Islamic Azad University

Journal of
Optoelectronical Nanostructures

Summer 2018 / Vol. 3, No. 3

Effect of Relative Phase on the Stability of Temporal Bright Solitons in a $\mathcal{PT}$-*Symmetric* NLDC

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(Received 15 Jun. 2018; Revised 10 Jul. 2018; Accepted 23 Aug. 2018; Published 15 Sep. 2018)

Abstract: In this paper we numerically investigate the effect of relative phase on the stability of temporal bright solitons in a Nano $\mathcal{PT}$- *Symmetric* nonlinear directional coupler (NLDC) by considering gain in bar and loss in cross. We also study the effect of relative phase on the output perturbed bright solitons energies, in the range of $\theta = 0^{\circ}$ to $\theta = 180^{\circ}$. By using perturbation theory three eigenfunctions and corresponding eigenvalues were derived analytically. These eigenvalues behave like equilibrium points and are not stable in all cases. Stability of these perturbed solitons under the effect of relative phase are examined and show that temporal bright solitons are almost unstable in the range of $\theta = 0^{\circ}$ to $\theta = 90^{\circ}$, but they keep their solitary shapes in the range of $\theta = 90^{\circ}$ to $\theta = 180^{\circ}$. In addition the evolution of normalized energies in these ranges are investigated.

Output pulse energy at bar and cross strongly depend on the relative phase. This effect in a $\mathcal{PT}$-*Symmetric* NLDC can be used for designing all-optical ultrafast self-switches and logic gates and Nano structures.

Keywords: Fiber Couplers, Nonlinear Optics, Photonics, Solitons, $\mathcal{PT}$-*Symmetry*, Nano structures.

1. INTRODUCTION

In 1998 Bender and Butcher found the unique remarkable phenomenon that even non-Hermitian Hamiltonians can still have completely real eigenvalue spectra if they respect Parity-Time (*PT*) symmetry. The concept has its roots in quantum mechanics where a $\mathcal{PT}$-*Symmetric* non-Hermitian Hamiltonian may have an entirely real spectrum of eigenvalues. In quantum mechanics, the parity reflection operator $\mathcal{P}$ and time-reversal $\mathcal{T}$ operator are defined by $x \to -x$ and $i \to -i$, respectively. Recently, it was recognized that a more particular species

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of $\mathcal{PT}$ (parity-time)-symmetric systems may be identified which remain invariant under the combination of parity and time-reversal symmetry operation. The necessary condition for a Hamiltonian to be $\mathcal{PT}$- *Symmetric* in nonlinear optics is that the $\mathcal{PT}$- *Symmetric* potential satisfies the condition V(x) =V*(-x). Recently, developing all-optical devices in modern optics and photonics that increase the speed of transmitting and processing information is so interested. In this field, $\mathcal{PT}$-*Symmetric* optical systems with considering gain and loss can bring novel functionalities. The presence of losses or gain in optical structures leads to non-Hermitian operators [1-4]. However, if such operators possess $\mathcal{PT}$-*Symmetry* condition, they have important role in real physical problems.

Theoretical proposals for optical $\mathcal{PT}$-*Symmetric* systems were formulated a decade ago [5-8]. Also, the first experimental observation of $\mathcal{PT}$-*Symmetric* effects in optics were seen in the wave guiding structures [9, 10].

The simplest configuration of the $\mathcal{PT}$-*Symmetric* optical system is a pair of coupled waveguides with gain and loss. In such systems, the optical refractive index should satisfy the following condition:

$$n(x,y) = n^*(-x,y)$$

This means that the absolute value of the gain and loss should be the same, and gain/loss regions should have mirror configurations with respect to the central symmetry point. Furthermore, two decades ago, the effect of nonlinearity on the beam dynamics in directional couplers composed of gain and loss was described theoretically [5, 11-14]. Generally, $\mathcal{PT}$-*Symmetric* optical systems demonstrate nontrivial, non-conservative wave interactions and phase transitions. These systems have important applications in signal filtering, all-optical switching, ultrafast communication systems, and logic gates [15-21].

