Adv. Math. Fin. App., 2025, **10**(3), P.335-350

Advances in Mathematical Finance & Applications
www.amfa.iau-arak.ac.ir
Print ISSN: 2538-5569
Online ISSN: 2645-4610
Doi: 10.71716/amfa.2025.31186851

Original Research

Constant Volatility Scaled and Semi-Constant Volatility Scaled Momentum in Tehran Stock Exchange

Mohammadreza Ghaseminejad*, Kamran Pakizeh

Department of Finance, Faculty of Financial Sciences, Kharazmi University, Tehran, Iran

ARTICLE INFO

Article history:
Received 2024-10-13
Accepted 2025-01-30

Keywords:
Momentum Strategy
Risk-Adjusted Return
Tehran Stock Exchange

ABSTRACT

Momentum strategies, due to their strong performance, are common investment methods designed based on the continuation of past asset performance. However, these strategies face sharp declines in high volatility conditions and market reversals. In this research, the impact of Constant Volatility Scaled Momentum (cMOM) and Semi-Constant Volatility Scaled Momentum (sMOM) strategies is examined using data from 100 stocks that constitute a significant portion of the Tehran Stock Exchange market value during the years 2013 to 2024. These strategies aim to reduce risk and improve risk-adjusted returns by adjusting for recent volatility. The results show that sMOM outperforms cMOM in factor-spanning tests and acts as a complement to traditional momentum. Moreover, its strong correlation with traditional momentum and its relative independence from market risk were confirmed in this study. These findings indicate that volatility adjustment does not always lead to performance improvement, and market conditions play a crucial role in the efficiency of these strategies. The results demonstrate that neither the constant volatility nor the semi-constant volatility scaled momentum strategies consistently outperform one another.

1 Introduction

Momentum strategies, which are based on the concept of following existing market trends, have become a cornerstone of investment approaches due to their straightforward implementation and strong performance across various time horizons. These strategies operate on the premise that assets which have recently exhibited strong performance will continue to do so, while those with poor performance will likely persist in underperforming [1,2]. Nevertheless, a significant limitation of traditional momentum strategies is their vulnerability to sharp losses during market reversals or periods of heightened

* Corresponding author. Tel.
E-mail address: *m.ghaseminejad7878@gmail.com*

volatility, potentially resulting in considerable drawdowns [3].One effective method for managing these fluctuations is the application of volatility adjustment techniques, which adjust the portfolio's risk exposure based on the inverse of its recent volatility to maintain a consistent level of risk under varying market conditions [4]. This approach, specifically designed to reduce risk and mitigate maximum drawdown, can enhance risk-adjusted returns. This study focuses on constant-volatility (cMOM) and semi-constant volatility (sMOM) scaled momentum strategies, which are specifically designed to address the large drawdowns often associated with momentum strategies. These strategies dynamically adjust portfolio exposure based on real-time volatility assessments, aiming to reduce risk and enhance downside protection during periods of market instability [5]. Examining the impact of these strategies in the Tehran Stock Exchange, which has its own unique characteristics and challenges, provides a valuable opportunity to study the dynamics of emerging markets. Given that financial markets in developing countries often experience higher volatility, strategies that contribute to risk reduction and more precise capital management gain added significance. By evaluating the performance of cMOM and sMOM strategies under such conditions, this research not only enhances the understanding and practical application of these strategies but also enables their comparison with other international markets. This information can assist investors and portfolio managers in making more informed decisions and improving investment strategies in similar markets.

2 Theoretical Fundamentals and Research Background

In our analysis of momentum strategies in the Tehran Stock Exchange, we specifically focus on constant-volatility scaled momentum (cMOM) and semi-constant volatility scaled momentum (sMOM) strategies. These innovative approaches, grounded in the principles proposed by Matthias and Hanauer [6], emphasize risk management through volatility scaling and aim to mitigate the significant risks associated with traditional momentum strategies, such as momentum crashes during market reversals.In the study of momentum strategies in financial markets, the foundational work of Fama and French on factor models plays a critical role in understanding the risk factors that drive stock returns. Specifically, Fama and French [7-9] introduced models that identify market, size, and value factors as key determinants of asset prices, providing a robust framework for analyzing investment strategies. Within this framework, the reviewed literature considers these factors to evaluate the performance of various momentum strategies, particularly the constant-volatility scaled momentum (cMOM) and semi-constant volatility scaled momentum (sMOM) strategies proposed by Matthias and Hanauer [6]. These authors, along with others such as Moreira and Muir [4], emphasize the importance of accounting for volatility as a key risk component when implementing momentum strategies. This methodological approach aligns with Fama and French's broader implication that market anomalies, such as momentum, should be assessed in the context of risk-adjusted returns. By utilizing Fama and French factor models [7-9], researchers can disentangle the specific effects of momentum from broader market movements, enabling a more precise evaluation of the true effectiveness and risk characteristics of these strategies across diverse market environments. This integration underscores the importance of factor-based analysis in the design and evaluation of advanced financial strategies, such as cMOM and sMOM, in the Tehran Stock Exchange, and provides a deeper understanding of risk management in momentum investing. The literature indicates that while traditional momentum strategies can be profitable, they are susceptible to sudden and severe drawdowns, often referred to as "momentum crashes." As detailed by Daniel and Moskowitz [3], these crashes are particularly pronounced during periods of intense market stress. This vulnerability is attributed to the high volatility experienced by momentum portfolios during downturns.

