Original Research

Advancing Organizational Efficiency: A Novel Dynamic Network DEA for Strategic Carry-Over Allocation

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Abstract

Carryover activities in dynamic DEA play a crucial role in transmitting information between periods and maintaining the continuity of resources from one period to the next within organizational processes. These variables reflect the impact of past decisions on current and future performance. In the literature of Dynamic DEA, carryovers typically connect two consecutive periods. However, existing models are limited in that they do not account for carryover variables that may persist beyond the immediate subsequent period, nor do they provide decision-makers the flexibility to strategically allocate resources over multiple periods. This paper introduces a novel dynamic network DEA (DNDEA) model within the Slack-Based Measure (SBM) framework, which optimizes the allocation of carryovers beyond consecutive periods and addresses this gap by enabling discretionary allocation by Decision-Makers (DM). Our model not only enhances overall efficiency evaluation but also identifies maximum inefficiency within a network system across multiple evaluation periods. To validate the proposed model, we perform a numerical example focused on the performance assessment of Iranian bank branches using Dynamic DEA techniques. Our findings reveal significant variations in efficiency among the branches, with DMU4 achieving the highest efficiency score of 1.000 and DMU10 the lowest at 0.617. Notably, the proposed model demonstrates superior discriminative power compared to the Dynamic Network Slack-Based Measure (DNSBM), providing more precise insights into resource allocation and operational inefficiencies.

Keywords- Dynamic Network DEA (DNDEA), Carry over activities, allocation, SBM.

INTRODUCTION

Data Envelopment Analysis (DEA), introduced by Charnes, Cooper, and Rhodes [1], has emerged as a powerful mathematical tool for evaluating the performance of homogeneous Decision-Making Units (DMUs). Among its various applications, resource allocation stands out prominently. According to the study conducted by Sadeghi and Dehnokhalaji [2], utilizing DEA for allocation in the literature, typically fall into two main categories. The first category addresses the allocation of fixed costs to DMUs. Fixed costs encompass the total overhead expenses incurred for shared infrastructure across the subunits of a DMU. This challenge is common in organizational budgeting and costing, involving the equitable distribution of overhead costs among different departments [3]. In contrast, the second category focuses on the allocation of input resources, such as funds and personnel, to DMUs. In these studies, the Decision Maker (DM) allocates available resources to units or sub-units with the aim of achieving specific objectives ([2]; [4], [5]). These investigations contribute valuable insights into effective resource management strategies within organizational contexts. In today's dynamic and competitive economic environment, it's clear that markets are constantly changing. However, existing allocation methods don't fully address these changes. We need to discuss how these methods can adapt to evolving market conditions over time. Dynamic DEA is a powerful tool for capturing these changes. DNDEA models can analyze how DMUs' internal processes and efficiencies change over time to assist decision-makers in adjusting resource allocations to stay competitive. Dynamic Network DEA models, DNDEA, analyze the internal processes of DMUs and monitor fluctuations in both period efficiency and divisional efficiency of DMUs over time. Incorporating DNDEA principles into resource allocation frameworks could improve the flexibility and responsiveness of allocation strategies to changing market conditions. Carryover activities are essential in dynamic DEA, showing how past decisions impact present and future performance. They provide valuable insights into performance evaluation over time by revealing the continuity of resources and outcomes. In Dynamic DEA literature, carryovers are typically seen as variables or activities passed from the current period to the next [6]. However, in real-world situations, some carryover variables in the production process extend beyond the immediate next period. We refer to these variables as discretionary allocation carryovers or simply discretionary carryovers. For instance, in banking sector, Loan Loss Reserves are such carryovers [7].

Banks set aside reserves to cover potential losses from loans that might default in the future. If actual losses in a period are lower than expected, these reserves can be moved to future periods to improve the bank's efficiency. Another example is deferred tax assets or liabilities. Banks may have deferred tax assets or liabilities due to differences between accounting and tax laws. If the tax impact of certain transactions or events cannot be understood in the current period, these assets or liabilities can be carried forward to future periods when they are expected to be realized or addressed. Additionally, when a bank agrees with a borrower to postpone loan repayment to a future period with a specific interest rate, it's another instance of carryover allocation. Moreover, when a bank negotiates to extend the payment of taxes to a future period, it delays the tax obligation from the current period to a later one. This adjustment impacts financial planning and resource allocation of organizations across multiple periods, making it a form of carryover. There are numerous examples in other industries as well. For instance, in manufacturing, carrying raw materials inventory from one production cycle to the next can significantly affect production efficiency and cost management. Therefore, strategically allocating carryover activities and tactically distributing resources, considering both the present and the future, is crucial for effective resource management within organizations. In this study, we present a novel dynamic network DEA model to measure the overall, period, and divisional efficiencies of DMUs in SBM framework. The proposed model integrates the excesses or shortfalls of carryover variables into the objective function. A key contribution is the introduction of discretionary carryover variables in DNDEA models, allowing strategic allocation to future periods based on the decision of the DM. This approach aims to enhance resource management efficiency in dynamic environments. The rest of this study is organized as follows: Section 2 provides an overview of prior research and theoretical discussions on DNDEA. Following this, Section 3 outlines the proposed method. Subsequently, Section 4 presents a case study in banking industry to demonstrate the application of the proposed model. Finally, Section 5 summarizes the key findings and concludes the paper.

LITERATURE REVIEW

In this section, we begin by reviewing key studies in Dynamic Network DEA (DNDEA), followed by a focused review of DNDEA applications in the banking industry. Lastly, we highlight some of the prominent models in DNDEA, discussing their limitations and how our proposed model addresses these shortcomings, while emphasizing its strengths in providing more flexible and comprehensive efficiency evaluations.

I. Key Studies in Dynamic Network DEA

DNDEA models address the complexity of efficiency evaluation by integrating multiple dynamic stages connected through network structures in each period. This involves comparing a predetermined number of static models [7], aiding in comprehensive analysis. Through DNDEA, we can observe changes in overall efficiency, dynamic adjustments in divisional efficiency, potential enhancements, and efficiency estimates derived from a holistic assessment considering interactions between periods and divisions [8]. The innovative approach of DNDEA allows us to delve into the traditional black box and model interactions among DMUs across different time periods. Network DEA, in particular, stands out for its examination of inefficiency sources within DMUs. However, incorporating dynamic considerations into network systems poses significant challenges [9]. While network modeling offers a theoretical framework for exploring the internal structure of DMUs, dynamic modeling elucidates connections between periods through carryover activities. The pioneering work of Chen [10] introduced dynamic effects into DNDEA, revolutionizing efficiency assessment. Since then, researchers have applied DNDEA across various sectors like healthcare [11], banking [12] and [13], transportation [14], education [15], research, and energy [16]. One notable model in the literature of DNDEA is by Tone and Tsutsui [17], building on earlier SBM approaches. They proposed their DNSBM model for evaluating period, divisional, and overall efficiency in a unified framework. Samavati, Badiezadeh and Saen [18] presented a DNDEA model capable of assessing both optimistic and pessimistic efficiency and effectiveness. Their model accommodates undesirable outputs and enables the ranking of sustainable supply chains based on optimistic and pessimistic efficiency and effectiveness. Omrani and Soltanzadeh [19]propose a relational dynamic NDEA (DNDEA) model to evaluate the efficiencies of interrelated processes over time within companies. They highlight the shortcomings of traditional NDEA models in assessing dynamic effects within production processes. The proposed model is applied to measure the efficiency of eight Iranian airlines across multiple connected periods, yielding comparisons with dynamic DEA and network DEA models. Lobo et al. [20] introduced a DNDEA model to develop an evaluation framework for assessing the efficiency of university hospitals.

