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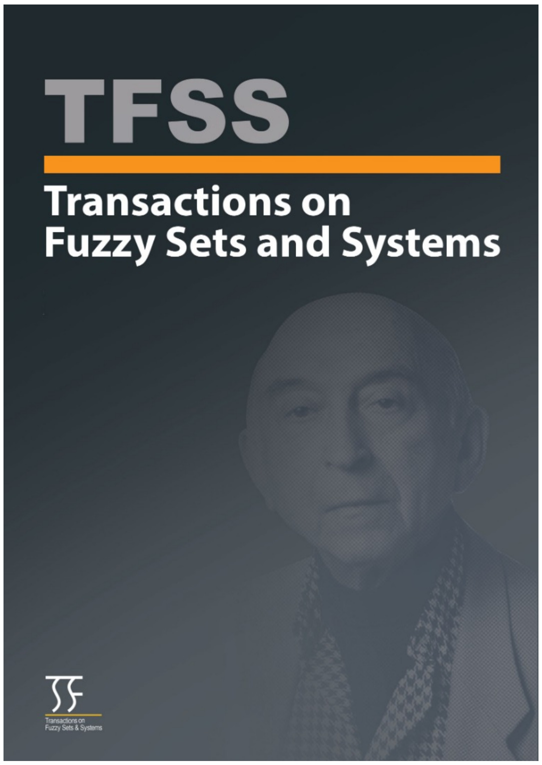

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Author(s):

Mithat Zeydan, Department of Industrial Engineering, Istanbul Medeniyet University, Istanbul, Turkey. E-mail: mithat.zeydan@medeniyet.edu.tr

Murat Güngör, Department of Industrial Engineering, Istanbul Medeniyet University, Istanbul, Turkey. E-mail: murat.gungor@medeniyet.edu.tr

Burak Urazel, Department of Electrical and Electronics Engineering, Eskişehir Osmangazi University, Eskişehir, Turkey. E-mail: burazel@ogu.edu.tr

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A TOPSIS-Based Improved Weighting Approach with Evolutionary Computation

Mithat Zeydan*, **Murat Güngör**, **Burak Urazel**

Abstract. Although optimization of weighted objectives is ubiquitous in production scheduling, the literature concerning the determination of weights used in these objectives is scarce. Authors usually suppose that weights are given in advance, and focus on the solution methods for the specific problem at hand. However, weights directly settle the class of optimal solutions, and are of utmost importance in any practical scheduling problem. In this study, we propose a new weighting approach for single machine scheduling problems. First, factor weights to be used in customer evaluation are found by solving a nonlinear optimization problem using the covariance matrix adaptation evolutionary strategy (CMAES) under fuzzy environment that takes a pairwise comparison matrix as input. Next, customers are sorted using the technique for order of preference by similarity to ideal solution (TOPSIS) by means of which job weights are obtained. Finally, taking these weights as an input, a total weighted tardiness minimization problem is solved by using mixed-integer linear programming to find the best job sequence. This combined methodology may help companies make robust schedules not based purely on subjective judgment, find the best compromise between customer satisfaction and business needs, and thereby ensure profitability in the long run.

AMS Subject Classification 2020: 90B50; 90B35
Keywords and Phrases: Covariance matrix adaptation evolutionary strategy, Technique for order of preference by similarity to ideal solution, Weighted single machine scheduling, Mixed-integer linear programming.

1 Introduction

Companies should develop customer-focused strategies for being one step ahead in today's competitive market. The primary rule, which is an overwhelming and daunting task, is getting to know and identifying customers better. This not only helps companies fulfill their expectations, but also facilitates prioritization. Actually, some customers are more valuable than others. In production scheduling, this is reflected in the practice of assigning weights to orders or jobs. Each jobs contribution to the objective function thereby depends on its weight.

Although optimization of weighted objectives is ubiquitous in production scheduling, the literature concerning the determination of weights used in these objectives is scarce. Authors usually suppose that weights are given in advance, and focus on the solution methods for the specific problem at hand. However, weights directly settle the class of optimal solutions, and are of utmost importance in any practical scheduling problem.

