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Spatial soliton pairs in an unbiased photovoltaic-photorefractive crystal circuit

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Abstract: Optical separate spatial dark and bright soliton pairs in steady-state case in one dimension, for a series circuit consisting of two-photon photorefractive (PR) crystal are investigated. Each crystal can be supported the spatial soliton, and at least one must be photovoltaic. The two solitons are known collectively as separate spatial soliton pairs with dark–dark, bright–dark and bright–bright. Results show that when an optical wave has a spatial extent much less than the width of the crystal, only the dark soliton can effect on the other soliton by light-induced current, while the bright soliton doesn't have such an effect. In fact when a crystal supports a bright soliton, the light-induced current is so small that the crystal cannot act as a current source, whereas when a crystal supports a dark soliton, the light-induced current is strong enough to affect the other soliton in the other crystal. Numerical results confirm that the two solitary states remain invariant under propagation. We also show that these solitons are stable under a small perturbation.

Keywords: nonlinear optics, separate photorefractive soliton, two-photon solitons.

1. Introduction

During the last decade, the optical spatial solitons based on photorefractive effect (PR) have attracted much special interest. These PRspatial solitons can be formed at low light intensity and are potentially useful for all optical switching, beam steering, and optical interconnects. So far, several different types of photorefractive soliton, such as, quasi-steady-state solitons [1, 2] and in steady-state screening solitons [3-8], photovoltaic (PV) solitons [8-10] and screening–photovoltaic (SP) solitons [8, 11, 12] have been predicted and observed. The investigations on PR, soliton pairs and soliton interaction were concerned with only one piece of PR crystal. Recently, a new type of soliton pair, named

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separate spatial soliton pair, which is formed in two PR crystals connected by electrode leads in a chain was also predicted [3, 13, 14]. All of these results are based on the single-photon photorefractive effect. Recently, Castro-Camus et al. [15] provided a model of the two-photon photorefractive effect. The Castro-Camus model includes a valence band (VB), a conduction band (CB), and an intermediate allowed level (IL). The intermediate allowed level is used to maintain a quantity of excited electrons from the valence band by photons with energy $\hbar\omega_1$ as gating beam. These electrons are then excited again to the conduction band by another photon with energy $\hbar\omega_2$. The signal beam can induce a spatial dependent charge distribution that gives rise to nonlinear changes of refractive index in the medium. Based on this model, the two-photon PR crystals can be generated, incoherently coupled soliton pairs [16-18] that result from the two- photon PR effect. Consequently, it would be of interesting to explore whether separate spatial soliton pairs can be realized in two- photon PR crystals as well. In this paper, bright-dark steady-state spatial solitons are predicted in one dimension for a series circuit consisting of two two-photon PR crystals with at least one PV crystal, and each crystal can support a spatial soliton separately. The two solitons are known collectively as separate spatial soliton pairs with dark–dark, bright–dark and bright–bright. Moreover, the stability of these solitons against perturbation also has been investigated simply.

2. Theoretical model

Consider two crystals P and $\hat{P}$ one of which should be PV. These two crystals are connected electronically in a chain by electrode leads, as shown in Fig. 1. The two optical beams I_2 and $\hat{I}_2$ are respectively propagating in a PR crystal P and $\hat{P}$ with two-photon PR effect along the *z*-axis and are permitted to diffract only along the *x* direction (parallel to c-axis). The distance of electrode and the area for PR crystals $P(\hat{P})$ are $W(\hat{W})$ and $S(\hat{S})$ respectively. Crystals are illuminated by the gating beam. We express the optical field of the incident beams for crystal P in terms of slowly varying envelope ϕ, i.e. $E = \hat{x}\exp(ikz)\phi(x,z)$ where $k = k_0 n_e = (2\pi/\lambda_0)n_e$, n_e is the unperturbed extraordinary index of refraction and λ_0 is the free-space wavelength. Under these conditions, the soliton wave beams satisfy the following envelope evolution equation [12, 13]:

$$i\phi_z + \frac{1}{2}\phi_{xx} - \frac{k_0}{2}\left(n_e^3 r_{33} E_{sc}\right)\phi = 0 \qquad \mathbf{(1)}$$

Where $\phi_z = \partial\phi/\partial z$, $\phi_{xx} = \partial^2\phi/\partial x^2$, r_{33} is electro-optic coefficient, E_{sc} is space charge field in the crystal. The relatively parameters of crystal $\hat{P}$ is denote by adding ^ above the variable. The space charge field in Eq. (1) can be obtained from the set of rate, current, and Poisson's equations proposed by Castro-Camus and Magana [15] and by taking a method similar to that described in Refs [12, 13], Space charge field for case ↑↓ will be obtained by Eq. (2) for crystal P and by adding ^ above the variables in E_{sc} for crystal $\hat{P}$.