Wu et al. [21] assess the efficiency of 26 international airlines from 2019 to 2022 using DNDEA methodology. The model evaluates overall, period-specific, and stage-specific efficiencies, accounting for dynamic effects between consecutive periods. Huang and Wang [22] improve the DEA game cross-efficiency model to a dynamic network DEA game cross-efficiency model to assess the efficiency of high-tech industries in China from 2011 to 2015. They find that China's high-tech industry has about 45% room for improvement, with uneven development among regions. R&D efficiency shows an inverted U-shaped trend, while commercialization efficiency follows a U-shaped trend, with R&D efficiency having a more significant impact on overall efficiency. The study provides recommendations to enhance efficiency based on these findings. Liu et al. [23] develop a novel evaluation method combining dynamic network DEA, cross-efficiency evaluation, and Shannon entropy to assess bus transit benefits in 33 Chinese cities from 2016 to 2019. Their study finds that most cities exhibit ineffective bus transit benefits, with a focus on improving service effectiveness rather than production efficiency yielding better results. Additionally, smaller cities demonstrate higher benefits compared to larger ones, and geographical variation in benefits is evident. The findings offer valuable insights for decision-making and public transit management. Anouze et al. [24] propose a novel analytical framework based on dynamic-network data envelopment analysis (DEA) to evaluate national innovation systems (NIS). Applied to 23 oil-producing countries and compared to the Global Innovation Index, this framework measures NIS efficiency, creates a new index, and suggests policies to improve performance. The study identifies Korea and Sweden as top performers, with Germany and Ukraine leading in knowledge development and Romania and Singapore in knowledge commercialization. The framework also offers targeted policies to enhance the innovation systems of less efficient countries. In this subsection, we review key studies on dynamic DEA. The following subsection focuses on notable research applying dynamic network DEA specifically within the banking industry

- DNDEA Applications in the Banking Industry

In the banking sector, numerous studies have examined various aspects of performance and efficiency. In the following, we highlight some notable research. Lahouel et al. [25] assessed the efficiency of 114 European banks from 2010 to 2019 using a three-stage DNSBM model. Their study analyzes the carryover characteristics of non-performing loans in the initial stage of the production process and the carryover characteristics of net operating income in the final stage. Deposits are treated as intermediate products in the first stage, while loans and securities are considered intermediate products in the second stage. Boussemart et al. [26] introduced an approach accounting for carryover activities in the production process, decomposing overall efficiency into two sub-efficiencies. These sub-efficiencies reflect distinct aspects of banking operations: economic efficiency, which focuses on economic performance, and credit risk efficiency, which addresses stability performance. Zhou et al. [27] developed a multi-period, multi-stage DEA model to evaluate the efficiency of Chinese commercial banks, accounting for unused assets, shared inputs, and non-performing loans as undesirable outputs. Their study found that while all banks were generally inefficient, inefficiencies varied across different stages, highlighting the need for a reasonable business scale and the benefits of a three-stage framework for accurate efficiency assessment. Fukuyama and Tan [28] . In their study, broke down the overall efficiency into five distinct components: innovation efficiency, primary business stability efficiency, strategic management stability efficiency, profitability efficiency, and corporate social responsibility efficiency. In another study,

Fukuyama and Tan [29] further decomposed overall efficiency into input efficiency, output efficiency, and stability efficiency. Beyond efficiency decomposition, they introduced market power in deposits and loans as intermediate products. Additionally, they incorporated loan loss provisions into the production process as a beneficial intermediate product for the first time. Li et al. [33] presented a novel DNDEA approach for assessing the performance of Chinese listed banks from 2014 to 2018. Chen [34]. Proposed a DNDEA model to measure the efficiency of financial institutions, particularly focusing on Taiwanese bank branches. Their study examines branch efficiency considering various factors like region and branch type. Wanke et al. [35] utilized DNDEA to explore the regulatory and cultural diversity within the Middle East and North Africa (MENA) banking industry. Their Dynamic Network DEA model scrutinizes the relationships among crucial financial indicators and integrates carry-over effects. Findings reveal diverse impacts of bank characteristics on efficiency levels across various indicators, with cultural and regulatory factors dominating at the national level. Shabani and Shirazi [36] proposed a mixed-integer DEA model to evaluate the performance of commercial bank branches under dynamic competitive conditions. Their model incorporates cross-shared and serial-shared resources, demonstrating that the weighting of periods has little impact on overall efficiency. The study emphasizes the importance of managing shared resources effectively in performance evaluation.

In the current subsection, we present a review of key studies that apply DNDEA models in the banking industry. In the following subsection, we will examine the limitations of existing DNDEA models and highlight the strengths of our proposed model, focusing on how it addresses these gaps and provides a more comprehensive approach to evaluating efficiency.

- Limitations of Existing DNDEA Models and Strengths of the Proposed Model

Over the past decades, various models have been developed to address the dynamic nature of efficiency evaluation in DEA. Below is a summary of the key models in the literature, highlighting their primary contributions and limitations:

1. **Dynamic DEA (Tone and Tsutsui,** [6]**):** This model introduced the concept of carryover activities in dynamic DEA, where outputs or inputs from one period are carried over to the next. However, it only accounted for carryovers between consecutive periods, which limits its ability to capture the broader temporal interactions in resource allocation.
2. **Dynamic Network SBM (Tone and Tsutsui,** [17]**):** Building on the earlier Dynamic DEA model, this framework integrates network structures, allowing for more detailed efficiency evaluations across multiple divisions within DMUs. Despite its innovation, the model still restricts carryover activities to consecutive periods and does not offer decision-makers flexibility in strategic resource allocation.
3. **Dynamic Network DEA (Samavati et al.,** [18]**):** This model improved upon previous efforts by introducing optimistic and pessimistic efficiency measures. It allowed for the assessment of both positive and negative aspects of performance, yet it did not address the discretionary allocation of carryovers beyond immediate periods.
4. **Relational Dynamic Network DEA (Omrani and Soltanzadeh,** [19]**):** This model extended the evaluation of efficiency by considering interrelated processes over time, focusing on connected periods. While effective for some industries, its rigid structure for carryover activities limits its applicability in contexts where discretionary carryover allocations are needed.
5. **Dynamic Network DEA (Fukuyama & Weber,** [37]**):** Fukuyama and Weber introduced a dynamic network DEA model to evaluate the performance of Japanese banks, specifically focusing on loan loss provisions as a key carryover variable. The model assesses both production efficiency and financial stability over time, taking into account the interconnected nature of different operational stages within the banks. By examining both period-specific and overall efficiency, the model provides a comprehensive view of how past decisions affect future performance. However, despite its innovative application to the banking sector, the model remains limited by its focus on consecutive period carryovers, without offering the flexibility needed for discretionary allocation of resources across multiple periods.