Lin et al. [14] consider a hybrid flow shop scheduling problem with dynamic reentrant characteristics substantiated by the complexities in a repairing company. A genetic algorithm is applied to obtain near-optimal

*Corresponding Author: Mithat Zeydan, Email: mithat.zeydan@medeniyet.edu.tr, ORCID: 0000-0001-9459-146X
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schedules, while the analytic hierarchy process (AHP) is used both to fulfill multiple criteria concerning the problem and to speed up the genetic algorithm's convergence. Deliktas et al. [6] propose an integrated approach for single machine scheduling with sequence-dependent setup times. In the first stage, job weights are determined by using AHP. In the second stage, a mixed-integer nonlinear programming model is built by considering three objective functions, namely the weighted number of tardy jobs, total weighted completion time, and makespan with sequence-dependent setup times. nemli [16] aims to create an algorithm to support the decision maker in the scheduling of customer orders for a box packaging production company in a make-to-order environment. In the first stage, the weighted tardiness of the orders is minimized, where the weights are determined by AHP, based on the knowledge and experience of experts. Ortiz-Barrios et al. [17] propose an integrated and enhanced method of a dispatching algorithm for scheduling flexible job shops based on fuzzy AHP and the technique for order of preference by similarity to ideal solution (TOPSIS). Fuzzy AHP is used to calculate the criteria weights under uncertainty, and TOPSIS is later applied to rank the eligible operations. Utku et al. [22] develop a mixed-integer programming model to minimize total lateness and total completion time of jobs in an automotive company. AHP is used to determine the weights of the two objectives.

Ignorance of weight determination in scheduling literature might be partly attributed to the gap between the theory and practice of scheduling. Stoop and Wiers [21] give an overview of the problems related to the complexity of scheduling in practice. Alternative suggestions to improve scheduling are proposed. First a description of scheduling and how it relates to planning and sequencing is presented. Then a description of problems that cause the scheduling function in practice to be very complex, and also an overview of shop floor models and scheduling techniques are given. Next, the problem of measuring schedule performance is discussed. Then possible solutions to the problems discussed are provided. Wiers [23] gives an overview of the applicability of techniques and the role of humans in production scheduling. He indicates that most of the literature reports give little indication of whether the system has been implemented in manufacturing practice, and for those systems that have been implemented, what types of implementation problems were encountered. The success of scheduling techniques in practice can only improve when researchers are aware of the implementation pitfalls through learning from each other's experiences. McKay and Wiers [15] argue that the gap between theory and practice in production scheduling has been confounded by the traditional view of scheduling as sequencing. This definition has focused researchers on the sequencing issue at the expense of the larger scheduling problem faced by practitioners dealing with the problems of partiality, temporality, and predictiveness. Namely, a scheduling process generates partial solutions for partial problems; anticipates, reacts to, and adjusts for disturbances in the process and environment; and is sensitive to and adjusts to the meaning of time in the production situation. The authors present an extended view of scheduling that unifies the traditional definition used in operations research and a number of key aspects of real-world scheduling. Dudek et al. [7] claim in the context of flow shop scheduling that scores of person-years of research time have been wasted on an intractable problem of little practical consequence. Although Gupta and Stafford [9] disagree with the viewpoint expressed by Dudek et al. [7], they admit that the mathematical theory of flow shop scheduling suffers from too much abstraction and too little application.

The purpose of this paper is to propose a new weighting approach with evolutionary computation for single machine scheduling problems. First, a pairwise comparison matrix that shows the relative importance of the criteria to be used in assessing customers is formed by having recourse to expert opinion, and criteria weights are determined by solving a nonlinear optimization problem via covariance matrix adaptation evolutionary strategy (CMAES) under fuzzy environment. Second, customer orders are ranked according to these criteria with TOPSIS. Finally, orders are sequenced so as to minimize total weighted tardiness by mixed-integer linear programming, where TOPSIS performance scores are taken as input.

The remainder of the paper is organized as follows. Section 2 presents the workflow regarding the application of the new approach. Section 3 defines the total weighted tardiness minimization problem, presents its mixed-integer linear program and suggests one possible way of handling the problem of quoting due dates for new orders in this context. Section 4 gives the numerical solution for the case study in a Turkish textile firm. Finally, we summarize our findings in Section 5.

2 TOPSIS-Based Improved Weighting Approach with Evolutionary Algorithm

Workflow of the proposed approach consists in (1) forming a pairwise comparison matrix for the criteria to be used in assessing customers, (2) determination of the weights of these criteria by solving an optimization problem via CMAES taking the pairwise comparison matrix as input, and (3) finding customers' scores with respect to these weights by using TOPSIS. The steps will be explained in detail below.

