



Multi-Objective Optimization of Reactive Power Distribution in Power System

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| Article info | Abstract |
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| <p>Keywords: Optimal reactive power distribution stochastic and intelligent algorithms active power loss reduction voltage stability improvement NSGA-II</p> <p>Article history: Received: 31 Aug 2024 Accepted: 26 Sep 2024</p> | <p>Optimal reactive power distribution is a critical aspect of economic and secure operation of power systems. This problem falls within the category of power system optimization problems where a specific objective function is optimized subject to a set of constraints and control variables. Due to the non-linear nature of this problem and the existence of multiple local optima, deterministic methods are not suitable for solving it. Therefore, stochastic and intelligent algorithms must be employed. Control variables in this problem include generator voltages, transformer tap positions, and reactive power compensation devices such as reactors and capacitors. Three objective functions, namely, minimizing active power losses, improving voltage profile, and maximizing voltage stability, are considered both individually and in a multi-objective manner. The primary algorithms investigated in this thesis include Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Colonial Competitive Algorithm (ICA), and Differential Evolution (DE). Additionally, weighted sum method and Non-dominated Sorting Genetic Algorithm II (NSGA-II) are employed for multi-objective optimization.</p> |

1. Introduction

Achieving a cost-effective and reliable power system is a primary objective for power system operators. To attain this goal, optimal planning and operation of power systems have been the focus of extensive research. One of the key tools to achieve this objective is optimal reactive power distribution, which significantly impacts the reliable and economic

operation of power systems. This problem is a sub-problem of optimal power flow calculations, first introduced by Carpentier in 1962. Optimal reactive power distribution is a power system optimization problem that aims to optimize a specific objective function subject to a set of constraints and control variables. In this problem, the objective function can be minimizing losses, improving voltage stability, reducing voltage deviations, and so on. By adjusting

reactive power sources such as capacitors and reactors, transformer tap positions, and generator bus voltages, while satisfying constraints such as power flow equations, voltage limits, and generator reactive power limits, these objectives can be achieved. This study focuses on minimizing losses, improving voltage stability, and reducing voltage deviations. Since generator reactive power outputs, bus voltage magnitudes, and angles are continuous variables, while transformer tap positions and outputs of shunt reactors and capacitors are discrete, the optimal reactive power problem can be modeled using non-linear programming and may have multiple local optima. This makes finding the global or near-global optimal solution challenging. The importance of electrical energy is undeniable. Due to its ease of conversion to other energy forms, ease of transmission, easy control, and environmental considerations, electrical energy has found more applications than other types of energy. Supplying the required electrical energy to customers at the lowest cost and with the best possible quality is the main objective of a power system. In electrical networks, losses are one of the biggest problems that affect power generation, transmission, and distribution. Therefore, reducing losses and improving voltage profile have been the main goals of power system designers. Voltage control (reactive power control) is an essential issue in both emergency and normal operating conditions of a power system. In emergency conditions, it can increase voltage stability margins by identifying voltage instability limits and thus enhance system security. In normal conditions, it can lead to reliable operation of the network with high power quality. Reactive power plays a significant role in power system voltage security. Insufficient reactive power in the system can cause undesirable voltage drops at some buses. If the system cannot overcome this voltage deficiency, it may lead to voltage instability in the entire network. Insufficient reactive power, which leads to voltage collapse, is known as a major cause of widespread power outages. The reactive power transmitted over a transmission line depends on the voltage difference between the line ends. Also, increasing the magnitude of the sending end bus voltage increases the reactive power injected from the bus. The reactive power generated by a generator depends on its excitation, and by changing the generator's driving force, the amount of reactive power

generated or consumed can be adjusted. In an interconnected system, it can be seen by performing load flow studies in different conditions that injecting reactive power into a bus raises the voltage of all buses and has the most significant impact on the voltage of that bus.