In light of this, Grobys et al. [10], examined risk-managed industry momentum strategies across various sectors, demonstrating that industry-specific momentum strategies can help diversify some of the inherent risks associated with broader market momentum strategies. Their findings suggest that risk management approaches can be more resilient under different market conditions, complementing the core hypotheses of the cMOM and sMOM strategies proposed by Matthias and Hanauer [6], which advocate that adjusting investment exposure based on market volatility can lead to more stable returns. Moreover, Moreira and Muir [4] demonstrate that volatility-scaled momentum strategies can significantly enhance the Sharpe ratios of momentum portfolios while reducing both the frequency and severity of negative anomalies (momentum crashes) without compromising high returns. They argue that the effectiveness of momentum can be substantially improved through dynamic position scaling based on realized volatility. This feature aligns with the logic of constant-volatility and semi-constant volatility scaled momentum strategies.The concept of volatility scaling has also been expanded by Barroso and Santa-Clara [5], who show that adding a layer of risk management significantly reduces the skewness and kurtosis of momentum strategy return distributions, effectively lowering the probability of severe losses while enhancing overall performance. This idea underpins the constant-volatility and semi-constant volatility scaled momentum strategies, as these strategies aim to constrain investment volatility to a manageable level, delivering more predictable and stable returns for investors. Evidence has highlighted the sensitivity of momentum profits to market conditions, suggesting that these profits are more pronounced when markets continue in the same state rather than transition to a different one [11]. This finding builds on earlier work by Cooper, Gutierrez, and Hameed [12], who demonstrated that momentum profits are largely confined to periods following UP markets. Asem and Tian empirically investigate the effects of market reversals and find that momentum profits significantly decline when markets transition from an UP state to a DOWN state [11], supporting the behavioral theories proposed by Daniel, Hirshleifer, and Subrahmanyam [13] and Hong and Stein [14]. These models argue that investor overconfidence and risk aversion dynamics drive momentum, predicting higher profits when markets continue in the same state. However, the results also reveal unexpected large momentum profits following DOWN markets, challenging prior findings and providing new insights into the underlying mechanisms of momentum profits in varying market conditions.Carhart [15] demonstrates that common factors in stock returns and investment expenses almost completely explain the persistence in mutual fund performance. Using a four-factor model that includes market risk, size, book-to-market, and momentum, Carhart shows that mutual funds following momentum strategies do not achieve superior returns once expenses and transaction costs are considered. The study concludes that persistence in mutual fund performance is largely driven by costs and common factor exposures, rather than managerial skill. This evidence challenges the notion of skilled fund managers consistently outperforming the market.Cederburg et al. [16] investigate the performance of volatility-managed portfolios using a comprehensive sample of 103 equity strategies. Their findings suggest that while volatility-managed portfolios tend to produce positive alphas in spanning regressions, they do not systematically outperform their unmanaged counterparts in direct performance comparisons. Moreover, the out-of-sample performance of these portfolios generally fails to replicate the in-sample results, indicating substantial structural instability in the underlying spanning regressions. These results imply that the benefits of volatility management may be less practical for real-time investors than previously suggested. Volatility weighting is an effective risk management technique applied to momentum strategies. Du Plessis and Hallerbach [17] explore two main forms of volatility weighting: weighting a strategy by its own volatility and weighting each underlying

asset by its volatility. Their empirical findings, based on U.S. industry portfolios, demonstrate that volatility weighting can significantly enhance the Sharpe ratio of both time-series and cross-sectional momentum strategies. They also introduce a novel form of volatility weighting, termed dispersion weighting, which uses cross-sectional dispersion as a measure of volatility. The results indicate that, while traditional volatility weighting improves performance, dispersion weighting offers additional benefits by stabilizing returns and reducing downside risk. Momentum strategies, which involve buying recent winners and shorting recent losers, inherently exhibit time-varying factor exposures due to the performance of common risk factors during the ranking period. Grundy and Martin [18] find that, even after adjusting for dynamic risk exposures, momentum profits remain remarkably stable across various subperiods, suggesting that factor models can explain a significant portion of the return variability but not the mean returns of these strategies. Their analysis also shows that strategies based on stock-specific return components are more profitable than those based on total returns, indicating that the momentum effect is primarily driven by stock-specific information rather than industry or cross-sectional factors.
Asness et al. [19] investigate the performance of value and momentum strategies across eight diverse asset classes, including equities, bonds, currencies, and commodities. Their findings reveal consistent and significant return premia for both value and momentum strategies in every asset class, with strong comovement of returns across asset classes. They also demonstrate that value and momentum are negatively correlated with each other, both within and across asset classes. The study introduces a three-factor model incorporating global funding liquidity risk to explain these patterns, highlighting the importance of examining value and momentum jointly rather than in isolation.Wang and Yan [20] examine the performance of downside volatility-managed portfolios compared to those scaled by total volatility. They find that strategies scaled by downside volatility exhibit superior performance, as evidenced by significantly higher alphas and Sharpe ratios. The enhanced performance is attributed primarily to the ability of downside volatility to predict future returns, allowing for better return timing. This suggests that managing downside risk, rather than total volatility, provides additional benefits for investors, particularly in real-time trading and across a broad range of equity factors and anomaly portfolios. Bekaert and Wu [21] investigate the asymmetry in volatility and risk in equity markets, focusing on both firm-level and market-level dynamics. Their study provides evidence that the negative correlation between returns and volatility is primarily driven by volatility feedback rather than leverage effects. Using data from the Nikkei 225 stocks, they find that conditional covariances respond more strongly to negative market shocks, which enhances volatility feedback at the firm level. These findings suggest that changes in conditional volatility are more significant for predicting risk premiums than previously assumed, challenging the traditional view that leverage effects are the primary driver of volatility asymmetry.Blitz, Hanauer, and Vidojevic [22] investigate the idiosyncratic momentum anomaly, demonstrating that it is distinct from conventional momentum and cannot be explained by established asset pricing factors. Their analysis shows that idiosyncratic momentum delivers robust returns across various international markets, and its performance remains stable even when controlling for factors such as crash risk and investor overconfidence. Unlike conventional momentum, idiosyncratic momentum is less sensitive to market reversals and exhibits lower transaction costs due to its unique risk-return profile. These findings suggest that idiosyncratic momentum represents a separate and valuable dimension in the cross-section of expected stock returns. Chui, Titman, and Wei [23] explore the relationship between cultural differences and momentum profits across global markets. Using Hofstede's individualism index, they find that momentum profits are significantly higher in countries with high levels of individualism, which is associated with greater overconfidence and self-attribution bias. Their results

