0.91)	(0.42,0.39,1)	(0.11,0.16,0.83)	(0.27,0.27,0.91)	(0.42,0.39,1)	(0.11,0.16,0.83)	(0.27,0.27,0.91)	(0.93,0.41,1)
32	(0.31,0.39,1)	(0.11,0.06,0.95)	(0.21,0.23,0.98)	(0.31,0.39,1)	(0.11,0.06,0.95)	(0.21,0.23,0.98)	(0.31,0.39,1)	(0.11,0.06,0.95)	(0.21,0.23,0.98)	(0.93,0.34,1)
33	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.93,0.45,1)
34	(0.31,0.41,0.98)	(0.11,0.16,0.71)	(0.21,0.28,0.84)	(0.31,0.41,0.98)	(0.11,0.16,0.71)	(0.21,0.28,0.84)	(0.31,0.41,0.98)	(0.11,0.16,0.71)	(0.21,0.28,0.84)	(0.4,0.39,0.94)
35	(0.38,0.45,0.98)	(0.12,0.16,0.55)	(0.25,0.3,0.77)	(0.38,0.45,0.98)	(0.12,0.16,0.55)	(0.25,0.3,0.77)	(0.38,0.45,0.98)	(0.12,0.16,0.55)	(0.25,0.3,0.77)	(0.39,0.45,0.95)
36	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.39,0.45,1)
37	(0.42,0.48,1)	(0.11,0.16,0.93)	(0.26,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.93)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.93)	(0.27,0.32,0.96)	(0.39,0.41,1)
38	(0.42,0.45,0.97)	(0.11,0.16,0.65)	(0.27,0.3,0.81)	(0.42,0.45,0.97)	(0.11,0.16,0.65)	(0.27,0.3,0.81)	(0.42,0.45,0.97)	(0.11,0.16,0.65)	(0.27,0.3,0.81)	(0.9,0.45,0.92)
39	(0.25,0.31,1)	(0.11,0.16,0.78)	(0.18,0.23,0.89)	(0.25,0.31,1)	(0.11,0.16,0.79)	(0.18,0.23,0.9)	(0.25,0.31,1)	(0.11,0.16,0.78)	(0.18,0.23,0.89)	(0.94,0.31,1)
40	(0.42,0.49,1)	(0.08,0.16,0.96)	(0.25,0.32,0.98)	(0.42,0.49,1)	(0.1,0.16,0.96)	(0.26,0.32,0.98)	(0.42,0.49,1)	(0.1,0.16,0.96)	(0.26,0.32,0.98)	(0.94,0.45,1)
41	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.94,0.41,1)
42	(0.32,0.39,0.98)	(0.13,0.16,0.89)	(0.22,0.28,0.93)	(0.32,0.39,0.98)	(0.13,0.16,0.89)	(0.22,0.28,0.93)	(0.32,0.39,0.98)	(0.13,0.16,0.89)	(0.22,0.28,0.93)	(0.93,0.4,0.99)
43	(0.42,0.49,0.99)	(0.11,0.16,0.91)	(0.27,0.32,0.95)	(0.42,0.49,0.99)	(0.11,0.16,0.91)	(0.27,0.32,0.95)	(0.42,0.49,0.99)	(0.11,0.16,0.91)	(0.27,0.32,0.95)	(0.93,0.45,0.99)
44	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.93,0.45,1)
45	(0.38,0.48,1)	(0.06,0.09,0.97)	(0.22,0.29,0.99)	(0.38,0.48,1)	(0.06,0.09,0.97)	(0.22,0.29,0.99)	(0.38,0.48,1)	(0.06,0.09,0.97)	(0.22,0.29,0.99)	(0.93,0.41,0.99)
46	(0.42,0.48,1)	(0.11,0.16,0.73)	(0.27,0.32,0.86)	(0.42,0.48,1)	(0.11,0.16,0.73)	(0.27,0.32,0.86)	(0.42,0.48,1)	(0.11,0.16,0.73)	(0.27,0.32,0.86)	(0.94,0.41,1)
47	(0.32,0.38,1)	(0.11,0.16,0.92)	(0.22,0.27,0.96)	(0.32,0.38,1)	(0.11,0.16,0.92)	(0.22,0.27,0.96)	(0.32,0.38,1)	(0.11,0.16,0.92)	(0.22,0.27,0.96)	(0.94,0.35,0.99)
48	(0.36,0.55,0.98)	(0.11,0.16,0.7)	(0.24,0.36,0.84)	(0.36,0.55,0.98)	(0.11,0.16,0.7)	(0.24,0.36,0.84)	(0.36,0.55,0.98)	(0.11,0.16,0.7)	(0.24,0.36,0.84)	(0.42,0.5,0.97)
49	(0.42,0.48,1)	(0.11,0.16,0.86)	(0.27,0.32,0.93)	(0.42,0.48,1)	(0.11,0.16,0.86)	(0.27,0.32,0.93)	(0.42,0.48,1)	(0.11,0.16,0.86)	(0.27,0.32,0.93)	(0.45,0.45,0.99)
50	(0.31,0.46,0.98)	(0.11,0.16,0.49)	(0.21,0.31,0.74)	(0.31,0.46,0.98)	(0.11,0.16,0.49)	(0.21,0.31,0.74)	(0.31,0.46,0.98)	(0.11,0.16,0.49)	(0.21,0.31,0.74)	(0.44,0.42,0.96)
51	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.96)	(0.27,0.32,0.98)	(0.46,0.45,1)
52	(0.53,0.58,0.96)	(0.11,0.16,1)	(0.32,0.37,0.98)	(0.53,0.58,0.96)	(0.13,0.16,1)	(0.33,0.37,0.98)	(0.53,0.58,0.96)	(0.13,0.16,1)	(0.33,0.37,0.98)	(0.52,0.31,1)

53	(0.26,0.36,0.99)	(0.13,0.17,0.66)	(0.19,0.26,0.82)	(0.26,0.36,0.99)	(0.13,0.17,0.66)	(0.19,0.26,0.82)	(0.26,0.36,0.99)	(0.13,0.17,0.66)	(0.19,0.26,0.82)	(0.48,0.3,0.99)
54	(0.42,0.48,0.99)	(0.11,0.16,0.31)	(0.27,0.32,0.65)	(0.42,0.48,0.99)	(0.11,0.16,0.31)	(0.27,0.32,0.65)	(0.42,0.48,0.99)	(0.11,0.16,0.31)	(0.27,0.32,0.65)	(0.76,0.45,0.99)
55	(0.42,0.49,0.99)	(0.11,0.16,0.77)	(0.27,0.32,0.88)	(0.42,0.49,0.99)	(0.11,0.16,0.77)	(0.27,0.32,0.88)	(0.42,0.49,0.99)	(0.11,0.16,0.77)	(0.27,0.32,0.88)	(0.94,0.45,0.98)
56	(0.42,0.48,1)	(0.12,0.16,0.47)	(0.27,0.32,0.73)	(0.42,0.48,1)	(0.12,0.16,0.47)	(0.27,0.32,0.73)	(0.42,0.48,1)	(0.12,0.16,0.47)	(0.27,0.32,0.73)	(0.44,0.35,1)
57	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.49,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.59,0.45,1)
58	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.59,0.41,1)
59	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.59,0.45,1)
60	(0.42,0.48,1)	(0.11,0.16,0.98)	(0.27,0.32,0.99)	(0.42,0.48,1)	(0.11,0.16,0.98)	(0.27,0.32,0.99)	(0.42,0.48,1)	(0.11,0.16,0.98)	(0.27,0.32,0.99)	(0.59,0.45,1)
61	(0.37,0.36,1)	(0.11,0.16,0.58)	(0.24,0.26,0.79)	(0.37,0.36,1)	(0.11,0.16,0.58)	(0.24,0.26,0.79)	(0.37,0.36,1)	(0.11,0.16,0.58)	(0.24,0.26,0.79)	(0.59,0.34,1)
62	(0.42,0.48,1)	(0.11,0.06,0.92)	(0.26,0.27,0.96)	(0.42,0.48,1)	(0.11,0.06,0.92)	(0.26,0.27,0.96)	(0.42,0.48,1)	(0.11,0.06,0.92)	(0.26,0.27,0.96)	(0.59,0.45,1)
63	(0.42,0.49,1)	(0.12,0.16,0.78)	(0.27,0.32,0.89)	(0.42,0.49,1)	(0.12,0.16,0.78)	(0.27,0.32,0.89)	(0.42,0.49,1)	(0.12,0.16,0.78)	(0.27,0.32,0.89)	(0.77,0.45,0.99)
64	(0.42,0.48,1)	(0.04,0.06,0.96)	(0.23,0.27,0.98)	(0.42,0.48,1)	(0.04,0.06,0.96)	(0.23,0.27,0.98)	(0.42,0.48,1)	(0.04,0.06,0.96)	(0.23,0.27,0.98)	(0.94,0.41,1)
65	(0.32,0.39,0.99)	(0.11,0.16,0.8)	(0.22,0.27,0.9)	(0.32,0.39,0.99)	(0.11,0.16,0.8)	(0.22,0.27,0.9)	(0.32,0.39,0.99)	(0.11,0.16,0.8)	(0.22,0.27,0.9)	(0.36,0.34,1)
66	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.33,0.45,1)
67	(0.32,0.4,1)	(0.06,0.09,0.95)	(0.19,0.25,0.97)	(0.32,0.4,1)	(0.06,0.09,0.95)	(0.19,0.25,0.97)	(0.32,0.4,1)	(0.06,0.09,0.95)	(0.19,0.25,0.97)	(0.33,0.36,1)
68	(0.42,0.49,1)	(0.04,0.06,0.99)	(0.23,0.27,0.99)	(0.42,0.49,1)	(0.04,0.06,0.99)	(0.23,0.27,0.99)	(0.42,0.49,1)	(0.04,0.06,0.99)	(0.23,0.27,0.99)	(0.33,0.45,1)
69	(0.36,0.38,0.99)	(0.12,0.16,0.84)	(0.24,0.27,0.91)	(0.36,0.38,0.99)	(0.12,0.16,0.84)	(0.24,0.27,0.92)	(0.36,0.38,0.99)	(0.12,0.16,0.84)	(0.24,0.27,0.91)	(0.33,0.34,0.99)
70	(0.32,0.38,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.32,0.38,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.32,0.38,1)	(0.11,0.16,0.95)	(0.21,0.27,0.97)	(0.33,0.41,1)
71	(0.42,0.48,0.97)	(0.06,0.09,0.99)	(0.24,0.29,0.98)	(0.42,0.48,0.97)	(0.06,0.09,0.99)	(0.24,0.29,0.98)	(0.42,0.48,0.97)	(0.06,0.09,0.99)	(0.24,0.29,0.98)	(0.64,0.45,1)
72	(0.42,0.43,1)	(0.11,0.16,0.82)	(0.27,0.3,0.91)	(0.42,0.43,1)	(0.11,0.16,0.82)	(0.27,0.3,0.91)	(0.42,0.43,1)	(0.11,0.16,0.82)	(0.27,0.3,0.91)	(0.94,0.4,1)
73	(0.42,0.48,0.99)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,0.99)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.42,0.48,0.99)	(0.11,0.16,0.95)	(0.27,0.32,0.97)	(0.94,0.45,1)
74	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.11,0.16,0.92)	(0.27,0.32,0.96)	(0.94,0.45,1)
75	(0.42,0.39,1)	(0.11,0.06,0.96)	(0.26,0.23,0.98)	(0.42,0.39,1)	(0.11,0.06,0.96)	(0.26,0.23,0.98)	(0.42,0.39,1)	(0.11,0.06,0.96)	(0.26,0.23,0.98)	(0.94,0.34,1)
76	(0.32,0.38,0.98)	(0.11,0.16,0.97)	(0.22,0.27,0.97)	(0.32,0.38,0.98)	(0.11,0.16,0.97)	(0.21,0.27,0.97)	(0.32,0.38,0.98)	(0.11,0.16,0.97)	(0.22,0.27,0.97)	(0.94,0.34,1)
77	(0.42,0.48,1)	(0.06,0.09,1)	(0.24,0.29,1)	(0.42,0.48,1)	(0.06,0.09,1)	(0.24,0.29,1)	(0.42,0.48,1)	(0.06,0.09,1)	(0.24,0.29,1)	(0.94,0.45,1)
78	(0.42,0.38,0.99)	(0.11,0.16,0.95)	(0.27,0.27,0.97)	(0.42,0.38,0.99)	(0.11,0.16,0.95)	(0.27,0.27,0.97)	(0.42,0.38,0.99)	(0.11,0.16,0.95)	(0.27,0.27,0.97)	(0.94,0.32,1)
79	(0.32,0.38,0.99)	(0.12,0.16,0.92)	(0.22,0.27,0.96)	(0.32,0.38,0.99)	(0.12,0.16,0.92)	(0.22,0.27,0.96)	(0.32,0.38,0.99)	(0.12,0.16,0.92)	(0.22,0.27,0.96)	(0.94,0.31,0.99)
80	(0.42,0.48,1)	(0.11,0.16,0.87)	(0.27,0.32,0.93)	(0.42,0.48,1)	(0.11,0.16,0.87)	(0.27,0.32,0.93)	(0.42,0.48,1)	(0.11,0.16,0.87)	(0.27,0.32,0.93)	(0.94,0.41,1)
81	(0.42,0.48,1)	(0.11,0.16,0.75)	(0.27,0.32,0.88)	(0.42,0.48,1)	(0.11,0.16,0.75)	(0.27,0.32,0.88)	(0.42,0.48,1)	(0.11,0.16,0.75)	(0.27,0.32,0.88)	(0.94,0.41,1)
82	(0.31,0.36,0.99)	(0.11,0.16,0.82)	(0.21,0.26,0.91)	(0.31,0.36,0.99)	(0.11,0.16,0.82)	(0.21,0.26,0.91)	(0.31,0.36,0.99)	(0.11,0.16,0.82)	(0.21,0.26,0.91)	(0.38,0.33,0.99)
83	(0.42,0.48,1)	(0.12,0.16,0.93)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.12,0.16,0.93)	(0.27,0.32,0.96)	(0.42,0.48,1)	(0.12,0.16,0.93)	(0.27,0.32,0.96)	(0.33,0.41,1)
84	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.49,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.33,0.45,1)
85	(0.32,0.38,0.99)	(0.12,0.16,0.9)	(0.22,0.27,0.95)	(0.32,0.38,0.99)	(0.12,0.16,0.9)	(0.22,0.27,0.95)	(0.32,0.38,0.99)	(0.12,0.16,0.9)	(0.22,0.27,0.95)	(0.33,0.31,0.99)
86	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.26,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.33,0.41,1)
87	(0.42,0.48,1)	(0.12,0.16,0.85)	(0.27,0.32,0.92)	(0.42,0.48,1)	(0.12,0.16,0.85)	(0.27,0.32,0.92)	(0.42,0.48,1)	(0.12,0.16,0.85)	(0.27,0.32,0.92)	(0.33,0.45,0.99)
88	(0.42,0.48,1)	(0.11,0.16,0.9)	(0.26,0.32,0.95)	(0.42,0.48,1)	(0.11,0.16,0.9)	(0.26,0.32,0.95)	(0.42,0.48,1)	(0.11,0.16,0.9)	(0.26,0.32,0.95)	(0.33,0.41,1)
89	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.97)	(0.27,0.32,0.98)	(0.33,0.41,1)
90	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.42,0.48,1)	(0.11,0.16,0.95)	(0.27,0.32,0.98)	(0.33,0.41,1)
91	(0.37,0.41,1)	(0.11,0.16,0.87)	(0.24,0.29,0.93)	(0.37,0.41,1)	(0.11,0.16,0.87)	(0.24,0.29,0.93)	(0.37,0.41,1)	(0.11,0.16,0.87)	(0.24,0.29,0.93)	(0.33,0.41,1)
92	(0.38,0.45,1)	(0.11,0.16,0.71)	(0.24,0.3,0.85)	(0.38,0.45,1)	(0.11,0.16,0.71)	(0.24,0.3,0.85)	(0.38,0.45,1)	(0.11,0.16,0.71)	(0.24,0.3,0.85)	(0.33,0.45,1)
93	(0.42,0.48,1)	(0.11,0.16,0.83)	(0.27,0.32,0.91)	(0.42,0.48,1)	(0.11,0.16,0.83)	(0.27,0.32,0.91)	(0.42,0.48,1)	(0.11,0.16,0.83)	(0.27,0.32,0.91)	(0.33,0.41,1)
94	(0.42,0.43,1)	(0.11,0.16,0.85)	(0.27,0.3,0.92)	(0.42,0.43,1)	(0.11,0.16,0.85)	(0.27,0.3,0.92)	(0.42,0.43,1)	(0.11,0.16,0.85)	(0.27,0.3,0.92)	(0.33,0.4,1)
95	(0.42,0.48,1)	(0.11,0.07,0.66)	(0.27,0.27,0.83)	(0.42,0.48,1)	(0.11,0.07,0.66)	(0.27,0.27,0.83)	(0.42,0.48,1)	(0.11,0.07,0.66)	(0.27,0.27,0.83)	(0.33,0.04,1)
96	(0.42,0.49,1)	(0.11,0.16,0.93)	(0.26,0.32,0.97)	(0.42,0.49,1)	(0.11,0.16,0.93)	(0.26,0.32,0.97)	(0.42,0.49,1)	(0.11,0.16,0.93)	(0.26,0.32,0.97)	(0.33,0.45,1)
97	(0.37,0.41,1)	(0.11,0.16,0.8)	(0.24,0.29,0.9)	(0.37,0.41,1)	(0.11,0.16,0.8)	(0.24,0.29,0.9)	(0.37,0.41,1)	(0.11,0.16,0.8)	(0.24,0.29,0.9)	(0.33,0.41,1)
98	(0.42,0.48,1)	(0.12,0.16,0.97)	(0.27,0.32,0.99)	(0.42,0.48,1)	(0.12,0.16,0.97)	(0.27,0.32,0.99)	(0.42,0.48,1)	(0.12,0.16,0.97)	(0.27,0.32,0.99)	(0.33,0.41,1)
99	(0.39,0.45,0.99)	(0.12,0.17,0.26)	(0.25,0.31,0.63)	(0.39,0.45,0.99)	(0.12,0.17,0.26)	(0.25,0.31,0.63)	(0.39,0.45,0.99)	(0.12,0.17,0.26)	(0.25,0.31,0.63)	(0.47,0.15,0.94)
100	(0.42,0.48,1)	(0.11,0.16,0.66)	(0.27,0.32,0.83)	(0.42,0.48,1)	(0.11,0.16,0.66)	(0.27,0.32,0.83)	(0.42,0.48,1)	(0.11,0.16,0.66)	(0.27,0.32,0.83)	(0.59,0.41,1)
101	(0.42,0.41,0.99)	(0.11,0.16,0.87)	(0.27,0.29,0.93)	(0.42,0.41,0.99)	(0.11,0.16,0.87)	(0.27,0.29,0.93)	(0.42,0.41,0.99)	(0.11,0.16,0.87)	(0.27,0.29,0.93)	(0.77,0.36,0.98)
102	(0.42,0.49,1)	(0.11,0.16,0.69)	(0.27,0.32,0.85)	(0.42,0.49,1)	(0.11,0.16,0.69)	(0.27,0.32,0.85)	(0.42,0.49,1)	(0.11,0.16,0.69)	(0.27,0.32,0.85)	(0.94,0.45,1)
103	(0.35,0.4,1)	(0.11,0.16,0.8)	(0.23,0.28,0.9)	(0.35,0.4,1)	(0.11,0.16,0.8)	(0.23,0.28,0.9)	(0.35,0.4,1)	(0.11,0.16,0.8)	(0.23,0.28,0.9)	(0.94,0.4,1)
104	(0.42,0.41,1)	(0.11,0.16,0.8)	(0.27,0.29,0.9)	(0.42,0.41,1)	(0.11,0.16,0.8)	(0.27,0.29,0.9)	(0.42,0.41,1)	(0.11,0.16,0.8)	(0.27,0.29,0.9)	(0.94,0.41,0.99)
105	(0.38,0.41,1)	(0.06,0.09,0.94)	(0.22,0.25,0.97)	(0.38,0.41,1)	(0.06,0.09,0.94)	(0.22,0.25,0.97)	(0.38,0.41,1)	(0.06,0.09,0.94)	(0.22,0.25,0.97)	(0.94,0.41,1)

