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Inventory Problems and the Parametric Measure m_λ

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(This paper is dedicated to Professor "John N. Mordeson" on the occasion of his 91st birthday.)

Abstract. The credibility theory was introduced by B. Liu as a new way to describe the fuzzy uncertainty. The credibility measure is the fundamental notion of the credibility theory. Recently, L.Yang and K. Iwamura extended the credibility measure by defining the parametric measure m_λ (λ is a real parameter in the interval $[0, 1]$ and for $\lambda = 1/2$ we obtain as a particular case the notion of credibility measure).

By using the m_λ- measure, we studied in this paper a risk neutral multi-item inventory problem. Our construction generalizes the credibilistic inventory model developed by Y. Li and Y. Liu in 2019. In our model, the components of demand vector are fuzzy variables and the maximization problem is formulated by using the notion of m_λ–expected value.

We shall prove a general formula for the solution of optimization problem, from which we obtained effective formulas for computing the optimal solutions in the particular cases where the demands are trapezoidal and triangular fuzzy numbers. For $\lambda = 1/2$ we obtain as a particular case the computation formulas of the optimal solutions of the credibilistic inventory problem of Li and Liu. These computation formulas are applied for some m_λ- models obtained from numerical data.

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1 Introduction

Let us consider a company that produces several types of goods (items). It will be assumed that buyers can order in advance. An inventory problem is a mathematical model that describes the management of this company.

There are mathematical models in which the management activity of the company is carried out in a single period and models with several periods. The inventory models can also be classified according to the attitude towards risk of a decision-maker: there are models in which the decision maker has a risk-averse attitude and models in which his attitude is neutral.

The mathematical formulation of the inventory model starts from the following initial data (model parameters) : $c_1, \ldots, c_n$ are unit fixed costs per inventoried item, $d_1, \ldots, d_n$ are unit revenues per inventoried item and $h_1, \ldots, h_n$ are unit holding costs per inventoried item.

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The demands are mathematically modeled by the variables $D_1, \cdots, D_n$; the order quantities will be the variables $x_1, \cdots, x_n$. The total profit from the sale of the n types of goods will have the following expression:: $\pi(\vec{x}, \vec{D}) = \sum_{i=1}^{n}(d_i x_i - c_i - \frac{h_i x_i^2}{2D_i})$ (see [23], [18]).

In a neutral inventory problem, one will determine those values of $x_1, \cdots, x_n$ for which the total profit $\pi(\vec{x}, \vec{D}) = \sum_{i=1}^{n}(d_i x_i - c_i - \frac{h_i x_i^2}{2D_i})$ is maximal. When making the decision the risk is taken into account, the values of $x_1, \cdots, x_n$ will be determined so that at the same time the maximum profit is achieved, and the risk (represented by various mathematical concepts) to be minimal.

The formulation of an inventory problem depends on how the demands $D_1, \cdots, D_n$ are modeled, as well as on how profit maximization and risk minimization are evaluated. The classical treatment of inventory problems is a probabilistic one: the demands $D_1, \cdots, D_n$ are random variables and, for risk - neutral models, the objective function of the maximization problem is the expected value of the total profit. In the case of a risk - averse attitude of the decision maker, several ways to describe the risk were proposed: in [6], [7] by means of mean-variance models and in [29] by using the value-at-risk (VaR) as a risk measure. In [1], the coherent risk measures [2] have been used in defining the objective function of an inventory problem. Using the multi-item inventory system introduced by Luciano et al. [29] (called, shortly, LCP-model), [3] developed several inventory problems, with decision-makers having various positions towards risk: from a neutral attitude to risk-averse attitude, corresponding to variance, mean-absolute deviation (MAD) and conditional value-at-risk (CVaR) as risk measures.

The credibility theory, specially developed by Liu in [27], is another way to model the fuzzy uncertainty. Its fundamental concept is the credibilistic measure [28] and its main indicators are the credibilistic expected value and the credibilistic variance (cf. [27], [28]). From the literature dedicated to the credibilistic treatment of inventory problems we mention the papers: [17], [19], [23], [30]. In this paper we will have as the starting point the papers [25], [26], [24] of Li and Liu: the first one concerns a multi - item inventory problem in which the decision - maker is neutral and the second one is a risk - averse inventory model. In both papers, the demands and the total profits are fuzzy variables and the expected profit is the credibilistic expected value of total profit. In [26] appears a risk evaluated by the notion of absolute semi - deviation.

In [33], Yang and Iwamura introduced a new measure m_λ as a convex linear combination of a possibility measure Pos and its associated necessity measure Nec (λ is a parameter in the interval $[0, 1]$). By using the measure m_λ, in [11] the notions of the expected value $E_\lambda(\xi)$ and the variance $Var_\lambda(\xi)$ of a fuzzy variable ξ are defined. These two indicators retain some algebraic properties of the possibilistic indicators corresponding to [27]. In this way, the credibility theory is enlarged to a new theory that models the fuzzy uncertainty (this will be named m_λ - theory). An issue that arises naturally is an m_λ–theory leading to the development of different economic and financial themes. Papers [20], [21], [16], [15] introduce new credibilistic real options models, which are based on the optimism-pessimism measure and interval–valued fuzzy numbers. The model outcomes are compared to the original credibilistic real options model through a numerical case example in a merger and acquisition context. Paper [11] applies m_λ–theory in the study of optimal portfolios when assets returns are described by triangular or trapezoidal fuzzy variables.