suggest that cultural factors, particularly individualism, play a critical role in explaining the cross-country variation in momentum returns. These findings highlight the importance of psychological biases in shaping investor behavior and market outcomes. Watanabe et al. [24] examine the asset growth effect in international equity markets and find that firms with higher asset growth rates experience significantly lower future stock returns across 42 countries. This negative relationship is more pronounced in developed markets where stocks are more efficiently priced. The study suggests that the asset growth effect is driven by optimal investment behavior rather than market mispricing. These findings indicate that the link between asset growth and future returns is a robust global phenomenon, consistent with the predictions of rational asset pricing models.Blitz, Huij, and Martens [25] introduce the concept of residual momentum, which ranks stocks based on their residual returns rather than total returns. This approach significantly reduces the time-varying exposures to common risk factors observed in traditional momentum strategies. Their findings indicate that residual momentum delivers approximately double the Sharpe ratio of conventional momentum strategies, with higher risk-adjusted returns and lower volatility. The strategy also performs consistently over time and across business cycles, suggesting that residual momentum offers a more robust and less risk-exposed alternative to traditional momentum investing. Griffin, Ji, and Martin [26] analyze whether macroeconomic risk factors can explain momentum profits across global markets. Their findings indicate that neither unconditional models nor conditional forecasting models using lagged instruments can account for the observed momentum profits. They demonstrate that momentum strategies are economically significant and statistically robust in both good and bad economic states, with profits tending to reverse over long horizons. These results challenge traditional risk-based explanations for momentum, suggesting that the phenomenon is not driven by macroeconomic risk. During the years 2001 to 2005, Fadaei Nejad and Sadeghi demonstrated that momentum strategies in the Tehran Stock Exchange were able to generate excess returns for one-month, three-month, and six-month time horizons [27]. Similarly, Eslami Bidgoli et al., in their study of the market from 2004 to 2009, concluded that return continuation, or momentum, exists in stock returns over 3- to 12-month periods [28]. Additionally, Badri and Fathollahi (2014) found evidence of momentum in stock returns and a significant relationship between past and future returns in their analysis of the Tehran Stock Exchange [29].Davallou and Javadian (2017) examined the profitability of a novel momentum strategy based on 52-week high timing and concluded that the winner portfolio formed according to this strategy did not achieve higher returns compared to the loser portfolio [30]. Other findings suggest that cost stickiness has a positive impact on the relationship between institutional investors and passive institutional investors with conservatism [31]. Using the Huang and Salmon model, researchers examined the impact of herding behavior of institutional investors on the stock returns of companies listed on the Tehran Stock Exchange, and their research results showed that there is a relationship between these two variables. Other findings of this study showed that the relationship between herding behavior and stock returns is greater in larger companies than in smaller companies, and also in companies with higher financial leverage; it is greater than in companies with lower financial leverage [32]. The hypotheses have been developed through an extensive analysis of existing research and are outlined as follows:

Hypothesis 1: Constant-volatility and semi-constant volatility scaled momentum strategies outperform traditional momentum strategies in terms of return, risk, and risk-adjusted return by delivering higher alphas and stronger risk-adjusted performance.

Hypothesis 2: Volatility management improves the performance of momentum strategies.

3 Methodology

In this study, we first construct momentum strategy portfolios. The statistical population used in this research consists of data on stocks listed on the Tehran Stock Exchange from April 2013 to March 2024. The sample includes data from 100 stocks, representing the majority of the market value of the Tehran Stock Exchange during the period from April 2013 to March 2024. The stocks in the sample are ranked based on their formation period returns (the returns over the past 12 months, excluding the most recent two months) according to the method proposed by Matthias and Hanauer [6], from highest to lowest. The top 30% of the sample, representing the highest returns, are classified as the winner portfolio, while the bottom 30%, representing the lowest returns, are classified as the loser portfolio. These winner and loser portfolios are formed based on the market value of the stocks. The monthly return of the momentum factor is then calculated as follows:

$$MOM = (\sum_{i=1}^{n_{Winners}} (\frac{V_i^{Winners}}{\sum_{j=1}^{n_{Winners}} V_j^{Winners}} \times r_i^{Winners})) - \sum_{i=1}^{n_{Losers}} (\frac{V_i^{Losers}}{\sum_{j=1}^{n_{Losers}} V_j^{Losers}} \times r_i^{Losers})) \tag{1}$$

where: n is the number of stocks in the portfolio. W_i is the weight of stock i in the portfolio, calculated based on its market value relative to the total market value of the portfolio, defined as $W_i = \frac{V_i}{\sum_{j=1}^{n} V_j}$, where V_i represents the market value of stock i. r_i is the return of stock i.

It should be noted that the holding period is one month, and at the end of each holding period, the momentum factor return is calculated, and the winner and loser portfolios are reformed based on their formation period returns. Subsequently, we will address the following factors:

3.1 Steps for Constructing the Constant-Volatility Scaled Momentum Factor

1) Select your target population and sample.

2) Construct the standardized momentum factor as described.

