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Two-stage fuzzy network based on central resource allocation

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Abstract

This Evaluation of fuzzy networks with imprecise data is crucial. In this article, we propose fuzzy two-stage network models based on the structure of central resource allocation models. Firstly, we obtain the target for the fuzzy decision-making units in the two-stage network by using central resource allocation models, with a maximum of one two-phase model in each stage of the network. Then, we determine the overall target for the network. The probability function approach is used in the two-stage fuzzy network models to rephrase the proposed models and find the target. In conclusion, we calculate the target for Iranian airlines using fuzzy data and the proposed model.

Keywords: Data Envelopment Analysis; DEA network; Fuzzy linear programming, central resources allocation; target.

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1. Introduction

Data Envelopment Analysis (DEA) is a method of analyzing data by considering both input and output vectors. It calculates relative efficiency and can determine the target units of a decision-maker using either fixed or variable scale efficiency technology. The CCR models, developed by Charnes, Cooper, and Rhodes in 1978, were the precursors to the BCC models proposed by Banker et al. in 1984. The CCR and BCC models consider fixed and variable scale efficiency technology, respectively [1][2]. Although calculating the efficiency score of decision-making units is very important, following the principles of DEA to build the efficiency frontier and determine efficient and inefficient units is the main basis of modeling in DEA. As a result, many studies have been conducted to identify efficient hyperplanes in DEA. For example, in 1978, Charnes, Cooper, and Rhodes investigated constructive hyperplanes (hyperplanes corresponding to a strong efficient boundary) [3]. In 1996, Yu et al. proposed a method for analyzing efficient hyperplanes [4]. Jahanshahloo et al. (2007) used hyperplanes to obtain the members of the reference set [5]. In 2009, Jahanshahloo, Shirzadi, and Mirdehghan presented a collective model and a multiplicative BCC model to obtain strong efficient hyperplanes [6]. In 2022, Leo, Chen et al. developed a three-stage network DEA approach for performance evaluation of BIM application in construction projects [7]. Merris et al. (2022) measured and evaluated multi-function parallel network hierarchical DEA systems [8]. Khovini (2022) proposed a two-stage network DEA with shared resources and illustrated the drawbacks and measured the overall efficiency [9]. In 2023, Amiri et al. proposed a new fuzzy DEA network based on possibility and necessity measures for agile supply chain performance [10].

In the real world, accurately measuring data can be a difficult and sometimes impossible task. As a result, the technique of Fuzzy Data Envelopment Analysis (FDEA) was introduced to handle imprecise data. The concept of fuzzy and fuzzy sets was first defined by Max Block in 1937, and since then, there have been extensive studies in the field of Fuzzy Data Envelopment Analysis (FDEA). For example, Azadi (2015) also proposed a new fuzzy DEA model to evaluate the effectiveness and efficiency of suppliers in sustainable supply chain management [10]. Ebrahimnejad et al. (2016) developed a new method for solving fuzzy transportation problems [11], while Lozano (2020) proposed a fuzzy DEA slacks-based approach [12]. HassanzadehLotfi et al. (2020) investigated solving the fully fuzzy multi objective transportation problem based on the common set of weights in DEA [13]. Chen et al. (2021) studied a Fuzzy fault detection for Markov jump systems with partly accessible hidden information [14], and Wang et al. (2021) presented Wang et al. (2021) presented a fuzzy mid-term capacity and production planning model for manufacturing system with cloud-based capacity [15].

On the other hand, in the primary models of data envelopment analysis, to evaluate the decision-making units, the problem should be solved by their number, therefore, central resource allocation models (CRA) were presented, based on which the target of all DMUs can be modeled by solving only one Calculate the linear programming problem on the efficiency frontier. In fact, one of the advantages of central resource allocation models compared to traditional data envelopment analysis models is this feature. In a special case, in the input-oriented centralized allocation model, instead of reducing the input of each DMU, the total input consumption of all DMUs is reduced. Golani et al. in 1995 presented a model that assigns all input values to a decision-making unit without limiting the output changes of each unit [16]. After that, Lozano and Villa in 2004 presented central resource allocation models using data envelopment analysis. In their proposed model, the total output production does not decrease and by presenting a linear programming problem, the image of all units on the efficiency frontier. [17] Also, Lozano et al. in 2004 developed central resource allocation models by introducing correct variables and investigated the application of the proposed models by providing an example on the paper industry [18]. Then, in 2012, HosseinzadehLotfi et al. presented a centralized resource allocation model with random data and showed that in the proposed model, the total random input is reduced, but the proposed model is in a situation where the manager faces limited resources in total inputs or total outputs. It was not used [19].