1-1 Literature Review

Numerous mathematical methods have been employed to solve this problem. In [A2,A3], gradient-based optimization methods [A1] have been used to solve this problem. In recent years, the interior point method [A2] has been used to solve the ORPD problem [A4]. In [5], a quadratic programming approach is used for optimization. However, the aforementioned methods often face limitations when dealing with nonlinear problems, discrete variables, and problems with multiple local optima, and they may fail to find the global or near-global optimal solution. To overcome these limitations, simplifications have been made in all these methods. Nevertheless, these methods are ultimately inefficient for solving the optimal reactive power distribution problem in large-scale practical systems. In the last decade, many stochastic search methods have been developed to solve optimization problems. Genetic Algorithm (GA) [A3], Particle Swarm Optimization (PSO) [A4], Differential Evolution (DE) [A5], and Colonial Competitive Algorithm (ICA) [A6] are among these optimization methods [A6-A8]. The genetic algorithm is inspired by the methods used in the living world to optimize the adaptability of organisms to the environment. PSO is a swarm intelligence method that was developed by simulating social systems and is very effective in solving nonlinear problems. The differential evolution algorithm is similar to the genetic algorithm, and the only difference is the selection method. ICA is a new exploratory algorithm proposed by the Iranian Scientist Professor Caro Lucas, which is population-based and considers the optimization process as a search for the optimal solution using a population of colonizing and colonized countries [A7,A8]. Over the past two decades, the power industry has undergone fundamental changes in the way power is generated, transmitted, and distributed, known as restructuring. With the restructuring of the power industry, reactive power has been introduced as one of the most important ancillary services for the secure and reliable

operation of the power system. In recent years, reactive power markets have been defined to increase system reliability, create a proper competitive structure, and encourage producers to generate reactive power. However, reactive power is interrelated through various means, such as power flow equations, synchronous generator capability curves, and maximum power transfer limits of lines. In [A9], a multi-objective approach for reactive power control in a network in the presence of generation sources and FACTS devices is proposed. Although this reference achieves optimization objectives to some extent using FACTS devices, the optimization performed does not consider the costs of purchasing power from the grid and distributed generation sources, and the simulation is only performed for peak hours. In [A10], a reactive power control method using fuzzy optimization is proposed. Although the proposed method in this reference considers tap changers and capacitors, it models the network without distributed generation sources. Considering the importance of reactive power control and voltage reduction, many studies have been conducted in this area in recent years

1-2 Efforts Made

In this paper, the probabilistic and stochastic algorithms introduced above are investigated for optimal reactive power distribution. This problem is examined both as a single-objective and multi-objective problem.

2- Multi-Objective Optimization

Considering three objective functions, namely minimizing active power losses, reducing voltage deviations, and improving system stability, the optimal reactive power distribution problem in power systems becomes a multi-objective optimization problem. In this section, the weighted sum method and NSGA-II are used to solve this problem. In the weighted sum method, four algorithms, GA, PSO, ICA, and DE, will be used.

2-1 Multi-Objective Optimization Using Weighted Sum Method

The advantage of this method over methods that generate a set of solutions is its simplicity. However, one of the biggest drawbacks of this method is the inaccessibility to a portion of good solutions.

Additionally, determining the weighting coefficients is another drawback in this method.

In this method, the objective function is as shown in equation (1).

$$(1) \min Z = W_1 f_1 + W_2 f_2 + W_3 f_3 + \sum_{i \in N_V} \lambda_{Vi} (V_i - V_i^{lim}) + \sum_{i \in N_Q} \lambda_{Gi} (Q_{Gi} - Q_{Gi}^{lim})$$

where f_1 , f_2 and f_3 are active system losses, voltage deviations and L index. Of course, three objective functions are first normalized using equation (2) and then used in equation (1).

$$(2) \bar{f} = (f - f_{min}) / (f_{max} - f_{min})$$

Type equation here.

λ_{Vi} and λ_{Gi} are penalty coefficients that are considered equal to 0.1 and 0.2, respectively, to optimize the active losses of the system. The coefficients ... are considered with two scenarios as shown in Table 1.