2.1 Forming Pairwise Comparison Matrix

We assume that preference of criterion i over j is given by a triple $(x_{ij}^l, x_{ij}^m, x_{ij}^u)$. We call this a "fuzzy triangular number." Here the superscripts l, m, u stand for lower, middle, and upper, respectively. The middle coordinate x_{ij}^m may take an integer value in between 1 and 9. The equality $x_{ij}^m = 1$ implies that the criteria in question are equally important, whereas the equality $x_{ij}^m = 9$ implies that criterion i is extremely important compared to j. Unless $x_{ij}^m = 1$ or $x_{ij}^m = 9$, the first coordinate x_{ij}^l is 1 less than x_{ij}^m, and the third coordinate x_{ij}^u is 1 more than x_{ij}^m. If $x_{ij}^m = 1$, then all three coordinates are 1; if $x_{ij}^m = 9$, then all three coordinates are 9. Formally, pairwise comparison matrix formation using triangular numbers is composed of the following steps, where n denotes the number of criteria:

1. Form the tentative pairwise comparison matrix for the criteria:

$$\begin{pmatrix} (1,1,1) & (x_{12}^l, x_{12}^m, x_{12}^u) & \cdots & (x_{1n}^l, x_{1n}^m, x_{1n}^u) \\ (x_{21}^l, x_{21}^m, x_{21}^u) & (1,1,1) & \cdots & (x_{2n}^l, x_{2n}^m, x_{2n}^u) \\ \vdots & \vdots & \ddots & \vdots \\ (x_{n1}^l, x_{n1}^m, x_{n1}^u) & (x_{n2}^l, x_{n2}^m, x_{n2}^u) & \cdots & (1,1,1) \end{pmatrix}.$$

2. Perform defuzzification according to the formula

$$x_{ij} := \frac{x_{ij}^l + 4x_{ij}^m + x_{ij}^u}{6}.$$

3. Calculate the consistency index (CI) of the matrix (x_{ij}):

$$\mathrm{CI} := \frac{\lambda_{\max} - n}{n-1}.$$

   Here $\lambda_{\max}$ denotes the principal eigenvalue.

4. Calculate the consistency ratio (CR) of the matrix (x_{ij}):

$$\mathrm{CR} := \frac{\mathrm{CI}}{\mathrm{RI}}.$$

   Here RI is the random index associated with dimension n.

5. If CR is less than 0.1, then proceed to obtain criteria weights; otherwise, revise the pairwise comparison matrix.

2.2 Determination of Criteria Weights

Many methods exist for deriving preference values from judgment matrices [5]. Basically, the idea is to obtain weights w_i such that

$$\frac{w_i}{w_j} \approx x_{ij}$$

for all i, j where (x_{ij}) denotes the pairwise comparison matrix [3]. Let μ_{ij} be a piecewise linear function of weights defined as

$$\mu_{ij}(w_1, \ldots, w_n) = \begin{cases} \frac{(w_i/w_j)-x^l_{ij}}{x^m_{ij}-x^l_{ij}}, & w_i/w_j \le x^m_{ij}; \\ \frac{x^u_{ij}-(w_i/w_j)}{x^u_{ij}-x^m_{ij}}, & w_i/w_j > x^m_{ij}. \end{cases}$$

Note that μ_{ij} is an indicator of how well the weights w_j, w_j comply with the pairwise comparison value x_{ij}. We have

- $\mu_{ij} = 1$ if and only if $w_i/w_j = x^m_{ij}$,
- $\mu_{ij} \in (0, 1)$ for $w_i/w_j \in (x^l_{ij}, x^u_{ij}) \setminus \{x^m_{ij}\}$, and
- $\mu_{ij} \le 0$ whenever $w_i/w_j \le x^l_{ij}$ or $w_i/w_j \ge x^u_{ij}$.

Therefore, all μ_{ij} shall be as large as possible. One possible way towards this end is to maximize the minimum of the μ_{ij}. So we define

$$G(w_1, \ldots, w_n) := \min_{i<j} \mu_{ij}(w_1, \ldots, w_n).$$

Hence, weights can be determined by solving the following nonlinear optimization problem:

$$\begin{aligned} \text{maximize} \quad & G(w_1, \ldots, w_n) \\ \text{such that} \quad & w_1 + \cdots + w_n = 1. \end{aligned}$$

Note that, rewriting w_n in terms of $w_1, \ldots, w_{n-1}$, the problem can be converted into an unconstrained maximization problem. As in Zeydan et al. [24], we solve this by CMAES under fuzzy environment, which is a derivative-free stochastic global search algorithm developed recently [19]. It works iteratively by adapting the resulting search distribution to the contours of the objective function by updating the covariance matrix deterministically using information from evaluated points [19]. We refer the reader to Hansen [10] for details of the CMAES algorithm.

2.3 Ranking Customers with TOPSIS

Let there be m alternatives and n criteria indexed respectively by i and j. Criteria weights w_j are assumed to be given. Steps for ranking customers with the TOPSIS method can be stated as follows [11]:

1. Form the decision matrix (x_{ij}). (This is not to be confused with the matrix obtained in §2.1 after defuzzification.)