Table 2. Cross-fuzzy efficiency of the stages and the whole system using Hatami-Marbini's approach [39].

Branch	Average			Stage 1 is the leader			Stage 2 is the leader			Aggregation
	Stage 1	Stage 2	System	Stage 1	Stage 2	System	Stage 1	Stage 2	System	System
1	(0.25,0.26,0.31)	(0.11,0.13,0.13)	(0.18,0.19,0.22)	(0.25,0.26,0.31)	(0.25,0.26,0.31)	(0.11,0.13,0.13)	(0.18,0.19,0.22)	(0.25,0.26,0.31)	(0.25,0.26,0.31)	(0.11,0.13,0.13)
2	(0.22,0.25,0.26)	(0.05,0.05,0.05)	(0.13,0.15,0.16)	(0.22,0.25,0.26)	(0.22,0.25,0.26)	(0.05,0.05,0.05)	(0.13,0.15,0.16)	(0.22,0.25,0.26)	(0.22,0.25,0.26)	(0.05,0.05,0.05)
3	(0.3,0.33,0.42)	(0.04,0.04,0.04)	(0.17,0.18,0.23)	(0.3,0.33,0.42)	(0.3,0.33,0.42)	(0.04,0.04,0.04)	(0.17,0.18,0.23)	(0.3,0.33,0.42)	(0.3,0.33,0.42)	(0.04,0.04,0.04)
4	(0.36,0.47,0.47)	(0.05,0.06,0.06)	(0.21,0.26,0.26)	(0.36,0.47,0.47)	(0.36,0.47,0.47)	(0.05,0.06,0.06)	(0.21,0.26,0.26)	(0.36,0.47,0.47)	(0.36,0.47,0.47)	(0.05,0.06,0.06)
5	(0.09,0.1,0.1)	(0.07,0.07,0.07)	(0.08,0.09,0.09)	(0.09,0.1,0.1)	(0.09,0.1,0.1)	(0.07,0.07,0.07)	(0.08,0.09,0.09)	(0.09,0.1,0.1)	(0.09,0.1,0.1)	(0.07,0.07,0.07)
6	(0.15,0.15,0.19)	(0.09,0.1,0.1)	(0.12,0.13,0.15)	(0.15,0.15,0.19)	(0.15,0.15,0.19)	(0.09,0.1,0.1)	(0.12,0.13,0.15)	(0.15,0.15,0.19)	(0.15,0.15,0.19)	(0.09,0.1,0.1)
7	(0.62,0.82,0.88)	(0.38,0.4,0.4)	(0.5,0.61,0.64)	(0.62,0.82,0.88)	(0.62,0.82,0.88)	(0.38,0.4,0.4)	(0.5,0.61,0.64)	(0.62,0.82,0.88)	(0.62,0.82,0.88)	(0.38,0.4,0.4)
8	(0.13,0.17,0.18)	(0.01,0.01,0.01)	(0.07,0.09,0.09)	(0.13,0.17,0.18)	(0.13,0.17,0.18)	(0.01,0.01,0.01)	(0.07,0.09,0.09)	(0.13,0.17,0.18)	(0.13,0.17,0.18)	(0.01,0.01,0.01)
9	(0.1,0.13,0.13)	(0.01,0.01,0.01)	(0.05,0.07,0.07)	(0.1,0.13,0.13)	(0.1,0.13,0.13)	(0.01,0.01,0.01)	(0.05,0.07,0.07)	(0.1,0.13,0.13)	(0.1,0.13,0.13)	(0.01,0.01,0.01)
10	(0.44,0.55,0.55)	(0,0,0)	(0.22,0.28,0.28)	(0.44,0.55,0.55)	(0.44,0.55,0.55)	(0,0,0)	(0.22,0.28,0.28)	(0.44,0.55,0.55)	(0.44,0.55,0.55)	(0,0,0)
11	(0.85,1.13,1.18)	(0.76,0.78,0.78)	(0.81,0.96,0.98)	(0.78,1.04,1.09)	(0.85,1.13,1.18)	(0.76,0.78,0.78)	(0.81,0.96,0.98)	(0.78,1.04,1.09)	(0.85,1.13,1.18)	(0.76,0.78,0.78)
12	(0.68,0.95,1.02)	(0.05,0.05,0.07)	(0.36,0.5,0.54)	(0.65,0.91,0.98)	(0.68,0.95,1.02)	(0.05,0.05,0.07)	(0.36,0.5,0.54)	(0.65,0.91,0.98)	(0.68,0.95,1.02)	(0.05,0.05,0.07)
13	(0.21,0.25,0.25)	(0,0,0.04)	(0.11,0.12,0.14)	(0.21,0.25,0.25)	(0.21,0.25,0.25)	(0,0,0.04)	(0.11,0.12,0.14)	(0.21,0.25,0.25)	(0.21,0.25,0.25)	(0,0,0.04)
14	(0.37,0.43,0.44)	(0,0,0)	(0.19,0.22,0.22)	(0.37,0.43,0.43)	(0.37,0.43,0.44)	(0,0,0)	(0.19,0.22,0.22)	(0.37,0.43,0.43)	(0.37,0.43,0.44)	(0,0,0)
15	(0.2,0.2,0.23)	(0.09,0.09,0.09)	(0.14,0.15,0.16)	(0.2,0.2,0.23)	(0.2,0.2,0.23)	(0.09,0.09,0.09)	(0.14,0.15,0.16)	(0.2,0.2,0.23)	(0.2,0.2,0.23)	(0.09,0.09,0.09)

