Analytical Investigation of TM Surface Waves in 1D Photonic Crystals Capped by a Self-Focusing Left-Handed Slab

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Abstract: In this paper, the localized TM surface waves of a nonlinear self-focusing left-handed slab sandwiched between a uniform medium and a one-dimensional photonic crystal (1D PC) is analytically investigated. Our method is based on the first integral of the nonlinear Maxwell's equations. For the TM surface waves, the presence of two electric field components makes the analysis difficult. Therefore, we used the uniaxial approximation for the nonlinear slab. The considered 1D PC can be a left-handed PC (made of alternate left-handed (LH) and right-handed material) or a usual PC. In these structures, we find that the nonlinear LHM slab allows to control the dispersion properties of p-polarized surface waves. It is shown that there are no surface modes in the linear regime in the conventional PC and by increasing the intensity of electromagnetic field to higher value, the surface modes are emerged. To confirm our achievements, we also compared the results obtained by this method and the attenuated total reflection (ATR) method.

Key words: Photonic crystals, Nonlinear surface waves, Left-handed materials.

1. INTRODUCTION

In the past two decades, an extensive research has been directed to photonic crystals (PCs) [1-5], because they can modulate the propagation of photons and control the properties of electromagnetic light in the same way that semiconductor materials do in controlling the propagation of electrons. The development of PC materials have attracted a great deal of interest in the field of surface modes at the interfaces of such materials [6-9]. The most well-known feature of PCs is the photonic bandgap, inside which the electromagnetic waves are prohibited to propagation in all directions. However, when appropriately terminated, PCs can support surface modes with frequencies lying inside the photonic bandgap [6]. Surface electromagnetic waves (SWs) on 1DPCs were observed almost 40 years ago [10]. As it is well known, surface modes are a

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special type of waves localized at an interface between two different media, and they decay exponentially along the normal direction away from the surface into both media. Because of low losses and flexibility in designing photonic band gap (PBG) materials and their potential use in sensors, modulators, atom mirrors, study of the SWs in PCs have attracted a great deal of interest in the field of electronics and photonics [11-12]. Recently, we have studied nonlinear transverse electric (TE) polarized SWs in 1DPCs [13-16]. Contrary to the TE polarized SWs, which have only one component of the electric field, in the case of TM polarized SWs, the presence of both components of the electric field makes problem analysis difficult. Here, we study the properties of p-polarized nonlinear surface waves that can be excited at the interface between a uniform material and 1DPC capped by a nonlinear self-focusing LH slab. We investigate a possibility to control the dispersion properties of SWs by adjusting the intensity of electromagnetic field. We consider two types of 1DPCs, conventional photonic crystal (PC) and left-handed PC (made of alternate LHM and right-handed material) ones. The results of this paper revealed that, in the case of conventional PC, there are no p-polarizes SWs in the linear regime, while by adjusting the intensity of electromagnetic field at the surface of a self-focusing nonlinear LH slab, we obtain TM SWs. This paper is organized as follows. In Section 2, we introduce model of system under consideration. In Section 3, we study the effect of nonlinear self-focusing LH slab on surface states and explore a possibility to control the dispersion properties of surface waves. Finally, conclusions are given in Section 4.