$$E_{sc} = E_0 \frac{(I_{2\infty} + I_{2d})(I_2 + I_{2d} + \gamma_1 N_A / s_2)}{(I_2 + I_{2d})(I_{2\infty} + I_{2d} + \gamma_1 N_A / s_2)} + E_P \frac{s_2 (I_{2\infty} - I_2)(I_2 + I_{2d} + \gamma_1 N_A / s_2)}{(s_1 I_1 + \beta_1)(I_2 + I_{2d})} - \frac{D\gamma_1 N_A}{\mu s_2 (I_2 + I_{2d} + \gamma_1 N_A / s_2)(I_2 + I_{2d})} \frac{\partial I_2}{\partial x} \quad \textbf{(2)}$$

where I_2 and $\hat{I}_2$ are intensity of the soliton beam, $I_{2d} = \beta_2 / s_2$ and $\hat{I}_{2d} = \hat{\beta}_2 / \hat{s}_2$ are intensity of dark irradiation of crystal P and $\hat{P}$, respectively. The value of the field depends on the parameters of both crystals, including $I_{2\infty}$ and $\hat{I}_{2\infty}$ .On the other hand, E_{sc} and $\hat{E}_{sc}$ are not independent. They couple each other by the parameters g, $\hat{g}$, Γ and $\hat{\Gamma}$. These parameters are defined in [14].

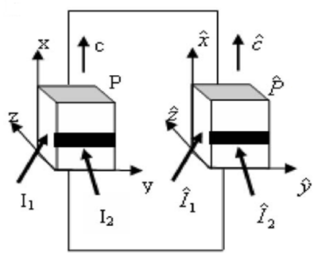


Fig.1. Illustration of the series two-photon PR crystal circuit consisting of two PR crystals. One crystal's c-axis is oriented in a right-handed screw sense but the other crystal c-axis is opposite handed screw sense (↑↓).

Now let us derive the evolution equation of the envelope of the optical beam. Considering first the crystal P, we can envelope evolution equation established by insertion of Eq. (2) into Eq. (1). It proves more convenient to study this equation in a normalized fashion. To do so, let us adopt the following dimensionless coordinates and variables i.e. $s = x/x_0$, $U = (2\eta_0 I_{2d}/n_e)^{-1/2} \phi$ and $\xi = z/(kx_0^2)$ .Where x_0 is an arbitrary spatial width, we can then show that the normalized envelope U obeys the following dynamical evolution equation:

$$iU_\xi+\frac{1}{2}U_{ss}-\frac{\beta(1+\rho)}{(1+\rho+\sigma)}\left(1+\frac{\sigma}{1+|U|^2}\right)U-\alpha\eta\frac{\left(\rho-|U|^2\right)\left(1+|U|^2+\sigma\right)}{1+|U|^2}U+\delta\frac{\sigma|U|_s^2}{\left(|U|^2+1\right)\left(|U|^2+1+\sigma\right)}U=0 \quad \textbf{(3)}$$

where $\alpha=(k_0x_0)^2\left(n_e^4r_{33}/2\right)E_P$, $\sigma=\gamma_1N_A/\beta_2$, $\beta=(k_0x_0)^2\left(n_e^4r_{33}/2\right)E_0$, $\eta=\beta_2/(s_1I_1+\beta_1)$, $\delta=(k_0x_0)^2\left(n_e^4r_{33}/2\right)D/(x_0\mu)$, and $\rho=I_{2\infty}/I_{2d}$. Similarly, we can obtain results for $\hat{P}$ adding ^ above the variables *U*.

We begin our analysis by considering dark-dark soliton pair i.e. both crystals support dark soliton, by using steps similar to those in Refs.[12,13]. Dark soliton solution in crystal *P* can be derived from Eq.(3) by expressing beam envelope *U* in the usual fashion $U=\rho^{1/2}y(s)\exp(iu\xi)$ where $y(s)$ is a normalized odd function of s that satisfies these boundary conditions: $y(0)=0$, $y(s\to\pm\infty)=1$ $y'(s\to\pm\infty)=y''(s\to\pm\infty)=0$, and all the derivatives of $y(s)$ vanish at infinity . Substitution of this form of *U* into Eq. (3) and ignoring the diffusion $(\delta=0)$, leads to the following equation:

$$\frac{d^2y}{d^2s}=2\left[u+\frac{\beta(1+\rho)}{(1+\rho+\sigma)}\right]y+\frac{2\beta\sigma(1+\rho)}{(1+\rho+\sigma)\left(1+\rho y^2\right)}-2\alpha\eta\sigma y-2\alpha\eta\rho y^3+\frac{2\alpha\eta\sigma(1+\rho)y}{1+\rho y^2} \quad \textbf{(4)}$$

