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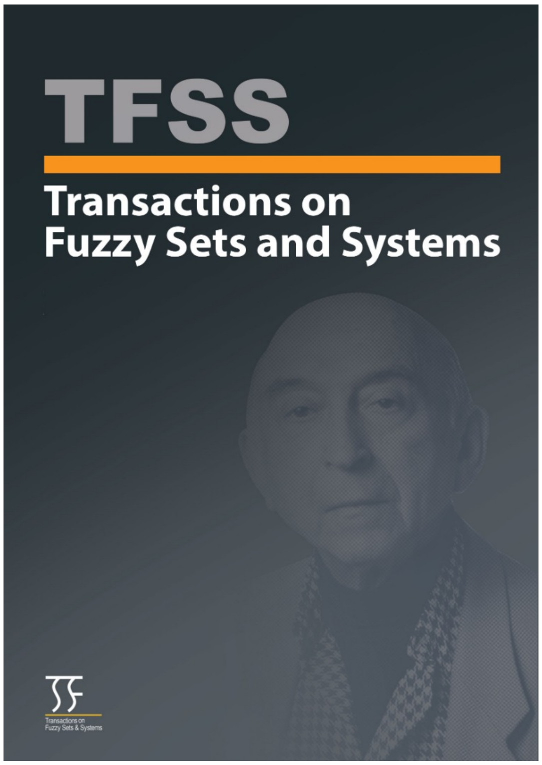

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Best States for Women to Work and Women's Peace and Security

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Best States for Women to Work and Women's Peace and Security

John N Mordeson , **Sunil Mathew*** , **Davender S Malik**

(This paper is dedicated to Professor "John N. Mordeson" on the occasion of his 91st birthday.)

Abstract. In [1], states are ranked with respect to the best states to work. In [2], states are ranked with respect to the peace and security for women. We determine the fuzzy similarity measure of these to rankings. We find the similarity to be high for one of the measures and very high for the other. We then break the United States into regions and determine the fuzzy similarity measure of these two rankings for each region. The fuzzy similarity here is medium for one measure and high for the other. Similarity plays a role in many fields. There exists many special definitions of similarity which have been used in different areas. We choose to use fuzzy similarity measures which seem appropriate in rankings. In fact, we develop some new measures.

AMS Subject Classification 2020: 03B52; 03E72
Keywords and Phrases: Women, Work, Peace and security, State rankings, Fuzzy similarity measures, Distance functions.

1 Introduction

It is stated in [3] that states have had to step up for workers and their families in the past few decades, as Congress has stalled on taking action. For example, while the federal minimum wage has been stuck at $7.25 an hour for 14 years, most states have mandated higher wages. In [3], The Best States to Work Index provides how the states rank overall and by policy area.

In [1], it is stated that since women make up the majority of the workforce-and-many are supporting families-this dimension considers how far the tipped minimum wage goes to cover the cost of living for a family of three (one wage earner and two children). In [1], The Best States for Working Women Index provides how the states rank overall and by policy area.

The U. S. Women, Peace and Security Index (WPSI) is a measurement of women's rights and opportunities in the United states. It examines how women's legal protections vary by state, and how their rights and opportunities vary based on their race. The index incorporates three basic dimensions of women's well-being: inclusion, justice, and security. Inclusion includes economic, social, and political aspects, justice includes formal laws and informal discrimination, and security includes the family, community, and societal levels.

In [1], states are ranked with respect to the best states to work. In [2], states are ranked with respect to the peace and security for women. The rankings can be found in Tables 1 - 6. We determine the fuzzy similarity measure of these two rankings. We find the similarity to be high. We then break the United States into regions and determine the fuzzy similarity measure of these two rankings for each region. Similarity plays

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a role in many fields. There exists many special definitions of similarity which have been used in different areas. We choose to use fuzzy similarity measures which seem appropriate in rankings. In particular, we use the t-norm algebraic product and the t-conorm, algebraic sum.

Let X be a set with n elements. We let $\mathcal{FP}(X)$ denote the fuzzy power set of X. We let $\wedge$ denote minimum and $\vee$ maximum. For two fuzzy subsets μ, ν of X, we write $\mu \subseteq \nu$ if $\mu(x) \leq \nu(x)$ for all $x \epsilon X$. If μ is a fuzzy subset of X, we let μ^c denote the complement of μ, i.e., $\mu^c(x) = 1 - \mu(x)$ for all $x \epsilon X$.