In this paper we shall study a multi - item risk neutral inventory problem in the framework of an m_λ - theory. We shall assume that the demands $D_1, \cdots, D_n$ are fuzzy variables and the criterion used in determination of the order quantities $x_1, \cdots, x_n$ is the maximization of the m_λ - expected value $\sum_{i=1}^{n}[d_i x_i - c_i - \frac{h_i x_i^2}{2} E_\lambda(\frac{1}{D_i})]$ of the total profit. We shall prove a general formula for computing the solution of optimization problem, of which we will then get formulas for effective computation of inventory problem solution whenever the demands are trapezoidal or triangular fuzzy numbers. For $\lambda = \frac{1}{2}$ we shall obtain as a particular case the credibilistic inventory problem of [25], as well as the form of its solution.

The paper is structured as follows. Section 2 contains introductory material on possibility and necessity measures, credibility measure and m_λ - measure, as well as on their relationship. In Section 3 we present the definition of the m_λ - expected value and some of its basic properties. Section 4 deals with the construction of a risk neutral inventory model whose objective function is defined by using the notion of m_λ - expected value. By using the linearity of m_λ–expected operator $E_\lambda(.)$, a general formula for the solution of the maximization problem is obtained. In Section 5 we proved some explicit formulas for this solution in the particular cases when the demands are trapezoidal and triangular fuzzy numbers. The proofs of these formulas are based on the form of m_λ–expected value $E_\lambda(A)$ of a trapezoidal fuzzy number A (see Proposition 3.4). Section 6 highlights how by applying the percentile method of Vercher et al. [32] we can build an inventory problem starting from a dataset. In this inventory problem, the components of a demand vector are trapezoidal fuzzy numbers, such that one can apply the formulas from Section 4 to compute the solution of the optimization problem.

2 Preliminaries

Let X be a universe whose elements can be individuals, objects, states, alternatives, etc. An events A is a subset of X: the set of events will be the family $P(X)$ of subsets of X. The complement of the event A will be denoted by A^c.

In this paper we shall assume that the elements of the universe X are real numbers ($X \subseteq \mathbb{R}$). A fuzzy variable will be an arbitrary function $\xi : X \to \mathbb{R}$.

The notions of possibility measure and necessity measure can be introduced both axiomatically and through a possibility distribution (cf. [34], [10], [13]).

A possibility measure on X is a function $Pos : \mathcal{P}(X) \to [0,1]$ such that

$(Pos1)$ $Pos(\emptyset) = 0$; $Pos(X) = 1$;

$(Pos2)$ $Pos(\bigcup_{i\in I} A_i) = sup_{i\in I} Pos(A_i)$, for any family $(A_i)_{i\in I}$ of events.

A necessity measure on X is a function $Nec : \mathcal{P}(X) \to [0,1]$ such that

$(Nec1)$ $Nec(\emptyset) = 0$; $Nec(X) = 1$;

$(Nec2)$ $Nec(\bigcap_{i\in I} A_i) = inf_{i\in I} Nec(A_i)$, for any family $(A_i)_{i\in I}$ of events.

The notions of possibility measure and necessity measure are dual: to each possibility measure Pos one can assign a necessity measure $Nec(A) = 1 - Pos(A^c)$ and, vice-versa, to each necessity measure Nec one can assign a possibility measure $Pos(A) = 1 - Nec(A^c)$.

Given a possibility measure Pos on the universe X, for any parameter $\lambda \in [0,1]$ consider the function $m_\lambda : P(X) \to [0,1]$ defined by

$$m_\lambda(A) = \lambda Pos(A) + (1-\lambda) Nec(A), \tag{1}$$

for any event A;

(Nec is here the necessity measure associated with Pos).

This new measure was introduced by Yang and Iwamura in [33] as a convex linear combination of Pos and Nec by means of the weight λ. If $\lambda = \frac{1}{2}$ then one obtains the notion of credibility measure in the sense of Liu's monograph [27]:

$$Cred(A) = \frac{1}{2}(Pos(A) + Nec(A)), \tag{2}$$

for any event A.

A possibilistic distribution on X is a function $\mu : X \to [0,1]$ such that $sup_{x\in X} = 1$; μ is normalized if $\mu(x) = 1$ for some $x \in X$.

Let us fix a possibility distribution $\mu : X \to [0,1]$. Then one can associate with μ a possibility measure Pos and a necessity measure Nec by taking

$$Pos(A) = sup_{x \in A}\mu(x) \tag{3}$$

for any event A;

$$Nec(A) = inf_{x \in A}\mu(x) \tag{4}$$

for any event A.

Then for each parameter $\lambda \in [0,1]$, the measure m_λ defined by (1) will have the following form:

$$m_\lambda(A) = \lambda sup_{x \in A}\mu(x) + (1-\lambda) inf_{x \in A}\mu(x), \tag{5}$$

for any event A.