In our previous paper we numerically investigate the stability of temporal bright solitons propagate in a $\mathcal{PT}$-*Symmetric* NLDC regardless the effect of relative phase. By using the analytical solutions of perturbed eigenfunctions and corresponding eigenvalues the stability of temporal bright solitons is studied numerically. Three perturbed eigenfunctions corresponding to the two eigenvalues are examined for stability. The results show that the two degenerate eigenfunctions are unstable while other one is stable which have important result that the eigenfunctions are equilibrium function but not stable for all cases [11].

In this paper we numerically studied the effect of relative phase on the stability of perturbed temporal bright solitons and output perturbed soliton's normalized energies.

2. THEORY

Nonlinear coupled equation for a *𝒫𝒯-Symmetric* NLDC with gain in bar and loss in cross is considered as:

$$iu_z + u_{\tau\tau} + 2|u|^2 u = -v + i\gamma u,$$
$$iv_z + v_{\tau\tau} + 2|v|^2 v = -u - i\gamma v. \tag{1}$$

Where, u and v are the normalized slowly varing amplitude variables at bar and cross, z and τ indicate the length of fiber and normalized time, respectively. The coefficients of $u_{\tau\tau}$ and $v_{\tau\tau}$ are normalize to unity, hence, Kerr nonlinearity coefficients are the same in the bar and cross.

The first term in the right hand side of Eq.(1), is related to the coupling between the modes of two fiber waveguides and γ s stand for the gain in one fiber (bar) and loss in the other (cross).

According to the analytical investigation of Melmod et.al [22], coupled NLSE equations for a *𝒫𝒯- Symmetric* NLDC with gain and loss have a soliton solutions in the following form:

$$u = sech(a\tau)e^{i(\Omega z)},$$
$$v = sech(a\tau)e^{i(\Omega z - \theta)} \tag{2}$$

Where, the relative phase $\theta = arg(u) - arg(v)$ is determined by relation:

$$tan\theta = \left[2H(\gamma_1 - \gamma_2) - 1\right]\frac{\sqrt{\gamma_1\gamma_2}}{\sqrt{1-\gamma_1\gamma_2}} \tag{3}$$

$H(x)$, is unit step function.

To confirm the *𝒫𝒯-Symmetry* condition the absolute value of gain should be the same as the absolute value of loss. So we consider $\gamma_1 = \gamma_2 = \gamma$ and since $\gamma < 1$ is a necessary condition for existing soliton, we can take $\gamma = sin\theta$ by considering $0^\circ < \theta < 180^\circ$.

Stability of solitons in a *𝒫𝒯-Symmetric* NLDC was determined by perturbation method. The perturbed eigenfunctions and their corresponding eigenvalues were obtained as [19]:

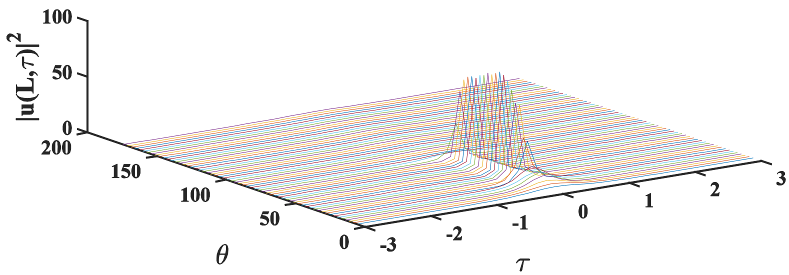
$$y_1 = sech(a\tau), \qquad \lambda_1 = 0$$
$$y_2 = sech^2(a\tau), \qquad \lambda_2 = -3 \tag{4}$$
$$y_3 = sech(a\tau)tanh(a\tau), \quad \lambda_3 = 0$$

The obtained eigenvalues are equilibrium points which may be stable or unstable. Usually the stability of these points are investigated numerically by adding the perturbed eigenfunctions to the initial input solitons. The evolution and stability of perturbed bright solitons and their energies in a $\mathcal{PT}$- *Symmetric* NLDC regardless to the relative phase were determined in our previous paper [11]. Now, according to the Eq. (3), we numerically investigate the effect of relative phase on the stability of these perturbed bright solitons and their energies.