While these models have advanced the field of dynamic efficiency assessment, they fail to account for situations where carryover variables extend beyond consecutive periods. Moreover, they do not allow decision-makers the flexibility to allocate resources strategically across multiple periods. This paper addresses these limitations by introducing a novel Dynamic Network DEA model that incorporates discretionary carryover activities. This model enables decision-makers to allocate resources across multiple periods, enhancing overall and period-specific efficiency evaluations. In this section, we reviewed various studies on DNDEA in the banking sector. In the next section, we will present our proposed model.

THE PROPOSED MODEL

In this section, we present our novel DNDEA model within the SBM framework. As discussed earlier, the proposed model allocates carryover variables to one of the subsequent periods in a manner that maximizes overall inefficiency.

I. Notations

We consider n DMUs $(j = 1, \dots, n)$ consisting of K divisions $(k = 1, \dots, K)$ over T time periods $(t = 1, \dots, T)$. Let m_k and r_k denote the numbers of inputs and outputs for division k, respectively. The link from division k $(divk)$to $divh$ is denoted by (k, h), and the set of all such links is represented by L_{kh}. Table 1 and Table 2 represent the notations for data and variables utilized in this study, respectively.

TABLE 1
DATA NOTATION

data	symbol	definition
input	x_{ip}^{tk}	Input resource i to Division k (Divk) of DMUj at time period t
output	y_{rp}^{tk}	Output product r from Divk of DMUj at time period t
link	$z_{dp}^{t(k,h)\beta}$	link d from category β between interconnected Divk and h of DMUp in period t. β stands for free, fixed, as-input, and as-output
Carry-over	$z_{ckj}^{tcarry\alpha}$	Carry-over c from category α of DMUj at Divk from period t

TABLE 2
VARIABLE NOTATION

variable	symbol	definition
Input slack	s_{ip}^{tk-}	Slack of input i of DMUp for Divk at period t.
Output slack	s_{rp}^{tk+}	Slack of output r of DMUp for Divk at period t.
Link slack	$s_{dp}^{t(k,h)\beta}$	Slack of link d between Divk and Divh of DMUp at period t. β stands for "fixed", "free", "as-input", and" as-output".
Carry-over slack	$s_{cp}^{(t,f)k\alpha}$	Slack of carry-over c for Divk of DMUp from period t to period f. α stands for free, fixed, good and bad.
intensity	λ_j^{tk}	Intensity of DMUj corresponding to Divk at period t.
Carry-over allocator	$t_{kc\alpha}^{(t,f)}$	a binary variable that equals 1 if the carry-over c in category α from Divk in period t is assigned to the corresponding division in period f; otherwise, it equals zero.
Carry-over value control	$zz_{cj}^{tcarry\_bad}$	A free variable which controls the value of the corresponding carry over.
symbol	$h_k^{(a,t)}$	The number of all carry overs from Divk in period a to Divk in period t ($\forall a < t$).
symbol	$h_k^{(t,a)}$	The number of all carry overs from Divk in period t to Divk in period a ($\forall a,\ t < a \le T$).

In this subsection, we have defined the notations for various elements of our dynamic network DEA model, including inputs, outputs, linking variables, and carryovers. These notations are essential for understanding how the model integrates and evaluates resource allocation and efficiency across multiple periods. With these notations established, the next section will proceed to the detailed formulation of the dynamic network DEA model. This will include the mathematical representation of the objective function and constraints, demonstrating how the notations are applied to optimize carryover allocation and measure efficiency.

II. Model Formulation

In the current subsection, we present our novel DNDEA model within the SBM framework. As discussed previously, our proposed model allocates carryover activities to one of the subsequent periods in a manner that maximizes overall inefficiency. Equations (1-14) illustrate the proposed model.

$$\psi_p^* = Min \frac{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 - \frac{1}{m_k + l_{(k,h)in} + nbad_k} \left(\sum_{i=1}^{m_k} \frac{s_{ip}^{tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{s_{dp}^{t(f,k)as-input}}{z_{dp}^{t(f,k)as-input}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{s_{cp}^{(t,f)kbad}}{z_{cj}^{tcarry_{bad}}}\right)\right]}{\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[1 + \frac{1}{m_k + l_{(k,h)out} + ngood_k} \left(\sum_{i=1}^{m_k} \frac{s_{ip}^{tk+}}{y_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{s_{dp}^{t(k,h)as-output}}{z_{dp}^{t(k,h)as-output}} + \sum_{c=1}^{h_k^{(t,t+1)}} \frac{s_{cp}^{(t,f)kgood}}{z_{cj}^{tcarry\_good}}\right.\right.} \quad (1)$$

$$\text{s.t. } \sum_{j=1}^{n} \lambda_j^{tk} x_{ij}^{tk} + s_{ip}^{tk-} = x_{ip}^{tk} \qquad (k = 1, \ldots, K), (i = 1, \ldots, m_k) \quad (2)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} y_{rj}^{tk} - s_{rp}^{tk+} = y_{rp}^{tk} \qquad (k = 1, \ldots, K), (r = 1, \ldots, r_k) \quad (3)$$

$$\sum_{j=1}^{n} \lambda_j^{th} z_{dj}^{t(k,h)in} + s_{dp}^{t(k,h)as-input} = z_{dp}^{t(k,h)as-input} \qquad (d \in l_{(k,h)in}), \forall (k,h) \quad (4)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} z_{dj}^{t(k,h)out} - s_{dp}^{t(k,h)as-output} = z_{dp}^{t(k,h)as-output} \qquad (d \in l_{(k,h)out}), \forall (k,h) \quad (5)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} z_{dj}^{t(k,h)fixed} = z_{dp}^{t(k,h)fixed} \qquad (d \in l_{(k,h)fix}), \forall (k,h) \quad (6)$$

$$\sum_{j=1}^{n} \lambda_j^{tk} z_{dj}^{t(k,h)} = \sum_{j=1}^{n} \lambda_j^{th} z_{dj}^{t(k,h)} \qquad (d = 1, \dots, l_{(k,h)}), \forall (k,h) \tag{7}$$

$$\sum_{j=1}^{n} \lambda_j^{kf} z_{kcj}^{tcarry\_\alpha} - M(1 - t_{kc\alpha}^{(t,f)}) \leq \sum_{j=1}^{n} \lambda_j^{kt} z_{cj}^{tcarry\_\alpha} \leq \sum_{j=1}^{n} \lambda_j^{kf} z_{cj}^{tcarry\_\alpha} + M(1 - t_{kc\alpha}^{(t,f)}) \qquad \forall f > t\ , \alpha \in \{bad, good, free, fixed\} \tag{8}$$

$$\sum_{f=t+1}^{T} t_{kc\alpha}^{(t,f)} = 1 \qquad \forall f > t, (k = 1, \dots, K), \alpha \in \{bad, good, free, fixed\} \tag{9}$$

$$\sum_{j=1}^{n} \lambda_j^{kt} z_{cj}^{tcarry\_fixed} = z_{cp}^{tfixed} \tag{10}$$

$$\sum_{j=1}^{n} \lambda_j^{kt} z_{cj}^{tcarry\_good} - s_{cp}^{(t,f)kgood} = z_{cj}^{tcarry\_good} \tag{11}$$

$$\sum_{j=1}^{n} \lambda_j^{kf} z_{cj}^{tcarry\_bad} + s_{cp}^{(t,f)kbad} = zz_{cj}^{tcarry\_bad} \tag{12}$$

$$z_{cj}^{tcarry\_bad} - M\left(1 - t_{kcbad}^{(t,f)}\right) \leq zz_{cj}^{tcarry_{bad}} \leq z_{cj}^{tcarry_{bad}} + M\left(1 - t_{kcbad}^{(t,f)}\right) \tag{13}$$

$$t_{k\alpha}^{(t,f)} = \{0,1\}; zz_{cj}^{tcarry_{bad}}: free, \lambda_j^k \geq 0, s_{rp}^{k+} \geq 0, s_{ip}^{k-} \geq 0, s_{dp}^{(k,h)-} \geq 0, s_{dp}^{(k,h)+} \geq 0 \tag{14}$$

Equation (1) presents the objective function in SBM framework which measures non-oriented overall efficiency, where W^t and W^k denote the weights for period t and division, respectively. Equations (2) and (3) represent input and output constraints, respectively, for $divk$ of $DMUj$ in period t. Regarding linking activities, we consider four scenarios named "free-link value case", "fixed-link value case", "as-input value case" and "as-output value case", each offering distinct characteristics. For comprehensive understanding, see Tone and Tsutsui [17].