16	(0.37,0.49,0.52)	(0.05,0.06,0.06)	(0.21,0.27,0.29)	(0.37,0.49,0.52)	(0.37,0.49,0.52)	(0.05,0.06,0.06)	(0.21,0.27,0.29)	(0.37,0.49,0.52)	(0.37,0.49,0.52)	(0.05,0.06,0.06)
17	(0.99,1.25,1.3)	(0.37,0.55,0.55)	(0.68,0.9,0.93)	(0.98,1.23,1.28)	(0.99,1.25,1.3)	(0.37,0.55,0.55)	(0.68,0.9,0.93)	(0.98,1.23,1.28)	(0.99,1.25,1.3)	(0.37,0.55,0.55)
18	(0.2,0.22,0.23)	(0.04,0.06,0.06)	(0.12,0.14,0.14)	(0.21,0.22,0.23)	(0.2,0.22,0.23)	(0.04,0.06,0.06)	(0.12,0.14,0.14)	(0.21,0.22,0.23)	(0.2,0.22,0.23)	(0.04,0.06,0.06)
19	(0.28,0.33,0.33)	(0.13,0.15,0.15)	(0.21,0.24,0.24)	(0.28,0.33,0.33)	(0.28,0.33,0.33)	(0.13,0.15,0.15)	(0.21,0.24,0.24)	(0.28,0.33,0.33)	(0.28,0.33,0.33)	(0.13,0.15,0.15)
20	(0.15,0.19,0.2)	(0.1,0.1,0.1)	(0.12,0.15,0.15)	(0.15,0.19,0.2)	(0.15,0.19,0.2)	(0.1,0.1,0.1)	(0.12,0.15,0.15)	(0.15,0.19,0.2)	(0.15,0.19,0.2)	(0.1,0.1,0.1)
21	(0.41,0.41,0.44)	(0.05,0.06,0.06)	(0.23,0.23,0.25)	(0.41,0.41,0.44)	(0.41,0.41,0.44)	(0.05,0.06,0.06)	(0.23,0.23,0.25)	(0.41,0.41,0.44)	(0.41,0.41,0.44)	(0.05,0.06,0.06)
22	(0.28,0.3,0.33)	(0,0,0)	(0.14,0.15,0.17)	(0.28,0.3,0.33)	(0.28,0.3,0.33)	(0,0,0)	(0.14,0.15,0.17)	(0.28,0.3,0.33)	(0.28,0.3,0.33)	(0,0,0)
23	(0.17,0.18,0.19)	(0.08,0.08,0.08)	(0.12,0.13,0.14)	(0.17,0.18,0.19)	(0.17,0.18,0.19)	(0.08,0.08,0.08)	(0.12,0.13,0.14)	(0.17,0.18,0.19)	(0.17,0.18,0.19)	(0.08,0.08,0.08)
24	(0.34,0.38,0.38)	(0.02,0.03,0.03)	(0.18,0.2,0.2)	(0.34,0.38,0.38)	(0.34,0.38,0.38)	(0.02,0.03,0.03)	(0.18,0.2,0.2)	(0.34,0.38,0.38)	(0.34,0.38,0.38)	(0.02,0.03,0.03)
25	(0.18,0.18,0.18)	(0.03,0.05,0.05)	(0.11,0.12,0.12)	(0.18,0.18,0.18)	(0.18,0.18,0.18)	(0.03,0.05,0.05)	(0.11,0.12,0.12)	(0.18,0.18,0.18)	(0.18,0.18,0.18)	(0.03,0.05,0.05)
26	(0.27,0.32,0.34)	(0.13,0.15,0.15)	(0.2,0.24,0.25)	(0.27,0.32,0.34)	(0.27,0.32,0.34)	(0.13,0.15,0.15)	(0.2,0.24,0.25)	(0.27,0.32,0.34)	(0.27,0.32,0.34)	(0.13,0.15,0.15)
27	(0.17,0.19,0.24)	(0.19,0.21,0.21)	(0.18,0.2,0.23)	(0.17,0.19,0.24)	(0.17,0.19,0.24)	(0.19,0.21,0.21)	(0.18,0.2,0.23)	(0.17,0.19,0.24)	(0.17,0.19,0.24)	(0.19,0.21,0.21)
28	(0.21,0.23,0.3)	(0.06,0.06,0.06)	(0.14,0.15,0.18)	(0.21,0.23,0.3)	(0.21,0.23,0.3)	(0.06,0.06,0.06)	(0.14,0.15,0.18)	(0.21,0.23,0.3)	(0.21,0.23,0.3)	(0.06,0.06,0.06)
29	(0.68,0.71,0.71)	(0.15,0.17,0.17)	(0.42,0.44,0.44)	(0.68,0.71,0.71)	(0.68,0.71,0.71)	(0.15,0.17,0.17)	(0.42,0.44,0.44)	(0.68,0.71,0.71)	(0.68,0.71,0.71)	(0.15,0.17,0.17)
30	(0.49,0.62,0.96)	(0.14,0.16,0.16)	(0.32,0.39,0.56)	(0.49,0.61,0.95)	(0.49,0.62,0.96)	(0.14,0.16,0.16)	(0.32,0.39,0.56)	(0.49,0.61,0.95)	(0.49,0.62,0.96)	(0.14,0.16,0.16)
31	(0.31,0.34,0.34)	(0.11,0.13,0.13)	(0.21,0.23,0.23)	(0.31,0.34,0.34)	(0.31,0.34,0.34)	(0.11,0.13,0.13)	(0.21,0.23,0.23)	(0.31,0.34,0.34)	(0.31,0.34,0.34)	(0.11,0.13,0.13)
32	(0.19,0.22,0.23)	(0.04,0.04,0.04)	(0.12,0.13,0.13)	(0.19,0.22,0.23)	(0.19,0.22,0.23)	(0.04,0.04,0.04)	(0.12,0.13,0.13)	(0.19,0.22,0.23)	(0.19,0.22,0.23)	(0.04,0.04,0.04)
33	(0.42,0.53,0.58)	(0.09,0.1,0.11)	(0.26,0.32,0.34)	(0.42,0.53,0.58)	(0.42,0.53,0.58)	(0.09,0.1,0.11)	(0.26,0.32,0.34)	(0.42,0.53,0.58)	(0.42,0.53,0.58)	(0.09,0.1,0.11)
34	(1.24,1.81,1.93)	(0.01,0.01,0.19)	(0.62,0.91,1.06)	(1.22,1.77,1.92)	(1.24,1.81,1.93)	(0.01,0.01,0.19)	(0.62,0.91,1.06)	(1.22,1.77,1.92)	(1.24,1.81,1.93)	(0.01,0.01,0.19)
35	(1.27,1.47,1.47)	(0.26,0.32,0.32)	(0.76,0.9,0.9)	(1.27,1.47,1.47)	(1.27,1.47,1.47)	(0.26,0.32,0.32)	(0.76,0.9,0.9)	(1.27,1.47,1.47)	(1.27,1.47,1.47)	(0.26,0.32,0.32)
36	(0.35,0.46,0.48)	(0.08,0.13,0.13)	(0.22,0.29,0.3)	(0.35,0.46,0.48)	(0.35,0.46,0.48)	(0.08,0.13,0.13)	(0.22,0.29,0.3)	(0.35,0.46,0.48)	(0.35,0.46,0.48)	(0.08,0.13,0.13)
37	(0.41,0.43,0.44)	(0.1,0.1,0.1)	(0.25,0.26,0.27)	(0.41,0.43,0.44)	(0.41,0.43,0.44)	(0.1,0.1,0.1)	(0.25,0.26,0.27)	(0.41,0.43,0.44)	(0.41,0.43,0.44)	(0.1,0.1,0.1)
38	(0.76,1.1,1.22)	(0.6,0.63,0.63)	(0.68,0.87,0.93)	(0.75,1.08,1.2)	(0.76,1.1,1.22)	(0.6,0.63,0.63)	(0.68,0.87,0.93)	(0.75,1.08,1.2)	(0.76,1.1,1.22)	(0.6,0.63,0.63)
39	(0.23,0.27,0.27)	(0.21,0.23,0.23)	(0.22,0.25,0.25)	(0.23,0.27,0.27)	(0.23,0.27,0.27)	(0.21,0.23,0.23)	(0.22,0.25,0.25)	(0.23,0.27,0.27)	(0.23,0.27,0.27)	(0.21,0.23,0.23)
40	(0.67,0.72,0.77)	(0.33,0.35,0.35)	(0.5,0.54,0.56)	(0.96,1.03,1.08)	(0.67,0.72,0.77)	(0.33,0.35,0.35)	(0.5,0.54,0.56)	(0.96,1.03,1.08)	(0.67,0.72,0.77)	(0.33,0.35,0.35)
41	(0.27,0.33,0.34)	(0.07,0.08,0.08)	(0.17,0.2,0.21)	(0.27,0.33,0.34)	(0.27,0.33,0.34)	(0.07,0.08,0.08)	(0.17,0.2,0.21)	(0.27,0.33,0.34)	(0.27,0.33,0.34)	(0.07,0.08,0.08)
42	(1.02,1.25,1.27)	(0.55,0.6,0.6)	(0.78,0.92,0.93)	(1.02,1.25,1.27)	(1.02,1.25,1.27)	(0.55,0.6,0.6)	(0.78,0.92,0.93)	(1.02,1.25,1.27)	(1.02,1.25,1.27)	(0.55,0.6,0.6)
43	(0.45,0.57,0.59)	(0.23,0.3,0.3)	(0.34,0.43,0.44)	(0.5,0.63,0.66)	(0.45,0.57,0.59)	(0.23,0.3,0.3)	(0.34,0.43,0.44)	(0.5,0.63,0.66)	(0.45,0.57,0.59)	(0.23,0.3,0.3)
44	(0.86,1,1.05)	(0.3,0.31,0.31)	(0.58,0.66,0.68)	(0.86,1,1.05)	(0.86,1,1.05)	(0.3,0.31,0.31)	(0.58,0.66,0.68)	(0.86,1,1.05)	(0.86,1,1.05)	(0.3,0.31,0.31)
45	(0.46,0.47,0.47)	(0.07,0.09,0.09)	(0.26,0.28,0.28)	(0.46,0.47,0.47)	(0.46,0.47,0.47)	(0.07,0.09,0.09)	(0.26,0.28,0.28)	(0.46,0.47,0.47)	(0.46,0.47,0.47)	(0.07,0.09,0.09)
46	(0.83,0.85,0.86)	(1.05,1.16,1.16)	(0.94,1,1.01)	(0.83,0.85,0.86)	(0.83,0.85,0.86)	(1.05,1.16,1.16)	(0.94,1,1.01)	(0.83,0.85,0.86)	(0.83,0.85,0.86)	(1.05,1.16,1.16)
47	(0.33,0.4,0.41)	(0.21,0.29,0.29)	(0.27,0.35,0.35)	(0.33,0.4,0.41)	(0.33,0.4,0.41)	(0.21,0.29,0.29)	(0.27,0.35,0.35)	(0.33,0.4,0.41)	(0.33,0.4,0.41)	(0.21,0.29,0.29)
48	(0.38,0.67,0.75)	(0.15,0.18,0.18)	(0.26,0.42,0.46)	(0.38,0.68,0.76)	(0.38,0.67,0.75)	(0.15,0.18,0.18)	(0.26,0.42,0.46)	(0.38,0.68,0.76)	(0.38,0.67,0.75)	(0.15,0.18,0.18)
49	(0.32,0.35,0.35)	(0.15,0.16,0.16)	(0.24,0.25,0.26)	(0.32,0.35,0.35)	(0.32,0.35,0.35)	(0.15,0.16,0.16)	(0.24,0.25,0.26)	(0.32,0.35,0.35)	(0.32,0.35,0.35)	(0.15,0.16,0.16)
50	(0.42,0.62,0.72)	(0.38,0.39,0.39)	(0.4,0.5,0.55)	(0.41,0.6,0.7)	(0.42,0.62,0.72)	(0.38,0.39,0.39)	(0.4,0.5,0.55)	(0.41,0.6,0.7)	(0.42,0.62,0.72)	(0.38,0.39,0.39)
51	(0.36,0.43,0.45)	(0.11,0.13,0.13)	(0.24,0.28,0.29)	(0.36,0.43,0.45)	(0.36,0.43,0.45)	(0.11,0.13,0.13)	(0.24,0.28,0.29)	(0.36,0.43,0.45)	(0.36,0.43,0.45)	(0.11,0.13,0.13)
52	(1.13,1.83,5.42)	(0.65,0.69,0.69)	(0.89,1.26,3.06)	(1.09,1.69,5.53)	(1.13,1.83,5.42)	(0.65,0.69,0.69)	(0.89,1.26,3.06)	(1.09,1.69,5.53)	(1.13,1.83,5.42)	(0.65,0.69,0.69)
53	(1.01,1.05,1.09)	(0.26,4.26,5.56)	(0.63,2.66,3.32)	(1,1.04,1.08)	(1.01,1.05,1.09)	(0.26,4.26,5.56)	(0.63,2.66,3.32)	(1,1.04,1.08)	(1.01,1.05,1.09)	(0.26,4.26,5.56)
54	(0.53,0.78,0.84)	(0.29,0.6,0.59)	(0.41,0.69,0.72)	(0.53,0.78,0.84)	(0.53,0.78,0.84)	(0.29,0.6,0.59)	(0.41,0.69,0.72)	(0.53,0.78,0.84)	(0.53,0.78,0.84)	(0.29,0.6,0.59)
55	(0.54,0.6,0.63)	(0.29,0.36,0.36)	(0.41,0.48,0.49)	(0.54,0.61,0.63)	(0.54,0.6,0.63)	(0.29,0.36,0.36)	(0.41,0.48,0.49)	(0.54,0.61,0.63)	(0.54,0.6,0.63)	(0.29,0.36,0.36)
56	(0.74,0.82,0.92)	(0.24,1.95,1.95)	(0.49,1.38,1.43)	(0.74,0.82,0.92)	(0.74,0.82,0.92)	(0.24,1.95,1.95)	(0.49,1.38,1.43)	(0.74,0.82,0.92)	(0.74,0.82,0.92)	(0.24,1.95,1.95)
57	(0.34,0.42,0.44)	(0.13,0.15,0.15)	(0.23,0.28,0.29)	(0.34,0.42,0.44)	(0.34,0.42,0.44)	(0.13,0.15,0.15)	(0.23,0.28,0.29)	(0.34,0.42,0.44)	(0.34,0.42,0.44)	(0.13,0.15,0.15)
58	(0,0,0.01)	(0.2,0.21,0.21)	(0.1,0.11,0.11)	(0,0,0.01)	(0,0,0.01)	(0.2,0.21,0.21)	(0.1,0.11,0.11)	(0,0,0.01)	(0,0,0.01)	(0.2,0.21,0.21)
59	(0.41,0.43,0.44)	(0.09,0.09,0.09)	(0.25,0.26,0.27)	(0.41,0.43,0.44)	(0.41,0.43,0.44)	(0.09,0.09,0.09)	(0.25,0.26,0.27)	(0.41,0.43,0.44)	(0.41,0.43,0.44)	(0.09,0.09,0.09)
60	(0.29,0.37,0.38)	(0.11,0.11,0.11)	(0.2,0.24,0.25)	(0.29,0.37,0.38)	(0.29,0.37,0.38)	(0.11,0.11,0.11)	(0.2,0.24,0.25)	(0.29,0.37,0.38)	(0.29,0.37,0.38)	(0.11,0.11,0.11)
61	(0.32,0.35,0.51)	(0.22,0.26,0.26)	(0.27,0.3,0.38)	(0.32,0.35,0.51)	(0.32,0.35,0.51)	(0.22,0.26,0.26)	(0.27,0.3,0.38)	(0.32,0.35,0.51)	(0.32,0.35,0.51)	(0.22,0.26,0.26)
62	(0.3,0.36,0.39)	(0.05,0.06,0.06)	(0.18,0.21,0.23)	(0.3,0.36,0.39)	(0.3,0.36,0.39)	(0.05,0.06,0.06)	(0.18,0.21,0.23)	(0.3,0.36,0.39)	(0.3,0.36,0.39)	(0.05,0.06,0.06)
63	(0.83,0.85,0.85)	(0.37,0.44,0.44)	(0.6,0.65,0.65)	(0.89,0.92,0.92)	(0.83,0.85,0.85)	(0.37,0.44,0.44)	(0.6,0.65,0.65)	(0.89,0.92,0.92)	(0.83,0.85,0.85)	(0.37,0.44,0.44)
64	(0.48,0.5,0.52)	(0.01,0.01,0.02)	(0.25,0.25,0.27)	(0.45,0.46,0.5)	(0.48,0.5,0.52)	(0.01,0.01,0.02)	(0.25,0.25,0.27)	(0.45,0.46,0.5)	(0.48,0.5,0.52)	(0.01,0.01,0.02)
65	(0.41,0.51,0.62)	(0.86,0.88,0.88)	(0.64,0.7,0.75)	(0.41,0.51,0.62)	(0.41,0.51,0.62)	(0.86,0.88,0.88)	(0.64,0.7,0.75)	(0.41,0.51,0.62)	(0.41,0.51,0.62)	(0.86,0.88,0.88)
66	(0.85,0.95,0.96)	(0.23,0.26,0.26)	(0.54,0.61,0.61)	(0.85,0.95,0.95)	(0.85,0.95,0.96)	(0.23,0.26,0.26)	(0.54,0.61,0.61)	(0.85,0.95,0.95)	(0.85,0.95,0.96)	(0.23,0.26,0.26)
67	(0.13,0.21,0.22)	(0.19,0.22,0.22)	(0.16,0.22,0.22)	(0.13,0.21,0.22)	(0.13,0.21,0.22)	(0.19,0.22,0.22)	(0.16,0.22,0.22)	(0.13,0.21,0.22)	(0.13,0.21,0.22)	(0.19,0.22,0.22)
68	(0.21,0.21,0.25)	(0,0,0.01)	(0.11,0.11,0.13)	(0.21,0.21,0.25)	(0.21,0.21,0.25)	(0,0,0.01)	(0.11,0.11,0.13)	(0.21,0.21,0.25)	(0.21,0.21,0.25)	(0,0,0.01)
69	(0.62,0.79,0.79)	(0.25,0.27,0.27)	(0.44,0.53,0.53)	(0.62,0.79,0.79)	(0.62,0.79,0.79)	(0.25,0.27,0.27)	(0.44,0.53,0.53)	(0.62,0.79,0.79)	(0.62,0.79,0.79)	(0.25,0.27,0.27)
70	(0.25,0.29,0.29)	(0.06,0.07,0.07)	(0.15,0.18,0.18)	(0.25,0.29,0.29)	(0.25,0.29,0.29)	(0.06,0.07,0.07)	(0.15,0.18,0.18)	(0.25,0.29,0.29)	(0.25,0.29,0.29)	(0.06,0.07,0.07)
71	(0.68,0.84,0.93)	(0.17,0.17,0.17)	(0.43,0.51,0.55)	(0.68,0.84,0.93)	(0.68,0.84,0.93)	(0.17,0.17,0.17)	(0.43,0.51,0.55)	(0.68,0.84,0.93)	(0.68,0.84,0.93)	(0.17,0.17,0.17)
72	(0.5,0.6,0.62)	(0.23,0.24,0.24)	(0.37,0.42,0.43)	(0.5,0.6,0.62)	(0.5,0.6,0.62)	(0.23,0.24,0.24)	(0.37,0.42,0.43)	(0.5,0.6,0.62)	(0.5,0.6,0.62)	(0.23,0.24,0.24)
73	(0.41,0.53,0.62)	(0,0,0)	(0.21,0.26,0.31)	(0.41,0.53,0.62)	(0.41,0.53,0.62)	(0,0,0)	(0.21,0.26,0.31)	(0.41,0.53,0.62)	(0.41,0.53,0.62)	(0,0,0)
74	(0.35,0.38,0.38)	(0.12,0.14,0.14)	(0.24,0.26,0.26)	(0.35,0.38,0.38)	(0.35,0.38,0.38)	(0.12,0.14,0.14)	(0.24,0.26,0.26)	(0.35,0.38,0.38)	(0.35,0.38,0.38)	(0.12,0.14,0.14)
75	(0.35,0.48,0.52)	(0.07,0.09,0.09)	(0.21,0.28,0.3)	(0.35,0.48,0.52)	(0.35,0.48,0.52)	(0.07,0.09,0.09)	(0.21,0.28,0.3)	(0.35,0.48,0.52)	(0.35,0.48,0.52)	(0.07,0.09,0.09)
76	(0.41,0.46,0.54)	(0.24,0.24,0.24)	(0.32,0.35,0.39)	(0.4,0.45,0.54)	(0.41,0.46,0.54)	(0.24,0.24,0.24)	(0.32,0.35,0.39)	(0.4,0.45,0.54)	(0.41,0.46,0.54)	(0.24,0.24,0.24)
77	(0.26,0.29,0.35)	(0.09,0.1,0.1)	(0.17,0.2,0.22)	(0.26,0.29,0.35)	(0.26,0.29,0.35)	(0.09,0.1,0.1)	(0.17,0.2,0.22)	(0.26,0.29,0.35)	(0.26,0.29,0.35)	(0.09,0.1,0.1)
78	(0.24,0.3,0.33)	(0.18,0.19,0.19)	(0.21,0.25,0.26)	(0.24,0.3,0.33)	(0.24,0.3,0.33)	(0.18,0.19,0.19)	(0.21,0.25,0.26)	(0.24,0.3,0.33)	(0.24,0.3,0.33)	(0.18,0.19,0.19)
79	(0.53,0.63,0.64)	(0.28,0.28,0.28)	(0.41,0.46,0.46)	(0.53,0.63,0.64)	(0.53,0.63,0.64)	(0.28,0.28,0.28)	(0.41,0.46,0.46)	(0.53,0.63,0.64)	(0.53,0.63,0.64)	(0.28,0.28,0.28)
80	(0.51,0.54,0.55)	(0.01,0.01,0.01)	(0.26,0.27,0.28)	(0.51,0.54,0.55)	(0.51,0.54,0.55)	(0.01,0.01,0.01)	(0.26,0.27,0.28)	(0.51,0.54,0.55)	(0.51,0.54,0.55)	(0.01,0.01,0.01)
81	(0.52,0.58,0.6)	(0,0,0.01)	(0.26,0.29,0.3)	(0.52,0.58,0.6)	(0.52,0.58,0.6)	(0,0,0.01)	(0.26,0.29,0.3)	(0.52,0.58,0.6)	(0.52,0.58,0.6)	(0,0,0.01)
82	(0,0,0.14)	(0.3,0.32,0.32)	(0.15,0.16,0.23)	(0,0,0.14)	(0,0,0.14)	(0.3,0.32,0.32)	(0.15,0.16,0.23)	(0,0,0.14)	(0,0,0.14)	(0.3,0.32,0.32)
83	(0.36,0.39,0.39)	(0.09,0.11,0.11)	(0.23,0.25,0.25)	(0.36,0.39,0.39)	(0.36,0.39,0.39)	(0.09,0.11,0.11)	(0.23,0.25,0.25)	(0.36,0.39,0.39)	(0.36,0.39,0.39)	(0.09,0.11,0.11)
84	(0.27,0.27,0.3)	(0.08,0.09,0.09)	(0.17,0.18,0.19)	(0.27,0.27,0.3)	(0.27,0.27,0.3)	(0.08,0.09,0.09)	(0.17,0.18,0.19)	(0.27,0.27,0.3)	(0.27,0.27,0.3)	(0.08,0.09,0.09)
85	(0.54,0.58,0.59)	(0.36,0.36,0.36)	(0.45,0.47,0.48)	(0.54,0.58,0.59)	(0.54,0.58,0.59)	(0.36,0.36,0.36)	(0.45,0.47,0.48)	(0.54,0.58,0.59)	(0.54,0.58,0.59)	(0.36,0.36,0.36)
86	(0.63,0.66,0.66)	(0.09,0.11,0.11)	(0.36,0.38,0.39)	(0.62,0.65,0.65)	(0.63,0.66,0.66)	(0.09,0.11,0.11)	(0.36,0.38,0.39)	(0.62,0.65,0.65)	(0.63,0.66,0.66)	(0.09,0.11,0.11)
87	(0.44,0.59,0.61)	(0.24,0.32,0.32)	(0.34,0.45,0.47)	(0.44,0.59,0.61)	(0.44,0.59,0.61)	(0.24,0.32,0.32)	(0.34,0.45,0.47)	(0.44,0.59,0.61)	(0.44,0.59,0.61)	(0.24,0.32,0.32)
88	(0.45,0.47,0.49)	(0.17,0.2,0.21)	(0.31,0.33,0.35)	(0.45,0.47,0.49)	(0.45,0.47,0.49)	(0.17,0.2,0.21)	(0.31,0.33,0.35)	(0.45,0.47,0.49)	(0.45,0.47,0.49)	(0.17,0.2,0.21)
89	(0.31,0.38,0.42)	(0.13,0.13,0.13)	(0.22,0.25,0.27)	(0.31,0.38,0.42)	(0.31,0.38,0.42)	(0.13,0.13,0.13)	(0.22,0.25,0.27)	(0.31,0.38,0.42)	(0.31,0.38,0.42)	(0.13,0.13,0.13)