According to [27], we say that the normalized possibility distribution μ is the membership function associated with a fuzzy variable ξ if for any event A we have

$$Pos(\xi \in A) = sup_{x \in A}\mu(x). \tag{6}$$

Then the following equalities hold:

$$Nec(\xi \in A) = inf_{x \in A}\mu(x); \tag{7}$$

$$m_\lambda(\xi \in A) = \lambda sup_{x \in A}\mu(x) + (1-\lambda) inf_{x \in A}\mu(x). \tag{8}$$

3 The Expected Value Associated with the Measure m_λ

We fix a parameter $\lambda \in [0,1]$ and assume that ξ is a fuzzy variable, μ is its membership function and m_λ is the measure defined in (5).

Following [11], the expected value of ξ w.r.t. the measure m_λ is defined by

$$E_\lambda(\xi) = \int_{-\infty}^{0} [m_\lambda(\xi \geq r) - 1]dr + \int_{0}^{\infty} m_\lambda(\xi \geq r)dr. \tag{9}$$

If $\lambda = \frac{1}{2}$ then one obtains the credibilistic expected value of ξ w.r.t. the credibility measure Cr defined in (2):

$$E_C(\xi) = \int_{0}^{\infty} Cr(\xi \geq r)dr - \int_{-\infty}^{0} Cr(\xi \leq r)dr. \tag{10}$$

The previous notion of credibilistic expected value was introduced by Liu and Liu in [28].

The following result shows that the expected operator $E_\lambda(\cdot)$ is linear.

Proposition 3.1. [11] *Let ξ_1, ξ_2 be two fuzzy variables such that $E_\lambda(\xi_1) < \infty$, $E_\lambda(\xi_2) < \infty$ and α, β are two non - negative real numbers. Then the following hold:*

$$E_\lambda(\xi_1 + \xi_2) = E_\lambda(\xi_1) + E_\lambda(\xi_2); \tag{11}$$

$$E_\lambda(\alpha\xi_1) = \alpha E_\lambda(\xi_1). \tag{12}$$

Lemma 3.2. *If $\xi > 0$ then $E_\lambda(\xi) = \int_0^\infty m_\lambda(\xi \geq r)dr$ and $E_\lambda(\xi) > 0$.*

According to [27], p.73, a trapezoidal fuzzy variable (= trapezoidal fuzzy number) $\xi = (r_1, r_2, r_3, r_4)$, with $r_1 \le r_2 \le r_3 \le r_4$, is defined by the following membership function:

$$\mu_\xi(x) = \begin{cases} \frac{x-r_1}{r_2-r_1} & r_1 \le x \le r_2, \\ 1 & r_2 \le x \le r_3, \\ \frac{x-r_3}{r_4-r_3} & r_3 \le x \le r_4, \\ 0 & otherwise. \end{cases} \tag{13}$$

If $r_2 = r_3$ then one obtains the triangular fuzzy number $\xi = (r_1, r_2, r_4)$.

Lemma 3.3. [11] *For any trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, r_4)$ we have:*

$$m_\lambda(\xi \le x) = \begin{cases} 1 & r_4 \le x, \\ \frac{\lambda(r_4-x)+x-r_1}{r_4-r_3} & r_3 \le x \le r_4, \\ \lambda & r_2 \le x \le r_3, \\ \frac{\lambda(x-r_1)}{r_2-r_1} & r_1 \le x \le r_2, \\ 0 & x \le r_1. \end{cases} \tag{14}$$

Proposition 3.4. [11] *For any trapezoidal fuzzy variable $\xi = (r_1, r_2, r_3, r_4)$ the expected value $E_\lambda(\xi)$ has the form*

$$E_\lambda(\xi) = (1-\lambda)\frac{r_1 + r_2}{2} + \lambda\frac{r_3 + r_4}{2}. \tag{15}$$

Corollary 3.5. *For any triangular fuzzy variable $\xi = (r_1, r_2, r_4)$ the expected value $E_\lambda(\xi)$ has the form*

$$E_\lambda(\xi) = (1-\lambda)\frac{r_1}{2} + \frac{r_2}{2} + \lambda\frac{r_4}{2}. \tag{16}$$

4 An Inventory Problem with Fuzzy Variables as Demands

This section concerns a risk - neutral multi - item inventory problem characterized by the following two hypotheses:

(I) the components of the demand vector are fuzzy variables;

(II) the objective function of the inventory model is defined by using the expected value operator $E_\lambda(\cdot)$ introduced in the previous section.

The inventory problem with n items has the following initial data:

- $c_1, \ldots, c_n$: unit fixed costs per inventoried item;
- $d_1, \ldots, d_n$: unit revenues per inventoried item;
- $h_1, \ldots, h_n$: unit holding costs per inventoried item;
- $\vec{D} = (D_1, \ldots, D_n)$: fuzzy demand vector in the inventory problem;
- $\vec{x} = (x_1, \ldots, x_n)$: order quantity vector in the inventory problem.

The components $D_1, \ldots, D_n$ of $\vec{D}$ are fuzzy variables. We shall assume that $c_i \ge 0$, $d_i \ge 0$ and $D_i > 0$, for all $i = 1, \ldots, n$.

Remark 4.1. *The initial data of the possibilistic inventory problem are similar to the probabilistic inventory problems from* [29], [3]*, the credibilistic inventory problems from* [25],[26] *and the possibilistic inventory problems from* [14].

We will further observe that the essential difference between the three types of models lies in the way of choosing the objective function of the optimization problem: the models in [29], [3] use the probabilistic expected value, those in [25], [26] use Liu credibilistic expected value [27] and those in [14] use the possibilistic expected value from [4].