Let A be a one-to-function of X onto $\{1, 2, ..., n\}$. Then A is called a **ranking** of X. Define the fuzzy subset μ_A of X by for all $x \epsilon X, \mu_A(x) = \frac{A(x)}{n}$. Then μ_A is called the **fuzzy subset associated** with A. For A a ranking of X, we have $\sum_{x \epsilon X} A(x) = \frac{n(n+1)}{2}$ and $\sum_{x \epsilon X} \mu_A(x) = \frac{n+1}{2}$ since $\sum_{x \epsilon X} A(x) = 1 + 2 + ... + n$.

Throughout the paper, A and B will denote rankings of a set X with n elements.

2 Distance Functions and Fuzzy Similarity Measures

Let $\mathcal{T}$ be a t-norm and $\mathcal{S}_T$ a t-conorm. Then $\mathcal{T}$ and $\mathcal{S}_T$ are called **dual** if for all $a, b \epsilon [0, 1], \mathcal{T}(a, b) = 1 - \mathcal{S}_T(1 - a, 1 - b)$. Clearly, $\wedge$ are $\vee$ dual.

Definition 2.1. [4] Let $\mathcal{T}$ and $\mathcal{S}$ be a t-norm and t -conorm, respectively. Define the function $d : [0, 1] \times [0, 1] \to [0, 1]$ by $\forall a, b \epsilon [0, 1]$,

$$d(a,b) = \left\{ \begin{array}{c} \mathcal{S}(a,b) - \mathcal{T}(a,b) \text{ if } a \neq b, \\ 0 \text{ if } a = b. \end{array} \right|$$

Consider (4) in the following result. Suppose $a \leq b \leq c$. We show $\mathcal{S}(a, c) - \mathcal{T}(a, c) \leq \mathcal{S}(a, b) - \mathcal{T}(a, b) + \mathcal{S}(b, c) - \mathcal{T}(b, c)$. This is equivalent to $\mathcal{S}(a, c) + \mathcal{T}(a, b) + \mathcal{T}(b, c) \leq \mathcal{S}(a, b) + \mathcal{S}(b, c) + \mathcal{T}(a, c)$. Now $\mathcal{S}(a, c) \leq \mathcal{S}(b, c)$ and $\mathcal{T}(a, b) \leq \mathcal{T}(a, c)$. Also, $\mathcal{T}(b, c) \leq b \wedge c \leq b \leq a \vee b \leq \mathcal{S}(a, b)$.

Theorem 2.2. *[4] Let $\mathcal{T}$ and $\mathcal{S}$ be a t-norm and t -conorm, respectively. Let d be defined as in Definition 2.1. Then d satisfies the following properties: $\forall a, b, c \epsilon [0, 1]$,*

(1) $0 \leq d(a, b) \leq 1$;

(2) $d(a, b) = 0$ *if and only if* $a = b$;

(3) $d(a, b) = d(b, a)$;

(4) $d(a, c) \leq d(a, b) + d(b, c)$ *if* $b \wedge c \leq b \leq a \vee b$.

Let $\mathcal{T}$ and $\mathcal{S}$ be a given t-norm and t-norm, respectively. Let d be defined as in Definition 2.1. Define $D : \mathcal{FP}(X) \times \mathcal{FP}(X) \to [0, 1]$ by all $(\mu, \nu) \epsilon \mathcal{FP}(X) \times \mathcal{FP}(X), D(\mu, \nu) = \sum_{x \epsilon X} d(\mu(x), \nu(x))$.

Define $S : \mathcal{FP}(X) \times \mathcal{FP}(X) \to [0, 1]$ as follows: $\forall (\mu, \nu) \epsilon \mathcal{FP}(X) \times \mathcal{FP}(X), S(\mu, \nu) = 1 - D(\mu, \nu)$.Then $S(\mu, \rho) = 1 - D(\mu, \rho) \geq 1 - D(\mu, \nu) - D(\nu, \rho) = S(\mu, \nu) - D(\nu, \rho) = S(\nu, \rho) - D(\mu, \nu)$. Thus $S(\mu, \rho) \leq S(\mu, \nu)$ and $S(\mu, \rho) \leq S(\nu, \rho)$ if $\mu \subseteq \nu \subseteq \rho$.

We have that $D_H(\mu, \nu) = \frac{1}{n} \sum_{i=1}^{n} |\mu(x_i) - \nu(x_i)| = \frac{1}{n} \sum_{i=1}^{n} ((\mu(x_i) \vee \nu(x_i) - \mu(x_i) \wedge \nu(x_i))$. This motivates the consideration of the following definition. Let $f(x_i) = (\mu(x_i) \oplus \nu(x_i) - \mu(x_i) \otimes \nu(x_i)$ if $\mu(x_i) \neq \nu(x_i)$ and $f(x_i) = 0$ if $\mu(x_i) = \nu(x_i)$.