The main goal of this article is to evaluate the effectiveness of two-stage fuzzy network modeling and the use of CRA structure in finding a suitable model for decision-making units. One of the key advantages of this article is the utilization of a two-phase fuzzy linear programming model to identify a target for fuzzy decision-making units. Additionally, the article highlights the importance of using the probability function approach to de-fuzzily the proposed models. In the second part, the basic concepts of CRA and the fuzzy probability function are briefly presented. In the third part, first and second stage models of the fuzzy network are proposed, then the general stage model and its de-fuzzification based on the probability function are presented. In the fourth section, a practical example is given and the conclusion is at the end.

2. Basic concepts

In this section, some materials related to non-radial CRA model, basic fuzzy concepts are given.

2.1. input - oriented non-radial CRA model

Assume n decision making units with m inputs $X_j=\left(x_{1j},x_{2j},\dots,x_{mj}\right)$ and s output $Y_j=\left(y_{1j},x_{2j},\dots,y_{sj}\right)$ is available and $X_j,Y_j\geq 0$ the non-radial CRA model consists of two stages in the input nature. In phase one, a different reduction factor is considered for each input. Suppose that the w_i priority factor is to reduce the overall consumption of the ith input and θ_i be the factor of reducing the overall consumption of the ith input. In this case we have

$$
\begin{aligned}
& z^*=Min \quad \sum_{i=1}^{m} w_i\theta_i && (1)\\
& s.t \quad \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}x_{ij}\leq\theta_i\sum_{j=1}^{n}x_{ij}, && i=1,\dots,m\\
& \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}y_{rj}\geq\sum_{j=1}^{n}y_{rj}, && r=1,\dots,s\\
& \sum_{j=1}^{n}\lambda_{jp}=1, && p=1,\dots,n\\
& \lambda_{jp}\geq 0,\ j=1,\dots,n,\ p=1,\dots,n,\theta\ free
\end{aligned}
$$

Model (1) is a linear programming problem with 1+2+…n variables and m + s + n constraints. If z* is the optimal value of model (1), then the second phase of the model is expressed as (2).

$$
\begin{aligned}
& Max\sum_{r=1}^{s}t_r && (2)\\
& s.t \quad \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}x_{ij}=z^*\sum_{j=1}^{n}x_{ij}, && i=1,\dots,m\\
& \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}y_{kj}=\sum_{j=1}^{n}y_{rj}+t_r, && r=1,\dots,s\\
& \sum_{j=1}^{n}\lambda_{jp}=1, && p=1,\dots,n\\
& \lambda_{jp}t_r\geq 0,\ j=1,\dots,n,\ p=1,\dots,n,r=1,\dots,s
\end{aligned}
$$

In model (2), the non-radial reduction of the first phase has been done, and on the other hand, the first set of constraints of the inputs are valid, so it does not need any slack input variables. After solving the model (2), an efficient point is obtained for the desired DMU. Therefore, the input and output of each point can be calculated as (3).

$$\hat{x}_{ip}=\sum_{j=1}^{n}\lambda^{*}{}_{jp}x_{ij} \qquad ,i=1,...,m, r=1,...,s, p=1,...,n \quad (3)$$
$$\hat{y}_{rp}=\sum_{j=1}^{n}\ \lambda^{*}{}_{jp}$$

Therefore, the advantage of CRA models compared to traditional DEA models is that, firstly, instead of solving a separate linear programming model for each DMU, all DMUs are evaluated simultaneously, and secondly, instead of reducing the inputs in the evaluated DMUs, the overall consumption The input of all DMUs is reduced.

3. Two-stage network with fuzzy data

In this section, first, the target of the fuzzy decision-making units for the two-stage network is obtained based on central resource allocation models with a maximum of one two-phase model in each stage of the network, then the target is determined for the overall state of the network. In two-stage fuzzy network models, the probability function approach has been used to de-phase the proposed models to find the target.