90	(0.4,0.49,0.5)	(0.15,0.15,0.15)	(0.27,0.32,0.32)	(0.4,0.49,0.5)	(0.4,0.49,0.5)	(0.15,0.15,0.15)	(0.27,0.32,0.32)	(0.4,0.49,0.5)	(0.4,0.49,0.5)	(0.15,0.15,0.15)
91	(1,1.2,1.2)	(0.29,0.29,0.29)	(0.64,0.74,0.74)	(1,1.2,1.2)	(1,1.2,1.2)	(0.29,0.29,0.29)	(0.64,0.74,0.74)	(1,1.2,1.2)	(1,1.2,1.2)	(0.29,0.29,0.29)
92	(0.78,0.87,0.89)	(0,0,0)	(0.39,0.44,0.45)	(0.78,0.87,0.89)	(0.78,0.87,0.89)	(0,0,0)	(0.39,0.44,0.45)	(0.78,0.87,0.89)	(0.78,0.87,0.89)	(0,0,0)
93	(0.99,1.18,1.19)	(0.27,0.29,0.29)	(0.63,0.73,0.74)	(0.99,1.18,1.19)	(0.99,1.18,1.19)	(0.27,0.29,0.29)	(0.63,0.73,0.74)	(0.99,1.18,1.19)	(0.99,1.18,1.19)	(0.27,0.29,0.29)
94	(0.67,0.72,0.78)	(0.25,0.3,0.3)	(0.46,0.51,0.54)	(0.67,0.72,0.78)	(0.67,0.72,0.78)	(0.25,0.3,0.3)	(0.46,0.51,0.54)	(0.67,0.72,0.78)	(0.67,0.72,0.78)	(0.25,0.3,0.3)
95	(0.36,0.44,0.45)	(0.13,0.14,0.14)	(0.25,0.29,0.3)	(0.36,0.44,0.45)	(0.36,0.44,0.45)	(0.13,0.14,0.14)	(0.25,0.29,0.3)	(0.36,0.44,0.45)	(0.36,0.44,0.45)	(0.13,0.14,0.14)
96	(1.16,1.52,1.6)	(0.17,0.19,0.19)	(0.66,0.86,0.9)	(1.16,1.52,1.6)	(1.16,1.52,1.6)	(0.17,0.19,0.19)	(0.66,0.86,0.9)	(1.16,1.52,1.6)	(1.16,1.52,1.6)	(0.17,0.19,0.19)
97	(0.49,0.62,0.62)	(0,0,0.06)	(0.24,0.31,0.34)	(0.49,0.62,0.62)	(0.49,0.62,0.62)	(0,0,0.06)	(0.24,0.31,0.34)	(0.49,0.62,0.62)	(0.49,0.62,0.62)	(0,0,0.06)
98	(0.47,0.55,0.57)	(0.18,0.18,0.18)	(0.32,0.36,0.37)	(0.47,0.55,0.57)	(0.47,0.55,0.57)	(0.18,0.18,0.18)	(0.32,0.36,0.37)	(0.47,0.55,0.57)	(0.47,0.55,0.57)	(0.18,0.18,0.18)
99	(1.21,1.23,1.27)	(1.15,1.28,1.43)	(1.18,1.25,1.35)	(0.74,1.22,1.75)	(1.21,1.23,1.27)	(1.15,1.28,1.43)	(1.18,1.25,1.35)	(0.74,1.22,1.75)	(1.21,1.23,1.27)	(1.15,1.28,1.43)
100	(0.57,0.69,0.69)	(0.18,0.22,0.22)	(0.37,0.46,0.46)	(0.57,0.69,0.69)	(0.57,0.69,0.69)	(0.18,0.22,0.22)	(0.37,0.46,0.46)	(0.57,0.69,0.69)	(0.57,0.69,0.69)	(0.18,0.22,0.22)
101	(0.75,0.91,0.97)	(0.53,0.54,0.54)	(0.64,0.73,0.76)	(0.74,0.88,0.94)	(0.75,0.91,0.97)	(0.53,0.54,0.54)	(0.64,0.73,0.76)	(0.74,0.88,0.94)	(0.75,0.91,0.97)	(0.53,0.54,0.54)
102	(0.59,0.71,0.71)	(0.17,0.22,0.24)	(0.38,0.46,0.47)	(0.59,0.71,0.71)	(0.59,0.71,0.71)	(0.17,0.22,0.24)	(0.38,0.46,0.47)	(0.59,0.71,0.71)	(0.59,0.71,0.71)	(0.17,0.22,0.24)
103	(0.94,1.1,1.11)	(0.02,0.02,0.1)	(0.48,0.56,0.6)	(0.94,1.1,1.11)	(0.94,1.1,1.11)	(0.02,0.02,0.1)	(0.48,0.56,0.6)	(0.94,1.1,1.11)	(0.94,1.1,1.11)	(0.02,0.02,0.1)
104	(1.65,1.79,1.79)	(0.46,0.49,0.49)	(1.05,1.14,1.14)	(1.65,1.79,1.79)	(1.65,1.79,1.79)	(0.46,0.49,0.49)	(1.05,1.14,1.14)	(1.65,1.79,1.79)	(1.65,1.79,1.79)	(0.46,0.49,0.49)
105	(1.41,1.64,1.64)	(0.23,0.29,0.29)	(0.82,0.97,0.97)	(1.41,1.64,1.64)	(1.41,1.64,1.64)	(0.23,0.29,0.29)	(0.82,0.97,0.97)	(1.41,1.64,1.64)	(1.41,1.64,1.64)	(0.23,0.29,0.29)

5. Conclusion

The aim of this study was to use the DEA technique in the banking industry. The problem assumptions were considered in such a way that the problem became closer to the real world. This is the reason for why we used the fuzzy data. The parameters of the problem were considered as TFN and the problem as fully fuzzy. To solve fully fuzzy problems, two different approaches were used. Alpha-cuts serve as a powerful tool for extracting crisp information from fuzzy sets, enabling precise analysis and control in various engineering applications. By utilizing alpha-cuts, engineers can effectively manage uncertainty and confidence level. Another method has high computational simplicity. In order that the proposed models do not have the defects of traditional models and to differentiate between efficient units, the cross efficiency technique was used. In order to be able to calculate the efficiency of the internal subsystems of each unit in order to make correct management decisions, we used network efficiency. A general two-stage network system was used, as it was compatible with the banking process. Using the combination of different approaches, several models were presented, each of which alone has all the mentioned conditions. Until now, due to the lack of a suitable model, it was not possible to measure the efficiency of the problem with these conditions. All the proposed models are linear, and some of them are able to calculate all the lower, median and upper bounds of the fuzzy number by solving one model. A real example from the banking industry was considered to test the proposed models.
Use of other approaches to fuzzy problems, such as Type II fuzzy, trapezoidal numbers is suggested for future research.

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Appendix

Table 4. Fuzzy data of application example.