Starting from above input data we will formulate a risk-neutral problem. Similar with [25], p. 132, the quantity $d_i x_i$ is the total revenue of the i^{th} item and the fuzzy variables $\frac{h_i x_i^2}{2}\frac{1}{D_i}$ is the holding cost of the i^{th} item.

We fix a parameter $\lambda \in [0,1]$, so we can use the expected value operator $E_\lambda(\cdot)$ defined in (9). According to Lemma 3.2, we remark that $E_f(\frac{1}{D_i}) > 0$, for all $i = 1, \ldots, n$.

The profit function of item i has the following form:

$$\pi_i(x_i, D_i) = d_i x_i - c_i - \frac{h_i x_i^2}{2}\frac{1}{D_i} \tag{17}$$

The total profit function of the possibilistic inventory problem has the following form:

$$\pi(\vec{x}, \vec{D}) = \sum_{i=1}^{n} \pi_i(x_i, D_i) = \sum_{i=1}^{n}(d_i x_i - c_i - \frac{h_i x_i^2}{2}\frac{1}{D_i}) \tag{18}$$

Then the optimization problem associated with the previous inventory model has the following form:

$$\begin{cases} \max\limits_{\vec{x}} E_\lambda(\pi(\vec{x}, \vec{D})) \\ \vec{x} \geq 0 \end{cases} \tag{19}$$

Remark 4.2. *The objective function in the optimization problem (19) is the expected value $E_\lambda(\pi(\vec{x}, \vec{D}))$ of the fuzzy variable $\pi(\vec{x}, \vec{D})$ (w.r.t. the measure m_λ).*

Remark 4.3. *For $\lambda = \frac{1}{2}$ we obtain as a particular case the credibilistic inventory problem studied in [25]:*

$$\begin{cases} \max\limits_{\vec{x}} E_\lambda(\pi(\vec{x}, \vec{D})) \\ \vec{x} \geq 0 \end{cases} \tag{20}$$

By applying Proposition 3.1 to (18), the expected value $E_\lambda(\pi(\vec{x}, \vec{D}))$ can be written

$$E_\lambda(\pi(\vec{x}, \vec{D})) = \sum_{i=1}^{n}[d_i x_i - c_i - \frac{h_i x_i^2}{2} E_\lambda(\frac{1}{D_i})] \tag{21}$$

hence the optimization problem (19) becomes

$$\begin{cases} \max\limits_{x_1,\ldots,x_n} \sum_{i=1}^{n}[d_i x_i - c_i - \frac{h_i x_i^2}{2} E_\lambda(\frac{1}{D_i})] \\ x_i \geq 0, i = 1, \ldots, n \end{cases} \tag{22}$$

The decision - maker aims to find the non - negative values $x_1, \ldots, x_n$ that maximize the expected total profit $E_\lambda(\pi(\vec{x}, \vec{D}))$.

In particular, setting $\lambda = \frac{1}{2}$ in (22) one obtains the credibilistic inventory problem from [25].

$$\begin{cases} \max\limits_{x_1,\ldots,x_n} \sum_{i=1}^{n}[d_i x_i - c_i - \frac{h_i x_i^2}{2} E_C(\frac{1}{D_i})] \\ x_i \geq 0, i = 1, \ldots, n \end{cases} \tag{23}$$

Proposition 4.4. *The optimization problem (22) has the following solution:*

$$x_i^* = \frac{d_i}{h_i E_\lambda(\frac{1}{D_i})}, \tag{24}$$

for $i = 1, \ldots, n$

Proof. In order to find the solution of the optimization problem (22) we write the first - order condition

$$\frac{\partial}{\partial x_i} \sum_{i=1}^{n} (d_i x_i - c_i - \frac{h_i x_i^2}{2} E_\lambda(\frac{1}{D_i})) = 0,$$

for $i = 1, \ldots, n$,
therefore by a simple computation we obtain the equations

$$d_i - h_i E_\lambda(\frac{1}{D_i}) x_i = 0, \tag{25}$$

for $i = 1, \ldots, n$.
We remind that $E_\lambda(\frac{1}{D_i}) > 0$ for $i = 1, \ldots, n$. Thus the solution of the optimization problem (13) will have the following form

$$x_i^* = \frac{d_i}{h_i E_f(\frac{1}{A_i})},$$

for $i = 1, \ldots, n$.
□

5 Solution Form when the Demands are Trapezoidal Fuzzy Variables

According to Proposition 4.4, in order to compute the values $(x_1^*, \ldots, x_n^*)$ of the solution of inventory problem (22) we need to compute the expected values $E_\lambda(\frac{1}{D_1}), \ldots, E_\lambda(\frac{1}{D_n})$. The computation of these expected values depends on the form of the fuzzy variables $D_1, \cdots, D_n$ and in most cases this operation seems to be very difficult. In this section we solve this problem whenever the demands $D_1, \cdots, D_n$ are trapezoidal or triangular fuzzy numbers. The formulas obtained for the computation of the optimal solutions $x_1^*, \ldots, x_n^*$ have simple algebraic forms which makes them very suitable from a computational point of view.

We will fix the parameter $\lambda \in [0, 1]$. The following proposition is a key result of this section: the application of the formula (26) will lead us to find the form of optimal solutions $x_1^*, \ldots, x_n^*$.