3) Calculate the realized daily volatility of the standardized momentum factor over the past 6 months (126 trading days). The realized volatility is obtained according to the method outlined by Barroso and Santa-Clara [5] as follows:

$$\sigma_{MOM.t}^2 = 21 \times \sum_{j=1}^{126} \frac{R_{MOM.d-j.t}^2}{126} \tag{2}$$

The above equation is executed from $j = 1$ to $j = 126$, where: $R_{MOM.d-1.t}^2$ represents the squared return of the standardized momentum strategy on the first day. Similarly, the squared daily returns of the momentum strategy up to day 126 are summed together. The variable $\sigma_{MOM.t}^2$ denotes the variance of the momentum strategy's returns up to time t. Specifically, for the constant-volatility scaled momentum strategy (cMOM), this variance is used to scale the momentum returns in order to achieve a target volatility level. By scaling momentum returns based on their historical volatility, the constant-volatility scaled momentum strategy aims to smooth returns and reduce the risk of large negative drawdowns, which are typically observed in unscaled momentum strategies. This approach contributes to effective risk management.

4) Selecting the Target Volatility Level. There are various methods for choosing $\hat{\sigma_t}$:

- The average historical volatility of the sample over a specific time period can be used.
- A fixed target volatility can be used.

In this study, the target volatility level is set using the average historical volatility of the past month.

5) Using the target volatility (Step 4) and the realized volatility (Step 3), the weight $W_{cMOM.t}$ is calculated as follows:

$$W_{cMOM.t} = \frac{\sigma_{target}}{\hat{\sigma_t}} \tag{3}$$

Based on Equation (3), the above ratio can be analyzed as follows:

1) When the ratio of the target volatility to the realized volatility equals one, it indicates that the risk levels of the standard momentum and the constant-volatility scaled momentum strategies are equivalent.

2) When the target volatility level is lower than the realized volatility, the scaling ratio will be less than one. This lower ratio indicates that the investment during that period carries more risk compared to the target risk. Consequently, less capital is allocated to it, reducing its high risk to the desired level, thereby effectively managing the momentum risk.

3) Conversely, if the target volatility level is higher than the realized volatility, the scaling ratio will be greater than one. This means that the investment during that period has less risk compared to the accepted risk level. A higher ratio indicates a larger allocation to the constant-volatility scaled momentum strategy compared to the standard momentum strategy. This ultimately leads to higher returns for investors, with the added benefit that the scaled momentum strategy carries less risk than the standard momentum strategy, as it utilizes variable capital for buying and selling instead of a fixed amount.

4) Finally, the return of the constant-volatility scaled momentum strategy in month t is calculated as follows:

$$R_{cMOM.t} = R_{MOM.t} \times W_{cMOM.t} \tag{4}$$

5) Repeat Steps 3 to 6 as the holding period ends and new data becomes available to obtain the returns of the constant-volatility scaled momentum factor.

3.2 Steps for Constructing the Semi-Volatility Scaled Momentum Factor

1) Select your target population and sample.

2) Construct the standardized momentum factor as previously described.

3) Calculate the downside volatility (semi-volatility) of the standardized momentum factor over the past 6 months (126 trading days). The downside volatility is obtained according to the method outlined by Matthias and Hanauer [6] as follows:

$$\hat{\sigma}_{MOM.t.semi} = 21 \times \sum_{j=1}^{126} \frac{R^2_{MOM.d-j.t} I_{|R_{MOM.d-j<0}|}}{126} \tag{5}$$

The above expression is similar to Step 3 of the constant-volatility scaled momentum factor, with the key difference being that only negative returns are used in the calculation of downside volatility.Semi-volatility is a measure of the downside risk of an investment. Unlike traditional volatility, which includes both upward and downward price movements, semi-volatility focuses only on negative returns. This makes it particularly useful for risk management, as investors are typically more concerned with downside risks than with upside potential.

4) Selecting the Target Volatility Level. Similar to the constant-volatility scaled momentum strategy, we also choose $\hat{\sigma}_{target}$ in this context.

5) Using the target volatility (Step 4) and downside realized volatility (Step 3), the weight $W_{sMOM.t}$ is calculated as follows:

$$W_{sMOM.t} = \frac{\sigma_{target}}{\hat{\sigma}_{t.semi}} \tag{6}$$

where $\hat{\sigma}_{t.semi}$ is obtained in Step 3.

Adjusting the weight in the semi-volatility scaled momentum strategy is designed to manage the strategy's exposure to market fluctuations. This approach aims to improve risk-adjusted returns.

6) Finally, the return of the semi-volatility scaled momentum factor in month t is calculated as follows:

$$R_{sMOM.t} = R_{MOM.t} \times W_{sMOM.t} \tag{7}$$

7) Repeat Steps 3 to 6 at the end of each holding period and as new data becomes available to obtain

The returns from the semi-volatility scaled momentum strategy. In financial research, the factor spanning test is a fundamental analytical tool used to assess the added value of incorporating new factors into an existing asset pricing model. The primary goal of this test is to determine whether adding a new factor provides additional explanatory power for asset returns beyond what is already captured by the existing factors. This is particularly important in the context of momentum strategies, where the objective is to examine whether volatility-scaled momentum strategies offer significant improvements over the traditional momentum factor. The factor spanning test involves regressing the returns of a new factor on the returns of existing factors and then examining the coefficients. If the coefficients are statistically significant, it indicates that the new factor provides unique information that the existing factors cannot explain. Conversely, if the coefficients are not significant, it suggests that the new factor does not offer additional explanatory power and is redundant within the existing model. This test is crucial for validating the effectiveness of volatility-scaled momentum strategies, as it ensures that they provide meaningful insights and improvements over traditional momentum strategies.In this study, we utilize the factor spanning test to rigorously evaluate the effectiveness of the proposed constant-volatility and semi-volatility scaled momentum strategies. Through this test, we aim to determine whether these strategies provide additional explanatory power and higher risk-adjusted returns compared to the traditional momentum factor (MOM). The factor spanning test is particularly important in this context, as it allows us to quantify the incremental benefits of each volatility-scaled strategy and confirm their significance and robustness. The following regressions are used for the factor spanning test:

$$MOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \tag{8}$$

$$MOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 CMOM_t + \epsilon_t \tag{9}$$

$$MOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 SMOM_t + \epsilon_t \tag{10}$$

$$CMOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \epsilon_t \tag{11}$$

$$CMOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \beta_5 SMOM_t + \epsilon_t \tag{12}$$

$$SMOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \epsilon_t \tag{13}$$

$$SMOM_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 MOM_t + \beta_5 CMOM_t + \epsilon_t \tag{14}$$