Branch	Personnel	ATM	POS and IPG	Internet banking	Deferred loan	Expensive deposit	Inexpensive deposit	Letter of guarantee	Loan
1	(5,5,6)	(2,2,2)	(145,148,151)	(691,727,733)	(1746,2562,2735)	(1796207,1882071,2062716)	(450420,477286,567345)	(2999,3550,3550)	(215652,241837,257533)
2	(7,7,7)	(3,4,4)	(39,42,43)	(490,502,527)	(7133,7383,7461)	(1846229,2089250,2587535)	(386418,401008,450358)	(21662,36441,92386)	(96793,100480,109998)
3	(5,5,5)	(1,1,1)	(78,80,97)	(609,673,677)	(9850,9988,11421)	(1804876,1861547,2233344)	(780263,864372,1012914)	(36236,36941,46184)	(74332,75470,145294)
4	(5,5,5)	(1,1,1)	(68,95,101)	(425,444,454)	(4352,4498,4580)	(2092562,2519762,4349813)	(314611,461404,767946)	(1818,2000,6567)	(100703,114715,143692)
5	(5,5,5)	(1,1,1)	(79,82,87)	(521,569,606)	(359,853,14522)	(675007,882247,894835)	(226627,253116,470432)	(1650,1850,1850)	(130995,141648,166484)
6	(6,6,6)	(5,5,5)	(74,74,78)	(419,436,460)	(778,1010,7577)	(1377316,1434120,1556106)	(360220,370062,436022)	(1063,15873,19053)	(177595,193792,240545)
7	(12,12,12)	(5,5,5)	(274,296,307)	(5173,5782,6136)	(186076,188381,197059)	(1120218,1678124,1919105)	(1069911,1419617,1498300)	(51226,57268,62539)	(1257440,1320538,1508103)
8	(5,5,5)	(1,1,1)	(56,60,62)	(351,378,389)	(240,250,330)	(1152394,1271431,1306157)	(358396,447295,477918)	(7026,10399,10823)	(91503,98582,102049)
9	(5,5,5)	(1,1,1)	(114,123,131)	(307,364,367)	(262,296,523)	(754906,756451,757201)	(254942,308430,350662)	(87,135,135)	(180782,194699,198335)
10	(7,7,7)	(3,3,3)	(98,112,139)	(880,917,949)	(482,864,13958)	(1708273,2279112,6093922)	(718488,883402,897972)	(13588,14908,19600)	(140021,153950,154734)
11	(12,12,12)	(6,6,6)	(85,85,89)	(974,999,1015)	(9329,237081,259710)	(3732626,3897467,4024463)	(1624344,2235872,2252227)	(175286,207728,247377)	(2510768,2573452,2714944)
12	(5,5,5)	(1,1,1)	(108,112,114)	(549,572,582)	(0,8,71)	(1578945,1581432,1826515)	(1050534,1510770,1584748)	(20060,21428,40060)	(135115,148534,203244)
13	(7,7,7)	(1,1,1)	(66,67,67)	(442,468,476)	(0,0,114)	(1075634,1206996,1549673)	(340593,413884,418556)	(4295,12843,20892)	(64444,72093,101009)
14	(5,5,5)	(1,1,1)	(63,67,68)	(446,475,486)	(87,184,270)	(1304398,1313141,1329225)	(632132,763077,950990)	(156596,161958,180614)	(60902,66538,69974)
15	(5,5,5)	(1,1,1)	(106,110,111)	(443,565,593)	(1162,2056,6422)	(1381225,1752338,1827967)	(484375,500826,579290)	(6240,6240,6240)	(175792,179050,189221)
16	(5,5,5)	(1,1,1)	(136,143,147)	(843,903,917)	(47864,48250,48925)	(1555341,1615784,1767281)	(927779,1177250,1351530)	(16192,40907,322162)	(173181,182850,188261)
17	(7,7,7)	(3,3,3)	(123,124,133)	(770,789,791)	(100789,100894,101436)	(2126676,2273410,2302944)	(1067964,1510127,1563726)	(464230,465398,714078)	(1209262,1827910,2228374)
18	(6,6,6)	(2,2,2)	(134,134,148)	(665,777,794)	(756,1627,1811)	(1134373,1200020,1231481)	(344232,365311,472861)	(46227,56722,145726)	(78398,110655,136892)
19	(7,7,7)	(4,4,4)	(50,53,59)	(443,454,498)	(755,769,928)	(1566475,1676859,2341361)	(534632,607285,632619)	(61186,94221,139942)	(264120,295424,409538)
20	(6,6,6)	(4,4,4)	(70,74,74)	(466,498,522)	(1172,1230,1262)	(1764611,1868629,1904220)	(371274,470752,690969)	(15387,25070,42953)	(188031,198932,253533)
21	(6,6,6)	(3,3,3)	(83,90,91)	(684,728,740)	(317,379,490)	(1488120,1531250,1664083)	(940372,943393,971580)	(32791,36002,47794)	(105639,107784,154298)
22	(6,6,6)	(2,2,2)	(145,147,155)	(726,796,805)	(692,744,799)	(1229475,1286301,1416338)	(500909,537682,624653)	(8902,9217,19402)	(131332,180044,181092)
23	(6,6,6)	(2,2,2)	(73,75,83)	(657,686,724)	(2135,2205,2297)	(886027,1034833,1059377)	(436095,478861,773382)	(14294,34359,83823)	(158980,159938,184694)
24	(6,6,6)	(2,2,2)	(110,112,113)	(807,830,845)	(30930,32345,32418)	(1392195,1530066,1920991)	(557519,608731,801766)	(620,620,2620)	(80265,109006,117361)
25	(6,6,6)	(1,1,1)	(65,69,76)	(583,606,613)	(5469,5469,5470)	(2222210,2381958,2401852)	(462538,465678,476977)	(20090,20090,20090)	(67183,105521,132254)
26	(6,6,6)	(2,2,2)	(115,116,119)	(526,541,559)	(15288,15695,15860)	(968493,974140,1042819)	(672406,786988,916768)	(8016,15633,20449)	(260384,300502,587671)
27	(6,6,6)	(1,1,1)	(72,85,92)	(358,375,392)	(12650,12866,12890)	(1583648,1959895,2125833)	(413745,456350,515639)	(5670,6875,21639)	(371423,401734,405026)
28	(6,6,6)	(1,1,1)	(211,215,224)	(691,786,799)	(732,46479,56904)	(911199,1022935,1244135)	(571058,620118,693454)	(523,3055,3741)	(202647,205959,246027)
29	(6,6,6)	(3,3,3)	(134,146,155)	(886,957,973)	(46018,46094,80289)	(2072444,2297643,4364199)	(862673,883962,1277777)	(105763,108765,175824)	(492047,559264,611557)
30	(10,10,10)	(1,1,1)	(76,88,154)	(383,435,797)	(1225,1511,73361)	(2479039,3155525,5415538)	(606648,762226,1030541)	(153419,172813,259520)	(287840,311858,792144)
31	(6,6,6)	(2,2,2)	(419,444,471)	(616,713,721)	(384,984,1614)	(978212,987237,1436951)	(505980,571720,686542)	(1167,1740,1975)	(213743,256498,327304)
32	(8,8,8)	(2,2,2)	(354,367,371)	(429,568,591)	(67,198,1279)	(1133845,1237430,1259326)	(476533,545829,671668)	(17897,25095,26415)	(145664,151176,157969)
33	(9,9,9)	(2,2,2)	(551,589,597)	(1227,1558,1661)	(587,648,1110)	(1479792,1563555,1745161)	(765428,961377,1191584)	(9106,23370,23874)	(177723,205630,227827)
34	(11,11,11)	(2,2,2)	(868,870,924)	(2522,3045,3184)	(1723,1998,3243)	(2011453,2529782,2785293)	(2032472,2974719,3847252)	(91768,150917,279652)	(761705,977669,1100428)
35	(11,11,11)	(3,3,3)	(652,668,711)	(2190,2549,2761)	(368,598,18382)	(1081177,1296126,3356570)	(1787986,2040313,2921342)	(175633,239008,539644)	(521096,645429,752088)
36	(8,8,8)	(2,2,2)	(259,275,299)	(1378,1524,1564)	(948,1153,1228)	(2189391,2347153,2417984)	(626037,830974,913698)	(15626,26387,34190)	(167825,249385,329829)
37	(9,9,9)	(3,3,3)	(405,470,496)	(1233,1538,1606)	(331,341,662)	(1118041,1502150,1548589)	(748156,768673,953713)	(600,1623,3623)	(181927,191757,243840)
38	(9,9,9)	(3,3,3)	(315,339,369)	(1597,1797,1890)	(1290,3675,6489)	(1839678,2018011,2280102)	(1153126,1665060,1918861)	(125030,915960,986163)	(1139883,1203569,1577924)
39	(7,7,7)	(3,3,3)	(308,315,322)	(508,630,643)	(5592,7812,8007)	(489731,525972,710189)	(434893,504288,589234)	(1280,1652,1685)	(430019,471289,704722)
40	(8,8,8)	(3,3,3)	31,14034,2220	(1968,2064,2132)	(29841,30637,34125)	(3173597,3318111,3677921)	(1393435,1416409,1504582)	(177666,269522,393876)	(626074,665882,743381)
41	(7,7,7)	(2,2,2)	(439,504,508)	(754,1066,1162)	(7704,7724,7809)	(520985,684751,744870)	(494569,589840,684691)	(3329,4229,5302)	(135819,157598,213272)
42	(15,15,15)	(3,3,3)	(476,505,531)	(2429,2747,2796)	(32475,32533,36220)	(3299914,3749248,5400760)	(1889276,2401675,2431716)	(79637,131324,172149)	(1093386,1183417,1215314)

43	(8,8,8)	(2,2,2)	(196,202,207)	(1156,1301,1406)	(19208,19461,31299)	(1267300,1347592,1511672)	(798816,1018260,1151512)	(58701,65095,66067)	(433017,558666,625534)
44	(12,12,12)	(4,4,4)	(848,852,890)	(3035,3443,3565)	(10872,12907,13128)	(2725370,2734319,2965774)	(1481561,1728192,1758943)	(62194,108638,110438)	(586104,602929,627910)
45	(8,8,8)	(2,2,2)	(677,751,766)	(943,1223,1350)	(36873,37749,39573)	(569264,812334,1069924)	(692681,709323,755140)	(2735,2958,3015)	(236310,289303,368073)
46	(13,13,13)	(7,8,8)	(468,507,520)	(2090,2519,2754)	(11993,14955,18454)	(1886960,2253739,2301368)	(1507359,1531938,1936341)	(4293,4335,20348)	(1991194,2209729,2309302)
47	(11,11,11)	(2,2,2)	(286,329,607)	(1075,1206,2045)	(5487,5649,6266)	(2555548,2880664,2917805)	(911578,1088887,1143591)	(16059,41436,41564)	(407957,556380,682186)
48	(5,5,5)	(2,2,2)	(97,101,106)	(728,836,838)	(525,761,959)	(1200986,1201330,1353938)	(412802,731309,816449)	(424403,870260,898130)	(280126,341843,618145)
49	(6,6,6)	(2,2,2)	(158,166,169)	(698,783,793)	(692,1032,1057)	(1122699,1271425,1602993)	(397610,430994,498901)	(44516,47662,62369)	(300971,318141,326764)
50	(5,5,5)	(1,1,1)	(151,160,164)	(714,807,851)	(616,741,807)	(944077,1003875,1390280)	(539968,692776,715566)	(276549,522126,568281)	(773044,778209,798884)
51	(7,7,7)	(2,2,2)	(220,225,238)	(1226,1303,1344)	(1079,1088,1092)	(2006519,2115698,2167109)	(648545,779250,862779)	(26533,28444,32791)	(218711,245727,294002)
52	(15,15,15)	(4,4,4)	(351,362,378)	(2074,2275,2404)	(167246,167609,205443)	(6017237,19549383,33950146)	(1399994,1729769,1997025)	(713302,1006331,1069435)	(1235204,1311962,1317250)
53	(15,16,16)	(2,3,3)	(155,158,346)	(869,938,2229)	(54239,56934,60198)	(2729106,2895008,3067428)	(1907013,1978351,2049483)	(188320,321412,333558)	(568818,9486557,14142049)
54	(8,8,8)	(4,5,5)	(431,495,512)	(1764,2073,2124)	(1118,1263,2480)	(2544270,2782947,2989410)	(956442,1415488,2466140)	(23570,26756,55407)	(555695,1144834,5548393)
55	(6,6,6)	(2,2,2)	(401,402,429)	(1339,1611,1748)	(2190,2206,2804)	(1738511,1914425,1952609)	(911905,1020898,1145176)	(83219,130393,142158)	(556836,691089,832721)
56	(10,10,10)	(3,4,4)	(820,863,910)	(2376,2704,2812)	(7892,8763,17310)	(2605008,2906851,3272613)	(1343808,1471606,1866528)	(35747,36814,48316)	(450104,3713949,7163291)
57	(5,5,6)	(2,2,3)	(351,379,389)	(1165,1348,1442)	(2245,2420,3804)	(1514685,1789442,1902290)	(559783,625700,658722)	(33235,96606,175622)	(238730,282395,344385)
58	(7,7,7)	(2,2,2)	(301,305,327)	(1076,1238,1269)	(904,2119,2272)	(1824725,2021913,2038589)	(521862,689084,689654)	(5295,5389,12786)	(388347,409096,409351)
59	(5,6,6)	(2,2,2)	(340,342,369)	(1039,1188,1189)	(562,588,630)	(1442746,1632758,1670444)	(734629,772131,812032)	(12283,20592,32727)	(174716,179463,199480)
60	(6,6,6)	(3,3,3)	(246,262,262)	(960,1056,1086)	(2118,2213,2323)	(2369044,2468414,2528117)	(516561,661267,680942)	(27253,36534,92498)	(205532,210186,257762)
61	(9,9,9)	(3,3,3)	(250,261,264)	(948,1071,1082)	(591,649,1009)	(905670,998429,1067311)	(639273,683577,734396)	(87517,256056,322050)	(410544,485862,618782)
62	(7,7,7)	(2,2,2)	(234,236,249)	(935,1120,1124)	(254,287,436)	(1403619,1587088,1702775)	(546957,650210,658658)	(24442,37400,69742)	(188372,220438,258805)
63	(8,8,8)	(3,3,3)	(228,785,795)	(965,2132,2211)	(4568,4911,5388)	(1633130,1848927,1858662)	(1466372,1502072,1548623)	(71042,77759,81419)	(708894,852053,1213230)
64	(7,7,7)	(3,3,3)	(275,294,294)	(1017,1122,1137)	(125,494,499)	(2734098,2902980,3254069)	(536686,536800,599770)	(2974,4181,22280)	(327901,346027,351750)
65	(13,14,14)	(2,2,2)	(246,276,461)	(849,985,1928)	(4260,14490,22873)	(2906762,2970804,3279410)	(998977,1244580,1389275)	(59355,118315,147702)	(1661834,1702005,1731342)
66	(8,8,8)	(3,3,3)	(453,492,496)	(1828,2048,2048)	(15891,16168,17732)	(2514002,2611101,3001684)	(1078332,1208868,1214230)	(132330,169524,206715)	(428926,490235,600426)
67	(11,11,11)	(2,2,2)	(545,551,574)	(1763,2019,2134)	(29269,45276,82582)	(1511071,1938446,2001451)	(947056,1217589,1218561)	(162546,266811,410387)	(656142,752447,814707)
68	(6,6,6)	(2,2,2)	(233,252,257)	(1032,1205,1216)	(443,464,472)	(3176817,3190932,4177447)	(498302,653040,673979)	(10226,218737,224096)	(203391,239238,254728)
69	(7,7,7)	(2,2,2)	(206,211,228)	(905,964,1026)	(5015,5047,5272)	(1103674,1186503,1253040)	(680825,771159,838418)	(273093,380692,599578)	(481920,508198,587923)
70	(5,5,5)	(1,1,1)	(188,201,219)	(819,963,995)	(488,779,915)	(1044585,1095945,1311681)	(388011,497937,540403)	(7880,9100,9580)	(115178,136557,180532)
71	(7,7,7)	(2,2,4)	(247,379,404)	(1343,2106,2171)	(95723,95852,95855)	(2360656,2605332,2939185)	(1225628,1522854,1591983)	(46259,51689,82919)	(572677,574897,613640)
72	(8,8,8)	(2,2,2)	(269,279,304)	(1476,1676,1691)	(827,1411,1602)	(1545353,1714049,1804555)	(803195,829859,1029728)	(120010,221839,453927)	(450531,461138,510770)
73	(7,8,8)	(2,2,4)	(202,323,342)	(940,1661,1680)	(3551,3631,3647)	(1885358,2097956,2449487)	(743427,954820,1209242)	(32538,42184,74654)	(218266,255742,256334)
74	(6,6,6)	(2,2,2)	(191,206,212)	(871,968,974)	(1256,1567,2081)	(1272885,1425996,1840807)	(431712,457466,530666)	(31776,38249,49258)	(234793,267735,313401)
75	(12,12,12)	(3,3,3)	(240,259,446)	(1221,1316,2308)	(244,585,863)	(2627144,3178584,3294282)	(856876,1166793,1275072)	(32511,33239,35742)	(246530,309279,368932)
76	(10,10,10)	(2,2,2)	(238,263,527)	(901,1140,2348)	(17664,27155,27734)	(2034222,2947722,3228952)	(1113302,1250862,1378544)	(30361,31425,36733)	(457374,471817,501056)
77	(5,7,7)	(2,2,2)	(286,286,310)	(1093,1241,1319)	(119538,124666,129055)	(1140770,1323987,1525821)	(469070,526148,585479)	(15343,26310,58989)	(292043,326211,331457)
78	(6,6,6)	(1,1,2)	(170,390,420)	(659,1669,1671)	(1293,2059,4374)	(2115766,2518713,2606755)	(659780,829658,1001875)	(11248,18366,20230)	(334438,362967,386566)
79	(12,13,13)	(2,2,2)	(464,488,735)	(2066,2359,3475)	(1148,2319,14428)	(2322119,2826443,2853375)	(1458673,1728217,2057801)	(1103,1715,56082)	(525659,528078,540793)
80	(8,8,8)	(2,2,2)	(766,812,845)	(1533,1837,1994)	(1524,1612,2065)	(1106736,1266656,1287228)	(915503,974176,1010811)	(1350,1400,2490)	(332711,370160,370380)
81	(7,7,7)	(2,2,2)	(817,834,868)	(1716,2126,2231)	(507,760,3334)	(1174117,1179017,1257936)	(943447,1051611,1077908)	(140,1198,7388)	(386481,411677,412932)
82	(11,11,11)	(1,1,1)	(330,363,704)	(907,1102,2346)	(2863,3117,3749)	(1114939,1320791,1477238)	(1118751,1388721,1456079)	(46932,50589,66139)	(568876,608191,610636)
83	(5,5,5)	(2,2,2)	(235,254,257)	(904,1058,1079)	(3349,3503,4419)	(919513,931214,1085755)	(503572,548579,629505)	(5786,5786,18520)	(180368,210857,211882)
84	(6,6,6)	(2,2,2)	(335,338,340)	(1025,1164,1219)	(4942,5046,6828)	(948012,1005071,1140108)	(458404,470726,691219)	(15511,64943,65290)	(148761,172232,182791)
85	(12,12,12)	(2,2,2)	(488,551,887)	(1430,1758,3305)	(9298,9424,12131)	(2487423,2555703,2615113)	(1467921,1571438,1645579)	(17350,19447,26378)	(680038,684946,694749)
86	(8,8,8)	(2,2,2)	(334,362,363)	(1700,1779,1839)	(1385,1437,1658)	(2229832,2291167,2571066)	(790502,828632,1003396)	(880,2780,4299)	(196842,226600,250793)
87	(7,7,7)	(2,3,3)	(316,334,360)	(1477,1688,1713)	(6835,7521,7546)	(1621044,1732280,1838934)	(802483,1056794,1057483)	(5889,21889,31762)	(468036,617178,722092)
88	(6,6,6)	(2,2,3)	(331,541,598)	(1116,2031,2051)	(1366,1565,1638)	(2029741,2329329,2410078)	(808844,841481,918271)	(1942,6212,7452)	(363591,437710,491789)
89	(7,7,7)	(2,2,2)	(477,479,486)	(1448,1659,1716)	(7276,16422,29636)	(1364779,1713972,1857298)	(551188,684980,698104)	(966,1695,1745)	(240115,240392,257635)
90	(7,7,7)	(2,2,2)	(570,610,622)	(1782,2057,2143)	(6860,8284,9329)	(1780117,1891027,1909343)	(724506,885221,891214)	(1892,2428,2428)	(283359,286391,299961)
91	(13,13,13)	(5,5,6)	1197,1362,1411	(1960,2662,2910)	(8593,10949,14727)	(1253205,1386699,1663463)	(1512431,1815564,2087350)	(1265,4800,6809)	(545451,556743,589072)
92	(8,8,8)	(3,3,3)	1039,1063,1101	(1800,2330,2675)	(163,331,413)	(986018,1107920,1309899)	(1193536,1339672,1697174)	(24994,28801,40726)	(329535,384590,385867)
93	(12,12,12)	(4,4,4)	(876,882,944)	(2215,2848,3191)	(831,910,1261)	(2195075,2361742,2412085)	(1785069,2130369,2222879)	(26233,26291,51954)	(505055,544493,621138)
94	(9,9,9)	(2,2,2)	(907,934,960)	(2102,2625,2911)	(2438,2462,3628)	(1822580,1916099,2033982)	(1396943,1617464,1715817)	(238520,248045,274860)	(469149,563138,633883)
95	(6,6,6)	(2,2,4)	(634,764,782)	(686,1054,1310)	(15,16,1557)	(918847,1050650,1101744)	(647492,789978,881376)	(1068,1634,2200)	(350121,357720,398738)
96	(13,13,13)	(8,8,9)	1403,1460,1523	(2455,3240,3591)	(518,647,814)	(2459512,3085910,3131577)	(1996051,2619691,2759383)	(22987,70792,117671)	(320568,375633,488621)
97	(10,10,10)	(4,4,4)	(504,554,579)	(1155,1411,1561)	(37,453,600)	(635483,908166,1068786)	(737209,942681,1070240)	(1060,2060,17746)	(161813,230213,273137)
98	(7,7,7)	(3,3,3)	(676,685,694)	(1300,1513,1660)	(30649,30768,33121)	(1500256,1653172,1711979)	(850575,991108,1060609)	(11081,12194,13161)	(334157,338220,346524)
99	(12,12,12)	(3,3,3)	(838,989,1006)	(1367,1801,2092)	(277,347,602)	(1654423,1731761,2202688)	(1856922,1887459,2846529)	(7410,313270,340150)	(2173365,2424106,2620422)
100	(8,8,8)	(2,2,2)	1101,1282,1290	(1764,2198,2466)	(506,536,565)	(978554,1129416,1423380)	(860263,1053826,1192147)	(0,0,2007)	(348520,419667,515678)
101	(14,14,14)	(2,2,4)	1008,1071,1582	(2294,2804,4414)	(16044,16712,26338)	(2256931,2507913,2794207)	(2249463,2735525,2767469)	(307643,396541,403186)	(1002636,1037697,1121001)
102	(8,8,8)	(3,3,3)	(593,673,683)	(946,1229,1441)	(1108,1183,23851)	(1050626,1107805,1309773)	(805585,987198,1195278)	(103687,105384,113118)	(381392,500007,588171)
103	(10,10,10)	(2,2,2)	1585,1723,1726	(2470,3346,3690)	(1366,2146,2290)	(1748803,1906719,2075505)	(1484425,1735413,1959897)	(25754,36508,83857)	(515640,613045,665484)
104	(17,17,18)	(5,5,5)	1505,1513,1590	(2987,4034,4443)	(5106,5193,5584)	(2479051,2537751,2656007)	(2510274,2723934,2868069)	(8042,12296,116420)	(866208,929824,1062649)
105	(14,14,14)	(4,6,6)	1833,2183,2186	(3116,4221,4960)	(51742,63228,63484)	(2232198,2480267,2567913)	(2141224,2492290,2674099)	(14514,18246,28770)	(780842,987222,1020182)