Proposition 5.1. *Let D be a trapezoidal fuzzy number $D = (r_1, r_2, r_3, r_4)$ such that $0 < r_1 \leq r_2 \leq r_3 \leq r_4$ then the expected value $E_\lambda(\frac{1}{D})$ has the following form*

$$E_\lambda(\frac{1}{D}) = \frac{\lambda}{r_2 - r_1} ln \frac{r_2}{r_1} + \frac{1-\lambda}{r_4 - r_3} ln \frac{r_4}{r_3} \tag{26}$$

Proof. Firstly we observe that the condition $0 < r_1$ means $D > 0$, hence one obtains $\frac{1}{D} > 0$. By using Lemma 3.3 we get the following equalities:

$$m_\lambda(\frac{1}{D} \geq r) = m_\lambda(D \leq \frac{1}{r}) = \begin{cases} 1 & r_4 \leq \frac{1}{r}, \\ \frac{\lambda(r_4 - \frac{1}{r}) + \frac{1}{r} - r_3}{r_4 - r_3} & r_3 \leq \frac{1}{r} \leq r_4, \\ \lambda & r_2 \leq \frac{1}{r} \leq r_3, \\ \frac{\lambda(\frac{1}{r} - r_1}{r_2 - r_1} & r_1 \leq \frac{1}{r} \leq r_2, \\ 0 & \frac{1}{r} \leq r_1. \end{cases}$$

which can be written as follows:

$$m_\lambda(\frac{1}{D} \geq r) = \begin{cases} 1 & r \leq \frac{1}{r_4}, \\ \frac{1}{r_4-r_3}[(1-\lambda)\frac{1}{r} + \lambda r_4 - r_3] & \frac{1}{r_4} \leq r \leq \frac{1}{r_3}, \\ \lambda & \frac{1}{r_3} \leq r \leq \frac{1}{r_2}, \\ \frac{\lambda}{r_2-r_1}[\frac{1}{r} - r_1] & \frac{1}{r_2} \leq r \leq \frac{1}{r_1}, \\ 0 & \frac{1}{r} \leq 0. \end{cases} \tag{27}$$

According to Lemma 3.2 we obtain

$$E_\lambda(\frac{1}{D}) = \int_0^\infty m_\lambda(\frac{1}{D} \geq r)dr = I_1 + I_2 + I_3 + I_4 \tag{28}$$

where I_1, I_2, I_3, I_4 have the following expressions:

$$I_1 = \int_0^{\frac{1}{r_4}} dr = \frac{1}{r_4}$$

$$I_2 = \frac{1}{r_4 - r_3}\int_{\frac{1}{r_4}}^{\frac{1}{r_3}} [\lambda(r_4 - \frac{1}{r}) + \frac{1}{r} - r_3]dr = \frac{1}{r_4 - r_3}[(1-\lambda)\ln\frac{r_4}{r_3} + (\lambda r_4 - r_3)(\frac{1}{r_3} - \frac{1}{r_4})]$$

$$I_3 = \lambda\int_{\frac{1}{r_3}}^{\frac{1}{r_2}} dr = \lambda(\frac{1}{r_2} - \frac{1}{r_3})$$

$$I_4 = \frac{\lambda}{r_2 - r_1}\int_{\frac{1}{r_2}}^{\frac{1}{r_1}} [\frac{1}{r} - r_1]dr = \frac{\lambda}{r_2 - r_1}[ln\frac{r_2}{r_1} - r_1(\frac{1}{r_1} - \frac{1}{r_2})]$$

Substituting in (28) these values of I_1, I_2, I_3, I_4 we get the formula (26).
□

Corollary 5.2. [25] *Let D be a trapezoidal fuzzy number $D = (r_1, r_2, r_3, r_4)$ such that $0 < r_1 \leq r_2 \leq r_3 \leq r_4$ then the credibilistic expected value $E_C(\frac{1}{D})$ has the following form*

$$E_C(\frac{1}{D}) = \frac{1}{2(r_2 - r_1)}ln\frac{r_2}{r_1} + \frac{1}{2(r_4 - r_3)}ln\frac{r_4}{r_3} \tag{29}$$

Proof. If we take $\lambda = \frac{1}{2}$ in (26) then we obtain the formula (29). □

Remark 5.3. *If in formula (26) one takes $r_2 = r_3$ then D is the triangular fuzzy number $D = (r_1, r_2, r_4)$ and the expected value $E_\lambda(\frac{1}{D})$ has the following form*

$$E_\lambda(\frac{1}{D}) = \frac{\lambda}{r_2 - r_1}ln\frac{r_2}{r_1} + \frac{1-\lambda}{r_4 - r_3}ln\frac{r_4}{r_2} \tag{30}$$

If in (30), we set $\lambda = \frac{1}{2}$ then we get the formula of the credibilistic expected value $E_\lambda(\frac{1}{D})$ from Theorem 2 of [25]:

$$E_C(\frac{1}{D}) = \frac{1}{2(r_2 - r_1)}ln\frac{r_2}{r_1} + \frac{1}{2(r_4 - r_2)}ln\frac{r_4}{r_2} \tag{31}$$