2. THEORETICAL MODEL

We study analytically the TM-polarized SWs in a nonlinear self-focusing LH slab sandwiched between a uniform medium and a 1DPC (Fig. 1). In the chosen coordinate system the slab with thickness d_s extends from $z = -d_s$ to 0 and the uniform material that is characterized by ε_0 and μ_0 is located at the left of $z = -d_s$. The parameters used for LH PC are: $\varepsilon_1 = -4$, $\mu_1 = -1$, $d_1 = 250\,\text{nm}$, $\mu_2 = 1$, $d_2 = 55\,\text{nm}$ for the LH PC. Also these parameters for the RH PC are: $\varepsilon_1 = 4$, $\mu_1 = 1$, $d_1 = 200\,\text{nm}$, $\varepsilon_2 = 6.25$, $\mu_2 = 1$, $d_2 = 165\,\text{nm}$. The order of magnitude of the lattice constant $d = d_1 + d_2$ depends on the desired spectral range. For the microwave region, d is on the order of centimeters, while d can be on the order of micrometers to nanometers for the optical range [17- 19].
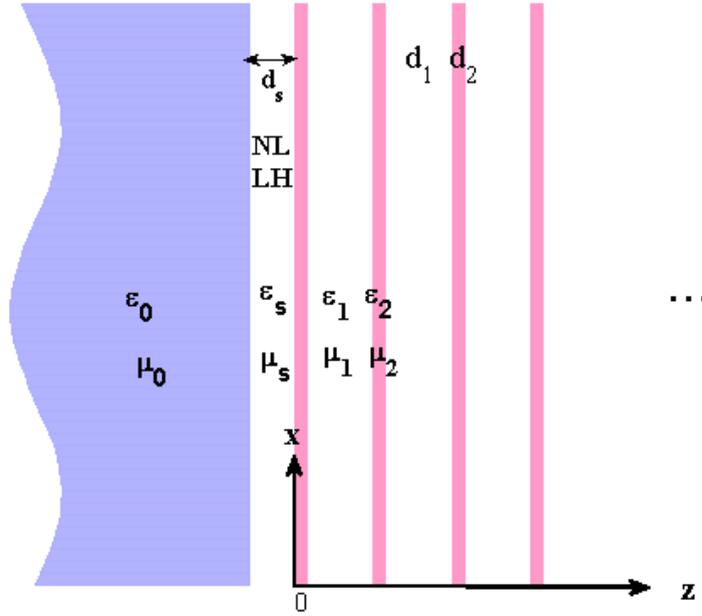
Fig. 1: Geometry of the problem. In our calculations we take the following values for LH PC: $\varepsilon_1 = -4$, $\mu_1 = -1$, $d_1 = 250\,\text{nm}$, $\varepsilon_2 = 6.25$, $\mu_2 = 1$, $d_2 = 55\,\text{nm}$ and slab layer characterized by ε_s, μ_s and thickness d_s; also the parameters used for conventional PC are: $\varepsilon_1 = 4$, $\mu_1 = 1$, $d_1 = 200\,\text{nm}$, $\varepsilon_2 = 6.25$, $\mu_2 = 1$, $d_2 = 165\,\text{nm}$.

The nonlinear LH slab is characterized by $\mu_s = -1$ and diagonal dielectric tensor ε_s^{TM}. For an isotropic material the nonlinear dielectric function can be expressed by

$$\varepsilon_s^{TM} = \begin{pmatrix} \varepsilon_s + \alpha_{xx}\left|E_x\right|^2 + \alpha_{xz}\left|E_z\right|^2 & 0 \\ 0 & \varepsilon_s + \alpha_{zz}\left|E_z\right|^2 + \alpha_{zx}\left|E_x\right|^2 \end{pmatrix} \quad (1)$$

Where ε_s is the linear part of the dielectric function and parameter α_{ij} describes Kerr-type nonlinearity. We use the approximation: $\alpha_{xx} = \alpha$, $\alpha_{zz} = \alpha_{xz} = \alpha_{zx} = 0$ [20, 21] for the LH slab and self-focusing characteristic ($\alpha < \cdot$) [22]. The propagation of monochromatic TM-polarized waves is given by

$$\vec{H} = H_y(z)e^{i(k_0\beta x - \omega t)}\hat{e}_y \tag{2}$$

$$\vec{E} = [E_x(z)\hat{e}_x + E_z(z)\hat{e}_z]e^{i(k_0\beta x - \omega t)}$$

Where ω is the angular frequency, $k = \dfrac{\omega}{c}$ is the vacuum wave number, and β is the normalized wave-number component along the interface. For simplicity it is assumed that the containing media are lossless, homogeneous and isotropic. It is well known that surface modes correspond to the localized solutions with the electromagnetic field decaying from the interface in both directions. In the left-side homogeneous medium the fields are decaying provided $\beta > \sqrt{\varepsilon_0 \mu_0}$. So, the solution of the Maxwell's equations for the TM SWs in a homogeneous medium ($z \leq -d_s$) that satisfies the boundary conditions at infinity is:

$$E_{xL}(z) = E_0 e^{q_0(z + d_s)} \tag{3}$$

Where E_0 is the x component of the electric field amplitude at $z = -d_s$, $q_0 = k\sqrt{\beta^2 - n_0^2}$ and $n_0 = \sqrt{\varepsilon_0 \mu_0}$. In the periodic structure, the waves are the Bloch modes

$$\psi(z) = \varphi(z)e^{iK_b z} \tag{4}$$

Where K_b is the Bloch wave number and $\varphi(z)$ is the Bloch function which is periodic with the period of the photonic structure [10]. In the periodic structure, the waves will be decaying provided that K_b is complex and this condition defines the spectral band gaps of an infinite multilayered structure. In the next stage, we analytically investigate the solution of the Maxwell's equation in the self-focusing LH slab. The stationary solution of the TM wave equation for the x component of the electric field in the nonlinear LH slab has the form $E_x(z) = E_s(z)e^{i(k\beta x - \omega t)}$ in which the amplitude field in the nonlinear slab modulated along z due to the nonlinearity. Using this form of stationary solution with the mentioned approximation we find that

$$\left[\frac{\partial^2}{\partial z^2} - \frac{\left(k^2\beta^2 - k^2\varepsilon_s\mu_s\right)}{\varepsilon_s}\left(\varepsilon_s + \alpha|E_s(z)|^2\right)\right]E_s(z) = 0 \tag{5}$$

Equation. (5) can be integrated once to give

$$(E_s')^2 - k_s^2 E_s(z) - k_s^2 \frac{\alpha}{2\varepsilon_s} E_s^4 = C \tag{6}$$

Where, the prime represents the differentiation with respect to z, $k_s^2 = k^2(\beta^2 - \varepsilon_s \mu_s)$ and C is the constant of integration (with respect to z). The quantity C which has arisen from the first integral is determined by applying the TM boundary conditions at the interfaces $z = -d_s$ and $z = 0$:

$$E_{xL}\big|_{z=-d_s} = E_s\big|_{z=-d_s} = E_0, \qquad H_L\big|_{z=-d_s} = H_s\big|_{z=-d_s} \tag{7}$$

$$E_s\big|_{z=0} = E_{x1}\big|_{z=0} = E_b, \qquad H_s\big|_{z=0} = H_1\big|_{z=0} \tag{8}$$

Where, E_{x1} and H_1 are the x components of the electric and magnetic field in the first layer of the PC [6]. Inserting Eqs. (7) and (8) into Eq. (6) gives

$$
\begin{aligned}
C &= E_0^2 \left[\left(\frac{\varepsilon_0}{\varepsilon_s}\right)^2 \left(\frac{k_s^2}{q}\right)^2 - k_s^2 - \frac{k_s^2}{2\varepsilon_s}\gamma_0 \right] = \\[2ex]
&\quad E_b^2 \left[\left(\frac{\varepsilon_1}{\varepsilon_s}\right)^2 \left(\frac{k_s^2}{k_1}\right)^2 R'^2 - k_s^2 - \frac{k_s^2}{2\varepsilon_s}\gamma_b \right]
\end{aligned}
\tag{9}
$$

Where $\gamma_0 = \alpha|E_0|^2$, $\gamma_b = \alpha|E_b|^2$, $R' = -i\left[\dfrac{B+(\lambda-A)}{B-(\lambda-A)}\right]$ is a real parameter

[18], A and B are the elements of the transfer matrix of the PC, and λ is the eigenvalue of the transfer matrix in the photonic bandgap [6, 10]. This equation exposes in a very clear manner the dependence of k on the intensity of electric field at the surface of the PC (γ_0). This enables one to find dispersion properties of nonlinear TM SWs provided that the x component of the electric field at the upper boundary of the slab (E_b) is determined. Equation. (8) is considered as the dispersion relation of nonlinear TM SWs. The solution of Eq. (5) can be written in terms of the Jacobi functions [13, 23]. Elliptic functions open up a window of solvable nonlinear (polynomial) differential equations, all of which are related to the physical problems and physical phenomena. There are, in fact, 12 Jacobi functions and their selection depends upon the sign of C.