Table 1: Multi-objective optimization coefficients in the IEEE 30-bus system

| W_3 | W_2 | W_1 | Coefficients |
|-------|-------|-------|--------------|
| 0/3 | 0/3 | 0/4 | Scenario 1 |
| 0/2 | 0/2 | 0/6 | Scenario 2 |

3- Multi-Objective Genetic Algorithm

Optimization is essentially the process of finding one or more solutions from a set of possible options (subject to constraints) with the aim of optimizing one or more criteria. Multi-objective optimization is a subset of multi-criteria decision-making (MCDM) methods, which involves searching among an infinite set of potential solutions. Multi-objective optimization arises from real-world decision-making problems where decision-makers face a set of conflicting and competing objectives and criteria. Unlike single-objective optimization problems, in these types of problems, due to the presence of multiple conflicting objectives, instead of a single solution, a set of solutions is obtained. The general form of multi-objective optimization problems is shown in equation (3)

$$\min F(x) = \{f_1(x), \dots, f_i(x), \dots, f_n(x)\}$$

$$\text{s.t. } g(x) \leq 0, h(x) = 0$$

$$x \in R$$

"g(x) includes inequality constraints and h(x) includes equality constraints of the problem."

In this section, we will introduce the multi-objective genetic algorithm based on non-dominated sorting. To understand this, we must first familiarize ourselves with the concept of dominance.

3.1 Concept of Dominance

In multi-objective optimization, to illustrate the concept of dominance, let's consider a two-objective optimization problem. As shown in Figure 1, point A dominates point C if A is not worse than C in any objective and A is better than C in at least one objective.

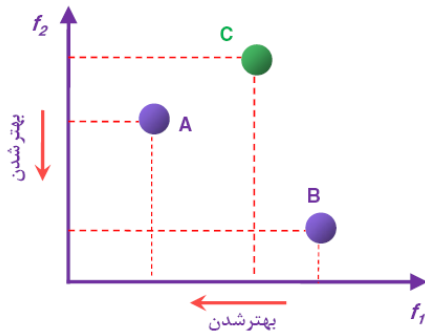


Figure 1: Points A and B dominate point C.

In multi-objective optimization, the goal is to find a set of good solutions known as the Pareto front. The advantage of this method over weighted sum methods is that weighted sum methods can never produce solutions within the green region shown in Figure 2.

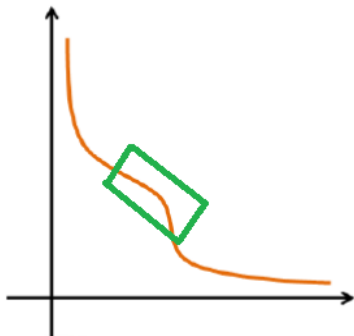


Figure 2: Pareto front in a two-objective optimization problem.

Depending on the type of multi-objective optimization problem, the shape of the Pareto front can vary, as illustrated in Figure 3. In this figure, the yellow curve represents the desired set of solutions

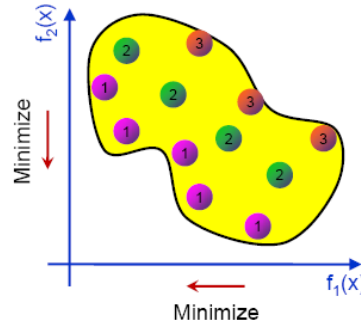


Figure 3: Pareto front in a two-objective optimization problem.

Now, considering the concept of dominance that we have learned, the steps of the NSGA-II algorithm can be described as follows.

3.2 Steps of the NSGA-II Algorithm

Step 1: Initial Population Generation As usual, the initial population is generated based on the problem's scale and constraints.

Step 2: Evaluation of the Generated Population The generated population is evaluated based on the defined objective functions.

Step 3: Non-dominated Sorting Using the non-dominated sorting method as shown in Figure 4, the population members are classified into different fronts. Members in the first front are completely non-dominated by other members of the current population. Members in the second front are dominated only by members of the first front, and this process continues for all other fronts until a rank is assigned to each member based on its front number.

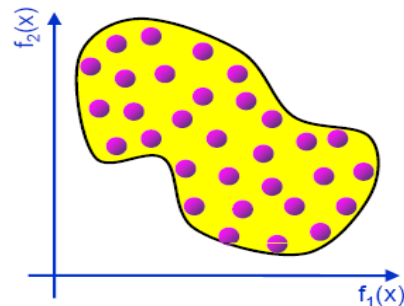


Figure 4: First and second steps in a two-objective optimization problem.