For all $\mu, \nu \epsilon \mathcal{FP}(X)$, define $D_{\otimes}(\mu, \nu) = \frac{1}{n} \sum_{i=1}^{n} f(x_i)$. Define $D_{\otimes}^{+}(\mu, \nu) = \frac{1}{n} \sum_{i=1}^{n} ((\mu(x_i) \oplus \nu(x_i) - \mu(x_i) \otimes \nu(x_i))$. Then $D_{\otimes}^{+}(\mu, \nu) = D_{\otimes}(\mu, \nu) + \sum_{x \epsilon X^+} ((\mu(x) \oplus \nu(x) - \mu(x) \otimes \nu(x))$, where $X^+ = \{x \epsilon X | \mu(x) = \nu(x)\}$. We note that $D_{\otimes}^{+}(\mu, \nu)$ does not satisfy (2) of Theorem 2.2.

Define $S_{\otimes}(\mu, \nu) = 1 - D_{\otimes}(\mu, \nu)$ and $S_{\otimes}^{+}(\mu, \nu) = 1 - D_{\otimes}^{+}(\mu, \nu)$.

We first wish to determine the smallest value $S^+_\otimes(\mu_A,\mu_B)$ can be for a given X. The smallest value $S^+_\otimes(\mu_A,\mu_B)$ can be determined from the largest value $D^+_\otimes(\mu_A,\mu_B)$ can be. Now $\sum_{i=1}^n(\mu_A(x_i)+\mu_B(x_i))$ is the fixed value $n+1$.Hence the largest value for $D^+_\otimes(\mu_A,\mu_B)$ is determined from the smallest $\sum_{i=1}^n \mu_A(x_i)\mu_B(x_i)$ since $\sum_{i=1}^n(\mu_A(x_i)\oplus\mu_B(x_i)-\mu_A(x_i)\otimes\mu_B(x_i))=\sum_{i=1}^n(\mu_A(x_i)+\mu_A(x_i)-\mu_A(x_i)\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i))$.

The rankings $A:1,...,i,...,n$ and $B:n,...,n-i+1,...1$ yield the smallest value for $\sum_{i=1}^n \mu_A(x_i)\mu_B(x_i)$. We have

$$\begin{aligned}\sum_{i=1}^n A(x_i)B(x_i) &= \sum_{i=1}^n i(n-i+1)\\ &= (n+1)\sum_{i=1}^n i-\sum_{i=1}^n i^2\\ &= \frac{(n+1)n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}\\ &= n[\frac{n^2+2n+1}{2}-\frac{2n^2+3n+1}{6}\\ &= n[\frac{1}{6}n^2+\frac{1}{2}n+\frac{1}{3}].\end{aligned}$$

Thus

$$\begin{aligned}\sum_{i=1}^n \mu_A(x_i)\mu_B(x_i) &= \frac{1}{n^2}n[\frac{1}{6}n^2+\frac{1}{2}n+\frac{1}{3}]\\ &= \frac{1}{6}n+\frac{1}{2}+\frac{1}{3n}.\end{aligned}$$

Hence

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^n(\mu_A(x_i)+\mu_B(x_i)-2\mu_A(x_i)\mu_B(x_i)) &= \frac{1}{n}\sum_{i=1}^n(\mu_A(x_i)+\mu_B(x_i)\\ &\quad -2\frac{1}{n}\sum_{i=1}^n \mu_A(x_i)\mu_B(x_i))\\ &= \frac{1}{n}[n+1-2(\frac{1}{6}n+\frac{1}{2}+\frac{1}{3n})]\\ &= 1+\frac{1}{n}-\frac{1}{3}-\frac{1}{n}-\frac{2}{3n^2}\\ &= \frac{2}{3}-\frac{2}{3n^2}.\end{aligned}$$

Thus the smallest value $S^+_\otimes(\mu_A,\mu_B)$ can be is $1-(\frac{2}{3}-\frac{2}{3n^2})=\frac{1}{3}+\frac{2}{3n^2}$.

We have just proved the following result.