Our calculations showed that the sign of C in the first photonic band gap is negative for both LH and conventional PCs. The solution of the nonlinear TM wave equation for the self-focusing LH slab ($\alpha < \cdot$) that satisfy the TM boundary conditions, have the following forms for the case of LH and conventional PCs, respectively:

$$E_{s1}(z) = \sqrt{a_1^2 + b_1^2}\, ds\left(z_{01} + \sqrt{a_1^2 + b_1^2}\,\sqrt{|\zeta|}(z + d_s)\Big| m_1 \right) \tag{10}$$

Where

$$a_1^2 = \frac{\left(\delta + \sqrt{\delta^2 + 4G}\right)}{2}, \qquad b_1^2 = \frac{\left(-\delta + \sqrt{\delta^2 + 4G}\right)}{2},$$

$$z_{01} = ds^{-1}\left(\frac{E_0}{\sqrt{a_1^2 + b_1^2}}\Big| m_1\right), \quad \zeta = \frac{k_s^2 \alpha}{2\varepsilon_s}, \quad \delta = \frac{k_s^2}{\zeta}, \quad G = \frac{|C|}{\zeta},$$

And $m_1 = \dfrac{a_1^2}{\left(a_1^2 + b_1^2\right)}$ is the nonlinear period of the Jacobi elliptic function ds.

$$E_{s2}(z) = b_1\, nc\left(z_{02} + \sqrt{a_1^2 + b_1^2}\,\sqrt{\zeta}(z + d_s)\Big| m_1 \right) \tag{11}$$

Where $z_{02} = nc^{-1}\left(\dfrac{E_0}{b_1}\Big| m_1\right).$

3. RESULTS AND DISCUSSION

In this work, we investigate the effect of the nonlinear self-focusing LH slab on the dispersion properties of the TM SWs using Eqs. (8-10). In Eq. (8) the dimensionless parameter γ_0, describes the intensity of electromagnetic field at the surface of LH slab. The dispersion properties of the nonlinear TM SWs are plotted in Fig. 1 in the first spectral gap on the plane (k, β) for different values of γ_0.
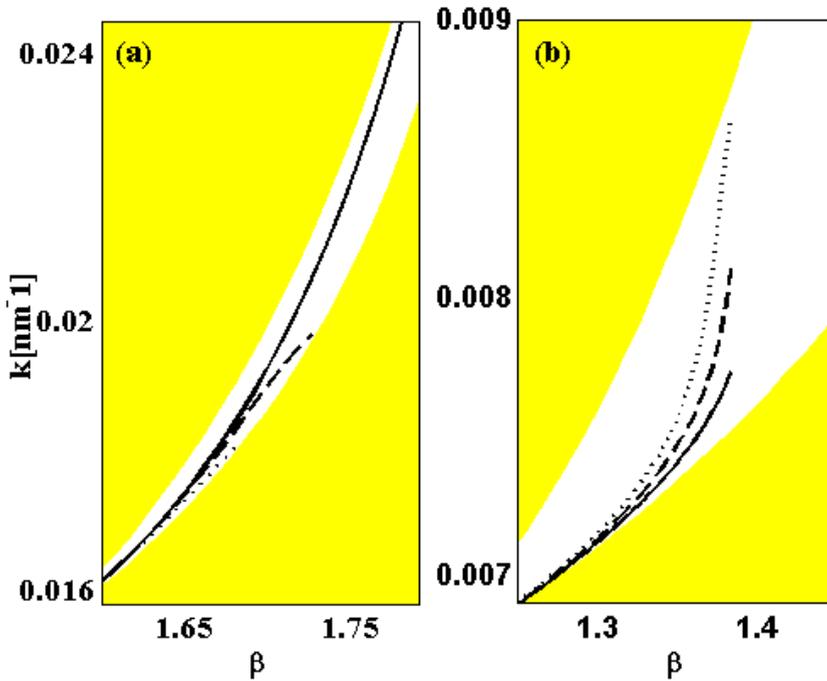
Fig. 2. Dispersion property of SWs for different values of γ_0 in the first spectral gap of PC. Unshaded area: First bandgap of PC. (a): LH PC and $\varepsilon_0 = 1$, $\mu_0 = 1$. Dotted, dashed and solid lines: dispersion of the SWs for $|\gamma_0| = 0.35, 0.25$ and $|\gamma_0| \to 0$ respectively. (b): Conventional PC and $\varepsilon_0 = -1$, $\mu_0 = -1$. Dotted, dashed and solid lines: dispersion of the SWs for $|\gamma_0| = 0.35, 0.3, 0.25$.