Using the boundary conditions at infinity, we can deduce from Eq.(4):

$$\left(\frac{dy}{ds}\right)^2=\left[-\left(\frac{2\beta\sigma}{(1+\rho+\sigma)}+2\alpha\eta\sigma\right)\right]\left[\left(y^2-1\right)-\frac{(1+\rho)}{\rho}\ln\left(\frac{1+\rho y^2}{1+\rho}\right)\right]-\alpha\eta\rho\left(y^2-1\right)^2 \quad \textbf{(5)}$$

By integrating Eq.(5) once, we get:

$$s=\pm\int_y^0\left\{\left[-\left(\frac{2\beta\sigma}{1+\rho+\sigma}+2\alpha\eta\sigma\right)\right]\left[\left(\tilde{y}^2-1\right)-\frac{1+\rho}{\rho}\ln\left(\frac{1+\rho\tilde{y}^2}{1+\rho}\right)\right]-\alpha\eta\rho\left(\tilde{y}^2-1\right)^2\right\}^{-1/2}d\tilde{y} \quad \textbf{(6)}$$

From which the normalized envelope $y(s)$ of the dark soliton can be obtained by the use of numerical integration procedures.

Similarly, by expressing the beam envelope $\hat{U}=\hat{\rho}^{1/2}\hat{y}(\hat{s})\exp(i\hat{u}\hat{\xi})$ we can obtain the result for crystal $\hat{P}$ as follows:

$$\hat{s}=\pm\int_{\hat{y}}^{0}\left\{\left[-\left(\frac{2\hat{\beta}\hat{\sigma}}{1+\hat{\rho}+\hat{\sigma}}+2\hat{\alpha}\hat{\eta}\hat{\sigma}\right)\right]\left[\left(\tilde{y}^2-1\right)-\frac{1+\hat{\rho}}{\hat{\rho}}\ln\left(\frac{1+\hat{\rho}\tilde{y}^2}{1+\hat{\rho}}\right)\right]\right.$$
$$\left.-\hat{\alpha}\hat{\eta}\hat{\rho}\left(\tilde{y}^2-1\right)^2\right\}^{-1/2}d\tilde{y} \tag{7}$$

Equations (6) and (7) describe that, at certain parameters of α, $\hat{\alpha}$, β and $\hat{\beta}$, steady-state one-dimensional profiles of dark–dark separate soliton pairs should exist in the series PR crystal circuit. The two dark solitons are coupled with each other electronically. Because g, $\hat{g}$, Γ and $\hat{\Gamma}$ depend on the parameters of both crystals, the soliton profile in one crystal not only depends on the parameters of that crystal, but also depends on the parameters of the other crystal. In other words, the character of any soliton in the separate soliton pairs depends on the parameters of two crystals. When the input optical light intensity of one crystal changes, soliton profiles in two crystals will change. The two dark solitons in the dark–dark soliton pairs can affect each other through light-induced current flowing from one crystal to the other.

For the bright-dark soliton pairs we assume the bright soliton formed in crystal P and dark soliton formed in crystal $\hat{P}$. Bright solitary solution can be derived from Eq.(3) by expressing the beam envelope U in the usual fashion $U=r^{1/2}y(s)\exp(i\nu\xi)$ in which ν represents a nonlinear shift of propagation constant , $y(s)$ is a normalized real function between $0\le y(s)\le 1$. By integrating the Eq.(3) under boundary conditions: $y(0)=1$, $y(s\to\pm\infty)=0$ and $\dot{y}(0)=0$ we found that:

$$\left(\frac{dy}{ds}\right)^2=[\ln(1+ry^2)-y^2\ln(1+r)][\frac{2\beta\sigma}{r\left(1+\sigma\right)}+$$
$$2\alpha\eta\frac{\sigma}{r}]+\alpha r\eta y^2(1-y^2) \tag{8}$$

By integrating Eq.(8) we obtain:

$$s=\pm\int_{y}^{1}\left\{\left[\frac{2\beta\sigma}{r\left(1+\sigma\right)}+2\alpha\eta\frac{\sigma}{r}\right]\left[\ln(1+r\tilde{y}^2)-\tilde{y}^2\ln(1+r)\right]\right.$$
$$\left.+\alpha r\eta\tilde{y}^2(1-\tilde{y}^2)\right\}^{-1/2}d\tilde{y} \tag{9}$$

The dark soliton profiles in the bright–dark soliton pair can be obtained by the use of a way similar to above and determined by Eq.(7).