2. Construct the normalized decision matrix (r_{ij}):

$$r_{ij} := \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x^2_{kj}}}.$$

3. Construct the weighted normalized decision matrix (v_{ij}):

$$v_{ij} := w_j \times r_{ij}.$$

4. Determine the positive and negative ideal rows $(v_1^+, \ldots, v_n^+)$ and $(v_1^-, \ldots, v_n^-)$.

5. Measure the distance of each alternative from the ideal rows:

$$d_i^+ := \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^+)^2}, \qquad d_i^- := \sqrt{\sum_{j=1}^{n}(v_{ij} - v_j^-)^2}.$$

6. Calculate the closeness of the alternatives to the ideal solution, namely the TOPSIS scores:

$$\text{Score}_i := \frac{d_i^-}{d_i^+ + d_i^-}.$$

3 Total Weighted Tardiness Minimization Problem on a Single Machine

Let there be n jobs to be processed on a single machine. We index jobs by j. Each job has a processing time p_j, due date d_j, and weight w_j. Preemptions are not allowed; in other words, processing of a job cannot be interrupted until it is completed. Let C_j denote the completion time of job j. Tardiness is defined as

$$T_j := \max\{C_j - d_j, 0\}.$$

Thus, tardiness equals lateness if the job is late, and it is zero otherwise. The question is to find a schedule that minimizes total weighted tardiness. In the common three-field notation, the problem is $1 \mid\mid \sum w_j T_j$ [8, 18]. The objective function is nondecreasing in completion times; i.e., it is regular. So there exists an optimal schedule in which the machine is never kept idle. Therefore, the problem amounts to finding the best job sequence with respect to total weighted tardiness. Table 1 shows the indices, parameters, and decision variables for $1 \mid\mid \sum w_j T_j$.

Table 1: Indices, parameters, and decision variables for $1 \mid\mid \sum w_j T_j$.

Symbol	Explanation
j	job index
n	number of jobs
p_j	processing time of job j
d_j	due date of job j
w_j	weight of job j
C_j	completion time of job j
T_j	tardiness of job j

3.1 Mixed-Integer Linear Programming Formulation

Minimization of total weighted tardiness on a single machine, which is strongly NP-hard in terms of computational complexity [13], has received much attention in the literature [1, 20, 4]. Branch-and-bound and dynamic programming approaches have been proposed to obtain optimal solutions. The problem can also be modeled as a mixed-integer linear program (MILP). One can build a model based on precedence or time-indexed decisions [2]. We shall focus on the former in this section. Let x_{jk} be a binary variable defined as follows:

$$x_{jk} = \begin{cases} 1, & \text{if job } j \text{ precedes job } k; \\ 0, & \text{otherwise.} \end{cases}$$

If $x_{jk} = 1$, then $C_j \leq C_k - p_k$; if $x_{jk} = 0$, then $C_k \leq C_j - p_j$. These conditional statements can be expressed respectively as

$$C_j \leq C_k - p_k + M(1 - x_{jk}),$$
$$C_k \leq C_j - p_j + M x_{jk},$$

where M is a sufficiently large number. Note that the first inequality becomes redundant when $x_{jk} = 0$, and the second becomes redundant when $x_{jk} = 1$. It is enough to define x_{jk} for each pair of jobs, so there are $\binom{n}{2} = n(n-1)/2$ such variables. Below is the MILP formulation of $1 \mid\mid \sum w_j T_j$ using precedence constraints:

$$\begin{aligned}
\text{minimize} \quad & \sum_j w_j T_j && && \text{(1a)} \\
\text{subject to} \quad & C_j \leq C_k - p_k + M(1 - x_{jk}) && \text{for all } j < k && \text{(1b)} \\
& C_k \leq C_j - p_j + M x_{jk} && \text{for all } j < k && \text{(1c)} \\
& T_j \geq C_j - d_j && \text{for all } j && \text{(1d)} \\
& C_j \geq p_j && \text{for all } j && \text{(1e)} \\
& C_j, T_j \geq 0 && \text{for all } j && \text{(1f)} \\
& x_{jk} \in \{0, 1\} && \text{for all } j < k. && \text{(1g)}
\end{aligned}$$

The objective (1a) is to minimize the sum of weighted tardiness. Constraints (1b) and (1c) relate the precedence decisions to jobs' completion times as explained above. Inequality (1d) must hold as an equality in view of the objective whenever job j is late. The next constraint (1e) guarantees that the completion time of the first job in the sequence is nonzero. Note that in the formulation there are $2n$ continuous variables, namely C_j and T_j, in addition to the $n(n-1)/2$ binary variables x_{jk}. There are $n^2 + n$ constraints in total (except binary and nonnegativity restrictions). The big M can be taken as the sum of all processing times.