Theorem 2.3. *Thus the smallest value $S^+_\otimes(\mu_A,\mu_B)$ can be is $1-(\frac{2}{3}-\frac{2}{3n^2})=\frac{1}{3}+\frac{2}{3n^2}$.*

Theorem 2.4 ([5], Theorem 3.5)**.** *If n is even, the smallest value $S_H(\mu_A,\mu_B)$ can be is $\frac{1}{2}$.*
If n is odd, the smallest value $S_H(\mu_A,\mu_B)$ can be is $\frac{1}{2}+\frac{1}{2n^2}$.

Example 2.5. Let $n = 3$. Consider the rankings $A : 1, 2, 3$ and $B : 3, 2, 1$. Then $S^+_\otimes(\mu_A, \mu_B) = 1 - \frac{1}{3}(\frac{6+6}{3} - 2\frac{3+4+3}{9}) = 1 - \frac{1}{3}(4 - \frac{20}{9}) = \frac{11}{27}$. Using the above result, $S^+_\otimes(\mu_A, \mu_B) = \frac{1}{3} + \frac{2}{3n^2}$, we obtain $\frac{1}{3} + \frac{2}{27} = \frac{11}{27}$.

Theorem 2.6. *[4] Let $\mathcal{T}$ and $\mathcal{S}_T$ be a dual t-norm and t -conorm, respectively. Let d be defined as in Definition 2.1. Then (4) of Theorem 2.2 holds.*

Recall that $X^+ = \{x\epsilon X | \mu_A(x) = \mu_B(x)\}$ for given μ_A, μ_B.

Theorem 2.7. *Let $s^+_\otimes$ be the smallest value $S^+_\otimes(\mu_A, \mu_B)$ can be. Then $s_\otimes = s^+_\otimes + \sum_{x\epsilon X^+}((\mu_A(x) \oplus_B (x) - \mu_A(x) \otimes \mu_B(x))$ is the smallest value $S_\otimes(\mu_A, \mu_B)$ can be, where $S_\otimes = 1 - D_\otimes$.*

Proof. Recall $D^+_\otimes(\mu_A, \mu_B) = D_\otimes(\mu_A, \mu_B) + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x))$. Now $S^+_\otimes = 1 - D^+_\otimes$ and $S_\otimes = 1 - D_\otimes$. Let $s_\otimes$ be the smallest value $S_\otimes(\mu_A, \mu_B)$ can be. Now $S^+_\otimes(\mu_A, \mu_B) = 1 - D^+_\otimes(\mu_A, \mu_B) = 1-(D_\otimes(\mu_A, \mu_B)+\sum_{x\epsilon X^+}((\mu_A(x)\oplus\mu_B(x)-\mu_A(x)\otimes\mu_B(x)) = S_\otimes(\mu_A, \mu_B)+\sum_{x\epsilon X^+}((\mu_A(x)\oplus\mu_B(x)-\mu_A(x)\otimes \mu_B(x))$. Now $s^+_\otimes = s + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x))$ for some s determine by $D_\otimes$. Then $s \geq s_\otimes$. Suppose $s > s_\otimes$. Then $s^+_\otimes = s + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x) > s_\otimes + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x))$, a contradiction. Thus $s = s_\otimes$. Hence $s^+_\otimes = s_\otimes + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x))$. □

Theorem 2.8. *The largest value $S^+_\otimes(\mu_A, \mu_B)$ can be is $\frac{2}{3} + \frac{1}{3n^2}$.*

Proof. We first find the smallest $D^+_\otimes(\mu_A, \mu_B)$ can be. This value is determined from the rankings $A : 1, 2, ..., n$ and $B : 1, 2, ..., n$. We have

$$\begin{aligned}
\frac{1}{n}\sum_{i=1}^{n}(\frac{i}{n} + \frac{i}{n} - 2\frac{i}{n}\frac{i}{n}) &= \frac{1}{n}\sum_{i=1}^{n}\frac{2i}{n} - \frac{2}{n}\sum_{i=1}^{n}\frac{i^2}{n^2} \\
&= \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{2}{n^3}\sum_{i=1}^{n} i^2 \\
&= \frac{2}{n^2}(\frac{n(n+1)}{2} - \frac{2}{n^3}(\frac{n(n=1)(2n+1)}{6} \\
&= \frac{n+1}{n} - \frac{1}{n^2}\frac{(n+1)(2n+1)}{3} \\
&= \frac{n+1}{n} - \frac{1}{3n^2}(2n^2 + 2n + 1) \\
&= 1 + \frac{1}{n} - \frac{2}{3} - \frac{1}{n} - \frac{1}{3n^2} \\
&= \frac{1}{3} - \frac{1}{3n^2}.
\end{aligned}$$