Eqs.(9) and (7) describe that, at certain parameters of α , $\hat{\alpha}$, β and $\hat{\beta}$, steady-state one-dimensional profiles of bright–dark separate soliton pairs which should exist in the series PR crystal circuit. We have $I_{2\infty} = 0$ and $\hat{I}_{2\infty} \neq 0$ and g , $\hat{g}$, Γ and $\hat{\Gamma}$ are independent on $\hat{I}_{2\infty}$. As a result, dark soliton can affect the bright one by the light-induced current, but the reverse is not true, i.e., changing the input intensity of the dark soliton can affect the bright soliton whereas changing the input intensity of the bright soliton cannot effect on the dark soliton. This behavior is proved by numerical methods.

For considering the bright–bright soliton pairs we can use steps similar to above and obtain the same with Eq.(9) for crystal P while the results for $\hat{P}$ can be obtained by adding ^ above the variables Eq.(9). These equations describe that, by certain parameters steady-state one-dimensional profiles of bright–bright separate, soliton pairs should exist in the series PR crystal circuit. $I_{2\infty} = 0$ and $\hat{I}_{2\infty} = 0$ and value of g , $\hat{g}$, Γ and $\hat{\Gamma}$ is zero. In the bright–bright soliton pairs, the solitons cannot affect each other by the light-induced current.

3. Numerical Simulations

A relevant example is provided for the bright–dark soliton pairs formed for Fig. 1(b) ↑↓. Let us consider two $LiNbO_3$ crystals used as P and $\hat{P}$ with the parameters that mentioned in Ref.[12,13]. We obtain curves according to Table 1 and 2. The normalized intensity profiles of the bright soliton in crystal P and dark soliton in crystal $\hat{P}$ can be obtained by solving Equations (9) and (7), as shown in Fig. 2(a) curve1 and Fig. 2(b) curve1 respectively. The input intensity of crystal $\hat{P}$ increases but the other parameters remain unchanged. According to tables not only does the dark soliton change as shown in Fig. 2(b) curve2 but also the bright soliton in crystal P changes as shown in Fig. 2(a) curve2.

Table 1: Data for bright soliton

curve No.	r	$\hat{\rho}$	α	β	$\hat{\Gamma} = g$	$\hat{g} = \Gamma$
1	1	1	-2.22	7.3982	0.3333	0
2	1	100	-2.22	21.7580	0.9803	0
3	100	1	-2.22	7.3982	0.3333	0

Table 2: Data for the dark soliton

curve No.	r	$\hat{\rho}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\Gamma} = g$	$\hat{g} = \Gamma$
1	1	1	-22.1953	7.3982	0.3333	0
2	1	100	-22.1953	21.7580	0.9803	0
3	100	1	-22.1953	7.3982	0.3333	0

It should be noted that when the input intensity of the crystal *P* increases while the other parameters remain unchanged, only the bright soliton in the crystal *P* changes as shown in Fig. 2(a) curve3, but dark soliton in other crystal does not change as shown in Fig. 2(b) curve3. Above results imply that, for the separate bright–dark spatial soliton pairs, the dark soliton can affect the profile of the bright soliton by the light-induced current, whereas the bright soliton cannot affect profile of the dark soliton. Moreover, in order to investigate the stability of the bright– dark soliton pairs, they have been solved numerically under neglecting the effects of diffusion process. The normalized envelopes given by Fig. 2 curve 1 have been used as the input beam profiles. As expected, our results confirm that the two solitary states remain invariant with propagation distance as shown in Fig. 3.