Table 5. Fuzzy ranking of the stages and the whole system using α-cut approach for α=0.5.

Branch	**Average**			**Stage 1 is the leader**			**Stage 2 is the leader**			**Aggregation**
	Stage 1	**Stage 2**	**System**	**Stage 1**	**Stage 2**	**System**	**Stage 1**	**Stage 2**	**System**	**System**
1	(5,21,28)	(46,54,70)	(31,32,36)	(5,28,30)	(14,46,50)	(22,30,36)	(5,40,42)	(46,52,65)	(30,34,36)	(1,46,66)
2	(5,105,105)	(1,5,51)	(1,105,105)	(5,105,105)	(3,8,14)	(2,105,105)	(5,105,105)	(1,6,19)	(1,105,105)	(1,12,102)
3	(96,100,103)	(1,19,80)	(65,92,94)	(96,102,103)	(3,32,78)	(64,94,94)	(96,98,103)	(1,40,82)	(64,92,94)	(1,60,97)
4	(75,83,93)	(8,68,83)	(12,65,78)	(74,83,93)	(8,14,51)	(12,63,78)	(75,83,93)	(8,47,80)	(12,66,78)	(1,61,88)
5	(5,99,102)	(29,41,50)	(46,93,94)	(5,99,101)	(14,42,50)	(46,93,95)	(5,99,101)	(24,46,50)	(46,91,95)	(1,34,100)
6	(5,84,92)	(6,27,48)	(40,78,85)	(5,84,92)	(7,14,48)	(40,78,84)	(5,85,92)	(7,20,48)	(40,78,85)	(1,23,80)
7	(5,10,18)	(17,88,93)	(10,50,58)	(5,7,18)	(17,93,96)	(10,49,58)	(5,14,18)	(17,85,96)	(10,50,58)	(7,8,47)
8	(70,95,97)	(13,89,97)	(9,99,103)	(70,95,97)	(13,86,97)	(9,99,103)	(70,95,97)	(13,86,97)	(9,99,103)	(30,45,101)
9	(45,73,84)	(10,64,78)	(62,78,79)	(45,73,84)	(74,78,78)	(63,78,79)	(45,74,84)	(13,75,78)	(64,78,79)	(30,75,90)
10	(62,90,94)	(11,40,50)	(6,86,87)	(62,90,92)	(11,14,40)	(6,86,86)	(62,89,92)	(11,35,76)	(6,86,86)	(25,30,94)
11	(57,70,92)	(29,92,100)	(43,80,97)	(57,70,92)	(29,94,100)	(43,80,97)	(57,70,92)	(29,91,100)	(43,80,97)	(66,69,72)
12	(2,2,105)	(28,101,104)	(2,42,76)	(2,2,105)	(28,104,104)	(2,42,76)	(2,2,105)	(28,104,104)	(2,42,76)	(1,59,100)
13	(5,99,103)	(15,103,105)	(8,104,104)	(5,100,103)	(15,104,105)	(8,104,104)	(5,100,103)	(15,105,105)	(8,104,104)	(4,49,105)
14	(96,100,102)	(1,101,105)	(44,102,103)	(96,100,102)	(3,101,102)	(44,102,103)	(96,100,102)	(1,101,101)	(44,102,103)	(49,93,96)
15	(76,77,91)	(27,45,47)	(26,75,85)	(76,77,91)	(24,27,67)	(26,74,85)	(76,77,91)	(27,30,54)	(26,74,85)	(49,71,99)
16	(69,77,104)	(9,86,97)	(66,95,100)	(69,77,104)	(9,84,94)	(66,95,100)	(68,77,104)	(9,94,95)	(66,95,100)	(43,49,59)
17	(50,68,101)	(58,92,99)	(59,69,90)	(50,68,101)	(58,90,99)	(59,68,90)	(50,68,101)	(58,89,99)	(59,69,90)	(58,67,95)
18	(5,14,87)	(22,26,63)	(17,20,81)	(5,14,87)	(22,49,67)	(17,17,81)	(5,14,87)	(9,22,51)	(17,18,81)	(1,33,97)
19	(58,104,104)	(23,66,70)	(67,98,102)	(58,104,104)	(14,70,81)	(67,98,102)	(58,104,104)	(20,70,81)	(67,98,102)	(68,81,97)
20	(53,84,93)	(4,34,79)	(29,77,88)	(53,85,93)	(3,34,61)	(29,77,88)	(53,84,93)	(4,34,62)	(29,77,88)	(13,82,97)
21	(76,90,92)	(16,36,85)	(11,86,87)	(76,90,93)	(16,83,85)	(11,87,87)	(76,90,93)	(16,83,85)	(11,87,87)	(51,92,97)
22	(5,17,25)	(17,37,76)	(18,30,42)	(5,17,25)	(10,38,79)	(13,31,44)	(5,20,47)	(17,37,79)	(19,30,44)	(3,20,97)
23	(52,98,98)	(24,52,70)	(48,91,93)	(52,98,98)	(14,23,52)	(48,91,92)	(52,98,102)	(25,52,70)	(48,93,94)	(24,97,98)
24	(1,89,93)	(6,11,84)	(3,84,99)	(1,89,94)	(1,18,85)	(1,85,99)	(44,91,94)	(7,12,84)	(4,84,99)	(19,93,97)
25	(5,97,101)	(1,29,46)	(1,92,95)	(5,97,99)	(3,14,27)	(2,92,93)	(1,97,99)	(1,28,81)	(1,92,92)	(17,95,97)
26	(83,88,94)	(16,18,64)	(61,83,89)	(83,88,94)	(14,47,64)	(61,83,89)	(83,88,94)	(19,54,64)	(61,83,89)	(76,79,97)
27	(66,75,82)	(22,43,70)	(35,65,77)	(66,75,82)	(4,37,42)	(35,63,77)	(66,76,82)	(23,43,65)	(35,65,77)	(21,46,75)
28	(68,74,81)	(10,90,94)	(5,96,98)	(68,74,81)	(10,85,93)	(5,96,98)	(68,74,79)	(10,93,93)	(5,96,98)	(39,42,86)
29	(10,19,99)	(8,39,85)	(4,49,55)	(14,19,99)	(10,39,95)	(4,50,57)	(6,19,99)	(9,39,87)	(4,49,55)	(4,41,67)
30	(51,69,89)	(14,27,66)	(40,63,67)	(51,69,89)	(4,66,67)	(40,63,64)	(51,69,89)	(17,66,79)	(40,65,67)	(33,70,87)
31	(1,24,70)	(48,51,80)	(28,62,79)	(1,24,70)	(14,54,79)	(32,62,79)	(5,35,70)	(39,55,80)	(32,62,79)	(33,43,65)
32	(5,72,85)	(40,78,100)	(32,80,101)	(5,72,85)	(40,71,99)	(32,80,101)	(5,73,85)	(40,72,101)	(32,80,101)	(5,33,76)
33	(33,45,69)	(13,44,75)	(16,41,42)	(44,45,69)	(10,44,75)	(33,41,42)	(47,50,69)	(11,44,76)	(21,42,42)	(21,33,50)
34	(65,87,95)	(47,56,92)	(58,82,94)	(65,87,95)	(58,67,92)	(58,82,94)	(65,87,95)	(36,59,92)	(58,82,94)	(64,70,104)
35	(54,58,91)	(12,80,101)	(46,53,101)	(54,58,91)	(14,15,101)	(45,53,101)	(54,58,91)	(15,70,101)	(46,53,101)	(6,71,102)
36	(28,33,54)	(10,31,71)	(16,24,38)	(28,30,54)	(14,30,76)	(22,23,43)	(22,40,54)	(9,31,77)	(17,24,43)	(8,35,71)
37	(5,21,41)	(6,54,83)	(14,46,50)	(30,43,44)	(54,67,83)	(22,46,50)	(5,23,40)	(13,54,83)	(18,46,50)	(5,46,71)
38	(10,53,98)	(25,70,99)	(10,44,99)	(10,53,98)	(14,26,99)	(11,44,99)	(10,53,98)	(29,65,99)	(11,44,99)	(5,40,105)
39	(5,102,103)	(29,77,87)	(87,97,101)	(5,102,103)	(14,48,87)	(87,97,101)	(5,102,103)	(34,65,87)	(87,97,101)	(28,78,89)
40	(1,10,20)	(23,33,90)	(5,27,52)	(1,7,20)	(14,33,90)	(6,27,51)	(1,10,20)	(33,54,90)	(11,27,51)	(5,21,27)