Thus the largest value $S^+_\otimes(\mu_A, \mu_B)$ can be is $1-(\frac{1}{3} - \frac{1}{3n^2}) = \frac{2}{3} + \frac{1}{3n^2}$. □

Consider Theorems 2.4, 2.7, and 2.8. Suppose that s denotes the smallest value for some fuzzy similarity measure S and l the largest. Define

$$\widehat{S}(\mu_A, \mu_B) = \frac{S(\mu_A, \mu_B) - s}{l - s}.$$

Then $\widehat{S}(\mu_A, \mu_B)$ varies between 0 and 1. For values between 0 and 0.2, we say that the fuzzy similarity is very low, between 0.2 and 0.4 low, between 0.4 and 0.6 medium, between 0.6 and 0.8 high, and between 0.8 and 1 very high. Some related work can be seen in [6].

3 United States

We determine fuzzy similarity measures for the rankings, best states for women and the peace and security index for the United States.

:

Table 1: United States

State	Women	WPSI	State	Women	WPSI
Oregon	1	18	Florida	26	30
California	2	15	Michigan	27	21
New York	3	8	Missouri	28	38
Washington	4	24	South Dakota	29	29
Connecticut	5	2	Indiana	30	34
Massachusetts	6	1	Ohio	31	25
New Jersey	7	11	Iowa	32	23
Nevada	8	35	Idaho	33	39
Colorado	9	14	Pennsylvania	34	17
Hawaii	10	10	Kentucky	35	47
Puerto Rico			Oklahoma	36	42
Illinois	11	13	Wisconsin	37	16
District of Columbia	12	3	North Dakota	38	20
Vermont	13	4	Kansas	39	26
Maine	14	9	Arizona	40	31
Rhode Island	15	5	Louisiana	41	51
New Mexico	16	40	Arkansas	42	49
Minnesota	17	12	West Virginia	43	46
Maryland	18	7	Utah	44	36
Virginia	19	27	Wyoming	45	43
Delaware	20	22	South Carolina	46	44
Alaska	21	28	Texas	47	41
Nebraska	22	19	Mississippi	48	50
Montana	23	32	Alabama	49	48
Tennessee	24	45	Georgia	50	37
New Hampshire	25	6	North Carolina	51	33

We consider $D_H(\mu_A,\mu_B) = \frac{1}{n}\sum_{x\epsilon X}|\mu_A(x) - \mu_B(x)|$. Here $n = 51$. We find $D_H(\mu_A,\mu_B) = \frac{1}{51}\frac{456}{51} = \frac{456}{2601} = 0.1753$. Thus $S_H(\mu_A,\mu_B) = 1 - D_H(\mu_A,\mu_B) = 0.8247$.

By Theorem 2.4, the smallest $S_H(\mu_A,\mu_B)$ can be is $\frac{1}{2} + \frac{1}{2n^2} = \frac{1}{2} + \frac{1}{5202} = 0.5002$. Thus $\widehat{S_H}(\mu_A,\mu_B) = \frac{0.8247-0.5002}{1-0.5002} = \frac{0.3245}{0.4998} = 0.6495$. The fuzzy similarity measure is high.

We now consider $D_\otimes(\mu_A,\mu_B) = \frac{1}{n}\sum_{x\epsilon X^+}(\mu_A(x)\oplus\mu_B(x) - \mu_A(x)\otimes\mu_B(x))$. We first see that $\mu_A(\text{Hawaii}) = \mu_B(\text{Hawaii})$ and $\mu_A(\text{South Dakota}) = \mu_B(\text{South Dakota})$. We find

$$\begin{aligned} D_\otimes(\mu_A,\mu_B) &= \frac{1}{51}\left(\frac{2574}{51} - 2\frac{41497}{51^2}\right) \\ &= \frac{2574}{2601} - \frac{82994}{132651} = 0.9896 - 0.6257 = 0.3639. \end{aligned}$$

Thus $S_\otimes(\mu_A,\mu_B) = 1 - 0.3639 = 0.6361$.

By Theorem 2.3, the smallest $S^+_\otimes(\mu_A, \mu_B)$ can be is $\frac{1}{3} + \frac{2}{3n^2} = \frac{1}{3} + \frac{2}{7803} = 0.3333 + .00003 = 0.3336$. Thus the smallest $S_\otimes(\mu_A, \mu_B)$ can be is $0, 3336 + 0.0062 + 0.0202 = 0.3600$.