Step 4: Crowding Distance Calculation A control parameter called crowding distance is calculated for each member in each front. This parameter measures the density of a solution with respect to its neighbors. As shown in Figure 5, a larger crowding distance value leads to better diversity and spread of the population members

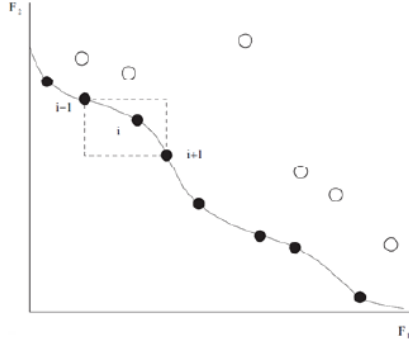


Figure 5: Crowding distance calculation in a two-objective optimization problem

$$(4) \quad d_j(k) = \sum_{i=1}^n \frac{f_i(k-1) - f_i(k+1)}{f_i^{\max} - f_i^{\min}}$$

Step 5: Parent Selection One common selection mechanism is binary tournament selection, where two individuals are randomly chosen from the population. The individual with a better fitness (often determined by rank or crowding distance) is selected as a parent.

Step 6: Crossover and Mutation Crossover and mutation operations are performed. Crossover involves exchanging genetic material between two parents to create offspring, while mutation introduces random changes to the offspring's genetic material.

Step 7: Termination Criteria If the termination criteria is not met (e.g., reaching a maximum number of generations or a satisfactory solution), the algorithm returns to step 2.

4. Simulation

We will now discuss the IEEE 30-bus system, a standard test system. This system will be used to demonstrate the application of various random search algorithms for solving single-objective and multi-objective optimization problems.

IEEE 30-bus System The IEEE 30-bus system, as depicted in Figure 6, consists of 13 control variables: 6 generator bus voltage magnitudes, 4 transformer tap settings, and 3 shunt compensators. The tapped transformers are located on lines 6-9, 4-12, 6-10, and 27-28. Transformer taps are discrete variables with a step size of 0.01 per unit. Buses 2, 5, 8, 11, and 13 contain generators, and bus 1 is the reference bus. Three shunt compensators are located on buses 3, 10, and 24, and they are also discrete variables with a step size of 0.01 per unit.

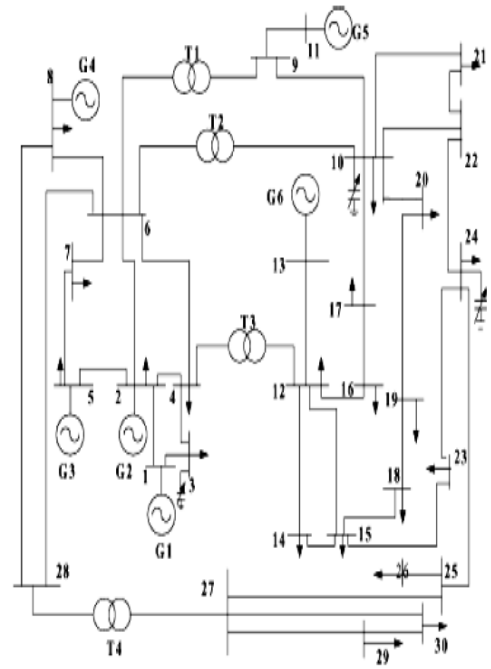


Figure 6: IEEE 30-bus system [6]

The limits of the system variables are given in Table 2. The initial conditions of the IEEE 30-bus system are as follows:

$$P_{load} = 2.834 \text{ pu} \quad Q_{load} = 1.262 \text{ pu} \quad S_{base} = 100 \text{ MW}$$

If the initial voltages of the generator buses and transformer taps are set at one per unit, the total generation and power losses will be as follows:

$$\sum P = 2.8937 \text{ pu} \quad \sum Q = 0.974 \text{ pu} \\ P_{loss} = 0.059743 \text{ pu} \quad Q_{loss} = 0.2585 \text{ pu}$$