3. NUMERICAL RESULTS

By applying three equilibrium perturbed eigenfunctions and corresponding eigenvalues, stability of temporal bright solitons are investigated numerically in a $\mathcal{PT}$- *Symmetric* NLDC with the existence of gain and loss.

The evolution of output perturbed temporal bright solitons in the range of $0^\circ < \theta < 180^\circ$ are illustrated in Figs. (1) - (3).



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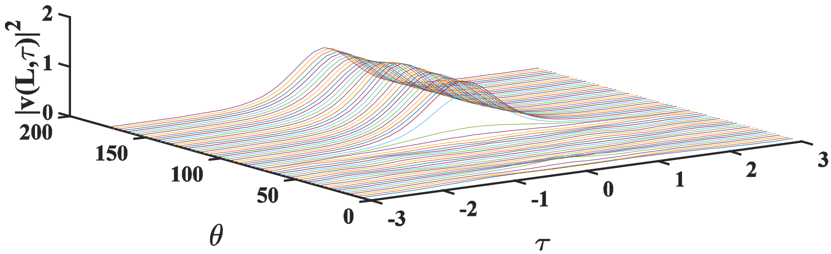

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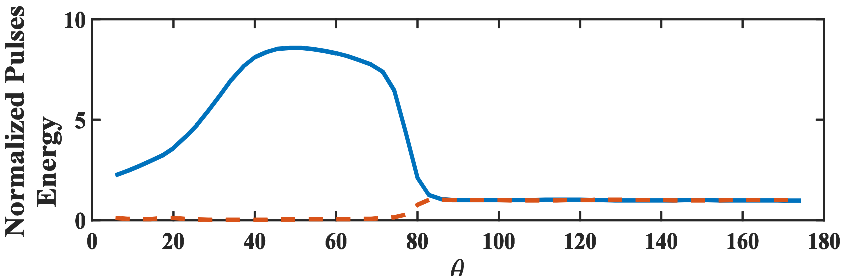
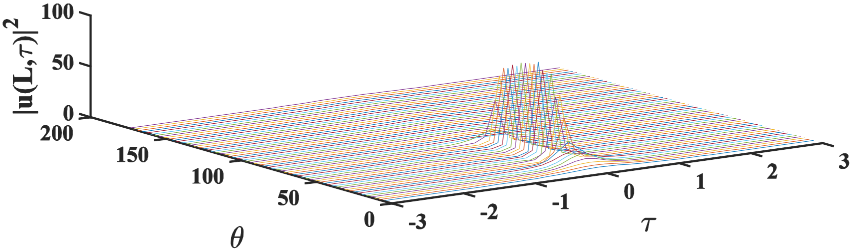


(c)

Fig. 1: (a) and (b): The effect of relative phase on output perturbed bright soliton of a $\mathcal{PT}$- *Symmetric* NLDC by applying $y_1 = sech(a\tau)$, $\lambda_1 = 0$ in bar and cross, respectively. (c) Evolution of perturbed solitons normalized energy (line for bar and dash-line for cross).

In Fig. (1), the firs perturbed eigenfunction $y_1 = sech(a\tau)$ and its corresponding eigenvalue $\lambda_1 = 0$ is applied. Figs. (1a) and (1b) show the output perturbed soliton of bar and cross for a $\mathcal{PT}$- *Symmetric* NLDC and in Fig. (1c) the evolution of their normalized energies due to the change of relative phase in the range of $0^\circ < \theta < 180^\circ$ is plotted, respectively. As we can see in Fig. (1a) and Fig. (1b), at first, output perturbed solitons in bar are highly amplified while in cross attenuation is happened and solitons are completely destroyed. After that these output pulses are stable and keep their solitray shapes. The evolution of normalized energy of output perturbed bright solitons is depicted in Fig. (1c). It shows that in (a) the range of $\theta = 0^\circ$ up to $\theta = 85^\circ$ the pulse is amplified in the bar and attenuated in cross, but up to $\theta = 180^\circ$ they are stable and do not have any changes.