By Theorem 2.8, the largest $S^+_\otimes(\mu_A, \mu_B)$ can be is $\frac{2}{3} + \frac{2}{3n^2} = \frac{2}{3} + \frac{2}{7803} = 0.6667 + 0.0003 = 0.6670$. Hence the largest $S_\otimes(\mu_A, \mu_B)$ can be is $0, 6670 + 0.0062 + 0.0202 = 0.6734$.

Thus $\widehat{S_\otimes} = \frac{06361-0.3600}{0.6734-0.3600} = \frac{0.2761}{0.3134} = 0.8810$. The fuzzy similarity measure is very high.

4 Regions

Suppose $\mu_A(x) = \mu_B(x) = 1$ for some $x \epsilon X$. Then $\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x) = \mu_A(x) + \mu_B(x) - 2_A(x)\mu_B(x) = 0$. Thus $S_\otimes(\mu_a, \mu_B) = S^+_\otimes(\mu_a, \mu_B)$ if this is the only x in X such that $\mu_A(x) = \mu_B(x)$. Thus we have $S_\otimes(\mu_A, \mu_B) = S^+_\otimes(\mu_A, \mu_B)$ for the following region.

Table 2: West

State	Women	WPSI
Oregon	1	4
California	2	3
Montana	3	7
Washington	4	5
Nevada	5	8
Colorado	6	2
Hawaii	7	1
Alaska	8	6
Idaho	9	10
Utah	10	9
Wyoming	11	11

Here $n = 11$. $S_H(\mu_A, \mu_B) = 1 - \frac{26}{121} = 1 - 0.2149 = 0.7851$. The smallest $S_H(\mu_A, \mu_B)$ can be is $\frac{1}{2} + \frac{1}{2n^2} = 0.5 + \frac{1}{242} = 0.5041$. Thus $\widehat{S_H}(\mu_A, \mu_B) = \frac{0.7851-0.5041}{1-0.5041} = \frac{0.2810}{0.4959} = 0.5666$. The fuzzy similarity measure is medium.

We first note that $\mu_A(\text{Wyoming}) = \mu_B(\text{Wyoming})$. We have that

$$\begin{aligned} D_\otimes(\mu_A, \mu_B) &= \frac{1}{11}(\frac{110}{11} - 2\frac{338}{121}) \\ &= \frac{110}{121} - \frac{676}{1331} = 0.9091 - 0.5079 = 0.4012. \end{aligned}$$

Thus $S_\otimes(\mu_A, \mu_B) = 1 - 0.4012 = 0.5988$. By Theorem 2.3, the smallest $S_\otimes(\mu_A, \mu_B)$ can be is $\frac{1}{3} + \frac{2}{3n^2} = \frac{1}{3} + \frac{2}{363} = 0.3333 + .00055 = 0.3355$.

By Theorem 2.8, the largest $S(\mu_A, \mu_B)$ can be is $\frac{2}{3} + \frac{2}{3n^2} = \frac{2}{3} + \frac{2}{363} = 0.6667 + 0.0055 = 0.6814 = 0.6612$. Thus $\widehat{S_\otimes} = \frac{0.5988-0.3355}{0.6612-0.335} = \frac{0.2233}{0.3257} = 0.6856$. The fuzzy similarity measure is high.

Table 3: Southwest

State	Women	WPSI
New Mexico	1	2
Oklahoma	2	4
Arizona	3	1
Texas	4	3

Here $n = 4$. $S_H = 1 - \frac{6}{16} = 1 - 0.3750 = 0.6250$. The smallest S_H can be is $\frac{1}{2} = 0$. Thus $\widehat{S_H} = \frac{0.6250-0.500}{1-0.5000} = \frac{0.1250}{0.5000} = 0.2500$. The fuzzy similarity measure is low.

We have that $D_\otimes(\mu_A, \mu_B) = \frac{1}{4}(\frac{20}{4} - 2\frac{25}{16}) = \frac{20}{16} - \frac{50}{64} = 1.25 - 0.7812 = 0.4688$. Hence $S_\otimes(\mu_A, \mu_B) = S_\otimes^+(\mu_A, \mu_B) = 1 - 0.4688 = 0.5412$. By Theorem 2.3, the smallest $S_\otimes^+(\mu_A, \mu_B)$ can be is $\frac{1}{3} + \frac{2}{3n^2} = \frac{1}{3} + \frac{2}{48} = 0.3333 + .0147 = 0.3480$

By Theorem 2.8, the largest $S(\mu_A, \mu_B)$ can be is $\frac{2}{3} + \frac{2}{3n^2} = \frac{2}{3} + \frac{2}{48} = 0.6667 + 0.0417 = 0.6814$.