From Fig. 2a, it is clear that the SWs exist in the linear regime in the LH PC. By increasing γ_0, the dispersion curves of the TM SWs approaches the lower edge of the band gap, so that the existence region of the SWs is limited. However in the conventional PC (Fig. 2b), there are no surface modes in the linear regime and by increasing the intensity of field γ_0 to higher values, the SWs are emerged. Our investigations indicate that the existence region of the SWs also depends on the thickness of the slab [see Fig. 3].
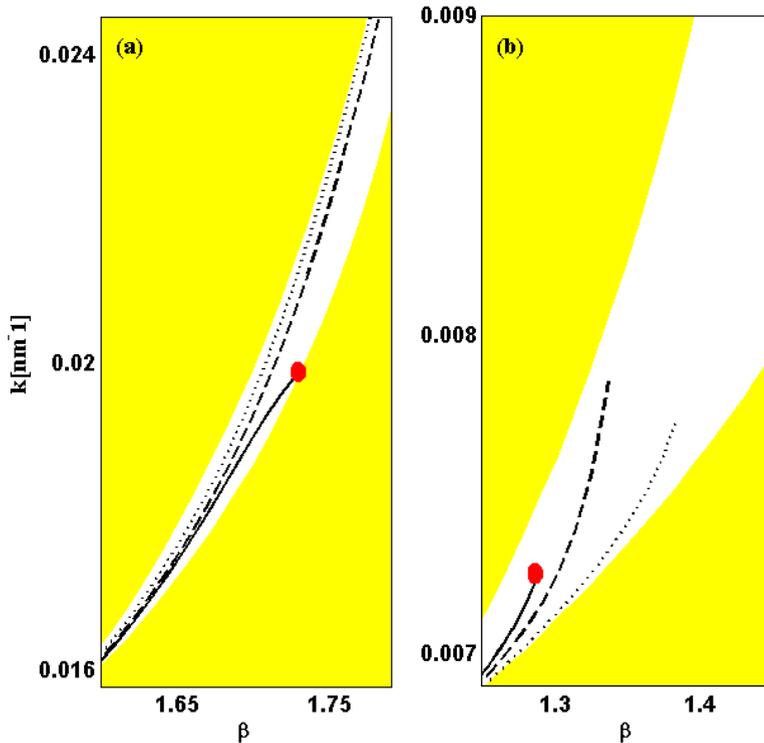
Fig. 3. Dispersion property of the SWs in the first spectral gap for $\left|\gamma_0\right| = 0.25$. Unshaded area: First band gap of PC. (a) LH PC: Dotted, dashed and solid lines: dispersion of the SWs for $d_s = 50\,nm, 112.5\,nm, 175\,nm$. (b) Conventional PC: Dotted, dashed and solid lines: dispersion of the SWs for $d_s = 200\,nm, 250\,nm, 275\,nm$.