- **Free-Link Value Case**

In this case, the linking variables are freely determined (discretionary), with an emphasis on maintaining continuity between input and output. If a linking activity is classified as a free-link, its deviation from the observed value does not directly influence the efficiency score and its impact is indirect, through the assumption of continuity of linking flows between divisions. This continuity assumption is presented in Equation (7).

- **Fixed-Link Value Case**

If a linking variable is beyond the control of the DM, it is considered as a fixed-link. Equation (6) outlines the constraint associated with fixed-link activities.

- **As-input link value case**

In this scenario, the linking variables are treated as inputs to the subsequent division, and any surpluses are factored into measuring input inefficiency. Equation (4) outlines the corresponding constraint for these activities.

- **As-output link value case**

In this case, the linking activities are treated as output from the preceding division, and any shortfalls being considered in the output inefficiency. Equation (5) represents the corresponding constraint with these activities. In this study carry-over activities are categorized into four distinct categories. For a comprehensive understanding, see Tone and Tsutsui [17]

i. Good or Desirable Carry-over:

This category encompasses favorable carry-over instances such as profits carried forward and net earned surplus transferred to the one of the subsequent periods. In our model, desirable carry-overs are considered outputs from the corresponding period, and their magnitudes are controlled to not fall below the observed values. Any shortfall in comparative carry-over within this category is considered as inefficiency.

ii. Bad or Undesirable Carry-over:

This category comprises unfavorable instances of carry-over such as losses carried forward, bad debts, and dead stock. In our model, undesirable carry-overs are treated as inputs to one of the next periods determined by the model, with their magnitudes constrained to not exceed the observed values. Any surplus in comparative carry-over within this category is considered as inefficiency.

iii. Free or Discretionary Carry-over:

The carry-overs in this category are managed freely by the DMU. Their value can be lower or higher than the observed value, and this variation does not directly influence the efficiency. They indirectly impact the efficiency score through the continuity assumption between periods.

iv. Fixed or Non-discretionary Carry-over:

The carry-overs in this category are beyond the control of the DMU and their values are fixed at the observed level. Like free carry-overs, fixed carry-overs influence the efficiency score through the continuity assumption between periods. The carry-over constraints are formulated as presented in Eq. (8-13). Equation (8) maintains the continuity assumption between periods t and f if there exists a connection between them. In the case $t_{kc\alpha}^{(t,f)}=1$ where, indicating the allocation of carry-over c from category α and division k in period t to period f, the replacement of $t_{kc\alpha}^{(t,f)}=1$ results in:

$$\sum_{j=1}^{n}\lambda_j^{kf}\, z_{kcj}^{tcarry\_\alpha} = \sum_{j=1}^{n}\lambda_j^{kt}\, z_{cj}^{tcarry\_\alpha} \tag{15}$$

In the scenario where $t_{kc\alpha}^{(t,f)}=0$, equation (8) becomes redundant. Equation (9) ensures that the carry over $z_{kcj}^{tcarry\_\alpha}$ from period t is allocated to only one of the subsequent periods. Equation (10) ensures that the carryovers in “fixed” category do not deviate from their observed values. Equation (11) illustrates that the carry over $z_{kcj}^{tcarry\_good}$ is considered as an output from period t and its shortfall from the observed values is measured by $s_{cp}^{(t,f)kgood}$. Equations (12) and (13) demonstrate that if the undesirable carry over $z_{kcj}^{tcarry\_bad}$ is allocated to period f ($t_{kc\alpha}^{(t,f)}=1$), it is regarded as an input to that period and any excess from the observed values is quantified by $s_{cp}^{(t,f)kbad}$. Due to the nonlinear nature of objective function (1) and the binary variables $t_{kc\alpha}^{(t,f)}$, the proposed model is a Mixed-integer nonlinear programming (MINLP) problems, In the next section, we apply the Charnes-Cooper transformation to linearize it.

3.3 Linearization of the proposed model.

The model described by equations (1-14) is nonlinear programming because of its nonlinear objective function. To simplify and linearize this model, we utilize the Charnes-Cooper transformation method [1]. Equation (15) introduces u as a factor modifying the original expression to achieve linearity.

$$u=\frac{1}{\sum_{t=1}^{T}W^t\sum_{k=1}^{K}W^k\left[1+\frac{1}{m_k+l_{(k,h)out}+ngood_k}\left(\sum_{i=1}^{m_k}\frac{s_{ip}^{tk+}}{y_{rp}^{tk}}+\sum_{d=1}^{l_{(f,k)}}\frac{s_{dp}^{t(k,h)as-output}}{z_{dp}^{t(k,h)as-output}}+\sum_{c=1}^{h_k^{(t,t+1)}}\frac{s_{cp}^{(t,f)kgood}}{z_{cj}^{tcarry\_good}}\right)\right]} \tag{15}$$

Applying $u>0$ to both the numerator and denominator of the objective function (1) keeps the variable ψ_p^* unchanged, while adjusting u effectively standardizes the denominator to 1. This leads to the reformulation of the model, resulting in equations (16) to (30).

$$\psi_p^* = \sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[\mathrm{u} - \frac{1}{m_k + l_{(k,h)in} + nbad_k} \left(\sum_{i=1}^{m_k} \frac{us_{ip}^{tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{us_{dp}^{\mathrm{t}(f,k)as-input}}{z_{dp}^{\mathrm{t}(f,k)as-input}} + \sum_{\mathrm{c}=1}^{h_k^{(\mathrm{t},t+1)}} \frac{us_{cp}^{(t,f)kbad}}{z_{cj}^{tcarry_{bad}}} \right) \right] \tag{16}$$

$$\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[u + \frac{1}{m_k + l_{(k,h)out} + ngood_k} \left(\sum_{i=1}^{m_k} \frac{us_{ip}^{tk+}}{y_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{us_{dp}^{\mathrm{t(k,h)}as-output}}{z_{dp}^{\mathrm{t(k,h)}as-output}} + \sum_{\mathrm{c}=1}^{h_k^{(\mathrm{t},t+1)}} \frac{us_{cp}^{(t,f)kgood}}{z_{cj}^{tcarry\_good}} \right) \right] = 1 \tag{17}$$

s.t. $\sum_{j=1}^{n} u\lambda_j^{tk} x_{ij}^{tk} + us_{ip}^{tk-} = ux_{ip}^{tk}$ $\quad (k = 1, \ldots, K), (i = 1, \ldots, m_k)$ (18)