3.2 Due Date Quotation Problem

Although the number of tardy jobs is a common performance criterion in practice, its minimization may be an unrealistic objective as it may lead to schedules with unacceptably late jobs. The same is true of weighted tardiness minimization: if a job has a small weight relative to others, it may be overly late in an optimal schedule. Therefore, it makes sense to assume deadlines $\overline{d_j}$ for real-life applications. Deadlines represent hard constraints: in any feasible schedule, all deadlines must be met. Mathematically, the inequality $C_j \leq \overline{d_j}$ must be satisfied for all j. We assume that deadlines are defined as translations of due dates by a specified constant δ:

$$\overline{d_j} = d_j + \delta.$$

Then inequalities $C_j \leq \overline{d_j}$ can be stated equivalently as

$$T_j \leq \delta.$$

Thus, a job is ideally to be completed by its due date, but if it somehow happens to be late, the lateness cannot exceed δ. In the three-field notation, we express this problem by $1 \mid \overline{d_j} = d_j + \delta \mid \sum w_j T_j$.

In scheduling literature it is often assumed that due dates are given beforehand. However, in many circumstances, determination of due dates itself is a problem: what due date should be assigned to a new customer order? This is called the due date quotation or management problem. A comprehensive literature survey thereon is given by Kaminsky and Hochbaum [12].

We consider due date quotation in the setting $1 \mid \overline{d_j} = d_j + \delta \mid \sum w_j T_j$. More precisely, let there be n jobs, already sequenced to minimize $\sum w_j T_j$ respecting the deadlines. Suppose a new customer order, namely the $(n+1)$st job, has arrived. Its processing time p_{n+1} and weight w_{n+1} are known. What due date d_{n+1} should be assigned to this job? There is an intrinsic trade-off to be faced here: if d_{n+1} is large, the existing schedule will not be affected much and the objective value $\sum_{j=1}^{n+1} w_j T_j$ will be small, but the new customer's satisfaction will be less; if d_{n+1} is small, the situation is the other way around.

Let $d_{\min}$ and $d_{\max}$ be the minimum and maximum possible due dates for the $(n+1)$st job. If it is sequenced first, it will be completed by p_{n+1}; if it is sequenced last, it will be completed by $\sum_{j=1}^{n+1} p_j$. So we naturally have $d_{\min} := p_{n+1}$ and $d_{\max} := \sum_{j=1}^{n+1} p_j$. For each possible due date $d \in [d_{\min}, d_{\max}]$ for the $(n+1)$st job, let $z^*(d)$ be the objective value $\sum_{j=1}^{n+1} w_j T_j^*(d)$ associated with the optimal sequencing of all $n+1$ jobs (we assume δ is large enough to guarantee that there always exists a feasible solution). As mentioned above, z^* is a nonincreasing function of d; that is, for all d, d',

$$d \le d' \;\; \Rightarrow \;\; z^*(d) \ge z^*(d').$$

It follows that in this context the due date quotation, in essence, is a multi-objective optimization problem. Namely, we are to find the best compromise between the due date d_{n+1} to be assigned and the objective value $z^*(d_{n+1})$ associated with it.

Now we discuss one way to do this. Let $\sum_{j=1}^{n} w_j T_j^*$ be the optimal total weighted tardiness for the existing n jobs, let $\sum_{j=1}^{n} w_j T_j^*(d)$ be the updated value of this sum after the arrival of the $(n+1)$st job given that its quoted due date is d, and let

$$\Delta z(d) := \sum_{j=1}^{n} w_j T_j^*(d) - \sum_{j=1}^{n} w_j T_j^*$$

denote the difference of these two sums. Clearly, as d gets smaller, $\Delta z(d)$ gets larger. Suppose, without loss of generality, that $\sum_{j=1}^{n} w_j = 1$. Then an increase of $\Delta z(d)$ by 1 means that the tardiness of each one of the existing n jobs has increased on average by 1 time unit. What is the utility of this in terms of assigning a better due date to the $(n+1)$st job? In other words, if the pairs of solutions $(d, \Delta z(d))$ and $(d', \Delta z(d) + 1)$ are equivalent, what is $d - d'$? Ultimately, this depends on the decision-maker, but a possible answer would be w_{n+1}. Then the due date quotation problem can be written concisely as

$$\min_{d} \;\; w_{n+1} d + \Delta z(d).$$

4 Application in a Textile Company

In this section, we present a numerical demonstration of the weighting approach introduced above with data obtained from a textile firm in Turkey. The problem is to find an optimal sequence of 11 customer orders that minimizes total weighted tardiness. Customers are to be assessed with respect to five criteria: profitability (%), average order quantity (meters), unit selling price (dollars per meter), payment performance (lateness per order), risk limit (dollars). First of all, managers from three distinct departments—production, marketing, and finance—are consulted in order to construct pairwise comparison matrices. Then an aggregate matrix has been built as shown in Table 2.