3.1 Fuzzy model of two-stage fuzzy network

Consider the two-stage network as shown in Figure (1):

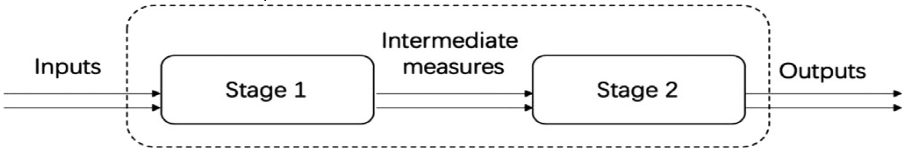

Figure 1. Two-stage fuzzy network

Considering the fuzzy vectors in the two-stage network, we consider the network fuzzy model based on CRA as follows.

$$
\begin{aligned}
& Min\, \theta^{1}_{CRA} && (4)\\
s.t\ \ & \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{x}_{ij} \le \theta^{1}_{CRA}\sum_{j=1}^{n}\tilde{x}_{ij}, && \forall i\\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{z}_{lj} \ge \sum_{j=1}^{n}\tilde{z}_{lj}, && \forall l\\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{z}_{lj} \le \sum_{j=1}^{n}\tilde{z}_{lj}, && \forall l\\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{y}_{rj} \ge \sum_{j=1}^{n}\tilde{y}_{rj}, && \forall r\\
& \sum_{j=1}^{n}\lambda_{jp}=1, && \forall p\\
& \mu_{j}\ge 0, && \forall j
\end{aligned}
$$

In phase (I), it is necessary to solve the fuzzy programming model (4) and in phase (II), taking into account the model of maximum auxiliary variables, it is a fuzzy programming model that is solved to find the appropriate pattern of decision-making units. Based on the idea of Lozano et al. [22], the model of decision-making units is obtained from the following relationship:

$$(\sum_{j=1}^{n}\lambda_{jp}\tilde{x}_{ij},\sum_{j=1}^{n}\lambda_{jp}\tilde{z}_{lj},\sum_{j=1}^{n}\lambda_{jp}\tilde{y}_{rj})$$
$$\forall p,\forall i,\forall r,\forall l. \qquad (5)$$

In model (4), the appropriate pattern for the first stage of the network is obtained. Of course, the model (4) is a fuzzy linear programming problem that is converted into a linear form by the approach of the probability function, and from the solution of the linear model, a suitable pattern can be depicted on the border that is built on the basis of CRA. Similarly, the second stage fuzzy network model based on CRA is proposed as follows.

$$Min\,\theta_{CRA}^{2} \tag{6}$$

$$
\begin{aligned}
s.t \quad & \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{x}_{ij} \le \sum_{j=1}^{n}\tilde{x}_{ij}, && \forall i \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{z}_{lj} \ge \sum_{j=1}^{n}\tilde{z}_{lj}, && \forall l \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{z}_{lj} \le \sum_{j=1}^{n}\tilde{z}_{lj}, && \forall l \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{y}_{rj} \ge \theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{y}_{rj}, && \forall r \\
& \sum_{j=1}^{n}\lambda_{jp} = 1, && \forall p \\
& \mu_{j} \ge 0, && \forall j
\end{aligned}
$$

3.2 The overall phase of the fuzzy network

The fuzzy network model of the overal stage based on CRA, considering and corresponding to the first and second stages of the network, is proposed as (7):

$$Min\,\theta_{CRA}^{1} \tag{7}$$

$$Min\,\theta_{CRA}^{2}$$

$$
\begin{aligned}
s.t & \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}\tilde{X}_{ij} \le \theta_{CRA}^{1}\times\theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{X}_{ij}, && \forall i \\
& \sum_{p=1}^{n}\sum_{j=1}^{n}\lambda_{jp}\tilde{Z}_{lj} \ge \theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}, && \forall l \\
& \sum_{p=1}^{n}\sum_{j=1}^{n}\mu_{jp}\tilde{Z}_{lj} \le \theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}, && \forall l \\
& \sum_{p=1}^{n}\sum_{j=1}^{n}\mu_{jp}\tilde{Y}_{rj} \ge \sum_{j=1}^{n}\tilde{Y}_{rj}, && \forall r \\
& \sum_{j=1}^{n}\lambda_{jp} = 1, && \forall p \\
& \sum_{j=1}^{n}\mu_{jp} = 1, && \forall p
\end{aligned}
$$