As one can see in the LH PC, the SWs corresponding to β>1.727 do not exist in the case of $d_s = 175\,nm$ (see solid line in Fig. 3(a)). Also there are no surface modes for β>1.288 in conventional PC with slab thickness $d_s = 275\,nm$ (see solid line in Fig. 3(b)). Accordingly, the dispersion of TM SWs can be controlled through the thickness of nonlinear LH slab. In order to compare our achievements, the reflectivity of the ATR geometry [24], is calculated using classical electromagnetic theory in the linear regime. The reflectivity of the ATR geometry has been plotted as a function of frequency (k=ω/c) for a typical longitudinal wave number in Fig. 4.
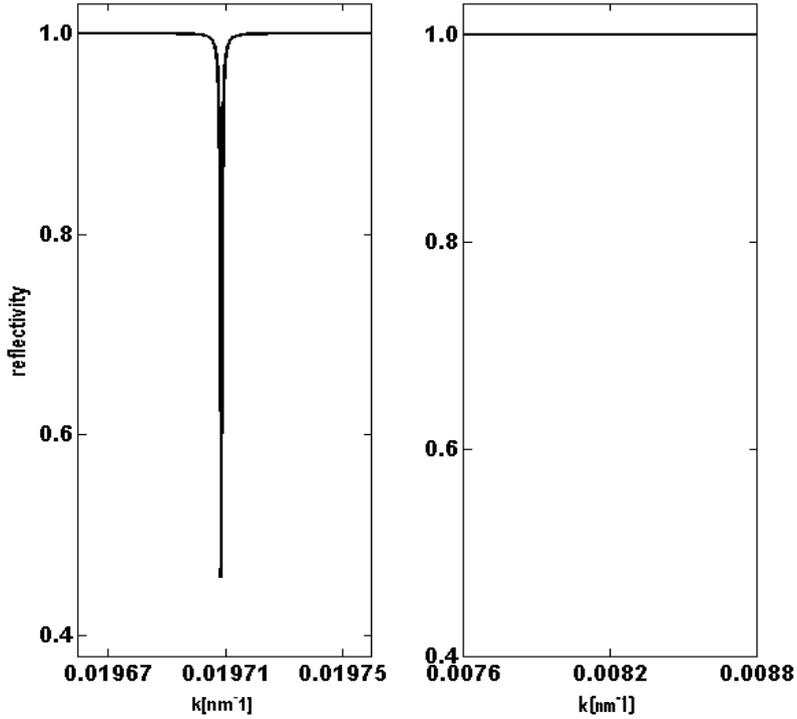
Fig. 4. The calculated ATR spectrum of the considered structure in the linear regime for (a) LH PC: $d_S = 175nm$ and $\beta = 1.7087$. (b) Conventional PC: $d_S = 200nm$ and $\beta = 1.337$.

In the case of ordinary PC, the different thicknesses of the slab was studied, and it was observed that, there are no surface modes in the linear regime (Fig. 4b and Fig. 5b). Our studies showed that the dispersion property of the TM SWs based on Eq. (8) when $|\gamma_0| \rightarrow 0$ confirm the ATR method results in the linear regime (see Fig. 5).
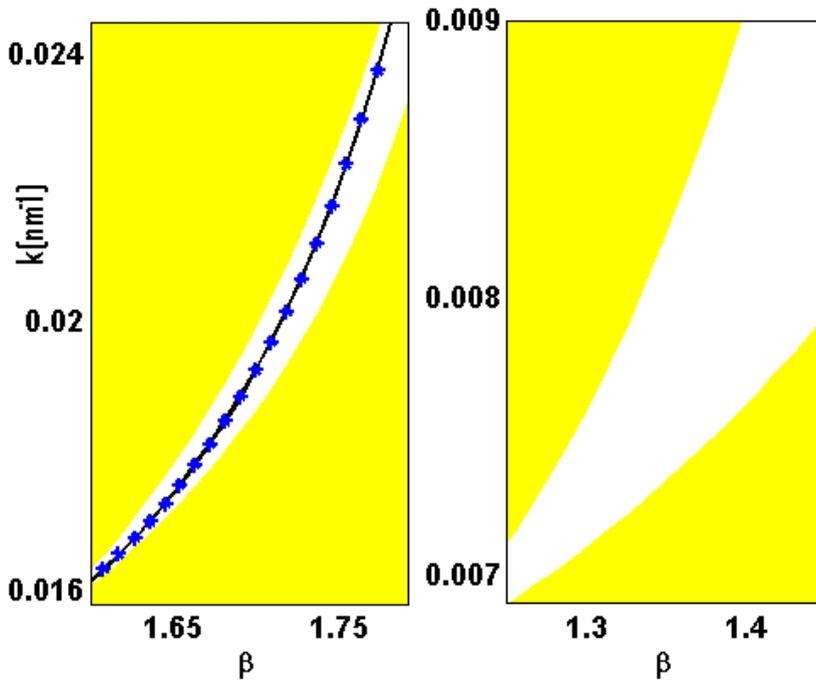
Fig. 5. Dispersion property of the SWs in the first spectral gap. (a): LH PC: $d_s = 175\,nm$ and (b) Conventional PC. Here the solid and marked lines show dispersion properties of SWs based on Eq. (9) when $\gamma_0 \to 0$ and the ATR method respectively.