Consistency ratio for the matrix in Table 2 turns out to be 0.089, so it is convenient to use this matrix as an input to CMAES to find criteria weights. We coded the 29 April 2014 version of CMAES algorithm using

Table 2: Aggregate pairwise comparison matrix for the five criteria.

	C1	C2	C3	C4	C5
C1	1.000	2.000	3.000	3.000	6.000
C2	0.556	1.000	2.000	2.000	7.000
C3	0.347	0.556	1.000	1.000	8.000
C4	0.347	0.556	1.000	1.000	8.000
C5	0.168	0.144	0.126	0.126	1.000

MATLAB 2018. Convergence of the method with respect to the objective function value and the weights are given in Figure 1. The resulting weights are

$$w_1 = 0.284, \quad w_2 = 0.335, \quad w_3 = 0.171 \quad w_4 = 0.164 \quad w_5 = 0.040.$$

Table 3 shows the decision matrix for the first step of the TOPSIS algorithm, namely numerical values associated with the 11 customers for the five aforementioned criteria. Tables 4 and 5 show the normalized and the weighted normalized versions thereof, respectively. Table 6 shows positive and negative ideal rows obtained from the weighted normalized decision matrix. Finally, Table 7 shows the distance of the alternatives (customers) to the ideal rows, their TOPSIS scores, and the relevant ranking.

Table 8 shows the data and the optimal solution of the single machine weighted tardiness minimization problem. The processing times and due dates are randomly generated following the weighted tardiness instance generation routine in the OR-Library maintained by John Beasley. We took the range of due dates (RDD) and the average tardiness factor (TF) parameters in this routine as 0.6. Weights are assumed to be the TOPSIS scores computed in Table 7. Solving the mixed-integer linear program (1), the optimal job sequence turns out to be (11, 8, 7, 2, 10, 5, 4, 1, 3, 6, 9) with an objective value of 204.19.

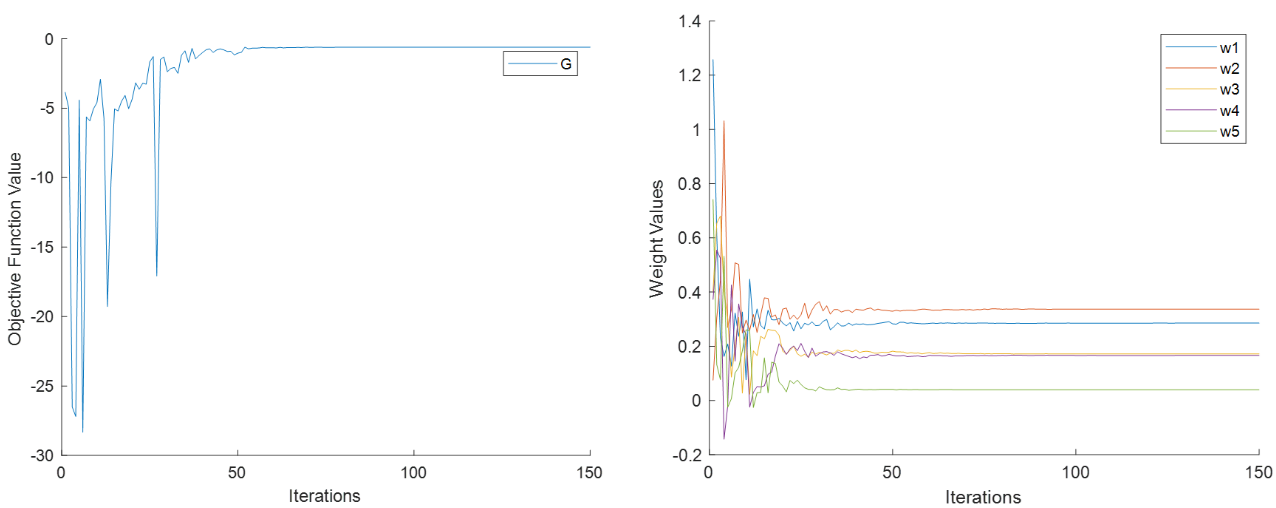

Figure 1: Convergence of CMAES with respect to the objective function value (on the left) and the weights (on the right).

Table 3: Decision matrix for TOPSIS.

Customer	Profitability	Average order quantity	Unit selling price	Payment performance	Risk limit
1	1	17976	2.67	2	200000
2	1	35766	2.29	29	100000
3	7	1966	1.95	10	50000
4	4	9306	1.63	9	15000
5	1	43721	1.55	18	50000
6	1	25030	2.20	20	200000
7	5	72609	2.19	2	50000
8	3	19444	2.52	34	30000
9	1	9515	1.81	7	200000
10	1	12961	5.81	7	150000
11	5	13921	3.76	2	200000

Table 4: Normalized decision matrix.