41	(5,23,47)	(9,42,57)	(19,26,53)	(5,23,45)	(44,57,81)	(27,36,53)	(5,23,35)	(27,48,57)	(23,27,53)	(26,40,57)
42	(68,75,96)	(1,36,69)	(61,66,68)	(68,75,96)	(3,14,69)	(61,67,69)	(69,75,96)	(2,39,69)	(61,66,68)	(29,61,90)
43	(4,17,78)	(21,31,65)	(4,14,60)	(4,17,78)	(4,65,78)	(3,20,60)	(4,17,78)	(27,65,70)	(7,14,60)	(21,37,81)
44	(5,6,7)	(23,62,80)	(7,10,56)	(4,5,7)	(6,19,60)	(3,6,54)	(5,7,10)	(20,45,62)	(6,10,56)	(21,37,52)
45	(5,49,59)	(19,92,92)	(14,57,68)	(5,21,59)	(19,90,95)	(14,56,68)	(5,44,59)	(19,88,95)	(14,56,68)	(37,50,86)
46	(5,33,50)	(38,59,91)	(25,31,91)	(5,17,50)	(14,22,91)	(13,23,91)	(5,20,47)	(44,45,91)	(23,25,91)	(8,43,49)
47	(61,73,81)	(35,59,60)	(58,72,72)	(61,73,81)	(14,28,62)	(58,71,72)	(61,73,79)	(33,45,60)	(58,72,72)	(9,69,89)
48	(3,65,88)	(59,60,94)	(2,62,95)	(3,65,88)	(14,64,94)	(2,62,95)	(3,65,88)	(54,65,94)	(2,62,95)	(2,68,99)
49	(4,5,30)	(59,69,74)	(11,31,72)	(4,5,30)	(14,69,74)	(12,22,72)	(4,5,48)	(39,67,74)	(12,38,72)	(27,64,82)
50	(52,86,90)	(29,59,102)	(41,81,102)	(52,86,90)	(14,60,102)	(41,81,102)	(52,86,90)	(24,61,102)	(41,81,102)	(34,66,101)
51	(35,40,60)	(7,32,72)	(15,25,39)	(35,42,60)	(32,67,73)	(25,36,41)	(27,32,60)	(7,32,74)	(15,25,41)	(12,53,63)
52	(1,1,100)	(1,10,37)	(1,1,20)	(1,1,100)	(1,3,14)	(1,1,20)	(1,1,100)	(1,3,23)	(1,1,20)	(60,64,87)
53	(87,90,101)	(2,2,98)	(82,91,98)	(87,90,101)	(2,2,98)	(82,91,98)	(87,90,101)	(1,2,98)	(82,91,98)	(61,91,94)
54	(33,35,82)	(51,58,104)	(31,36,104)	(35,39,82)	(59,67,104)	(22,36,104)	(23,32,82)	(39,60,104)	(26,36,104)	(27,45,96)
55	(14,16,80)	(40,47,89)	(10,18,89)	(14,16,80)	(14,53,89)	(11,19,89)	(10,16,80)	(30,54,89)	(7,19,89)	(11,12,97)
56	(33,48,53)	(3,70,103)	(5,31,103)	(39,48,53)	(5,14,103)	(5,22,103)	(44,48,53)	(4,54,103)	(5,34,103)	(58,65,68)
57	(4,5,9)	(32,52,55)	(6,12,51)	(4,5,9)	(14,33,55)	(5,10,51)	(4,5,9)	(24,41,55)	(4,10,51)	(11,12,53)
58	(5,25,33)	(36,55,56)	(27,34,52)	(5,23,25)	(14,55,56)	(15,33,52)	(5,35,35)	(56,57,65)	(33,34,52)	(40,48,53)
59	(33,35,55)	(5,24,80)	(13,19,43)	(35,39,55)	(14,24,80)	(19,22,45)	(35,35,55)	(5,24,80)	(13,19,45)	(12,33,53)
60	(5,28,40)	(18,21,81)	(13,22,44)	(5,28,42)	(14,18,68)	(13,33,39)	(5,23,27)	(6,18,71)	(13,14,39)	(12,14,53)
61	(5,62,89)	(32,62,100)	(54,84,100)	(5,62,89)	(14,67,100)	(54,84,100)	(5,62,89)	(27,70,100)	(54,84,100)	(3,53,72)
62	(5,21,50)	(63,87,99)	(49,57,74)	(5,23,50)	(63,87,98)	(48,56,74)	(5,23,40)	(63,87,98)	(48,57,74)	(21,22,53)
63	(8,10,57)	(4,51,88)	(3,10,88)	(7,8,57)	(6,14,88)	(3,6,88)	(8,15,57)	(5,39,88)	(3,12,88)	(31,43,88)
64	(5,21,48)	(30,102,104)	(23,65,75)	(5,23,44)	(31,99,102)	(24,66,75)	(5,35,40)	(30,100,102)	(23,65,75)	(10,29,36)
65	(72,74,75)	(20,51,83)	(64,71,85)	(72,75,76)	(10,35,83)	(66,71,85)	(71,72,75)	(22,45,83)	(63,71,85)	(62,71,75)
66	(5,6,64)	(21,40,45)	(6,8,38)	(5,7,64)	(14,38,45)	(6,8,38)	(5,10,64)	(24,36,45)	(7,7,38)	(32,59,92)
67	(5,67,71)	(47,89,91)	(37,90,96)	(5,66,71)	(47,88,91)	(37,90,96)	(5,67,71)	(47,89,91)	(37,90,96)	(37,67,92)
68	(14,21,65)	(12,102,102)	(7,64,67)	(7,21,65)	(12,99,103)	(7,65,67)	(6,21,65)	(12,103,103)	(7,64,67)	(12,30,92)
69	(66,72,83)	(16,51,77)	(61,76,77)	(66,72,83)	(6,17,77)	(61,76,77)	(66,72,83)	(18,45,77)	(61,76,77)	(73,85,92)
70	(5,77,79)	(14,42,77)	(33,69,76)	(5,78,79)	(14,43,70)	(33,70,76)	(5,79,79)	(12,42,68)	(33,69,76)	(18,57,92)
71	(25,46,97)	(14,92,95)	(21,56,59)	(25,45,97)	(14,88,92)	(21,57,59)	(24,44,97)	(14,91,92)	(21,57,59)	(5,7,47)
72	(11,49,58)	(40,70,81)	(16,47,81)	(11,49,59)	(14,66,81)	(15,48,81)	(11,49,59)	(45,69,81)	(16,48,81)	(16,63,70)
73	(21,45,79)	(36,57,59)	(27,34,35)	(23,44,79)	(14,31,36)	(15,24,34)	(23,52,79)	(36,39,54)	(23,30,34)	(16,27,55)
74	(5,21,41)	(47,53,61)	(22,31,54)	(5,30,41)	(14,57,61)	(22,34,55)	(5,32,50)	(39,56,61)	(26,33,54)	(16,21,56)
75	(48,67,71)	(26,88,98)	(22,50,100)	(44,67,71)	(26,88,102)	(22,49,100)	(40,67,72)	(26,88,99)	(22,49,100)	(16,48,78)
76	(78,86,94)	(25,26,59)	(41,74,79)	(78,86,94)	(25,43,67)	(42,75,79)	(78,86,94)	(25,32,54)	(41,75,79)	(16,63,74)
77	(5,17,41)	(7,90,96)	(4,55,60)	(5,17,41)	(1,85,97)	(2,55,60)	(5,32,35)	(6,86,97)	(3,55,60)	(16,27,69)
78	(28,73,79)	(41,51,59)	(30,39,72)	(28,73,79)	(14,41,52)	(31,39,71)	(47,73,79)	(41,53,77)	(31,39,73)	(16,74,83)
79	(79,80,81)	(7,17,59)	(59,70,70)	(79,80,81)	(9,14,59)	(59,70,71)	(77,80,81)	(8,16,59)	(59,70,70)	(16,84,85)
80	(5,28,30)	(36,40,73)	(23,27,71)	(5,28,49)	(14,36,73)	(25,39,71)	(5,19,32)	(30,42,73)	(22,24,71)	(16,44,53)
81	(44,47,54)	(33,40,90)	(33,38,90)	(30,43,54)	(14,34,90)	(22,35,90)	(5,49,54)	(30,38,90)	(35,39,90)	(16,28,42)
82	(74,88,95)	(27,70,82)	(82,83,88)	(74,88,95)	(14,25,82)	(82,83,88)	(74,88,95)	(26,54,82)	(82,83,88)	(74,79,91)
83	(5,17,35)	(13,53,59)	(21,22,49)	(5,23,35)	(13,14,53)	(15,21,49)	(5,23,40)	(13,53,54)	(21,30,49)	(37,46,76)

84	(5,6,6)	(32,34,35)	(6,9,28)	(5,6,7)	(6,30,35)	(6,7,28)	(5,6,6)	(30,35,36)	(5,8,28)	(12,16,76)
85	(77,81,84)	(15,32,67)	(64,71,73)	(77,81,84)	(14,14,67)	(65,69,73)	(77,81,84)	(14,29,67)	(63,71,73)	(76,84,92)
86	(21,35,59)	(21,77,82)	(16,31,45)	(23,35,59)	(14,21,62)	(15,16,37)	(35,35,59)	(21,63,77)	(16,34,37)	(31,56,76)
87	(21,45,51)	(17,51,76)	(22,27,74)	(30,44,51)	(6,16,76)	(15,22,74)	(23,27,51)	(16,45,76)	(22,26,74)	(10,76,80)
88	(30,50,56)	(36,68,74)	(22,47,62)	(30,50,56)	(14,63,68)	(22,47,62)	(32,50,56)	(30,64,68)	(26,47,62)	(36,41,76)
89	(5,21,28)	(23,25,44)	(18,20,27)	(5,21,28)	(23,46,67)	(15,18,29)	(5,23,27)	(23,36,50)	(18,23,29)	(51,76,76)
90	(5,40,55)	(30,38,70)	(29,31,38)	(5,48,55)	(20,37,67)	(28,30,36)	(1,35,55)	(21,38,54)	(28,30,31)	(42,48,76)
91	(5,63,64)	(40,59,72)	(53,57,70)	(5,63,64)	(14,41,72)	(53,57,70)	(5,63,64)	(45,54,72)	(53,57,70)	(32,55,76)
92	(5,55,61)	(8,84,93)	(45,56,92)	(5,55,61)	(82,84,93)	(46,56,92)	(5,55,61)	(8,84,93)	(45,56,92)	(12,41,76)
93	(28,40,63)	(20,73,79)	(22,40,80)	(28,42,63)	(14,72,80)	(33,40,80)	(18,25,63)	(20,73,79)	(19,40,80)	(35,76,77)
94	(15,59,71)	(32,59,75)	(15,48,75)	(15,58,71)	(29,67,75)	(14,47,75)	(15,58,71)	(37,45,75)	(15,47,75)	(62,73,76)
95	(1,20,41)	(61,96,96)	(37,68,96)	(1,17,41)	(65,96,96)	(38,68,96)	(1,23,35)	(66,96,96)	(38,68,96)	(5,76,104)
96	(16,47,52)	(3,51,86)	(3,47,48)	(16,47,52)	(14,51,89)	(12,47,50)	(16,26,47)	(3,51,89)	(3,47,50)	(12,15,76)
97	(5,61,63)	(14,65,86)	(51,55,86)	(5,61,63)	(10,77,86)	(51,55,86)	(5,61,63)	(13,78,86)	(51,55,86)	(5,45,76)
98	(35,40,46)	(9,20,47)	(15,19,31)	(23,35,46)	(11,14,20)	(15,15,18)	(27,42,46)	(10,20,70)	(15,20,34)	(38,38,76)
99	(55,57,85)	(1,10,105)	(43,51,105)	(56,57,85)	(1,12,105)	(43,52,105)	(56,57,85)	(1,11,105)	(43,52,105)	(62,103,103)
100	(5,22,40)	(19,67,97)	(20,24,97)	(5,22,30)	(78,82,97)	(22,26,97)	(5,17,22)	(18,82,97)	(15,26,97)	(26,48,52)
101	(12,64,86)	(39,51,71)	(17,54,73)	(12,64,86)	(14,39,71)	(16,54,73)	(12,64,86)	(43,45,71)	(17,54,73)	(44,65,98)
102	(3,6,56)	(40,43,95)	(8,8,93)	(3,7,56)	(14,45,95)	(6,9,93)	(3,6,56)	(30,49,95)	(5,9,93)	(10,12,54)
103	(5,66,67)	(49,77,84)	(60,63,83)	(5,67,67)	(14,56,84)	(60,63,83)	(5,66,67)	(58,74,84)	(60,63,83)	(2,12,60)
104	(5,13,62)	(28,59,85)	(13,52,84)	(5,13,62)	(14,21,85)	(13,52,84)	(5,13,62)	(31,75,85)	(13,52,84)	(12,53,83)
105	(5,60,60)	(49,87,98)	(45,69,89)	(5,60,60)	(49,92,98)	(45,69,89)	(5,60,60)	(49,94,98)	(45,69,89)	(12,27,39)

Table 6. Fuzzy ranking of the stages and the whole system using Hatami Marbini's approach [39].

Branch	**Average**			**Stage 1 is the leader**			**Stage 2 is the leader**			**Aggregation**
	Stage 1	**Stage 2**	**System**	**Stage 1**	**Stage 2**	**System**	**Stage 1**	**Stage 2**	**System**	**System**
1	(84,84,87)	(59,61,62)	(80,85,85)	(84,85,87)	(58,60,62)	(81,84,85)	(62,66,68)	(58,60,62)	(58,64,64)	(79,81,84)
2	(88,89,89)	(83,87,90)	(90,93,93)	(88,89,89)	(84,87,90)	(92,93,93)	(97,97,97)	(85,88,90)	(98,99,100)	(102,102,103)
3	(68,76,77)	(88,89,92)	(78,86,86)	(68,76,77)	(87,89,92)	(78,86,86)	(58,60,65)	(88,91,94)	(66,67,69)	(64,69,70)
4	(55,59,61)	(82,84,86)	(63,67,73)	(54,59,61)	(83,85,86)	(62,66,73)	(73,74,79)	(82,86,86)	(75,77,87)	(92,93,98)
5	(103,103,104)	(77,77,79)	(103,104,104)	(103,103,104)	(77,77,79)	(103,104,104)	(101,102,103)	(78,78,79)	(100,101,101)	(101,101,101)
6	(98,99,101)	(66,67,68)	(95,97,98)	(98,99,101)	(65,66,68)	(95,97,99)	(68,75,75)	(66,67,69)	(70,72,74)	(95,99,99)
7	(24,24,30)	(11,15,15)	(21,22,22)	(25,26,30)	(11,15,15)	(23,23,23)	(35,35,41)	(11,15,15)	(26,27,29)	(24,24,27)
8	(100,100,101)	(94,96,98)	(103,103,104)	(100,100,101)	(96,96,98)	(103,103,104)	(82,84,86)	(97,98,99)	(93,94,94)	(97,97,98)
9	(102,102,103)	(93,93,97)	(105,105,105)	(102,102,103)	(95,95,96)	(105,105,105)	(103,104,105)	(94,95,97)	(105,105,105)	(99,100,100)
10	(45,48,49)	(104,104,105)	(59,61,66)	(45,48,49)	(102,102,103)	(58,61,65)	(39,48,51)	(101,102,103)	(53,58,64)	(61,62,63)
11	(12,13,16)	(4,6,6)	(6,8,8)	(14,15,18)	(4,6,6)	(7,9,9)	(20,21,22)	(4,6,6)	(10,12,14)	(9,10,17)
12	(17,18,25)	(81,85,86)	(29,30,39)	(18,20,27)	(81,90,91)	(31,32,41)	(26,26,37)	(83,83,84)	(39,39,50)	(17,17,22)
13	(88,90,91)	(93,100,100)	(96,99,99)	(88,90,91)	(93,97,97)	(97,98,99)	(96,99,99)	(96,96,96)	(102,102,103)	(104,104,105)
14	(60,62,67)	(98,99,101)	(78,79,84)	(60,64,67)	(101,101,101)	(78,79,85)	(77,78,80)	(100,100,102)	(90,90,93)	(60,61,63)
15	(93,93,95)	(67,70,72)	(90,92,92)	(93,94,95)	(66,69,72)	(90,91,92)	(75,79,79)	(67,70,72)	(77,79,80)	(86,89,90)
16	(52,52,59)	(82,84,88)	(58,60,69)	(51,52,59)	(83,85,88)	(57,60,68)	(69,70,74)	(83,86,88)	(73,75,82)	(37,39,43)
17	(7,8,12)	(11,11,13)	(10,11,11)	(8,8,11)	(11,11,13)	(10,11,12)	(15,16,19)	(11,11,13)	(14,15,15)	(6,8,9)
18	(92,92,95)	(81,85,86)	(95,96,97)	(91,92,93)	(82,85,86)	(95,95,96)	(76,81,83)	(84,85,87)	(86,88,89)	(89,90,91)
19	(78,78,83)	(54,54,55)	(73,74,76)	(78,78,83)	(52,54,56)	(72,74,76)	(82,92,92)	(53,54,56)	(76,85,88)	(73,82,85)