Table 2: Variable limits in IEEE 30-bus system

| limitations of reactive power generation (pu) | | | | | | |
|---|--------------|-------------------|-------------------|--------------|--------------|--------------|
| 13 | 11 | 8 | 5 | 2 | 1 | bass |
| 0/155 | 0/15 | 0/53 | 0/6 | 0/480 | 0/596 | Q_G^{\max} |
| -0/078 | -0/075 | -0/265 | -0/3 | -0/24 | -0/298 | Q_G^{\min} |
| Limitations of tap transformer settings and voltage (pu) | | | | | | |
| T_k^{\min} | T_k^{\max} | V_{load}^{\min} | V_{load}^{\max} | V_G^{\min} | V_G^{\max} | |
| 0/95 | 1/05 | 0/95 | 1/05 | 0/9 | 1/1 | |
| Voltage limitations and reactive power production of reactive compensation sources (pu) | | | | | | |
| V_c^{\min} | V_c^{\max} | Q_c^{\min} | Q_c^{\max} | | | |
| 0/95 | 1/05 | -0/12 | 0/36 | | | |

The three buses which voltages have exceeded the permissible limits are as follows:
 $V_{26} = 0.932 \text{ pu}$ $V_{29} = 0.940 \text{ pu}$ $V_{30} = 0.928 \text{ pu}$

4-1 Scenario 1

Four algorithms, GA, PSO, ICA, and DE, are presented in Table 3. By comparing the results and examining Figures 7 and 8, it is evident that PSO and ICA algorithms have performed better. Moreover, based on Tables 3 and 4, all parameters have remained within their specified limits across all four methods

Table 3: Results of the weighted optimization algorithm applied to the IEEE 30-bus system

| DE | ICA | PSO | GA | objective function |
|--------|--------|--------|--------|----------------------------|
| 5/2971 | 5/2760 | 5/2982 | 5/3287 | Best system loss (mw) |
| 0/1542 | 0/1635 | 0/1483 | 0/1475 | Best voltage deviation |
| 0/1332 | 0/1330 | 0/1346 | 0/1367 | The best voltage stability |

Table 4: Control variable values in per unit after the weighted optimization algorithm.

| DE | ICA | PSO | GA | variable |
|--------|--------|--------|--------|----------|
| 1/0370 | 1/0429 | 1/0368 | 1/0342 | V_1 |
| 1/0292 | 1/0329 | 1/0271 | 1/0241 | V_2 |

| DE | ICA | PSO | GA | variable |
|--------|--------|--------|--------|-------------|
| 1/0037 | 1/0075 | 1/0059 | 1/0062 | V_5 |
| 1/0042 | 1/0054 | 1/0028 | 0/9911 | V_8 |
| 1/0014 | 1/0090 | 1/0077 | 1/0028 | V_{11} |
| 1/0247 | 1/0218 | 1/0297 | 1/0029 | V_{13} |
| 1/03 | 1/04 | 1/03 | 0/95 | T_{6-9} |
| 1/03 | 1/03 | 0/98 | 0/97 | T_{6-10} |
| 0/98 | 0/98 | 0/99 | 0/96 | T_{4-12} |
| 0/95 | 0/96 | 0/95 | 0/95 | T_{27-28} |
| -0/01 | -0/04 | -0/05 | 0/22 | Q_3 |
| 0/34 | 0/36 | 0/29 | 0/19 | Q_{10} |
| 0/13 | 0/13 | 0/11 | 0/11 | Q_{24} |

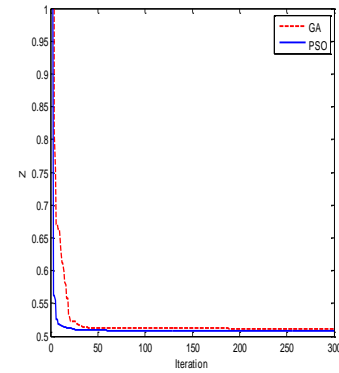


Figure 7: Convergence of GA and PSO algorithms in multi-objective optimization, Scenario 1, in the IEEE 30-bus system.