$\sum_{j=1}^{n} u\lambda_j^{tk} y_{rj}^{tk} - us_{rp}^{tk+} = y_{rp}^{tk}$ $\quad (k = 1, \ldots, K), (r = 1, \ldots, r_k)$ (19)

$\sum_{j=1}^{n} u\lambda_j^{th} z_{dj}^{t(k,h)in} + us_{dp}^{t(k,h)as-input} = uz_{dp}^{t(k,h)as-input}$ $\quad (d \in l_{(k,h)in}), \forall (k,h)$ (20)

$\sum_{j=1}^{n} u\lambda_j^{tk} z_{dj}^{t(k,h)out} - us_{dp}^{t(k,h)as-output} = uz_{dp}^{t(k,h)as-output}$ $\quad (d \in l_{(k,h)out}), \forall (k,h)$ (21)

$\sum_{j=1}^{n} u\lambda_j^{tk} z_{dj}^{t(k,h)fixed} = uz_{dp}^{t(k,h)fixed}$ $\quad (d \in l_{(k,h)fix}), \forall (k,h)$ (22)

$\sum_{j=1}^{n} u\lambda_j^{tk} z_{dj}^{t(k,h)} = \sum_{j=1}^{n} u\lambda_j^{th} z_{dj}^{t(k,h)}$ $\quad (d = 1, \ldots, l_{(k,h)}), \forall (k,h)$ (23)

$$\sum_{j=1}^{n} u\lambda_j^{kf} z_{kcj}^{tcarry\_\alpha} - Mu(1 - t_{kc\alpha}^{(t,f)}) \leq \sum_{j=1}^{n} u\lambda_j^{kt} z_{cj}^{tcarry\_\alpha} \leq \sum_{j=1}^{n} u\lambda_j^{kf} z_{cj}^{tcarry\_\alpha} + Mu(1 - t_{kc\alpha}^{(t,f)}) \quad \forall f > t, \ \alpha \in \{bad, good, free, fixed\} \tag{24}$$

$$\sum_{f=t+1}^{T} t_{kc\alpha}^{(t,f)} = 1 \quad \forall f > t, (k = 1, \ldots, K), \ \alpha \in \{bad, good, free, fixed\} \tag{25}$$

$$\sum_{j=1}^{n} u\lambda_j^{kt} z_{cj}^{tcarry\_fixed} = uz_{cp}^{tfixed} \tag{26}$$

$$\sum_{j=1}^{n} u\lambda_j^{kt} z_{cj}^{tcarry\_good} - us_{cp}^{(t,f)kgood} = uz_{cj}^{tcarry\_good} \tag{27}$$

$$\sum_{j=1}^{n} u\lambda_j^{kf} z_{cj}^{tcarry\_bad} + us_{cp}^{(t,f)kbad} = uzz_{cj}^{tcarry\_bad} \tag{28}$$

$$uz_{cj}^{tcarry\_bad} - M\left(u - ut_{kcbad}^{(t,f)}\right) \leq uzz_{cj}^{tcarry_{bad}} \leq uz_{cj}^{tcarry_{bad}} + M\left(u - ut_{kcbad}^{(t,f)}\right) \tag{29}$$

$$t_{k\alpha}^{(t,f)} = \{0,1\}; zz_{cj}^{tcarry_{bad}}: free, \lambda_j^k \geq 0, s_{rp}^{k+} \geq 0, s_{ip}^{k-} \geq 0, s_{dp}^{(k,h)-} \geq 0, s_{dp}^{(k,h)+} \geq 0 \tag{30}$$

Given that $zz_{cj}^{tcarry\_bad}$ is an unrestricted variable and u is positive ($u > 0$), the variable $ZZ_{cj}^{tcarry\_bad}$, defined as $ZZ_{cj}^{tcarry\_bad} = uzz_{cj}^{tcarry\_bad}$, maintains the same sign as $zz_{cj}^{tcarry\_bad}$. To transform constraints (29) into a linear form, we utilize the relationships outlined in equations (31) to (34).

$$u.t_{kc\alpha}^{(t,f)} = h_{kc\alpha}^{(t,f)} \quad (31)$$
$$h_{kc\alpha}^{(t,f)} \leq u \quad (32)$$
$$h_{kc\alpha}^{(t,f)} \leq M.t_{kc\alpha}^{(t,f)} \quad (33)$$
$$h_{kc\alpha}^{(t,f)} \geq u - M(1 - t_{kC\alpha}^{(t,f)}) \quad (34)$$

Now, in the last stage of linearization of the model, we define $us_{ip}^{tk-} = S_{ip}^{tk-}$, $us_{rp}^{tk+} = S_{rp}^{tk+}$, $us_{dp}^{t(k,h)as-input} = S_{dp}^{t(k,h)as-input}$, $us_{dp}^{t(k,h)as-output} = S_{dp}^{t(k,h)as-output}$، $us_{cp}^{(t,f)kgood} = S_{cp}^{(t,f)kgood}$, $us_{cp}^{(t,f)kbad} = S_{cp}^{(t,f)kbad}$, $u\lambda_j^{kf} = \Lambda_j^{kf}$.

As a result, we obtain the equivalent linear representation of the model, as described in equations (35)-(52).

$$\psi_p^* = \sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[\mathrm{u} - \frac{1}{m_k + l_{(k,h)in} + nbad_k} \left(\sum_{i=1}^{m_k} \frac{S_{ip}^{tk-}}{x_{ip}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{S_{dp}^{\mathrm{t}(f,k)as-input}}{z_{dp}^{\mathrm{t}(f,k)as-input}} + \sum_{\mathrm{c}=1}^{h_k^{(\mathrm{t,a})}} \frac{S_{cp}^{(t,a)kbad}}{z_{cj}^{tcarry_{bad}}} \right) \right] \quad (35)$$

$$\sum_{t=1}^{T} W^t \sum_{k=1}^{K} W^k \left[u + \frac{1}{m_k + l_{(k,h)out} + ngood_k} \left(\sum_{i=1}^{m_k} \frac{S_{ip}^{tk+}}{y_{rp}^{tk}} + \sum_{d=1}^{l_{(f,k)}} \frac{S_{dp}^{\mathrm{t(k,h)}as-output}}{z_{dp}^{\mathrm{t(k,h)}as-output}} + \sum_{\mathrm{c}=1}^{h_k^{(\mathrm{t,a})}} \frac{S_{cp}^{(a,t)kgood}}{z_{cj}^{tcarry\_good}} \right) \right] = 1 \quad (36)$$

s.t. $\sum_{j=1}^{n} \Lambda_j^{tk} x_{ij}^{tk} + S_{ip}^{tk-} = ux_{ip}^{tk} \qquad (k = 1,\ldots,K), (i = 1,\ldots,m_k) \quad (37)$

$$\sum_{j=1}^{n} \Lambda_j^{tk} y_{rj}^{tk} - S_{rp}^{tk+} = uy_{rp}^{tk} \qquad (k = 1,\ldots,K), (r = 1,\ldots,r_k) \quad (38)$$