20	(96,97,98)	(62,66,67)	(93,94,95)	(96,97,98)	(62,65,67)	(93,94,96)	(59,63,65)	(62,65,68)	(59,62,66)	(91,92,94)
21	(54,64,66)	(80,83,87)	(63,74,77)	(54,64,66)	(82,84,87)	(61,74,76)	(54,64,64)	(82,85,87)	(57,67,68)	(50,58,59)
22	(79,81,82)	(104,105,105)	(91,91,91)	(79,81,82)	(104,104,104)	(90,91,91)	(87,88,89)	(105,105,105)	(95,96,97)	(71,76,79)
23	(97,98,99)	(72,75,77)	(94,96,98)	(97,98,99)	(71,75,77)	(94,96,98)	(83,85,86)	(72,75,77)	(83,86,88)	(93,94,95)
24	(68,71,75)	(90,90,94)	(79,82,87)	(68,71,75)	(90,91,94)	(79,82,87)	(84,86,90)	(94,95,95)	(91,93,95)	(80,81,83)
25	(95,99,100)	(85,89,89)	(100,101,101)	(95,99,100)	(86,89,89)	(100,100,101)	(95,98,100)	(87,89,92)	(98,99,100)	(92,96,97)
26	(78,80,81)	(51,52,53)	(72,75,76)	(78,80,81)	(51,52,53)	(73,74,76)	(78,80,81)	(50,52,53)	(71,71,72)	(66,67,69)
27	(92,96,97)	(39,41,42)	(81,81,83)	(92,96,97)	(38,40,41)	(80,81,83)	(55,56,57)	(39,40,42)	(49,50,51)	(91,93,94)
28	(86,89,90)	(78,80,84)	(90,92,94)	(86,89,90)	(79,81,84)	(90,92,94)	(94,98,99)	(79,81,84)	(98,99,101)	(84,86,87)
29	(23,30,33)	(48,48,49)	(29,37,41)	(24,30,32)	(48,48,49)	(29,38,41)	(35,42,51)	(47,48,49)	(38,44,46)	(29,42,43)
30	(19,38,40)	(49,51,51)	(26,42,44)	(20,38,41)	(50,51,51)	(27,42,44)	(27,60,64)	(49,51,51)	(32,54,55)	(48,53,56)
31	(73,76,80)	(57,59,61)	(71,76,77)	(73,76,80)	(57,59,59)	(71,75,77)	(90,91,91)	(57,59,60)	(85,86,87)	(69,72,75)
32	(91,94,94)	(87,88,91)	(97,98,99)	(92,94,95)	(88,88,91)	(97,98,99)	(100,101,101)	(89,90,92)	(102,103,103)	(87,87,88)
33	(47,48,49)	(65,66,69)	(49,49,52)	(47,48,49)	(64,66,68)	(49,49,53)	(57,59,60)	(66,67,70)	(59,59,62)	(45,45,47)
34	(2,2,4)	(45,94,96)	(6,10,17)	(2,2,4)	(43,93,94)	(6,11,19)	(4,5,9)	(41,93,93)	(10,13,21)	(1,1,3)
35	(3,6,6)	(20,20,23)	(8,12,13)	(3,6,7)	(20,20,23)	(8,10,13)	(6,7,8)	(20,21,23)	(12,14,17)	(5,6,7)
36	(58,58,67)	(58,60,71)	(53,55,67)	(57,58,67)	(56,58,70)	(53,55,66)	(57,58,61)	(58,58,71)	(56,57,63)	(52,52,58)
37	(51,64,65)	(63,68,69)	(54,64,64)	(50,63,65)	(63,67,69)	(54,63,64)	(38,40,44)	(64,68,70)	(45,49,51)	(48,56,58)
38	(10,14,20)	(6,8,8)	(9,12,13)	(10,13,20)	(6,8,8)	(9,12,13)	(18,19,29)	(6,8,8)	(15,16,18)	(13,18,23)
39	(86,87,88)	(36,36,37)	(65,70,71)	(86,87,88)	(36,37,37)	(64,69,70)	(95,96,96)	(36,36,37)	(80,82,85)	(88,89,90)
40	(27,29,30)	(15,19,19)	(22,25,25)	(12,16,16)	(15,19,19)	(12,20,20)	(1,1,3)	(15,19,19)	(1,2,4)	(10,16,19)
41	(79,79,80)	(75,76,78)	(81,84,86)	(79,79,80)	(75,76,78)	(81,84,86)	(72,73,74)	(76,76,78)	(72,74,78)	(71,72,74)
42	(7,8,9)	(7,9,10)	(7,9,10)	(7,7,9)	(7,10,10)	(6,8,10)	(2,38,45)	(7,10,10)	(3,24,24)	(9,11,11)
43	(44,45,46)	(24,24,32)	(39,40,41)	(36,36,40)	(24,24,32)	(34,35,38)	(27,28,31)	(24,24,32)	(28,29,31)	(33,34,37)
44	(14,16,16)	(16,23,23)	(19,20,20)	(15,17,17)	(16,23,23)	(20,22,22)	(7,8,9)	(16,23,23)	(11,17,19)	(15,16,16)
45	(44,56,60)	(73,74,76)	(49,58,60)	(44,55,60)	(73,74,76)	(51,57,60)	(66,73,76)	(73,75,76)	(69,73,78)	(53,59,60)
46	(17,22,25)	(2,4,4)	(3,6,7)	(17,23,27)	(1,3,4)	(2,6,7)	(8,9,11)	(2,4,4)	(3,6,7)	(13,20,21)
47	(67,70,70)	(28,28,35)	(46,46,47)	(67,70,70)	(28,28,35)	(46,46,47)	(41,43,46)	(28,28,35)	(37,39,40)	(54,55,56)
48	(31,33,58)	(46,47,50)	(36,40,51)	(31,33,58)	(46,47,50)	(36,40,50)	(42,49,72)	(46,47,50)	(42,45,65)	(29,31,59)
49	(72,74,76)	(47,50,50)	(61,67,70)	(72,74,76)	(47,49,50)	(57,66,69)	(89,89,92)	(49,51,52)	(76,81,84)	(83,83,85)
50	(32,36,50)	(10,16,16)	(28,29,33)	(34,41,53)	(10,16,16)	(28,29,33)	(49,55,71)	(10,16,16)	(34,38,40)	(27,29,44)
51	(61,62,63)	(58,58,60)	(57,59,60)	(61,62,63)	(57,58,60)	(56,58,59)	(42,47,50)	(56,57,59)	(44,47,50)	(55,55,57)
52	(1,1,7)	(5,7,7)	(2,3,4)	(1,3,6)	(5,7,7)	(2,3,4)	(1,2,5)	(5,7,7)	(2,4,5)	(4,4,6)
53	(9,15,15)	(1,1,24)	(1,1,15)	(8,14,15)	(1,1,25)	(1,1,17)	(16,21,22)	(1,1,25)	(1,1,22)	(8,13,14)
54	(27,27,36)	(9,10,18)	(19,19,31)	(28,28,36)	(9,9,19)	(19,19,31)	(10,12,15)	(9,9,19)	(11,12,19)	(23,23,28)
55	(34,38,40)	(17,17,19)	(30,31,32)	(34,39,39)	(17,17,18)	(30,30,32)	(30,34,37)	(17,17,18)	(26,29,30)	(33,38,38)
56	(22,22,25)	(2,2,29)	(2,3,23)	(21,24,26)	(2,2,29)	(2,4,24)	(10,12,12)	(2,2,29)	(3,5,13)	(10,11,15)
57	(63,65,69)	(52,53,56)	(56,57,62)	(63,65,69)	(53,54,55)	(55,56,60)	(45,46,47)	(53,55,55)	(46,47,49)	(60,61,62)
58	(104,104,105)	(37,40,40)	(102,102,102)	(104,104,105)	(39,42,42)	(102,102,102)	(66,67,68)	(37,41,43)	(54,55,58)	(62,64,68)
59	(55,63,66)	(68,71,73)	(55,66,66)	(55,62,66)	(67,70,73)	(55,65,65)	(50,58,61)	(68,71,73)	(53,60,62)	(52,54,57)
60	(72,73,77)	(60,63,65)	(74,75,77)	(72,73,77)	(60,63,65)	(73,75,77)	(43,44,48)	(60,64,65)	(47,51,52)	(68,71,75)
61	(55,71,75)	(33,33,34)	(45,48,51)	(54,71,75)	(33,33,34)	(45,49,51)	(72,88,90)	(33,33,34)	(56,67,68)	(57,73,77)
62	(71,73,75)	(79,81,83)	(80,80,82)	(71,73,75)	(80,81,82)	(80,80,82)	(87,87,91)	(80,81,81)	(90,91,92)	(63,65,66)
63	(18,21,26)	(12,14,14)	(18,21,21)	(14,19,23)	(12,14,14)	(16,21,21)	(11,11,14)	(12,14,14)	(9,16,18)	(18,22,22)
64	(42,51,54)	(92,92,95)	(56,65,68)	(45,56,58)	(99,100,100)	(63,71,77)	(65,70,71)	(97,98,98)	(73,82,83)	(67,75,78)
65	(41,50,52)	(3,5,5)	(14,16,18)	(42,50,51)	(3,5,5)	(14,15,18)	(18,18,20)	(3,5,5)	(6,9,11)	(33,44,45)
66	(15,17,20)	(32,32,33)	(20,23,23)	(16,18,19)	(32,32,33)	(21,24,25)	(21,24,28)	(32,32,33)	(25,28,30)	(24,27,28)
67	(94,96,101)	(37,38,38)	(78,82,87)	(94,96,101)	(37,38,38)	(78,82,87)	(102,103,104)	(37,38,38)	(89,91,96)	(95,96,100)
68	(90,91,93)	(96,101,102)	(100,100,101)	(90,91,93)	(97,104,104)	(100,101,101)	(98,100,102)	(100,103,103)	(104,104,104)	(66,68,74)
69	(26,28,29)	(25,31,31)	(26,27,31)	(27,29,29)	(24,31,31)	(26,27,30)	(37,40,41)	(24,31,31)	(34,35,38)	(28,30,30)
70	(83,85,87)	(78,79,80)	(87,88,89)	(83,84,87)	(78,80,80)	(87,88,89)	(93,94,95)	(79,80,80)	(96,97,97)	(78,81,88)
71	(21,23,24)	(43,47,48)	(27,28,28)	(22,24,25)	(44,47,48)	(26,27,28)	(30,33,36)	(43,47,48)	(33,36,37)	(20,20,21)
72	(39,39,42)	(30,35,36)	(37,41,42)	(39,40,43)	(30,35,36)	(37,41,42)	(32,32,36)	(30,35,36)	(32,32,35)	(38,40,41)

73	(39,49,53)	(101,102,103)	(52,62,75)	(40,49,52)	(103,103,105)	(52,61,75)	(45,51,56)	(99,99,101)	(60,61,68)	(43,44,46)
74	(66,69,74)	(55,57,57)	(59,65,68)	(66,69,74)	(55,56,57)	(58,64,67)	(83,85,85)	(55,56,57)	(75,78,81)	(78,79,82)
75	(53,54,65)	(74,75,76)	(53,55,70)	(53,53,65)	(74,75,76)	(54,54,70)	(69,71,82)	(74,75,77)	(70,70,84)	(46,47,54)
76	(51,56,59)	(28,34,34)	(43,43,45)	(51,56,59)	(28,34,35)	(43,43,45)	(23,23,23)	(28,34,34)	(25,25,27)	(41,48,49)
77	(77,83,84)	(69,70,70)	(83,83,84)	(77,83,84)	(68,69,70)	(83,83,84)	(93,93,94)	(69,69,71)	(92,94,95)	(74,77,77)
78	(81,82,86)	(42,44,44)	(69,72,72)	(81,82,86)	(43,44,45)	(68,71,72)	(67,67,69)	(42,44,45)	(56,57,61)	(70,72,76)
79	(35,35,37)	(21,30,30)	(32,35,37)	(35,35,38)	(21,30,30)	(32,36,38)	(31,31,34)	(21,30,30)	(28,31,31)	(26,26,26)
80	(38,47,50)	(95,95,99)	(53,61,62)	(38,47,50)	(93,94,99)	(52,59,59)	(55,62,63)	(89,90,91)	(61,66,69)	(32,41,42)
81	(37,42,44)	(97,97,100)	(50,52,54)	(37,43,45)	(92,92,95)	(48,52,53)	(52,54,56)	(91,92,93)	(60,63,65)	(31,35,36)
82	(102,104,104)	(17,21,21)	(79,89,89)	(102,104,104)	(17,22,22)	(79,89,89)	(104,105,105)	(17,22,22)	(83,89,92)	(102,105,105)
83	(62,68,72)	(62,64,64)	(64,71,73)	(62,68,72)	(62,64,64)	(62,70,72)	(80,84,88)	(62,64,65)	(79,84,87)	(70,73,76)
84	(82,85,85)	(72,73,74)	(85,88,88)	(82,85,85)	(72,72,74)	(85,88,88)	(70,76,77)	(72,73,74)	(71,76,79)	(80,84,86)
85	(33,43,45)	(14,18,18)	(26,32,33)	(33,44,46)	(14,18,18)	(26,33,34)	(20,22,24)	(14,18,18)	(16,22,24)	(25,30,31)
86	(28,34,36)	(63,64,65)	(38,43,44)	(28,34,37)	(61,61,63)	(39,43,44)	(39,52,54)	(63,63,63)	(48,53,55)	(40,51,53)
87	(41,43,47)	(22,22,27)	(35,36,40)	(42,44,47)	(21,21,27)	(37,37,40)	(27,28,29)	(20,21,27)	(26,27,30)	(36,37,42)
88	(46,57,57)	(41,42,45)	(45,47,48)	(46,56,57)	(40,41,45)	(45,47,48)	(53,61,62)	(40,42,44)	(46,52,54)	(39,50,51)
89	(69,70,74)	(55,59,61)	(63,68,69)	(69,70,74)	(54,59,61)	(63,67,68)	(43,44,49)	(54,59,61)	(48,48,52)	(64,65,65)
90	(53,56,57)	(49,53,54)	(47,48,51)	(52,55,57)	(49,52,53)	(47,48,51)	(33,36,38)	(48,52,54)	(36,40,41)	(49,50,51)
91	(10,10,11)	(20,27,27)	(12,15,17)	(9,10,11)	(20,27,27)	(13,15,16)	(17,17,19)	(20,27,27)	(20,20,22)	(12,12,12)
92	(19,20,23)	(102,103,103)	(34,38,39)	(19,22,25)	(99,100,102)	(34,39,40)	(25,29,32)	(104,104,104)	(41,43,45)	(21,25,25)
93	(11,11,12)	(22,29,29)	(16,16,18)	(10,11,12)	(22,29,29)	(16,18,18)	(13,14,16)	(22,29,29)	(17,19,20)	(7,7,8)
94	(26,28,29)	(25,25,26)	(25,27,30)	(26,29,30)	(25,25,26)	(25,28,29)	(34,39,40)	(25,25,26)	(34,35,36)	(67,82,85)
95	(60,61,64)	(52,56,56)	(54,56,57)	(60,61,64)	(71,78,79)	(62,67,69)	(77,78,81)	(66,74,77)	(77,80,81)	(103,103,104)
96	(5,5,6)	(43,43,46)	(11,14,14)	(5,5,6)	(43,44,46)	(11,14,14)	(2,3,4)	(43,44,46)	(7,7,8)	(3,3,4)
97	(37,40,41)	(82,98,99)	(50,50,58)	(37,41,42)	(83,98,98)	(50,50,56)	(53,59,63)	(82,101,102)	(63,65,74)	(46,47,49)
98	(43,46,48)	(41,45,46)	(42,44,46)	(43,46,48)	(42,45,46)	(42,44,46)	(26,30,33)	(41,45,46)	(33,33,37)	(36,39,40)
99	(5,8,9)	(1,3,3)	(1,4,4)	(4,9,22)	(2,3,4)	(3,3,4)	(10,13,14)	(1,3,3)	(2,5,6)	(80,96,98)
100	(32,32,35)	(39,39,40)	(34,36,38)	(32,32,35)	(39,39,40)	(35,36,39)	(48,50,53)	(39,39,40)	(42,43,44)	(34,35,35)
101	(18,19,21)	(8,12,12)	(13,15,17)	(21,21,23)	(8,12,12)	(15,17,17)	(24,25,25)	(8,12,12)	(18,21,21)	(18,19,19)
102	(31,31,34)	(35,38,44)	(33,34,35)	(31,31,33)	(34,36,41)	(31,33,35)	(46,47,52)	(35,38,45)	(41,42,43)	(32,32,34)
103	(13,13,14)	(71,91,91)	(24,24,24)	(12,13,13)	(55,71,73)	(22,24,25)	(13,15,17)	(50,61,61)	(23,23,23)	(14,14,15)
104	(1,3,3)	(9,13,13)	(2,5,5)	(1,1,3)	(9,13,13)	(1,5,5)	(3,5,6)	(9,13,13)	(4,8,9)	(1,2,2)
105	(2,4,4)	(26,26,31)	(5,7,9)	(2,4,5)	(26,26,31)	(5,7,8)	(4,6,7)	(26,26,31)	(8,10,13)	(2,5,5)