We also compared our results in the nonlinear regime with the Runge-Kutta method that validates the above results.

For further study of nonlinear p-polarized surface modes, the transverse profile of some typical surface modes is plotted as a function of coordinate z in both LH and conventional PCs (Fig. 6).
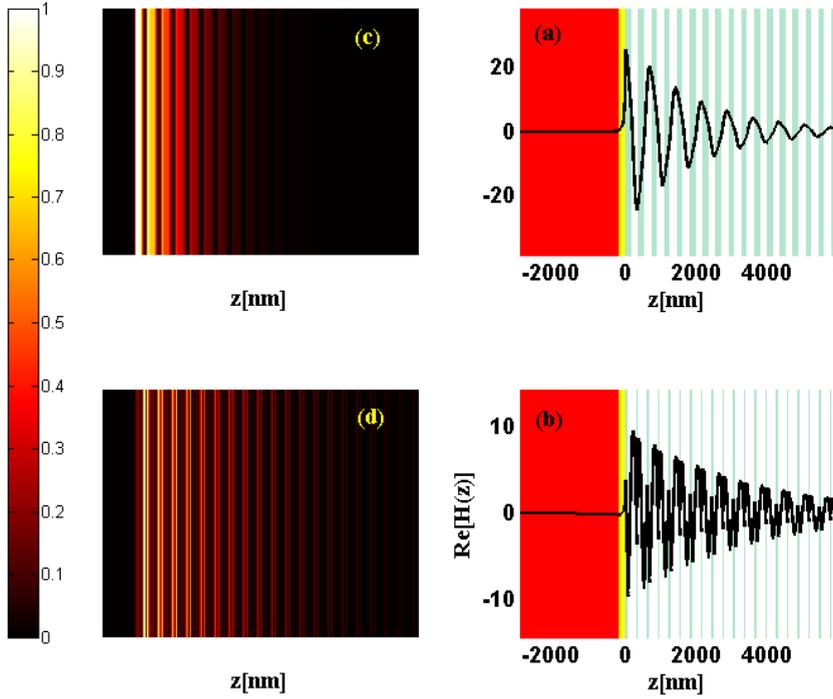
Fig. 6. The transverse profile of SWs versus coordinate z. (a): corresponds to the $d_s = 175\ nm$, $|\gamma_0| = 0.25$, $\beta = 1.7$, $k = 0.019 nm^{-1}$, $z_{01} = 1.81$ in LH PC and (b): $d_s = 200\ nm$, $|\gamma_0| = 0.3$, $\beta = 1.372$, $k = 0.00776 nm^{-1}$, $z_{02} = 1.59$ in conventional PC. (c) and (d) the intensity distributions for modes (a) and (b) respectively.

As mentioned above, in the case of LH PC by increasing the intensity of field γ_0 to higher values, the dispersion curves of the TM SWs moves to the lower edge of the band gap. So that, these modes are weaker localized at the interface of the LH PC (Figs. 6b, 6d). On the other hand in conventional PC, by increasing γ_0 to higher values, provides better localizations of the modes (Figs. 6a, 6c).

4. CONCLUSION

We have analytically investigated nonlinear p-polarized electromagnetic SWs of a nonlinear self-focusing LH slab sandwiched between a homogeneous medium and a semi-infinite 1DPC. Our method is based on the first integral of the nonlinear Helmholtz wave equation. For the transverse magnetic (TM) polarized waves, the presence of two electric field components makes the analysis difficult.

Therefore, the approximation $\alpha_{xx} = \alpha, \alpha_{zz} = \alpha_{xz} = \alpha_{zx} = 0$ was considered for the slab. It was shown that, the presence of a nonlinear self-focusing LH slab at the surface of PC allows us to control the dispersion properties of TM SWs. Furthermore, the ATR method in the linear regime confirmed our achievement based on the first integral of the nonlinear Maxwell's equations.

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