Where:

MOM_t represents the return of the momentum strategy in month t. $CMOM_t$ represents the return of the constant-volatility scaled momentum strategy in month t. $SMOM_t$ represents the return of the semi-volatility scaled momentum strategy in month t. $RMRF_t$ represents the market risk premium in month t, calculated as the excess return of the market portfolio over the risk-free rate $(R_m - R_f)$. SMB_t is the size factor in month t, representing the difference in average returns between small-cap and large-cap stock portfolios. In this study, firm size is defined as the market capitalization (stock price multiplied by the number of shares outstanding). Following the approach of Fama and French [7], the median market capitalization of the sample companies is used for this classification. HML_t is the book-to-market factor in month t, calculated as the difference in average returns between stocks with high book-to-market ratios and those with low book-to-market ratios. Based on the Fama and French methodology [7], 30% of companies are classified as low, 40% as medium, and 30% as high based on their book-to-market rankings. It is worth noting that, in the tables presented in the research findings section, following the notation used by Matthias and Hanauer [6], the factors $RMRF_t$, SMB_t and HML_t are collectively represented as FF_t for brevity. The Jobson-Korkie test is a statistical method used to compare the Sharpe ratios of two investment portfolios, allowing us to determine whether there is a significant difference in their risk-adjusted performance. The Sharpe ratio is a common metric in finance that adjusts returns for risk and is defined as the ratio of a portfolio's excess return over the risk-free rate to the standard deviation of the portfolio's excess return. Specifically, the Sharpe ratio (S) is defined as follows:

$$S = \frac{R_p - R_f}{\delta_p} \tag{15}$$

Where R_p is the average return of the portfolio, R_f is the risk-free rate, and δ_p is the standard deviation of the portfolio's excess return.

In the context of the Jobson-Korkie test, our goal is to test the null hypothesis H_0 of equality of the Sharpe ratios of two portfolios against the alternative hypothesis H_1 of inequality of their Sharpe ratios. Formally, the hypotheses are stated as follows:

- **Null Hypothesis H_0:** The Sharpe ratios of the two portfolios are equal.
- **Alternative Hypothesis H_1:** The Sharpe ratios of the two portfolios are not equal.

The test statistic for the Jobson-Korkie test is calculated based on the difference between the two Sharpe ratios, taking into account the correlation between the returns of the two portfolios and the number of observations. The test statistic z is given as follows:

$$\frac{S_1 - S_2}{\sqrt{\frac{2(1-\rho)}{T-1}}} \tag{16}$$

After performing the calculations, the test statistic is expressed as follows:

$$Z = \frac{\sigma_1\mu_2 - \sigma_2\mu_1}{\sqrt{\theta}} \tag{17}$$

$$\theta = \frac{1}{T}(2\sigma_1^2\sigma_2^2 - 2\sigma_1\sigma_2\sigma_{1,2} + \frac{1}{2}\mu_1^2\sigma_2^2 + \frac{1}{2}\mu_2^2\sigma_1^2 - \frac{\mu_1\mu_2}{2\sigma_1\sigma_2}\left(\sigma_{1,2}^2 + \sigma_1^2\sigma_2^2\right)) \tag{18}$$

Where S_1 and S_2 are the Sharpe ratios of the first and second portfolios, respectively. Z represents the test statistic for comparing the Sharpe ratios of the two populations, μ_1 is the mean return of Portfolio 1, μ_2 is the mean return of Portfolio 2, σ_1 is the standard deviation of Portfolio 1, σ_2 is the standard deviation of Portfolio 2, N is the total number of strategies calculated for each ranking and holding period, and $\sigma_{1,2}$ is the covariance between the returns of Portfolios 1 and 2.

3.3 Classification of Volatility-Scaled Momentum Strategies for Performance Evaluation

In this paper, the constant-volatility and semi-volatility scaled momentum strategies are divided into two distinct groups to systematically evaluate and compare their performance. Each group is characterized by specific configurations of realized volatility periods and target volatility levels. By classifying the volatility-scaled momentum strategies into these categories, the aim of this study is to precisely assess which configurations deliver better results in terms of risk, return, and risk-adjusted return.

3.3.1 Group1

Group One in the constant-volatility scaled momentum and semi-volatility scaled momentum strategies is defined by using the realized daily volatility over a 6-month period (126 trading days). From this point forward, the Group One volatility-scaled momentum strategies will be referred to with the addition of the number one at the end of their labels (cMOM1, sMOM1) for reference in this study.

3.3.2 Group2

Group Two in the constant-volatility scaled momentum and semi-volatility scaled momentum strategies is defined by using the realized daily volatility over a 3-month period (63 trading days). From this point forward, the Group Two volatility-scaled momentum strategies will be referred to with the addition of the number two at the end of their labels (cMOM2, sMOM2) for reference in this study.