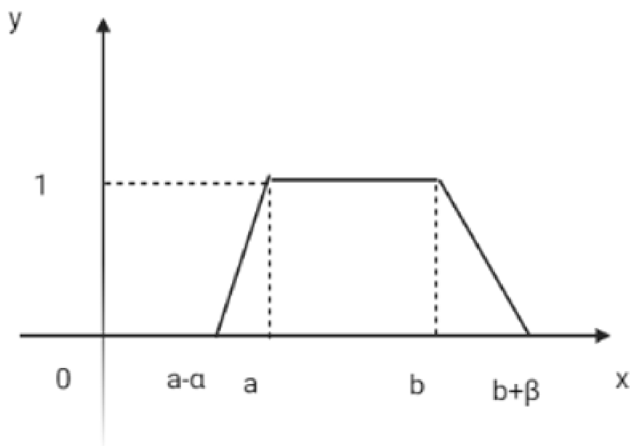

Figure 1: Trapezoidal fuzzy number

Remark 5.4. *Often in literature a trapezoidal fuzzy number D is given under the form $D = (a-\alpha, a, b, b+\beta)$, with $a, b \in \mathbb{R}$ and $\alpha, \beta \geq 0$ (Figure 1). Thus its membership μ_D has the form:*

$$\mu_D(x) = \begin{cases} 1 - \frac{a-x}{\alpha} & a-\alpha \leq x \leq a, \\ 1 & a \leq x \leq b, \\ 1 - \frac{x-b}{\beta} & b \leq x \leq b+\beta, \\ 0 & otherwise. \end{cases}$$

Assuming that $0 < a - \alpha$ we have $D > 0$ and the formula (26) becomes

$$E_\lambda(\frac{1}{D}) = \frac{\lambda}{\alpha} ln \frac{a}{a-\alpha} + \frac{1-\lambda}{\beta} ln \frac{b+\beta}{b} \tag{32}$$

Remark 5.5. *Assume that a triangular fuzzy number D is written under the form $D = (a-\alpha, a, a+\beta)$, with $a \in \mathbb{R}$ and $\alpha, \beta \geq 0$. If $0 < a - \alpha$ the formula (30) becomes*

$$E_\lambda(\frac{1}{D}) = \frac{\lambda}{\alpha} ln \frac{a}{a-\alpha} + \frac{1-\lambda}{\beta} ln \frac{a+\beta}{a} \tag{33}$$

The previous formulas (26), (30), (32) and (33) provide very computable expressions for the expected value $E_\lambda(\frac{1}{D})$ for the particular cases when D is a trapezoidal or a triangular fuzzy number.

By using these formulas we are now able to compute the solution $x_1^*, \cdots, x_n^*$ of the optimization problem (22) whenever the components $D_1, \ldots, D_n$ of demand vector are trapezoidal fuzzy numbers, respectively triangular fuzzy numbers.

Theorem 5.6. *Assume that the components $A_1, \ldots, A_n$ of demand vector $\vec{A}$ are trapezoidal fuzzy numbers $D_i = (a_i - \alpha_i, a_i, b_i, b_i + \beta_i)$, $i = 1, \ldots, n$, where $0 < a_i - \alpha_i \leq a_i \leq b_i \leq b_i + \beta_i$, for $i = 1, \ldots, n$. Then the solution of the optimization problem (13) has the following form*

$$x_i^* = \frac{d_i}{h_i[\frac{\lambda}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{1-\lambda}{\beta_i} ln \frac{b_i+\beta_i}{b_i}]}, \tag{34}$$

for all $i = 1, \ldots, n$.

Proof. By (32), for each $i = 1, \ldots, n$ we have

$$E_\lambda(\frac{1}{D_i}) = \frac{\lambda}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{1-\lambda}{\beta_i} ln \frac{b_i + \beta}{b_i}$$

If we substitute these values of $E_\lambda(\frac{1}{D_1}), \cdots, E_\lambda(\frac{1}{D_n})$ in (24) then we get the desired formula (34). □

Corollary 5.7. *If $D_1, \ldots, D_n$ are the triangular fuzzy numbers $D_i = (a_i - \alpha_i, a_i, a_i + \beta_i)$, $i = 1, \ldots, n$, where $0 < a_i - \alpha_i \leq a_i \leq b_i + \beta_i$, for $i = 1, \ldots, n$ then the solution of optimization problem (19) has the form*

$$x_i^* = \frac{d_i}{h_i[\frac{\lambda}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{1-\lambda}{\beta_i} ln \frac{a_i + \beta_i}{a_i}]}, \tag{35}$$

for all $i = 1, \ldots, n$.

Proof. If in (34) one sets $b = a$, then the formula (35) follows immediately. □

Now we shall write the formula (34) for the following particular values of λ:

(a) $\lambda = 1/3$ (the pessimistic case)

$$x_i^* = \frac{3d_i}{h_i[\frac{1}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{2}{\beta_i} ln \frac{b_i + \beta_i}{b_i}]}, \tag{36}$$

for all $i = 1, \ldots, n$.

(b) $\lambda = 1/2$ (the credibilistic case [25])

$$x_i^* = \frac{2d_i}{h_i[\frac{1}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{1}{\beta_i} ln \frac{b_i + \beta_i}{b_i}]}, \tag{37}$$

for all $i = 1, \ldots, n$.

(c) $\lambda = 2/3$ (the optimistic case)

$$x_i^* = \frac{3d_i}{h_i[\frac{2}{\alpha_i} ln \frac{a_i}{a_i - \alpha_i} + \frac{1}{\beta_i} ln \frac{b_i + \beta_i}{b_i}]}, \tag{38}$$

for all $i = 1, \ldots, n$.