Thus $\widehat{S_\otimes}(\mu_A, \mu_B) = \frac{0.5412-0.3480}{0.6814-0.3480} = \frac{0.1932}{0.3334} = 0.5895$. The fuzzy similarity measure is medium.

Table 4: Midwest

State	Women	WPSI
Illinois	1	2
Minnesota	2	1
Nebraska	3	4
Michigan	4	6
Missouri	5	12
South Dakota	6	10
Indiana	7	11
Ohio	8	8
Iowa	9	7
Wisconsin	10	3
North Dakota	11	5
Kansas	12	9

Here $n = 12$. $S_H(\mu_A, \mu_B) = 1 - \frac{38}{144} = 1 - 0.2639 = 0.7361$. The smallest S_H can be is $\frac{1}{2} = 0.5000$. Thus $\widehat{S_H}(\mu_A, \mu_B) = \frac{0.7361-0.5000}{1-0.5000} = \frac{0.2361}{0.5000} = 0.4722$. The fuzzy similarity measure is medium.

We first note that $\mu_A(\text{Ohio}) = \mu_B(\text{Ohio})$. We have that

$$
\begin{aligned}
D_\otimes(\mu_A, \mu_B) &= \frac{1}{12}(\frac{140}{12} - 2\frac{493}{144}) \\
&= \frac{140}{144} - \frac{986}{1728} \\
&= 0.9722 - 5706 \\
&= 0.4016.
\end{aligned}
$$

Thus $S_\otimes(\mu_A, \nu_B) = 1 - 0.4016 = 0.5984$. By Theorem 2.3, the smallest $S_\otimes(\mu_A, \mu_B)$ can be is $s_\otimes^+ + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x)) = \frac{1}{3} + \frac{2}{3n^2} + 0.00043 + 0.0371 = \frac{1}{3} + \frac{2}{432} + 0.0371 = \frac{1}{3} + 0.0036 + 0.0371 = 0.3333 = 0.0046 = 0.371 = 0.3750$, where $s_\otimes^+$ is the smallest $S_\otimes^+(\mu_A, \mu_B)$ can be.

By Theorem 2.8, the largest $S_\otimes(\mu_A, \mu_B)$ can be is $l_\otimes^+ + \sum_{x\epsilon X^+}((_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x)) = \frac{2}{3} + \frac{2}{3n^2} + 0.0370 + 0.0210 = \frac{2}{3} + \frac{2}{432} + 0.0307 = 0.6667 + 0.0046 + 0.0370 = 0.7083$, where $l_\otimes^+$ is the largest $S_\otimes^+(\mu_A, \mu_B)$ can be.

Thus $\widehat{S_\otimes}(\mu_A, \mu_B) = \frac{0.5984-0.3750}{0.7983-0.3750} = \frac{0.2234}{0.3333} = 0.6703$. The fuzzy similarity measure is high.

Table 5: Southeast

State	Women	WPSI
Puerto Rico		
Washington D. C.	1	1
Virginia	2	2
Tennessee	3	7
Florida	4	3
Kentucky	5	9
Louisiana	6	13
Arkansas	7	11
West Virginia	8	8
South Carolina	9	6
Mississippi	10	12
Alabama	11	10
Georgia	12	5
North Carolina	13	4

Here $n = 13$. $S_H(\mu_A, \mu_B) = 1 - \frac{42}{169} = 1 - 0.2485 = 0.7515$. The smallest $S_H(\mu_A, \mu_B)$ can be is $\frac{1}{2} + \frac{1}{2n^2} = 0.5 + \frac{1}{338} = 0.5030$. Thus $\widehat{S_H}(\mu_A, \mu_B) = \frac{0.7515-0.5030}{1-0.5030} = \frac{0.2485}{0.4970} = 0.5000$. The fuzzy similarity measure is medium.