4 Findings

4.1 Descriptive Statistics of Research Variables

Table 1: Descriptive Statistics of Various Strategies

	MOM	cMOM1	cMOM2	sMOM1	sMOM2
Avg. monthly Returns	0.98%	1.28%	1.13%	-0.07%	0.19%
Median	0.96%	0.78%	0.97%	0.75%	0.92%
standard deviation	0.07	0.09	0.08	0.08	0.07
Sharpe ratio (annualized)	-0.38	-0.18	-0.27	-0.74	-0.72
Skewness	0.40	1.59	1.09	-0.74	-0.31
Kurtosis	7.86	17.15	11.90	14.77	10.76
Minimum	-24.38%	-33.90%	-26.63%	-45.67%	-34.51%
Maximum	32.93%	56.11%	44.04%	40.76%	35.27%
Max. Drawdown	-34.28%	-38.20%	-34.71%	-73.12%	-62.71%

In the descriptive statistics analysis of various momentum strategies presented in Table 1, several key insights emerge. The average return for these strategies indicates that cMOM1 leads with the highest average return of 1.28%, although this return comes with the highest standard deviation of 0.09, suggesting a more volatile but higher-yielding strategy. Interestingly, both sMOM1 and sMOM2 have negative average returns, indicating poor performance of these strategies over the observed period.

The relatively high standard deviations of sMOM1 and sMOM2 compared to their average returns highlight a high level of risk without a commensurate benefit in terms of investment returns.In terms of risk-adjusted performance, as measured by the Sharpe ratio, all strategies exhibit negative ratios, indicating that none of them have provided returns that justify their risks relative to the risk-free asset.

However, cMOM1 and cMOM2 have less negative Sharpe ratios (-0.18 and -0.27, respectively), which may suggest relatively better risk-adjusted performance among the strategies, even though they are still negative.

The maximum drawdown reflects the potential losses from peak to trough during the observed period, with sMOM1 showing the highest potential loss at -73.12%. Consequently, while none of the strategies demonstrate a clear advantage, cMOM1 emerges as a more favorable option due to its highest average return and a less negative Sharpe ratio, suggesting that this strategy manages the risk-return balance better than the others, although it still carries significant risk, as evidenced by its maximum drawdown and negative Sharpe ratio.

4.2 Factor Spanning Test for Group One Volatility-Scaled Momentum Strategies

Table 2 : Factor Spanning Test (Alpha Value)

	FF+(MOM)	FF+(MOM)+cMOM1	FF+(MOM)+sMOM1
MOM	0.0162	0.0013	0.0095
cMOM1	0.0008		0.0063
sMOM1	-0.0092	-0.0098	

Table 3 : Factor Spanning Test (t-Statistic)

	FF+(MOM)	FF+(MOM)+cMOM1	FF+(MOM)+sMOM1
MOM	2.8050	0.0013	4.5057
cMOM1	0.3129		2.8804
sMOM1	-3.9113	-5.6906	

The results from Tables 2 and 3 indicate that the factors cMOM1 and sMOM1 do not consistently enhance the explanatory power of the traditional momentum factor (MOM) in various Fama-French models. The traditional momentum factor alone performs well and generates significant alpha, while adding cMOM1 does not lead to any substantial improvement, leaving the alpha very low and insignificant. However, the reduction in alpha from 0.0162 to 0.0013 in the model where cMOM1 is included suggests that cMOM1 explains a portion of the momentum-related returns. On the other hand, sMOM1 generally worsens model performance, showing a significantly negative alpha, although it can be somewhat useful in certain combined settings. Overall, neither cMOM1 nor sMOM1 outperforms the traditional momentum factor, underscoring the importance of model configurations in determining the effectiveness of volatility-scaled momentum strategies.

4.3 Factor Spanning Test for Group Two Volatility-Scaled Momentum Strategies

Table 4 : Factor Spanning Test (Alpha Value)

	FF+(MOM)	FF+(MOM)+cMOM2	FF+(MOM)+sMOM2
MOM	0.0162	0.0007	0.0077
cMOM2	0.0001		0.0028
sMOM2	-0.0070	-0.0071	

Table 5: Factor Spanning Test (t-Statistic)

	FF+(MOM)	FF+(MOM)+cMOM2	FF+(MOM)+sMOM2
MOM	2.8050	0.5224	4.5421
cMOM2	0.0652		2.4800
sMOM2	-3.9352	-5.2357	

The analysis of the volatility-scaled momentum strategies, cMOM2 and sMOM2, in relation to the traditional momentum factor (MOM), reveals distinct dynamics. While MOM alone shows strong performance in the Fama-French model, the introduction of cMOM2 significantly reduces the alpha and t-statistic of MOM, indicating that cMOM2 captures a substantial portion of MOM's explanatory power. In contrast, sMOM2 slightly reduces MOM's alpha but maintains a strong t-statistic, suggesting a complementary role rather than a complete replacement of MOM. The cMOM2 strategy itself shows almost no independent explanatory power, whereas sMOM2, in combination with the momentum strategy, demonstrates moderate effectiveness with positive alpha and a significant t-statistic. Overall, sMOM2 enhances the model more effectively than cMOM2, complementing the traditional momentum strategy without fully replacing it. This characteristic aligns with the findings of Matthias and Hanauer [6] regarding the benefits of volatility-scaled strategies under specific conditions.

4.4 Comparative Performance Analysis of Portfolios Using the Jobson-Korkie Test for Sharpe Ratios

Table 6: Jobson-Korkie Test Statistic

cMOM1	cMOM2	sMOM1	sMOM2
0.046	-0.043	4.353	4.271

Table 6 presents the t-statistics from the Jobson-Korkie test and the Sharpe ratios for the four mentioned strategies compared to a momentum strategy. While sMOM1 and sMOM2 show better significance based on their high t-statistics, the negative Sharpe ratios for all strategies indicate poor risk-adjusted performance. This suggests that despite higher statistical significance, the sMOM1 and sMOM2 strategies carry higher risk. The negative Sharpe ratios across all strategies reflect an inadequate balance between risk and return, indicating the need for a more detailed analysis of the risk profile and market conditions.