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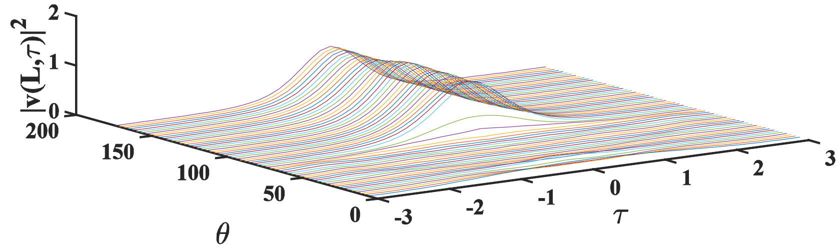

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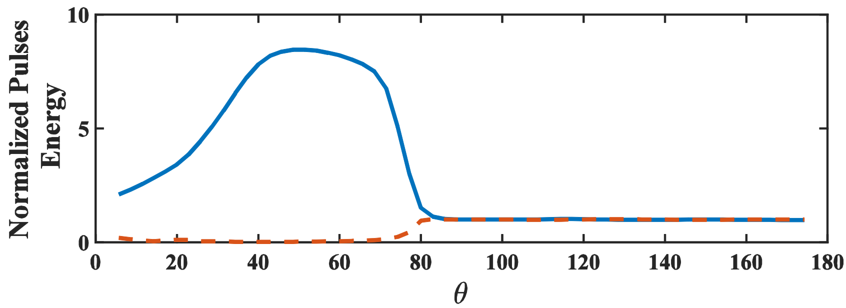


(c)

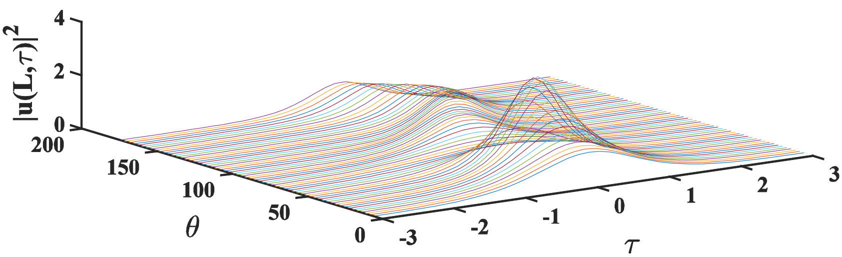
Fig. 2: (a) and (b): The effect of relative phase on output perturbed bright soliton of a $\mathcal{PT}$-*Symmetric* NLDC by applying $y_2 = sech^2(a\tau)$, $\lambda_2 = -3$ in bar and cross, respectively. (c) Evolution of perturbed solitons normalized energy (line for bar and dash-line for cross).

The effect of relative phase on the second perturbed eigenfunction, $y_2 = sech^2(a\tau)$ and the corresponding eigenvalue, $\lambda_2 = -3$, is presented in Fig. (2). Figs. (2a) and (2b) show the effect of relative phase on the output perturbed temporal bright solitons of a $\mathcal{PT}$- *Symmetric* NLDC. In Fig. (2c) the effect of relative phase on their normalized energies is presented. These figures are similar to the Fig. (1), such that the range of stability is the same, meanwhile the initial perturbed solitons have different eigenfunctions and eigenvalues.

Figs. show that the output perturbed bright solitons are amplified in the range of $0^\circ < \theta < 85^\circ$ for bar and attenuated in the cross, while in the range of $85^\circ < \theta < 180^\circ$ they keep their shapes and they are stable.

In Figs. (3), the third perturbed eigenfunction, $y_3 = sech(a\tau)tanh(a\tau)$ and its corresponding eigenvalue, $\lambda_3 = 0$ is added to the initial bright soliton and simulate the propagations for different relative phase. There exist three regions of relative phase with different behaviors in Fig. (3).

In region 1, $\theta = 0^\circ$ to 60° amplification and attenuation are observed in the bar and cross, respectively, but the rate of growth and drop are less than that in Figs. (1) and (2). In addition in environs of $\theta = 60^\circ$ to $\theta = 90^\circ$, opposite behavior is happened, in the bar attenuation is seen and in the cross amplification. After that, up to $\theta = 180^\circ$, despite some fluctuations output perturbed solitons are stable and keep their solitary shapes. Due to Fig. (3c), as we can see the evaluation of normalized energies are in agree with the results in Fig. (3a) and. (3b) and confirm them.