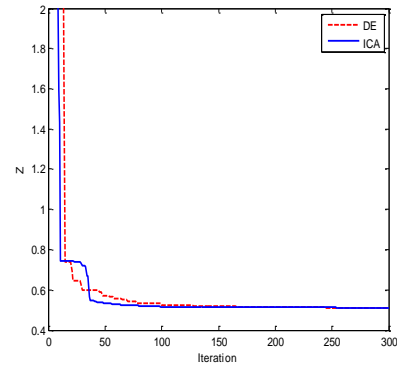


Figure 8: Convergence of ICA and DE algorithms in multi-objective optimization, Scenario 1, in the IEEE 30-bus system.

4-2 Scenario 2

The results of the four algorithms, GA, PSO, ICA, and DE, are presented in Table 5. By comparing the results and examining Figures 9 and 10, it is evident that the PSO and ICA algorithms have exhibited superior performance. Moreover, according to Table 6, all parameters have remained within their specified limits

in both methods. Another advantage of PSO and ICA compared to GA and DE is their faster convergence speed, as evident in Figures 7 to 10. Additionally, with the increase in the weighting factor of the loss function, we observe an improvement in the response of the active power loss in the system. Naturally, with the decrease in the voltage deviation and voltage stability coefficients, these two indices exhibit a weaker response in Scenario 2 compared to Scenario 1.

Table 5: Results of the weighted optimization algorithm applied to the IEEE 30-bus system.

| DE | ICA | PSO | GA | objective function |
|--------|--------|--------|--------|----------------------------|
| 5/2800 | 5/2448 | 5/1451 | 5/2530 | Best system loss (mw) |
| 0/1711 | 0/1686 | 0/2392 | 0/1840 | Best voltage deviation |
| 0/1307 | 0/1334 | 0/1352 | 0/1375 | The best voltage stability |

Table 6: Control variable values in per unit after the weighted optimization algorithm in the IEEE 30-bus system.

| DE | ICA | PSO | GA | variable |
|--------|--------|--------|--------|-------------|
| 1/0414 | 1/0418 | 1/0547 | 1/0396 | V_1 |
| 1/0280 | 1/0313 | 1/0448 | 1/0291 | V_2 |
| 1/0033 | 1/0066 | 1/0199 | 1/0078 | V_5 |
| 1/0038 | 1/0075 | 1/0205 | 1/0078 | V_8 |
| 0/9877 | 1/0038 | 0/9912 | 1/0338 | V_{11} |
| 1/0244 | 1/0250 | 1/0363 | 1/0314 | V_{13} |
| 1/02 | 1/02 | 1 | 0/99 | T_{6-9} |
| 1/06 | 1/02 | 1/01 | 1/02 | T_{6-10} |
| 0/98 | 0/98 | 1/01 | 0/99 | T_{4-12} |
| 0/96 | 0/96 | 0/97 | 0/96 | T_{27-28} |
| 0/01 | 0/02 | 0/10 | 0/03 | Q_3 |
| 0/36 | 0/32 | 0/23 | 0/18 | Q_{10} |
| 0/10 | 0/12 | 0/13 | 0/11 | Q_{24} |

By comparing the four optimization algorithms, GA, PSO, ICA, and DE, in both single-objective and multi-objective reactive power optimization, it is clear that the PSO and ICA algorithms have higher capabilities in solving this problem. Due to the problems and disadvantages of the weighted coefficient method in multi-objective optimization, the NSGA-II algorithm is used in the next step to solve the multi-objective optimization problem, which has the ability to provide a set of good solutions.

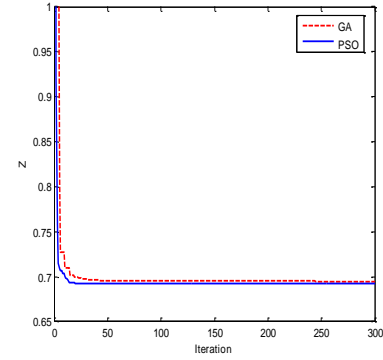


Figure 9: Convergence of GA and PSO algorithms in multi-objective optimization, Scenario 2, in the IEEE 30-bus system.