4.5 Correlation Coefficients for Volatility-Scaled Momentum Strategies

Table 7: Correlation Coefficients of Various Variables

	MOM	RMRF	SMB	HML	cMOM1	sMOM1	cMOM2	sMOM2
MOM	1.00	0.04	-0.35	-0.28	0.95	0.94	0.98	0.96
RMRF	0.04	1.00	-0.04	0.44	0.12	0.01	0.12	0.03
SMB	-0.35	-0.04	1.00	0.19	-0.38	-0.37	-0.36	-0.34
HML	-0.28	0.44	0.19	1.00	-0.25	-0.34	-0.26	-0.32
cMOM1	0.95	0.12	-0.38	-0.25	1.00	0.95	0.99	0.95
sMOM1	0.94	0.01	-0.37	-0.34	0.95	1.00	0.95	0.99
cMOM2	0.98	0.12	-0.36	-0.26	0.99	0.95	1.00	0.97
sMOM2	0.96	0.03	-0.34	-0.32	0.95	0.99	0.97	1.00

The correlation matrix indicates strong positive correlations between the momentum factor (MOM) and the volatility-scaled momentum strategies, suggesting that these scaled strategies closely follow the traditional momentum while being adjusted for risk. Low correlations between momentum and the market factor (RMRF) highlight momentum's independence from market risk, which is a key benefit for diversification. Negative correlations with SMB indicate that momentum tends to favor larger stocks, while stronger negative correlations with HML reflect an increased divergence between momentum and value strategies [19]. Overall, the enhanced momentum strategies maintain the core dynamics of traditional momentum and improve risk-adjusted returns without significantly altering exposure to size and value factors.

5 Discussion and Conclusions

In comparing the results of this study in the Tehran Stock Exchange with international research, several key similarities and differences emerge. The findings are consistent with those of Matthias and Hanauer [6], Barroso and Santa-Clara [5], and Moreira and Muir [4], where volatility-adjusted strategies typically show higher returns accompanied by greater risk, as observed in cMOM1, which exhibits higher returns but also higher volatility. Similar to the observations in these studies, particularly the Barroso and Santa-Clara paper [5], the volatility-adjusted approach improves returns but still faces significant drawdowns and negative Sharpe ratios in the Tehran market. Interestingly, the negative returns of the sMOM strategies highlight the limitations of volatility adjustment in certain markets, aligning with Grobys [10], who noted the inefficacy of volatility management in some subsets. The less negative Sharpe ratios for the cMOM strategies, similar to the results of Matthias and Hanauer [6], suggest that these strategies perform better in risk adjustment compared to others. However, the absence of a clearly superior strategy in this study aligns with the broader literature, which points to the ongoing trade-off between risk and return in momentum strategies, even with volatility adjustments. This comparison indicates that while volatility adjustment can lead to higher returns, its effectiveness depends on market conditions and does not always result in positive risk-adjusted performance. Comparing these results with the factor spanning test reveals several key insights. Similar to the findings of Matthias and Hanauer [6], the traditional momentum factor alone continues to exhibit strong explanatory power, while volatility-scaled factors such as cMOM and sMOM present mixed outcomes. The inability of cMOM1 and sMOM1 to improve the traditional momentum model is consistent with Barroso and Santa-Clara [5], who demonstrated that not all volatility-scaled momentum strategies outperform traditional models, particularly in specific market conditions. The capacity of cMOM2 to capture some of MOM's explanatory power aligns with Moreira and Muir [4] on the effectiveness of volatility scaling. In contrast, the complementary role of sMOM2, rather than fully replacing the momentum strategy, reflects the varying behavior of these strategies under different conditions, as highlighted by Grobys et al. [10]. Ultimately, the results emphasize that while volatility-scaled strategies can add value in certain configurations, they often act as complements rather than substitutes for traditional momentum, underscoring the importance of model specifications. Among the strategies evaluated, sMOM2 demonstrates better performance, complementing MOM rather than completely replacing it and providing a more effective enhancement to the model compared to cMOM2.In comparing the results of the Jobson-Korkie test and the correlation matrix with related international studies, several notable similarities emerge. Consistent with the findings of Matthias and Hanauer [6], the strong correlation between the scaled strategies and traditional momentum in this study indicates that these strategies effectively track momentum trends. This is also in line with Barroso and Santa-Clara [5], who demonstrated that volatility-scaled strategies remain dependent on momentum, albeit with different risk adjustments. In this section, the performance of volatility-scaled momentum strategies in the Tehran Stock Exchange was compared with international studies, revealing significant similarities and differences. Consistent with the findings of Matthias [6], Barroso and Santa-Clara [5], and Moreira and Muir [4], the volatility-scaled strategies demonstrated higher returns and greater risk but were still accompanied by drawdowns and negative Sharpe ratios. In the factor spanning tests, sMOM2 outperformed cMOM2, acting as a complement to traditional momentum. This strategy also showed a strong correlation with traditional momentum and relative independence from market risk.

References

[1] Jegadeesh N, Titman S. Returns to buying winners and selling losers: implications for stock market efficiency, *The Journal of Finance*, 1993;48(1):65-91.

[2] Jegadeesh N, Titman S. Profitability of momentum strategies: an evaluation of alternative explanations, *The Journal of Finance*, 2001;56(2):699-720. doi: 10.1111/0022-1082.00342.

[3] Daniel K, Moskowitz TJ. Momentum crashes, *Journal of Financial Economics*, 2016;122(2):221-47. doi: 10.1016/j.jfineco.2015.12.002.

[4] Moreira A, Muir T. Volatility-managed portfolios, *The Journal of Finance*, 2017;72(4):1611-44. doi: 10.1111/jofi.12513.