$$\sum_{j=1}^{n} \Lambda_j^{th} z_{dj}^{t(k,h)in} + S_{dp}^{t(k,h)as-input} = uz_{dp}^{t(k,h)as-input} \qquad (d \in l_{(k,h)in}), \forall (k,h) \quad (39)$$

$$\sum_{j=1}^{n} \Lambda_j^{tk} z_{dj}^{t(k,h)out} - S_{dp}^{t(k,h)as-output} = uz_{dp}^{t(k,h)as-output} \qquad (d \in l_{(k,h)out}), \forall (k,h) \quad (40)$$

$$\sum_{j=1}^{n} \Lambda_j^{tk} z_{dj}^{t(k,h)fixed} = uz_{dp}^{t(k,h)fixed} \qquad (d \in l_{(k,h)fix}), \forall (k,h) \quad (41)$$

$$\sum_{j=1}^{n} \Lambda_j^{tk} z_{dj}^{t(k,h)} = \sum_{j=1}^{n} \Lambda_j^{th} z_{dj}^{t(k,h)} \qquad (d = 1,\ldots,l_{(k,h)}), \forall (k,h) \quad (42)$$

$$\sum_{j=1}^{n} \Lambda_j^{fk} z_{kcj}^{tcarry\_\alpha} - M(u - h_{kc\alpha}^{(t,f)}) \leq \sum_{j=1}^{n} \Lambda_j^{tk} z_{cj}^{tcarry\_\alpha} \leq \sum_{j=1}^{n} \Lambda_j^{fk} z_{cj}^{tcarry\_\alpha} + M(u - h_{kc\alpha}^{(t,f)}) \qquad \forall f > t, \ \alpha \in \{bad, good, free, fixed\} \quad (43)$$

$$\sum_{f=t+1}^{T} t_{kc\alpha}^{(t,f)} = 1 \qquad \forall f > t, (k = 1,\ldots,K), \ \alpha \in \{bad, good, free, fixed\} \quad (44)$$

$$\sum_{j=1}^{n} \Lambda_j^{tk} z_{cj}^{tcarry\_fixed} = uz_{cp}^{tfixed} \quad (45)$$

$$\sum_{j=1}^{n} \Lambda_j^{tk} z_{cj}^{tcarry\_good} - S_{cp}^{(t,f)kgood} = uz_{cj}^{tcarry\_good} \quad (46)$$

$$\sum_{j=1}^{n} \Lambda_j^{fk} z_{cj}^{tcarry\_bad} + S_{cp}^{(t,f)kbad} = ZZ_{cj}^{tcarry\_bad} \quad (47)$$

$$uz_{cj}^{tcarry\_bad} - M\left(u - h_{kCbad}^{(t,f)}\right) \le ZZ_{cj}^{tcarry_{bad}} \le uz_{cj}^{tcarry_{bad}} + M\left(u - h_{kCbad}^{(t,f)}\right) \quad (48)$$

$$h_{k\alpha}^{(t,f)} \le u \quad (49)$$

$$h_{k\alpha}^{(t,f)} \le M.t_{k\alpha}^{(t,f)} \quad (50)$$

$$h_{k\alpha}^{(t,f)} \ge \mathrm{u} - M(1 - t_{kC\alpha}^{(t,f)}) \quad (51)$$

$$t_{k\alpha}^{(t,f)} = \{0,1\}; ZZ_{cj}^{tcarry_{bad}}: free, \Lambda_j^{tk} \ge 0, s_{rp}^{k+} \ge 0, s_{ip}^{k-} \ge 0, s_{dp}^{(k,h)-} \ge 0, s_{dp}^{(k,h)+} \ge 0 \quad (52)$$

In this subsection, we established linear model of the proposed model. In the next section, we will introduce an empirical study aimed at validating the proposed model.

III.EMPIRICAL STUDY

In this section, utilizing the proposed model, we calculate the overall, divisional, and period efficiencies of 10 Iranian bank branches spanning the years 1396 to 1402. Initially, drawing from existing literature and expert consultations, we conceptualize the banks as a network structure comprising three divisions. Each division is characterized by inputs, outputs, links, and carryovers as presented in Table 3. Loan loss reserves (LLR) and non-performing loans (NPL) are categorized as carryover variables. As previously discussed, these carryovers can be allocated to future periods to enhance the bank's overall efficiency. Loan loss reserves, considered a good carryover, are under the control of the current period and can be strategically allocated to improve objective function. Conversely, non-performing loans are regarded as bad carryovers, constituting bad links carried over to the next period. The allocation of NPL is not discretionary. It's important to note that, in compliance with privacy policies, the names of the branches under evaluation are not disclosed. Furthermore, the related data have been gathered from multiple sources, including the bank's records unit, the Statistical Center of Iran, online resources, and insights provided by experts.

TABLE 3
FACTORS OF PRODUCTION UTILIZED IN THIS STUDY.

Input	input 1 consumed at the first stage in period t	Personnel expenses
Input	input 2 consumed at the first stage in period t	Fixed assets
Intermediate variable	Intermediate variable which is output from the first stage in period t and input to the second stage in the same period	Total deposits
Intermediate variable	Intermediate variable which is output from the second stage in period t and input to the third stage in the same period	Gross loans
Intermediate variable	Intermediate variable which is output from the second stage in period t and input to the third stage in the same period	Total securities investment
Output	Output from the third stage in period t	Income
carry over	The generated carry over form the 3rd stage in period t and is considered as good carry over to one of the subsequent periods.	Loan loss reserves (LLR)
carry over	The generated carry over form the 3rd stage in period t and is considered as bad carry over to one of the subsequent periods.	Non-Performing Loans (NPLs)-bad loans

- Efficiency evaluation of bank branches

In this section, we apply our proposed model to evaluate the relative efficiency of bank branches. Using the Generalized Algebraic Modeling System (GAMS), we solve the proposed model to determine efficiency bounds for branches and allocate carryovers. The resulting outcomes are presented in Tables 4, showcasing the efficiencies of 10 branches. Figure 4 compares the overall and period scores during 1396 to 1402.

TABLE 4
OVERALL AND PERIOD EFFICIENCY OBTAINED BY THE PROPOSED MODEL.

DMU	Overall score	Rank	Overall Efficiency in 1396	Overall Efficiency in 1397	Overall Efficiency in 1398	Overall Efficiency in 1399	Overall Efficiency in 1400	Overall Efficiency in 1401	Overall Efficiency in 1402
DMU1	0.902	5	0.924	1.000	1.000	0.819	0.788	0.945	0.861
DMU2	0.916	4	0.893	0.935	0.924	0.966	0.809	1.000	0.851
DMU3	0.792	7	0.809	0.914	0.777	0.641	0.966	0.599	0.473
DMU4	1.000	1	1.000	1.000	0.998	0.903	0.978	1.000	0.988
DMU5	0.996	2	0.893	0.914	0.998	1.000	1.000	1.000	1.000
DMU6	0.613	9	0.693	0.730	0.819	0.473	0.609	0.378	0.609
DMU7	0.663	8	0.851	0.667	0.641	0.588	0.641	0.525	0.641
DMU8	0.950	3	1.000	0.971	0.998	0.935	0.893	1.000	0.893
DMU9	0.831	6	1.000	0.870	0.830	0.809	0.777	0.651	0.777
DMU10	0.617	10	0.725	0.515	0.714	0.672	0.473	0.672	0.462
average	0.828	-	0.878	0.8516	0.869	0.7806	0.793	0.777	0.755

According to the results presented in the Table 4, the overall efficiency scores range from 0.613 to 1.000, indicating significant variability in performance across the bank branches. DMU4 achieves the highest overall efficiency score of 1.000, indicating that it is utilizing its resources optimally to generate outputs. On the other hand, DMU10 has the lowest overall efficiency score of 0.617, suggesting inefficiencies in resource utilization. DMU4 is ranked first, indicating its superior performance compared to other branches, while DMU10 is ranked last, reflecting its relatively poor efficiency compared to other branches.