Model (7) is a fuzzy nonlinear programming problem and has two objectives. Therefore, we convert model (7) into a fuzzy linear programming problem with the appropriate variable. It should be noted that model (7) in case of maximizing the variables an auxiliary related to the adverbs means that in the second phase, it can calculate the appropriate model of the decision-making units based on the fuzzy probability function. Now, by changing the variable, the model (7) can be converted into the (8).

$$Min\,\theta_{CRA}^{ALL} \tag{8}$$

$$
\begin{aligned}
s.t & \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{X}_{ij} \le \theta_{CRA}^{ALL}\sum_{j=1}^{n}\tilde{X}_{ij}, && \forall i \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{Z}_{lj} \ge \theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}, && \forall l \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Z}_{lj} \le \theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}, && \forall l \\
& \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Y}_{rj} \ge \sum_{j=1}^{n}\tilde{Y}_{rj}, && \forall r \\
& \sum_{j=1}^{n}\lambda_{jp} = 1, && \lambda_{jp} \ge 0 \\
& \sum_{j=1}^{n}\mu_{jp} = 1, && \mu_{jp} \ge 0
\end{aligned}
$$

Model (8) is a linear model. If an optimal solution is model (8), then the relative efficiency of the second stage and the relative efficiency of the first stage are defined. Therefore, considering the boundary image method, the overall efficiency and the relative efficiency of both stages of the model (8) are obtained.

3.3 overall step of fuzzy network with fuzzy probability function approach

In this section, we consider the overal phase model of the fuzzy network without introducing the generality of the gap reasoning, and the probability function approach is used to de-fuzzify it. Model (8) is a linear and fuzzy model that must be de-fuzzified to solve it, so the fuzzy probability function approach of Charz and Cooper [23] is used.

$$Min\,\theta_{CRA}^{ALL} \tag{9}$$

$$s.t\ \pi(\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{X}_{ij}-\theta_{CRA}^{ALL}\sum_{j=1}^{n}\tilde{X}_{ij}\le 0)\ge\alpha_i,\forall i$$

$$\pi(\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{Z}_{lj}-\theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}\ge 0)\ge\beta_l,\ \forall l$$

$$\pi(\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Z}_{lj}-\theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj}\le 0)\ge\gamma_l,\ \forall l$$

$$\pi(\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Y}_{rj}-\sum_{j=1}^{n}\tilde{Y}_{rj}\ge 0)\ge\omega_r,\qquad \forall r$$

$$\sum_{j=1}^{n}\lambda_{jp}=1,\qquad \lambda_{jp}\ge 0$$

$$\sum_{j=1}^{n}\mu_{jp}=1,\qquad \mu_{jp}\ge 0$$

In model (9) $\alpha_i,\beta_i,\gamma_l$ and ω_r the predicted acceptable levels for the first to fourth constraints are all in the range of [0, 1]. In this model, the value of the objective function is a minimum value. While all the constraints are met at the level of the predicted probabilities. This model is a fuzzy model and to solve this model, it should be de-fuzzified. This is possible due to the emergence of acceptable levels of $\alpha_i,\beta_i,\gamma_l$ and ω_r in the above model. Considering that the fuzzy variables are convex and normal, therefore, for every possible level $\tilde{X}_{ij}$ and $\tilde{Z}_{lj}$ which are in the interval $\alpha_i,\beta_i,\gamma_l$ and ω_r [0,1], Lemma 1 holds. Therefore, by applying Lemma 1 on model (9), it is possible to find the upper and lower bounds of the alpha level sets corresponding to the first to fourth constraints in the mentioned model and write this model in the following form:

$$Min\,\theta_{CRA}^{ALL} \tag{10}$$

$$s.t(\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{X}_{ij}-\theta_{CRA}^{ALL}\sum_{j=1}^{n}\tilde{X}_{ij})_{\alpha_i}^{l}\le 0,\qquad \forall i$$

$$(\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}\tilde{Z}_{lj}-\theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj})_{\beta_l}^{u}\ge 0,\qquad \forall l$$

$$(\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Z}_{lj}-\theta_{CRA}^{2}\sum_{j=1}^{n}\tilde{Z}_{lj})_{\gamma_l}^{l}\le 0,\qquad \forall l$$

$$(\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}\tilde{Y}_{rj}-\sum_{j=1}^{n}\tilde{Y}_{rj})_{r}^{u}\ge 0,\qquad \forall r$$