6 A Numerical Example

In order to solve the optimization problems associated with some inventory models we should know the form of the variables $D_1, \cdots, D_n$ and of the (probabilistic, credibilistic, possibilistic, etc.) indicators that appear in models. In the examples of credibilistic inventory problems from [25], [26] the expressions of $D_1, \cdots, D_n$ are assumed to be trapezoidal fuzzy numbers.

In general, the mathematical expressions of $D_1, \cdots, D_n$ are not known, but through measurements can be found different values of them. In the numerical example of possibilistic inventory problem from [14] it started from a data table, then the method of Vercher et al. [32] was applied to determine the concrete form of fuzzy numbers $D_1, \cdots, D_n$.

In this section we will present the solution of an m_λ-inventory problem in which the initial information on the variables $D_1, \cdots, D_n$ (which in our case are trapezoidal numbers) is given in the form of a numerical table. In order to obtain the trapezoidal numbers that describe the demands $D_1, \cdots, D_n$ we will apply the sample percentile method of Vercher et al. [32].

Table 1: Data on demand vector

Item1	Item2	Item3	Item4	Item5	Item6	Item7	Item8	Item9	Item10
35	20	30	25	28	33	18	18	31	20
30	30	50	25	32	37	27	28	33	27
15	35	28	36	25	20	33	17	25	31
25	35	40	35	35	40	35	20	30	37
25	28	25	32	50	37	28	19	35	35
28	25	42	27	45	28	28	37	35	37
31	27	36	35	43	35	35	27	22	35
30	24	39	28	27	35	24	37	27	25
44	33	44	28	32	22	39	30	29	25
37	34	37	44	44	32	47	36	28	37
23	17	22	33	32	29	31	31	45	19

Table 2: Trapezoidal fuzzy numbers

A_1	A_2	A_3	A_4	A_5
(28,30,9,10.5)	(27,30,8.5,5)	(36,39,12.5, 5)	(28,32,3,4)	(32,35,6,10)

A_6	A_7	A_8	A_9	A_{10}
(32,35,11,2)	(28,33,7,6)	(27,30,9.5,7)	(29,31,5.5,4)	(27,35,7.5,2)

We continue with the presentation of the values of basic parameters d_i, c_i, d_i, so the inventory problem is entirely defined. Finally, we apply the formulas (36)- (38) in order to obtain the optimal solutions of the model.

Our inventory problem has a demand vector of size 10. Table 1 contains the data we have on demand vector.

In column i of Table 1 are placed the known values of item i. In a probabilistic inventory model, the above columns will contain values of random variables. In this case the maximization problem of the model will be obtained by usual statistical methods.

Under the hypothesis that the 10 items are modeled by trapezoidal fuzzy numbers, one has to convert the data from the above table in 10 such fuzzy numbers (each column is assigned to a trapezoidal fuzzy number).

Let's present shortly the percentile method of Vercher et al. [32], by which to a data set of real numbers $x_1, \ldots, x_m$ one assigns a trapezoidal fuzzy number $A = (a, b, \alpha, \beta)$.

Let us denote by P_k the k-the percentile of the sample $x_1, \ldots, x_m$. Then the trapezoidal fuzzy number $A = (a, b, \alpha, \beta)$ will be determined by the formulas:

$$a = P_{40}, b = P_{60}, \alpha = P_{40} - P_5, \beta = P_{95} - P_{60} \tag{39}$$

By applying Vercher et al.'s method [32] to each of the columns of Table 1 obtains the trapezoidal fuzzy numbers in Table 2.

The trapezoidal fuzzy numbers $A_1, \ldots, A_{10}$ obtained from Table 1 will be the components of the demand vector of a risk neutral multi–item inventory problem. This inventory problem will be defined by the data in the first five columns of Table 3:

Columns two, three and four of Table 3 contain the unit fixed costs, unit revenues and holding costs of the model. The trapezoidal fuzzy numbers from the fifth column make up the demand vector in the m_λ–inventory

Table 3: The elements of the inventory problem

Item	d_i	c_i	h_i	$A_i = (a_i, b_i, \alpha_i, \beta_i)$	$x_i^*(\lambda = 1/3)$	$x_i^*(\lambda = 1/2)$	$x_i^*(\lambda = 2/3)$
1	12	2	0.5	(28,30,9,10.5)	718.21	668.76	627.44
2	11	1	0.6	(27,30,8.5,5)	518.19	486.88	459.14
3	14	3	0.5	(36,39,12.5, 5)	1019.75	961.42	909.4
4	10	4	0.8	(28,32,3,4)	387.92	371.9	357.14
5	11	5	0.9	(32,35,6,10)	432.03	409.19	388.64
6	10	3	0.9	(32,35,11,2)	355.13	336.3	319.37
7	12	2	0.5	(28,33,7,6)	743.93	696.25	654.32
8	15	1	0.6	(27,30,9.5,7)	710.45	661.32	618.54
9	13	3	0.7	(29,31,5.5,4)	563.24	541.63	521.61
10	13	4	0.9	(27,35,7.5,2)	437.88	405.88	378.24

problem. In fact, for distinct parameters $\lambda \in [0, 1]$ we obtain distinct inventory problems. We consider the three inventory models (a)-(c) corresponding to the parameters 1/3, 1/2 and 2/3. By applying the formulas (36)-(38) we obtain the solutions of the three optimization problems. These solutions are placed in the last three columns of Table 3.