We first note that μ_A(Washington D. C.) $= \mu_B$(Washington D. C.), μ_A(Virginia) $= \mu_B$(Virginia), and μ_A(West Virginia) $= \mu_B$(West Virginia). We have that

$$\begin{aligned}
D_{\otimes}(\mu_A, \mu_B) &= \frac{1}{14}(\frac{140}{14} - 2\frac{629}{196}) \\
&= \frac{140}{196} - \frac{1258}{2744} \\
&= 0.7143 - 4585 \\
&= 0.2558.
\end{aligned}$$

Thus $S_{\otimes}(\mu_A, \mu_B) = 1 - 0.2558 = 0.7442$. By Theorem 2.3, the smallest $S_{\otimes}(\mu_A, \nu_B)$ can be is $s_{\otimes}^{+} + \sum_{x \epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x)) = \frac{1}{3} + \frac{2}{3n^2} + 0.00043 + 0.0210 = \frac{1}{3} + \frac{2}{507} + 0.0253 = \frac{1}{3} + 0.0039 + 0.0252 = 0.3333 + 0.3625$, where $s_{\otimes}^{+}$ is the smallest $S_{\otimes}^{+}(\mu_A, \mu_B)$ can be.

By Theorem 2.8, the largest $S_{\otimes}(\mu_A, \mu_B)$ can be is $l_{\otimes}^{+}(\mu_A, \mu_B) + \sum_{x \epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_a(x) \otimes \mu_B(x)) = \frac{2}{3} + \frac{2}{3n^2} + 0.0043 + 0.0210 = \frac{2}{3} + \frac{2}{507} + 0.0253 = 0.6667 + 0.0039 + 0.0253 = 0.6958$, where $l_{\otimes}^{+}$ is the largest $S_{\otimes}^{+}(\mu_A, \mu_B)$ can be.

Thus $\widehat{S_{\otimes}}(\mu_A, \mu_B) = \frac{0.6422-0.3625}{0.6958-0.3625} = \frac{0.2797}{0.3333} = 0.8392$. The fuzzy similarity measure is very high.

Table 6: Northeast

State	Women	WPSI
New York	1	7
Connecticut	2	2
Massachusetts	3	1
New Jersey	4	9
Vermont	5	3
Maine	6	8
Rhode Island	7	4
Maryland	8	6
Delaware	9	11
New Hampshire	10	5
Pennsylvania	11	10

Here $n = 11$. $S_H(\mu_A, \mu_B) = 1 - \frac{35}{121} = 1 - 0.2893 = 0.7107$. The smallest $S_H(\mu_A, \mu_B)$ can be is $\frac{1}{2} + \frac{1}{2n^2} = 0.5 + \frac{1}{242} = 0.5041$. Thus $\widehat{S_H}(\mu_A, \mu_B) = \frac{0.7107-0.5041}{1-0.5041} = \frac{0.2066}{0.4959} = 0.4166$. The fuzzy similarity measure is medium.

We first note that μ_A(Connecticut) $= \mu_B$(Connecticut). We have that

$$\begin{aligned} D_{\otimes}(\mu_A, \mu_B) &= \frac{1}{11}(\frac{128}{11} - 2\frac{444}{121}) \\ &= \frac{128}{121} - \frac{888}{1331} \\ &= 1.0579 - 0.6672 \\ &= 0.3907. \end{aligned}$$

Thus $S_{\otimes}(\mu_A, \mu_B) = 1 - 0.3907 = 0.6093$. By Theorem 2.3, the smallest $S_{\otimes}(\mu_A, \mu_B)$ can be is $s^+_{\otimes} + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x)) = \frac{1}{3} + \frac{2}{3n^2} + 0.0271 = \frac{1}{3} + \frac{2}{363} + 0.0271 = \frac{1}{3} + 0.0055 + 0.0271 = 0.3659$, where $s^+_{\otimes}$ is the smallest $S^+_{\otimes}(\mu_A, \mu_B)$ can be.

By Theorem 2.8, the largest $S_{\otimes}(\mu_A, \mu_B)$ can be is $l^+_{\otimes} + \sum_{x\epsilon X^+}((\mu_A(x) \oplus \mu_B(x) - \mu_A(x) \otimes \mu_B(x)) = \frac{2}{3} + \frac{2}{3n^2} + 0.0271 = 0.6667 + 0.0033 + 0.0271 = 0.6993$, where $l^+_{\otimes}$ is the largest $S^+_{\otimes}(\mu_A, \mu_B)$ can be.

Thus $\widehat{S_{\otimes}}(\mu_A, \mu_B) = \frac{0.6093-0.3659}{0.6993-0.3659} = \frac{0.2434}{0.3334} = 0.7301$.The fuzzy similarity measure is high.

5 Conclusion

In this paper, we used two fuzzy similarity measures of the rankings best states for women to work and the peace and security of women. We accomplished this for the United States in general and for various regions of the U. S We found the similarity to be medium to high for one fuzzy similarity measures and high to very high for another. Additional results on the best places for women to work can be found in [5].

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