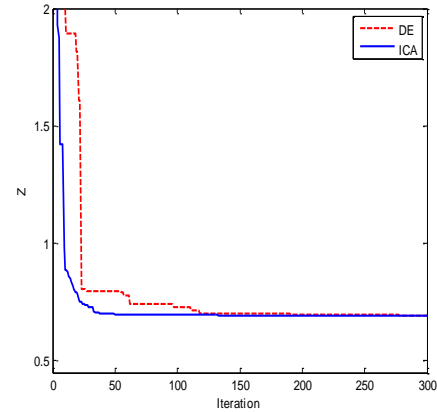


Figure 10: Convergence of ICA and DE algorithms in multi-objective optimization, Scenario 2, in the IEEE 30-bus system.

2-2 Multi-objective optimization using NSGA-II

Unlike the weighted coefficient method, this method does not require normalization of the objective functions. Moreover, this method provides a set of good solutions, which is called a Pareto front. The initial settings in this method are considered exactly like the genetic algorithm. The number of chromosomes in the first layer is considered to be 40. Figure 11 shows the Pareto front of the multi-objective reactive power optimization problem in the standard IEEE 30-bus system. The advantage of this method over the weighted coefficients is the diversity of the system's output response. In this method, as in the previous methods, all parameters in all responses are within the permissible limits.

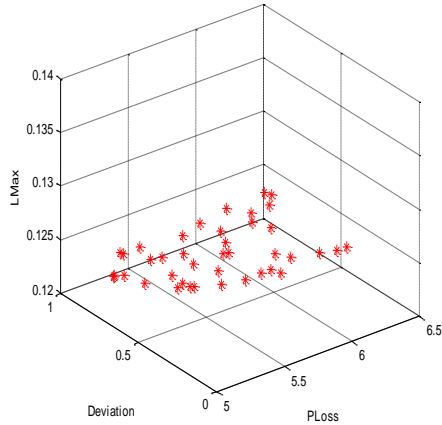


Figure 11: Pareto front in multi-objective optimization using NSGA-II, in the IEEE 30-bus system.

Table 7 shows four sample solutions generated for solving the multi-objective reactive power optimization problem by the NSGA-II algorithm. By carefully examining the objective function values in this table, it is clear that the concept of non-domination is well observed in these data. One of the advantages of the NSGA-II method compared to optimization using weighted coefficients is the generation of a set of desirable solutions by this algorithm in a single program run. However, its disadvantages include:

- Difficult computer program implementation.
- Reduced program speed.
- High computer memory consumption.

Table 7: Results of running the NSGA-II algorithm in the IEEE 30-bus system

| NSGAI | | | | |
|----------|----------|----------|----------|----------------------------|
| Answer 4 | Answer 3 | Answer 2 | Answer 1 | objective function |
| 5/0162 | 5/0840 | 5/5751 | 5/5411 | Best system loss (mw) |
| 0/5013 | 0/5697 | 0/1438 | 0/6379 | Best voltage deviation |
| 0/1251 | 0/1263 | 0/1344 | 0/1214 | The best voltage stability |

5. Conclusion

The problem of optimal reactive power distribution in power systems has a growing impact on the economic and reliable performance of power systems. The two main objectives in this problem are to reduce losses and create a suitable voltage profile. Since there is a possibility of voltage collapse in large-scale transmission systems, voltage security is also considered as the third objective in this problem. It is clear that this problem is a non-linear, multi-modal optimization problem with a combination of discrete and continuous variables that obtains the optimal value of control variables for a network state, parameters, and load using reactive power compensation sources such as capacitors and reactors, transformer taps, and generator bus voltage regulation, while satisfying constraints such as load flow constraints, bus voltage constraints, and generator reactive power output. In this paper, different reactive power optimization algorithms were investigated, and then the drawbacks of traditional methods such as derivative-based methods and linear programming methods were mentioned. To overcome these problems, the most important of which is the existence of a non-linear system with many local optimal points and the existence of discrete variables, it was preferred to use random search methods. In this paper, GA, PSO, ICA, and DE methods were investigated in the single-objective optimization problem, and the weighted coefficient method and NSGA-II algorithm were investigated in the multi-objective.

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