[5] Barroso P, Santa-Clara P. Momentum has its moments, *Journal of Financial Economics*, 2015;116(1):111-20.

[6] Hanauer MX, Windmüller S. Enhanced momentum strategies, *Journal of Banking & Finance*, 2023;148:106712. doi: 10.1016/j.jbankfin.2022.106712.

[7] Fama EF, French KR. Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics*, 1993;33(1):3-56. doi: 10.1016/0304-405X(93)90023-5

[8] Fama EF, French KR. Size, value, and momentum in international stock returns, *Journal of Financial Economics*, 2012;105(3):457-72. doi: 10.1016/j.jfineco.2012.05.011.

[9] Fama EF, French KR. International tests of a five-factor asset pricing model, *Journal of Financial Economics*, 2017;123(3):441-63.

[10] Grobys K, Ruotsalainen J, Äijö J., Risk-managed industry momentum and momentum crashes, *Quantitative Finance*, 2018;18(4):649-61. doi: 10.1080/14697688.2017.1420211.

[11] Asem E, Tian GY. ,Market dynamics and momentum profits, *Journal of Financial and Quantitative Analysis*, 2010;45(6):1549-62. doi: 10.1017/S0022109010000542.

[12] Cooper MJ, Gutierrez RC Jr, Hameed A., Market states and momentum, *The Journal of Finance*, 2004;59(3):1345-65. doi: 10.2139/ssrn.299927.

[13] Daniel K, Hirshleifer D, Subrahmanyam A., Investor psychology and security market under and overreactions, *The Journal of Finance*, 1998;53(6):1839-86.

[14] Hong H, Stein JC. , A unified theory of underreaction, momentum trading, and overreaction in asset markets, *The Journal of Finance*, 1999;54(6):2143-84.

[15] Carhart MM., On persistence in mutual fund performance, *The Journal of Finance*, 1997;52(1):57-82. doi: 10.1111/j.1540-6261.1997.tb03808.x.

[16] Cederburg S, O'Doherty MS, Wang F, Yan XS. , On the performance of volatility-managed portfolios, *Journal of Financial Economics*, 2020;136(2):219-38. doi: 10.1016/j.jfineco.2020.04.015.

[17] du Plessis J, Hallerbach WG. , Volatility weighting applied to momentum strategies, *The Journal of Alternative Investments*, 2016;19(3):40-58. doi: 10.3905/jai.2017.19.3.040.

[18] Grundy BD, Martin JS. ,Understanding the nature of the risks and the source of the rewards to momentum investing, *The Review of Financial Studies*, 2001;14(1):29-78. doi: 10.1093/rfs/14.1.29.

[19] Asness CS, Moskowitz TJ, Pedersen LH. ,Value and momentum everywhere, *The Journal of Finance*, 2013;68(3):929-85. doi: 10.1111/jofi.12021.

[20] Wang F, Yan XS. ,Downside risk and the performance of volatility-managed portfolios, *Journal of Banking & Finance*, 2021;131:106198. doi: 10.1016/j.jbankfin.2021.106198.

[21] Bekaert G, Wu G., Asymmetric volatility and risk in equity markets, *The Review of Financial Studies*, 2000;13(1):1-42. doi: 10.1093/rfs/13.1.1.

[22] Blitz D, Hanauer MX, Vidojevic M. ,The idiosyncratic momentum anomaly, *International Review of Economics & Finance*, 2020;69:932-57. doi: 10.1016/j.iref.2020.05.008.

[23] Chui ACW, Titman S, Wei KCJ., Individualism and momentum around the world, *The Journal of Finance*, 2010;65(1):361-92. doi: 10.1111/j.1540-6261.2009.01532.x.

[24] Watanabe A, Xu Y, Yao T, Yu T. ,The asset growth effect: insights from international equity markets, *Journal of Financial Economics*. 2013;108(2):529-63.

[25] Blitz D, Huij J, Martens M., Residual momentum, *Journal of Empirical Finance*, 2011;18(3):506-21. doi: 10.1016/j.jempfin.2011.01.003.

[26] Griffin J, Ji X, Martin J., Momentum investing and business cycle risk: evidence from pole to pole, *The Journal of Finance*, 2003;58(6):2515-46.

[27] Fadaei Nejad ME, Sadeghi M., Examining the profitability of momentum and reversal strategies, *Payam Management*, 2006;17(18):7-31, (in Persian).

[28] Eslami Bidgoli G, Nabavi Chashmi SA, Yahyazadehfar M, Iekani S., Examining the profitability of momentum investment strategies in the Tehran Stock Exchange, *Quarterly Journal of Quantitative Studies in Management*, 2010;1:49-78, (in Persian).

[29] Badri A, Fathollahi F., Return momentum: evidence from the Tehran Stock Exchange, *Journal of Investment Knowledge*, 2014;3(9):1-20, (in Persian).

[30] Davallou,M., Javadian, B., Highest 52-week price momentum timing: evidence from the Tehran Stock Exchange, *Financial Knowledge of Securities Analysis*, 2017;35:63-77 ,(in Persian).

[31] Mohamadi,M., Zanjirdar,M., On the Relationship Between Different Types of Institutional Owners and Accounting Conservatism with Cost Stickiness, *Journal of Management Accounting and Auditing Knowledge*, 2018;7(28): 201-214

[32] Zanjirdar, M., Khojasteh,S. ,The impact of investors' herding behavior on the stock returns using Huang and Solomon model, *Quarterly Journal of Fiscal and Economic Policies*, 2017;4(15): 115-134

[33] Zanjirdar, M., Moslehi Araghi, M., The Impact of Changes in Uncertainty, Unexpected Earning of Each Share and Positive or Negative Forecast of Profit Per Share in Different Economic Condition, Quarterly Journal of Fiscal and Economic Policies,2016;4(13): 55-76.