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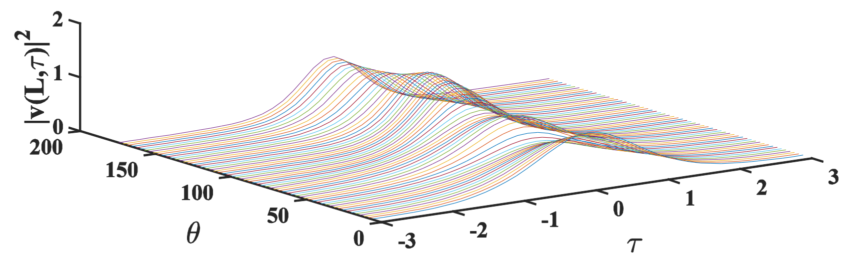

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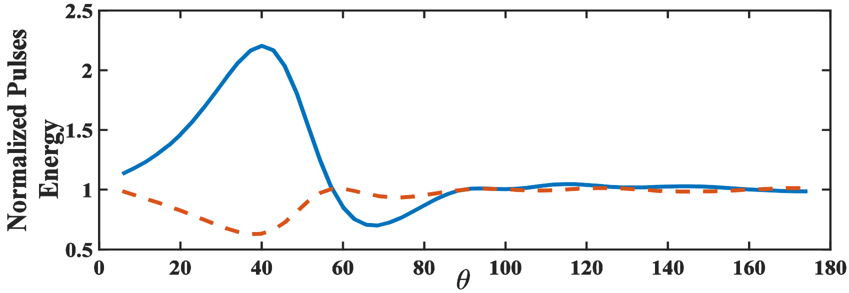

(c)

Fig. 3: (a) and (b): The effect of relative phase on output perturbed bright soliton of a $\mathcal{PT}$- *Symmetric* NLDC by applying $y_3 = sech(a\tau)tanh(a\tau)$, $\lambda_3 = 0$ in bar and cross, respectively. (c) Evolution of perturbed solitons normalized energy (line for bar and dash-line for cross).

4. CONCLUSION

In this paper, we examine the effect of relative phase on the stability of temporal bright solitons in a $\mathcal{PT}$-*Symmetric* NLDC by considering gain in bar and loss in cross. In addition, the evolution of output perturbed temporal bright solitons energies are investigated. Stability analysis shows that there exist three perturbed eigenfunctions and corresponding eigenvalues which are equilibrium functions and points. Numerical studies can prove the stability or instability of equilibrium points. We studied the stability of soliton respect to relative phase in the range of $0^\circ < \theta < 180^\circ$. Numerical results show that the effect of relative phase on the first and second perturbed bright solitons are the same. In the range of $\theta = 0^\circ$ to $\theta = 90^\circ$, all these output perturbed solitons are mostly unstable, amplification in bar and attenuation in cross is happened. While, for the last output perturbed bright solitons in the environs of $\theta = 60^\circ$ up to $\theta = 90^\circ$, attenuation in bar and amplification in cross is happened. After that, in the range of $\theta = 90^\circ$ up to $\theta = 180^\circ$ all of the perturbed solitons are stable and keep their solitary shapes. The numerical studies on evolution of normalized energies of these output perturbed temporal bright solitons are more precise and confirm these results.

As follows from the results summarized above, the study of the effect of relative phase can be used in designing all-optical switches, ultrafast optical communication systems and logic gates.

REFERENCES

[1] C. M., Bender and S. Boettcher, *Real spectra in non- Hermitian Hamiltonians having PT- symmetry*, Phys. Rev. Lett. 80 (1998) 5243.

[2] C. M., Bender; D. C., Brody, and H. F., Jones, *Complex extension of quantum mechanics*, Phys. Rev. Lett. 89 (2002) 270401–240404.

[3] C. M., Bender, *Making sense of non-Hermitian hamiltonians*, Rep. Prog. Phys. 70 (2007) 947–1018.