Examining the period efficiency trends, it can be seen that some branches exhibit consistent performance over the years, while others show fluctuations in efficiency scores (See Figure 1). For example, DMU5 maintains high efficiency scores across all years, indicating consistent performance. In contrast, DMU6 shows fluctuating efficiency scores, suggesting inconsistency in performance over time. The average overall scores for all branches range from 0.761 to 0.884 across the years 1396 to 1402. These averages provide insights into the overall efficiency trend of the banking sector during this period. The trend shows some variation, with efficiency improving in certain years and declining in others. Furthermore, as a result of solving the model, the optimal allocation of carryovers for each branch during the periods under consideration can be determined. This allocation strategy aims to enhance overall efficiency by strategically distributing carryovers across different periods. Table 5 indicates the optimal allocation of the carryover LLR across the years 1396-1402 for DMU1.

TABLE 5
THE OPTIMAL ALLOCATION OF THE CARRY OVER LLR ACROSS THE YEARS 1396-1402

Year	1399	1400	1401	1402
1396	*	-	-	-
1397	-	-	*	-
1398	-	*	-	-
1399	-	-	-	*
1400	-	-	*	-
1401	-	-	-	*

According to Table 5, the optimal allocation of the LLR for DMU1 is as follows: the LLR from 1396 is allocated to 1399, from 1397 to 1401, from 1398 to 1400, from 1399 to 1402, from 1400 to 1401, and from 1401 to 1402.

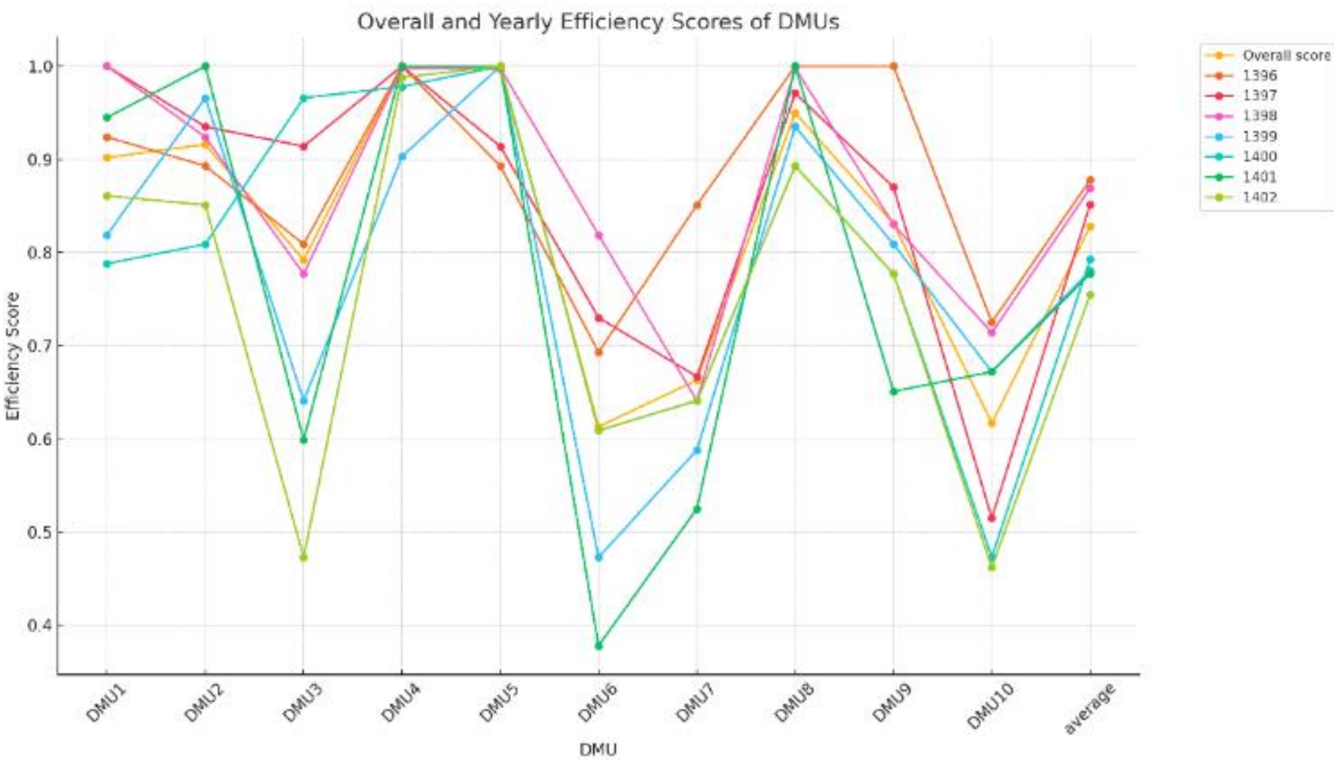

FIGURE 1
COMPARISON OF OVERALL AND PERIOD EFFICIENCY SCORES FOR DMUS FROM 1396 TO 1402.

IV Comparison of the Proposed Model and DNSBM

In this subsection, we compare the results obtained from the proposed model with those from the Dynamic Network Slack-Based Measure (DNSBM) proposed by Tone and Tsutsui [17] to validate the effectiveness of our model. Table 6 represents the results obtained by the DNSBM. Figure 2 compares the period and overall scores computed by DNSBM.

TABLE 6
THE SCORES OBTAINED BY THE DNSBM.

DMU	Overall score	rank	Overall Efficiency in 1396	Overall Efficiency in 1397	Overall Efficiency in 1398	Overall Efficiency in 1399	Overall Efficiency in 1400	Overall Efficiency in 1401
DMU1	1.000	1	1.000	1.000	0.809	0.782	0.981	0.868
DMU2	0.923	2	0.945	0.954	1.000	0.769	1.000	0.859
DMU3	0.829	5	0.904	0.787	0.679	1.000	0.639	0.493
DMU4	1.000	1	1.000	1.000	1.000	1.000	1.000	1.000
DMU5	0.853	4	0.934	1.000	1.000	1.000	1.000	1.000
DMU6	0.623	7	0.63	0.849	0.507	0.702	0.428	0.609
DMU7	0.871	3	0.687	0.631	0.498	0.695	0.625	0.641
DMU8	1.000	1	0.971	1.000	0.889	0.82	1.000	0.893
DMU9	1.000	1	0.88	0.81	0.71	0.677	0.551	0.7827
DMU10	0.755	6	0.525	0.734	0.658	0.455	0.692	0.482
average	0.8854	-	0.8476	0.8765	0.775	0.79	0.7916	0.76277

Comparing the overall scores obtained by the proposed model and DNSBM it can be concluded that the proposed model is more discriminative than DNSBM, and the overall scores obtained by the proposed model are lower than those of DNSBM (See Figure 2).