$$\sum_{j=1}^{n}\lambda_{jp}=1,\qquad \lambda_{jp}\ge 0$$

$$\sum_{j=1}^{n}\mu_{jp}=1,\qquad \mu_{jp}\ge 0$$

Due to the fuzziness of the variables of the model (10), we can write the formula (11):

$$Min\,\theta_{CRA}^{ALL} \tag{11}$$

$$s.t\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}(\tilde{X}_{ij})_{\alpha_i}^{l}-\theta_{CRA}^{ALL}\sum_{j=1}^{n}(\tilde{X}_{ij})_{\alpha_i}^{l}\le 0,\forall i$$

$$\sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}(\tilde{Z}_{lj})_{\beta_l}^{u}-\theta_{CRA}^{2}\sum_{j=1}^{n}(\tilde{Z}_{lj})_{\beta_l}^{u}\ge 0,\ \forall l$$

$$\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}(\tilde{Z}_{lj})_{\gamma_l}^{l}-\theta_{CRA}^{2}\sum_{j=1}^{n}(\tilde{Z}_{lj})_{\gamma_l}^{l}\le 0,\ \forall l$$

$$\sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}(\tilde{Y}_{rj})_{r}^{u}-\sum_{j=1}^{n}(\tilde{Y}_{rj})_{r}^{u}\ge 0,\qquad \forall r$$

$$\sum_{j=1}^{n}\lambda_{jp}=1,\qquad \lambda_{jp}\ge 0$$

$$\sum_{j=1}^{n}\mu_{jp}=1,\qquad \mu_{jp}\ge 0$$

Model (11) is still a fuzzy linear programming problem. In this model, the fuzzy variables and fuzzy numbers are trapezoidal and according to theorem 1, any trapezoidal fuzzy number that can be represented as the upper and lower bounds of alpha-level sets can be by using the related α-cuts, it was de-fuzzified and converted into a definite number. Therefore, the model (11) can be removed from the fuzzy state and converted into a non-fuzzy programming model. Therefore, according to theorem 1, model (11) can be written as (12):

$$Min\,\theta_{CRA}^{ALL} \tag{12}$$

$$s.t \begin{cases} \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}(x_{ij1}+(x_{ij2}-x_{ij1})\alpha_i)- \\ \theta_{CRA}^{ALL}\sum_{j=1}^{n}(x_{ij1}+(x_{ij2}-x_{ij1})\alpha_i)\le 0 \end{cases}, \quad \forall i$$

$$\begin{cases} \sum_{j=1}^{n}\sum_{p=1}^{n}\lambda_{jp}(z_{lj4}-(z_{lj4}-z_{lj3})\beta_l)- \\ \theta_{CRA}^{2}\sum_{j=1}^{n}(z_{lj4}-(z_{lj4}-z_{lj3})\beta_l)\ge 0 \end{cases}, \quad \forall l$$

$$\begin{cases} \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}(z_{lj1}+(z_{lj2}-z_{lj1})\gamma_l)- \\ \theta_{CRA}^{2}\sum_{j=1}^{n}(z_{lj1}+(z_{lj2}-z_{lj1})\gamma_l)\le 0 \end{cases}, \quad \forall l$$

$$\begin{cases} \sum_{j=1}^{n}\sum_{p=1}^{n}\mu_{jp}(y_{rj4}-(y_{rj4}-y_{rj3})\omega_r)- \\ \sum_{j=1}^{n}(y_{rj4}-(y_{rj4}-y_{rj3})\omega_r)\ge 0 \end{cases}, \quad \forall r$$

$$\sum_{j=1}^{n}\lambda_{jp}=1, \qquad \lambda_{jp}\ge 0$$

$$\sum_{j=1}^{n}\mu_{jp}=1, \qquad \mu_{jp}\ge 0$$

Therefore, the model (12) becomes a parametric linear programming problem. and targets are obtained for the overall state of the network.

4. Numerical example

This section, we analyze the performance of 24 non-life insurance companies in Taiwan where the operation of each company includes two distinct processes; (i) premium acquisition and (ii) profit generation. The inputs of the first process are operating expenses (x_1) and insurance expenses (x_2) to produce the two intermediate measures; direct written premiums (z_1) and reinsurance premiums (z_2). All these intermediate measures are then consumed by the second process to produce the two final outputs; underwriting profit (y_1) and investment profit (y_2). The fuzzy data has been created based on the data of 2001 and 2002 to deal with imprecision to some appropriate extent. The data are shown in Table 1.