[4] Y. Kivshar, G. P. Agrawal, *Optical Solitons from fibers to photonic crystals*, Academic Press, (2003).

[5] E. Rueter, K. G. Makris, R. El-Ganainy, D. N. Christodoulides, M. Segev and D. Kip, *Observation of parity-time symmetry in optics*, Nat. Phys. 6 (2010) 192.

[6] R. El., Ganainy, K. G., Makris, D. N., Christodoulides, and Z. H., Musslimani, *Theory of coupled optical PT-symmetric structures*, Opt. Lett. 32 (2007) 2632–2634.

[7] A., Ruschhaupt, F., Delgado and J. G., Muga, *Physical realization of PT-symmetric potential scattering in a planar slab waveguide*, J. Phys. A., 38 (2005) L171–L176.

[8] A. A., Zyablovsky, A. P., Vinogradov, A. A., Pukhov, A. V., Dorofeenko and A. A., Lisyansky, "PT-symmetry in optics", Phys. Usp. 57 (2014) 1063–1082.

[9] L. Feng, Z. J. Wong, R. M. Ma, Y. Wang and X. Zhang, *Single-mode laser by parity-time symmetry breaking*, Science, 346 (2014) 972.

[10] A. A., Sukhorukov, Z. Y., Xu, and Y. S., Kivshar, *Nonlinear suppression of time reversals in PT-symmetric optical couplers*, Phys. Rev. A. 82 (2010) 043815–043818.

[11] L. Safaei, M. Hatami and M. Borhani Zarndi, *Numerical analysis of stability for temporal Bright solitons in a PT-symmetric NLDC*, J. Optoelectronical Nanostructures, 2. 2 (2017) 69-78.

[12] L. Safaei, M. Hatami and M. Borhani Zarndi, *Stability of Temporal Dark Soliton in PT-Symmetric Nonlinear Fiber Couplers in Normal Dispersion Regime*, J Laser Opt Photonics, 3 (2016) 141.

[13] I. V., Barashenkov and M., Gianfreda, *An exactly solvable PT-symmetric dimer from a hamiltonian system of nonlinear oscillators with gain and loss*, J. Phys. A. 47 (2014) 282001–282018.

[14] L. Safaei, M. Hatami and M. Borhani Zarndi, *PT-Symmetric Nonlinear Directional Fiber Couplers with Gain and Loss for Ultrashort Optical Pulses*, J. Laser Opt. Photonics, 4 (2017) 155.

[15] X. Chen and J. Yang, *A direct perturbation theory for solitons of the derivative nonlinear Schrodinger equation and the modified nonlinear Schrodinger equation*, Phys. Rev. E. 65 (2002) 066608.

[16] X., Lü and W. X., Ma, *Study of lump dynamics based on a dimensionally reduced Hirota bilinear equation*, Nonlinear Dyn. 85 (2016) 1217.

[17] A., Lupu, H., Benisty and A. Degiron, *Using optical PT-symmetry for switching applications, Photonics Nanostruct*, Fundam. Appl. 12 (2014) 305–311.

[18] S. Suchkov, A. Sukhorukov, J. Huang, S. Dmitriev, C. Lee and Y. Kivshar, *Nonlinear switching and solitons in PT-symmetric photonic systems*, Laserand Photonics Rev. 10 (2015).

[19] N. V., Alexeeva, I. V., Barashenkov, A. A., Sukhorukov and Y. S., Kivshar, *Optical solitons in PT-symmetric nonlinear couplers with gain and loss*, Phys. Rev. A. 85 (2012) 063837–13.

[20] C. Q., Dai, and J. F. Zhang., *Controllable dynamical behaviors for spatiotemporal bright solitons on continues wave background*, Nonlinear Dyn., (2013).

[21] Y., Zhu, W. Qin, J. Li, and et al., *Recurrence behavior for controllable excitation of rogue waves in a two-dimensional pt-symmetric couple*. Nonlinear Dyn. 88 (2017) 1883.

[22] R., Driben and B. A., Malomed, *Stability of solitons in parity-time-symmetric couplers*, Opt. Lett. 36 (2011) 4323-4325.