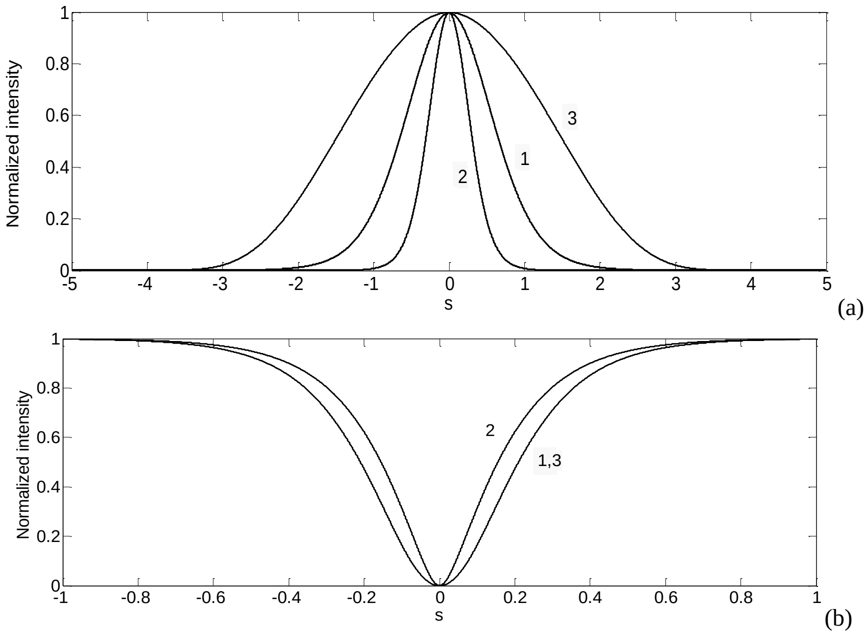
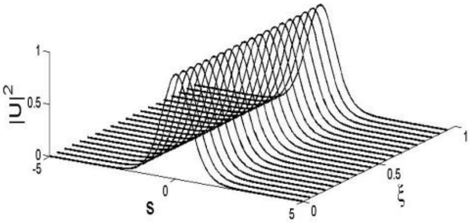
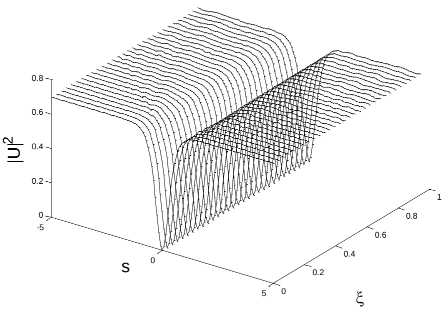


Fig. 2. Bright–dark two-photon separate soliton pairs in an unbiased series $LiNbO_3$ crystal circuit, (a) Bright soliton profiles in the crystal *P*, (b) Dark soliton profiles in the crystal $\hat{P}$.



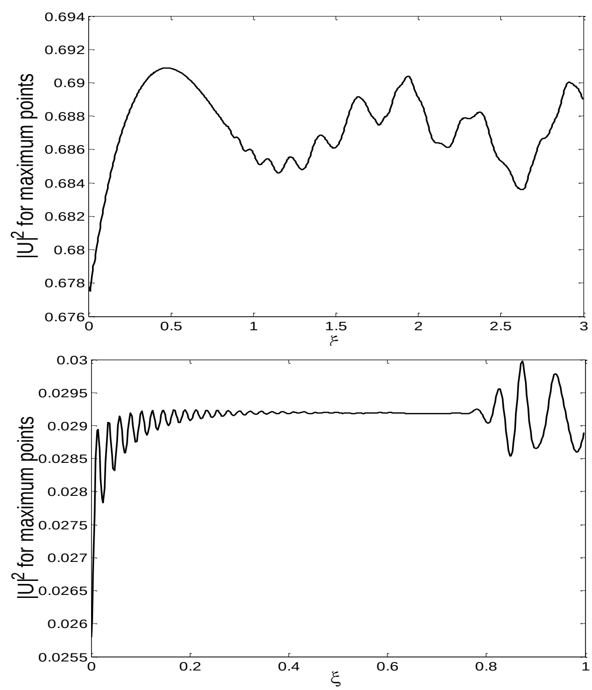
(a)



(b)

Fig. 3. Stable propagation of the, (a) bright and, (b) dark soliton of bright–dark soliton pairs.

Figure 4 depicts maximum soliton peak versus propagation distance, when the bright and dark soliton in pair when its intensity beam has been perturbed by 30% in its amplitude. Evidently, the pairs exhibits robustness, don't break up, and instead oscillate around the soliton solution although the strength of dark soliton against the perturbation is more than bright soliton.



(a)

(b)

Fig.4. Maximum soliton peak versus propagation distance. For r=1 and $\beta = 21.7580$, bright soliton, (a) and for ρ=1 and $\hat{\beta} = 21.7580$, dark soliton,(b) when its amplitude is perturbed by 30% at the input.

4. Conclusion

We investigate the optical separate spatial soliton pairs in a series two-photon photovoltaic-photorefractive crystal circuit in one dimension. Results show that when an optical wave has a spatial extent much less than the width of the crystal, only the dark soliton can affect the other soliton, while the bright soliton can't affect. In fact when a crystal supports a bright soliton, the light-induced current is so small that the crystal cannot act as a current source, whereas when a crystal supports a dark soliton, the light-induced current is strong enough to affect the other one. Numerical results confirm that the two solitary states remain invariant with propagation. We also show that these solitons are stable under small perturbation.

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