5. Conclusion

In general, the main goal of this paper is to model the two-stage fuzzy network and use the CRA model to evaluate fuzzy networks with imprecise data. In this article, using a two-phase fuzzy linear programming model, the pattern of fuzzy decision-making units is obtained, and based on the structure of central resource allocation models, two-stage fuzzy network models are proposed. First, the target of the fuzzy decision-making units for the two-stage network is obtained, then the target is determined for the overall state of the network. In two-stage fuzzy network models, the probability function approach is used to de-phase the proposed models to find the target.

Table 1. Trapezoidal fuzzy numbers of 24 insurance companies in Taiwan

	x_1	x_2	z_1	z_2	y_1	y_2
1	(1113,1178,1178,1256)	(636,673,673,717)	(7041,7451,7451,7934)	(809,856,856,912)	(930,984,984,1049)	(644,681,681,726)
2	(1305,1381,1381,1472)	(1278,1352,1352,1441)	(9469,10020,10020,10.681)	(1712,1812,1812,1932)	(1160,1228,1228,1309)	(788,834,834,889)
3	(1112,1117,1117,1255)	(559,592,592,631)	(4513,4776,4776,5091)	(529,560,560,597)	(227,293,293,312)	(622,658,658,701)
4	(568,601,601,641)	(561,594,594,633)	(2999,3174,3174,3383)	(351,371,371,395)	(234,248,248,264)	(167,177,177,189)
5	(6331,6699,6699,7141)	(3167,3351,3351,3572)	(35335,37362,37362,39.680)	(1657,1753,1753,1869)	(7419,7851,7851,8369)	(3709,3925,3925,4184)
6	(2483,2627,2627,2800)	(631,668,668,712)	(9211,9747,9747,10.390)	(900,952,952,1015)	(1619,1713,1713,1826)	(392,415,415,442)
7	(1853,1942,1942,2047)	(1377,1443,1443,1521)	(10193,10685,10685,11262)	(613,643,643,678)	(2136,2239,2239,2350)	(419,439,439,463)
8	(3615,3789,3789,3994)	(1787,1873,1873,1974)	(16473,17267,17267,18.199)	(1082,1134,1134,1195)	(3720,3899,3899,4110)	(593,622,622,656)
9	(1495,1567,1567,1652)	(906,950,950,1001)	(10945,11473,11473,12.093)	(521,546,546,575)	(995,1043,1043,1099)	(252,264,264,278)
10	(1243,1303,1303,1373)	(1238,1298,1298,1368)	(7832,8210,8210,8653)	(481,504,504,531)	(1619,1697,1697,1789)	(529,554,554,584)
11	(1872,1962,1962,2068)	(641,672,672,708)	(6890,7222,7222,7612)	(613,43,643,678)	(1418,1486,1486,1566)	(17,18,18,19)
12	(2473,2592,2592,2732)	(620,650,650,685)	(9000,9434,9434,9943)	(1067,1118,1118,1178)	(1502,1574,1574,1652)	(867,909,909,958)
13	(2481,2609,2609,2739)	(1301,1368,1368,1436)	(13239,13921,13921,14.617)	(771,811,811,852)	(3432,3609,3609,3789)	(212,223,223,234)
14	(1328,1369,1369,1466)	(940,988,988,1037)	(7034,7396,7396,7766)	(442,465,465,488)	(1332,1401,1401,1471)	(316,332,332,349)
15	(2077,2184,2184,2293)	(619,651,651,684)	(9911,10422,10422,10.943)	(712,749,749,786)	(3191,3355,3355,3523)	(528,555,555,583)
16	(1152,1211,1211,1272)	(395,415,415,436)	(5331,5606,5606,5886)	(382,402,402,422)	(812,854,854,897)	(187,197,197,207)
17	(1382,1453,1453,1526)	(1032,1085,1085,1139)	(7318,7695,7695,8080)	(325,3452,342,359)	(2990,3144,3144,3301)	(353,371,371,390)
18	(720,757,757,795)	(520,547,547,574)	(3453,3631,3631,3813)	(947,995,995,1045)	(658,692,692,727)	(155,163,163,171)
19	(151,159,159,167)	(173,182,182,191)	(1083,1141,1141,1196)	(458,483,483,506)	(493,519,519,544)	(44,46,46,48)
20	(138,145,145,152)	(50,53,53,56)	(300,316,316,331)	(124,131,131,137)	(337,355,355,372)	(25,26,26,27)
21	(80,84,84,88)	(25,26,26,27)	(214,225,225,236)	(38,40,40,42)	(48,51,51,53)	(6,6,6,6)
22	(14,15,15,16)	(9,10,10,10)	(49,52,52,54)	(13,14,14,15)	(78,82,82,86)	(4,4,4,4)
23	(51,54,54,57)	(27,28,28,29)	(233,245,245,257)	(47,49,49,51)	(1,1,1,1)	(17,18,18,19)
24	(155,163,163,171)	(223,235,235,246)	(452,476,476,499)	(611,644,644,675)	(135,142,142,149)	(15,16,16,17)