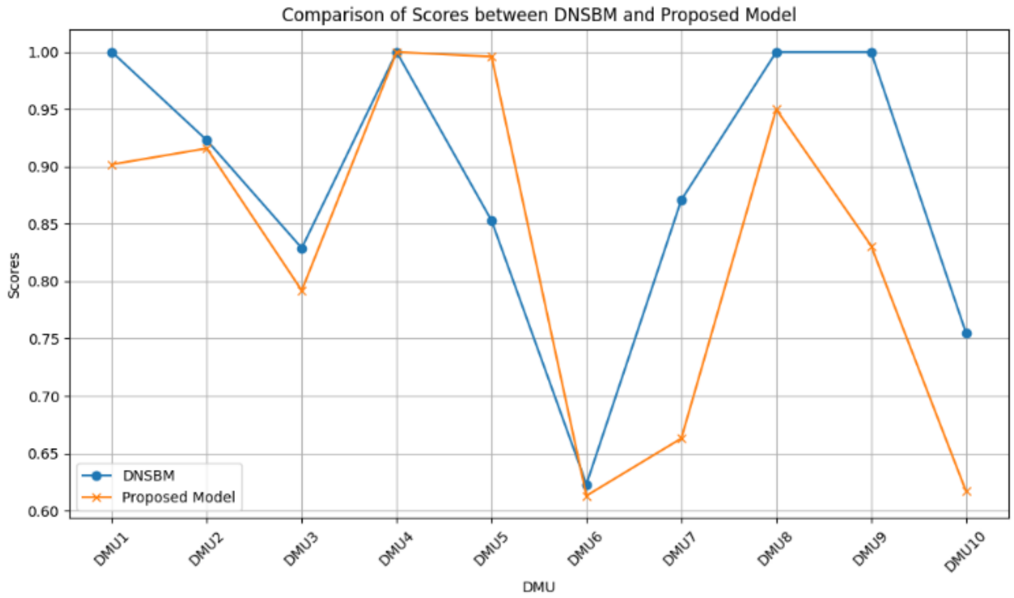

FIGURE 2
COMPARISON OF THE OVERALL SCORES OBTAINED BY PROPOSED MODEL AND DNSBM.

Observing the average scores obtained from both models (see Figure 3), it's evident that the trends display a high degree of similarity. However, there's a noticeable gap among the scores, likely due to differing assumptions in the allocation of carry-over variables. Overall, the scores obtained from the proposed model are lower than those of DNSBM. Notably, both models show the lowest average score in 1402, while the highest average score is recorded in 1396.

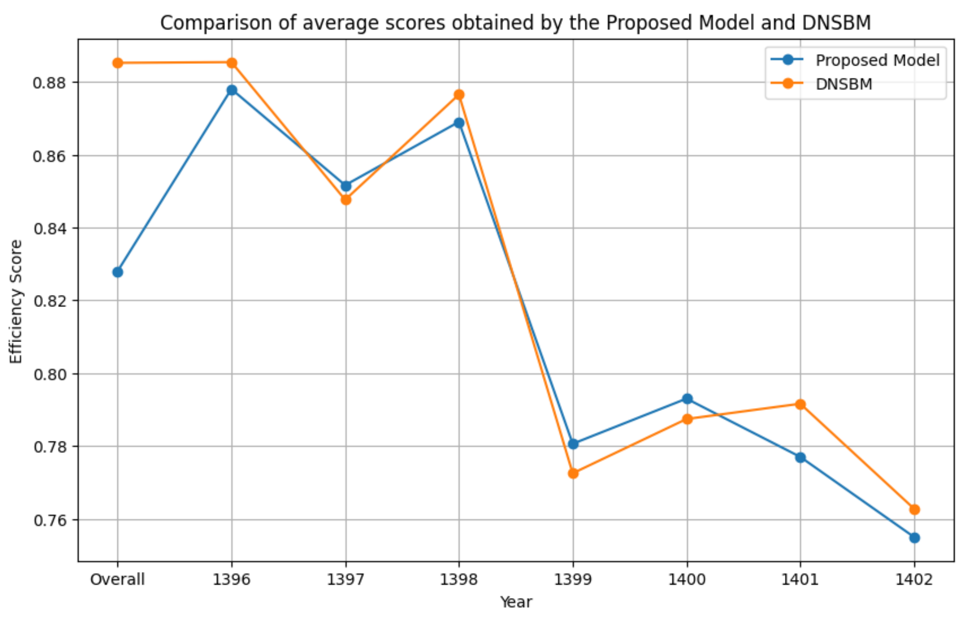

FIGURE 3
COMPARISON OF THE AVERAGE OVERALL SCORES OBTAINED BY PROPOSED MODEL AND DNSBM.

In this subsection, we conducted a comparison between the scores derived from the proposed model and DNSBM. The findings reveal that the proposed model not only demonstrates higher discriminative power but also offers an optimal strategy for allocating carry-over activities, thereby providing valuable insights.

CONCLUSION

In this study, we investigated the dynamic allocation of carry-over variables within the framework of resource management utilizing a novel DNDEA model. By conducting a thorough review of prior literature, we illuminated the importance of allocating discretionary carry-over activities to improve organizational performance evaluation across various time periods. The proposed dynamic network DEA model not only calculates the overall, divisional, and period efficiencies of DMUs with network structure but also effectively integrates inefficiencies related to carry-over activities. by strategic allocation of carry-over variables across time periods, the proposed model measures the overall scores. This dynamic allocation not only enhances organizations' adaptability and responsiveness to changing market conditions but also strengthens their competitive edge. To elucidate the proposed approach and demonstrate its effectiveness, we conducted an empirical study evaluating the performance of 10 Iranian bank branches from the years 1396 to 1402. The model results uncover inefficiencies across various time periods and clarifies their origins within the system. The analysis revealed significant performance variability, with DMU4 achieving the highest overall efficiency score of 1.000 and DMU10 the lowest at 0.617. Our model highlighted that loan loss reserves (LLRs) can be strategically allocated to improve efficiency, while non-performing loans (NPLs) are challenging to manage due to their nature as bad carryovers. The efficiency trends showed that some branches, like DMU5, maintained high performance consistently, whereas others, such as DMU6, exhibited fluctuations. Comparison between the proposed model and DNSBM reveals that the proposed model demonstrates superior discriminative power, adept at pinpointing inefficiency sources. Its lower scores compared to DNSBM enhance its discriminative capability.

Additionally, the proposed model effectively reveals optimal resource allocation, providing invaluable insights for organizational efficiency enhancement. For future research, allocating carry-overs to future periods and determining the optimal division to receive them could be explored. This investigation could shed light on efficient resource allocation strategies and enhance organizational performance. This study has several limitations that should be acknowledged. First, the research is based on data from a limited number of bank branches (10) over a specific period (1396 to 1402). This narrow scope may not fully represent the broader variability or long-term trends in bank performance, potentially affecting the generalizability of the findings to other sectors or regions. Second, the DNDEA model relies on several assumptions, such as treating non-performing loans as bad carryovers, which may not capture all nuances of different financial environments or banking regulations. Lastly, the model does not account for external factors like economic conditions, regulatory changes, or technological advancements that could impact bank performance and influence efficiency scores and resource allocation strategies. These limitations suggest that while the model provides valuable insights, further research is needed to address these constraints and enhance the robustness of the findings.

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