Remark 6.1. Regarding the last three columns of Table 3, it is noticed that with the increase of the parameter λ ($\frac{1}{3} < \frac{1}{2} < \frac{2}{3}$) the solution values of the optimization problem decrease. The theoretical argument of this fact is given by Proposition 8.2 in the Appendix.

7 Conclusion

In the work we studied a new inventory model whose construction is based on the parametric measure m_λ (introduced by Yang and Iwamura in [33]) and on the notion of m_λ–expected value (introduced by Dzouche et al. in [11]). More precisely, in this inventory model, the demands and the total profit are fuzzy variables and the objective function of the optimization problem is the m_λ–expected value of total profit. It was found the general form of the solution of the optimization problem and when the demands are trapezoidal or triangular fuzzy variables computationally efficient forms of the solution have been found.

An open problem is finding the calculation formulas for the optimal solutions also when the demands are represented by other types of fuzzy variables: discrete repartitions, Erlang fuzzy variables, etc.

The inventory model in the paper is risk–neutral. Another open problem is the study of risk–averse inventory models in the framework of m_λ–theory. It would also be interesting to treat some mean–value inventory model, in which besides maximizing the m_λ–expected value of the total profit to be required to minimize the m_λ–variance of the total profit (the notion of m_λ–variance has been defined in [11]). Defining a notion of mean–absolute deviation in the context of an m_λ–theory would lead to an inventory model in which the risk is eventually represented by this indicator.

Continuing the research line from [25], [26], in paper [24] is investigated an inventory problem in which the components of the demand vector are type-2 fuzzy variables. This model is studied with the techniques of Liu's credibility theory [27]. It arises naturally a question of extending this model to m_λ–theory, so that giving the parameter λ the value $\frac{1}{2}$ to obtain as a particular case some results of [24].

The newsvendor problem is a core concept in inventory management dealing with stochastic demand. Traditionally, it centers on a single goal: either minimizing expected costs or maximizing expected profits.

A mean–variance model for the newsvendor problem is presented in paper [8]. A newsvendor problem is studied in which the maximization of expected profit and the minimization of risk, expressed by the profit variance, are required. It would be interesting to formulate and study a newsvendor problem in which the expected profit is expressed by m_λ–expected value and the risk of profit by m_λ–variance (according to Definition 2 of [11]).

8 Appendix

One asks the question of how the solutions of the optimization problem (20) vary depending on the parameter λ. We will give a solution to this problem in case when the demands $D_1, \ldots, D_n$ are trapezoidal fuzzy numbers.

Lemma 8.1. *Assume that ξ is a trapezoidal fuzzy variable. If $\lambda_1 \le \lambda_2$ then $E_{\lambda_1}(\xi) \le E_{\lambda_2}(\xi)$.*

Proof. See Proposition 1 of [11]. □

Let λ_1, λ_2 be two parameters in the interval $[0, 1]$. We consider the two inventory problems with the same input data, but with different objective functions of the optimization problems, defined by the expected operators $E_{\lambda_1}(\xi)$ and $E_{\lambda_2}(\xi)$, respectively.

We denote by $x_1^*, \ldots, x_n^*$ the solution of the optimization problem corresponding to $E_{\lambda_1}(\xi)$ and with $y_1^*, \ldots, y_n^*$ the solution of the optimization problem corresponding to $E_{\lambda_2}(\xi)$.

Proposition 8.2. *Assume that the demands $D_1, \ldots, D_n$ are trapezoidal fuzzy variables. If $\lambda_1 \le \lambda_2$ then $x_i^* \ge y_i^*$ for any $i = 1, \ldots, n$.*

Proof. Assume that $\lambda_1 \le \lambda_2$. By Proposition 4.4, the solutions $x_1^*, \ldots, x_n^*$ and $y_1^*, \ldots, y_n^*$ are written in the following form:

$$x_i^* = \frac{d_i}{h_i E_{\lambda_1}(\frac{1}{D_i})} \tag{40}$$

for $i = 1, \ldots, n$.

$$y_i^* = \frac{d_i}{h_i E_{\lambda_2}(\frac{1}{D_i})} \tag{41}$$

for $i = 1, \ldots, n$.

Applying Lemma 8.1 for any $i = 1, \ldots, n$ the following implications hold:

$$\lambda_1 \le \lambda_2 \Rightarrow E_{\lambda_1}(\frac{1}{D_i}) \le E_{\lambda_2}(\frac{1}{D_i}) \Rightarrow \frac{1}{E_{\lambda_2}(\frac{1}{D_i})} \le \frac{1}{E_{\lambda_1}(\frac{1}{D_i})} \tag{42}$$

By (40)-(42) for any $i = 1, \ldots, n$ we will have:

$$x_i^* - y_i^* = \frac{d_i}{h_i}(\frac{1}{E_{\lambda_2}(\frac{1}{D_i})} - \frac{1}{E_{\lambda_1}(\frac{1}{D_i})}) \ge 0.$$

We conclude that $x_i^* \ge y_i^*$ for any $i = 1, \ldots, n$. □

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