Table 2. The results of the first stage

	X_1	X_2	z_1	z_2
1	820.437	469.112	5563.147	641.740
2	1139.000	650.800	7740.800=z	889.600
3	1139.000	650.800	7740.800	889.600
4	1139.000	650.800	7740.800	889.600
5	1139.000	650.800	7740.800	889.600
7	1139.000	650.800	7740.800	889.600
7	1139.000	650.800	7740.800	889.600
8	1139.000	650.800	7740.800	889.600
9	1139.000	650.800	7740.800	889.600
10	1139.000	650.800	7740.800	889.600
11	1139.000	650.800	7740.800	889.600
12	1139.000	650.800	7740.800	889.600
13	1139.000	650.800	7740.800	889.600
14	139.000	650.800	7740.800	889.600
15	1139.000	650.800	7740.800	889.600
16	162.372	180.535	1228.495	500.060
17	154.200	176.600	1174.000	496.800
18	154.200	176.600	1174.000	496.800
19	154.200	176.600	1174.000	496.800
20	154.200	176.600	1174.000	496.800
21	154.200	176.600	1174.000	496.800
22	154.200	176.600	1174.000	496.800
23	154.200	176.600	1174.000	496.800
24	154.200	176.600	1174.000	496.800

Table 3. The results of the second stage

	z_1	Z_2	Y_1	Y_2
1	25485.651	1298.732	6234.566	2643.534
2	10115.400	726.800	3455.800	571.800
3	10115.400	726.800	3455.800	571.800
4	10115.400	726.800	3455.800	571.800
5	10115.400	726.800	3455.800	571.800
6	10115.400	726.800	3455.800	571.800
7	10115.400	726.800	3455.800	571.800
8	36145.800	1695.400	8161.800	4080.400
9	14650.537	693.753	3351.634	1652.866
10	50.200	13.400	84.400	4.000
11	50.200	13.400	84.400	4.000
12	50.200	13.400	84.400	4.000
13	50.200	13.400	84.400	4.000
14	50.200	13.400	84.400	4.000
15	50.200	13.400	84.400	4.000
16	50.200	13.400	84.400	4.000
17	50.200	13.400	84.400	4.000
18	50.200	13.400	84.400	4.000
19	50.200	13.400	84.400	4.000
20	50.200	13.400	84.400	4.000
21	50.200	13.400	84.400	4.000
22	50.200	13.400	84.400	4.000
23	50.200	13.400	84.400	4.000
24	50.200	13.400	84.400	4.000

Table 4. Results from the overall network stage

	X_1^1	X_2^1	z_1^1	z_2^1	z_1^2	z_2^2	y_1^2	y_2^2
1	829.769	474.434	5626.943	649.002	25485.651	1298.732	6234.566	2643.534
2	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
3	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
4	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
5	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
6	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
7	1139.000	650.800	7740.800	889.600	10115.400	726.800	3455.800	571.800
8	1139.000	650.800	7740.800	889.600	36145.800	1695.400	8161.800	4080.400
9	1139.000	650.800	7740.800	889.600	14650.537	693.753	3351.634	1652.866
10	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
11	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
12	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
13	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
14	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
15	1139.000	650.800	7740.800	889.600	50.200	13.400	84.400	4.000
16	153.040	175.212	1164.699	492.798	50.200	13.400	84.400	4.000
17	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
18	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
19	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
20	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
21	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
22	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
23	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000
24	154.200	176.600	1174.000	496.800	50.200	13.400	84.400	4.000

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