Customer	Profitability	Average order quantity	Unit selling price	Payment performance	Risk limit
1	0.0877	0.1768	0.2839	0.0364	0.4459
2	0.0877	0.3518	0.2435	0.5284	0.2229
3	0.6134	0.0193	0.2074	0.1822	0.1114
4	0.3508	0.0915	0.1733	0.1639	0.0334
5	0.0877	0.4301	0.1648	0.3279	0.1114
6	0.0877	0.2462	0.2340	0.3644	0.4459
7	0.4385	0.7142	0.2329	0.0364	0.1114
8	0.2631	0.1912	0.2680	0.6195	0.0668
9	0.0877	0.0936	0.1925	0.1275	0.4459
10	0.0877	0.1275	0.6179	0.1275	0.3344
11	0.4385	0.1369	0.3999	0.0364	0.4459

Table 5: Weighted normalized decision matrix.

Customer	Profitability	Average order quantity	Unit selling price	Payment performance	Risk limit
1	0.0250	0.0595	0.0488	0.0060	0.0175
2	0.0250	0.1185	0.0418	0.0878	0.0087
3	0.1752	0.0065	0.0356	0.0302	0.0043
4	0.1001	0.0308	0.0298	0.0272	0.0013
5	0.0250	0.1449	0.0283	0.0545	0.0043
6	0.0250	0.0829	0.0402	0.0605	0.0175
7	0.1251	0.2406	0.0400	0.0060	0.0043
8	0.0750	0.0644	0.0460	0.1029	0.0026
9	0.0250	0.0315	0.0330	0.0211	0.0175
10	0.0250	0.0429	0.1062	0.0211	0.0131
11	0.1251	0.0461	0.0687	0.0060	0.0175

Table 6: Positive and negative ideal rows.

	Profitability	Average order quantity	Unit selling priceprice	Payment performance	Risk limit
v^+	0.1752	0.2406	0.1062	0.0060	0.0175
v^-	0.0250	0.0065	0.0283	0.1029	0.0013

Table 7: Distance to ideal rows and composite indices of the customers.

Customer	d_i^+	d_i^-	Score	Ranking
1	0.2421	0.1135	0.3192	7
2	0.2199	0.1140	0.3414	5
3	0.2460	0.1670	0.4044	4
4	0.2370	0.1093	0.3156	8
5	0.2007	0.1466	0.4221	3
6	0.2339	0.0896	0.2770	9
7	0.0840	0.2727	0.7645	1
8	0.2330	0.0785	0.2520	10
9	0.2680	0.0871	0.2453	11
10	0.2487	0.1192	0.3240	6
11	0.2043	0.1512	0.4253	2

Table 8: Data and optimal solution of the single machine weighted tardiness minimization problem.

Customer	p_j	d_j	w_j	C_j^*	T_j^*
1	50	393	0.3192	378	0
2	90	215	0.3414	224	9
3	58	416	0.4044	436	20
4	50	332	0.3156	328	0
5	41	330	0.4221	278	0
6	79	214	0.2770	515	301
7	74	151	0.7645	134	0
8	44	179	0.2520	60	0
9	82	150	0.2453	597	447
10	13	386	0.3240	237	0
11	16	68	0.4253	16	0

5 Conclusion

In this paper, we proposed a novel bottom-up approach for solving weighted single machine scheduling problems. First, a pairwise comparison matrix that shows the relative importance of the criteria to be used in evaluating customers is formed through expert opinion, and criteria weights are calculated by optimizing a nonlinear function via the covariance matrix adaptation evolutionary strategy (CMAES) under fuzzy environment. Second, customer orders are sorted with respect to these criteria with the technique for order of preference by similarity to ideal solution (TOPSIS). Finally, orders are sequenced by mixed-integer linear programming with the objective of minimizing total weighted tardiness, where TOPSIS scores are taken as weights. This combined methodology may help companies make robust schedules not based purely on subjective judgment, find the best compromise between customer satisfaction and business needs, and thereby ensure profitability in the long run. As a topic of future study, it is worthwhile to investigate how the proposed methodology works in practice for due date quotation as discussed in Section 3.2.

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References

[1] Abdul-Razaq TS, Potts CN, Van Wassenhove LN. A survey of algorithms for the single machine total weighted tardiness scheduling problem. *Discrete Applied Mathematics*. 1990; 26(2-3): 235-253. DOI: http://doi.org/10.1016/0166-218X(90)90103-J

[2] Baker KR, Trietsch D. *Principles of Sequencing and Scheduling*. Wiley; 2009.

[3] Blanquero R, Carrizosa E, Conde E, Messine F. On weights estimation in multiple criteria decision analysis. In: International Workshop on Global Optimization. 2005. p.53-56.

[4] Cheng TCE, Ng CT, Yuan JJ, Liu ZH. Single machine scheduling to minimize total weighted tardiness. *European Journal of Operational Research*. 2005; 165(2): 423-443. DOI: http://doi.org/10.1016/j.ejor.2004.04.013

[5] Choo EU, Wedley WC. A common framework for deriving preference values from pairwise comparison matrices. *Computers & Operations Research.* 2004; 31(6): 893908. DOI: http://doi.org/10.1016/S0305-0548(03)00042-X

[6] Deliktas D, Torkul O, Ustun O, Kiris S. An integrated approach for single machine scheduling with sequence-dependent setup times. *International Journal of the Analytic Hierarchy Process.* 2015; 7(1): 50-67. DOI: http://doi.org/10.13033/ijahp.v7i1.291

[7] Dudek RA, Panwalkar SS, Smith ML. The lessons of flowshop scheduling research. *Operations Research.* 1992; 40(1): 7-13. DOI: http://doi.org/10.1287/opre.40.1.7

[8] Graham RL, Lawler EL, Kenstra JK, Rinnooy Kan AHG. Optimization and approximation in deterministic sequencing and scheduling: A survey. *Annals of Discrete Mathematics.* 1979; 5: 287-326. DOI: http://doi.org/10.1016/S0167-5060(08)70356-X

[9] Gupta JND, Stafford Jr EF. Flowshop scheduling research after five decades. *European Journal of Operational Research.* 2006; 169(3): 699-711. DOI: http://doi.org/10.1016/j.ejor.2005.02.001

[10] Hansen N. The CMA Evolution Strategy: A Tutorial. *HAL open science* [Preprint] 2005. HAL Id: hal-01297037 [Accessed: May 3rd, 2024].

[11] Hwang CL, Yoon K. *Multiple Attribute Decision Making: Methods and Applications.* Springer-Verlag; 1981.

[12] Kaminsky P, Hochbaum D. Due date quotation models and algorithms. In: Leung JYT. (ed.) *Handbook of Scheduling: Algorithms, Models, and Performance Analysis.* Chapman & Hall/CRC; 2004. DOI: http://doi.org/10.1201/9780203489802

[13] Lenstra JK, Rinnooy Kan AHG, Bruker P. Complexity of machine scheduling problems. *Annals of Discrete Mathematics.* 1977; 1: 343-362. DOI: http://doi.org/10.1016/S0167-5060(08)70743-X

[14] Lin D, Lee CKM, Wu Z. Integrating analytical hierarchy process to genetic algorithm for reentrant flow shop scheduling problem. *International Journal of Production Research.* 2012; 50(7): 1813-1824. DOI: http://doi.org/10.1080/00207543.2011.561884

[15] McKay KN, Wiers VCS. Unifying the theory and practice of production scheduling. *Journal of Manufacturing Systems.* 1999; 18(4): 241-255. DOI: http://doi.org/10.1016/S0278-6125(00)86628-5

[16] Önemli BC. *Customer Order Scheduling Problem for the Box Packaging Industry: An Application.* Master's thesis; 2019.

[17] Ortiz-Barrios M, Petrillo A, De Felice F, Jaramillo-Rueda N, Jimenez-Delgado G, Borrero-Lopez L. A dispatching-fuzzy AHP-TOPSIS model for scheduling flexible job-shop systems in industry 4.0 context. *Applied Sciences.* 2021; 11(11): 5107. DOI: http://doi.org/10.3390/app11115107

[18] Pinedo ML. *Scheduling: Theory, Algorithms, and Systems.* Springer; 2012.

[19] Rios LM, Sahinidis NV. Derivative-free optimization: A review of algorithms and comparison of software implementations. *Journal of Global Optimization.* 2013; 56(3): 1247-1293. DOI: http://doi.org/10.1007/s10898-012-9951-y

[20] Sen T, Sulek JM, Dileepan P. Static scheduling research to minimize weighted and unweighted tardiness: A state-of-the-art survey. *International Journal of Production Economics.* 2003; 83(1): 1-12. DOI: http://doi.org/10.1016/S0925-5273(02)00265-7

[21] Stoop PPM, Wiers VCS. The complexity of scheduling in practice. *International Journal of Operations & Production Management*. 1996; 16(10): 37-53. DOI: http://doi.org/10.1108/01443579610130682

[22] Utku, DH, Özyiğit K, Farizoğlu EY. A mixed-integer programming model for the job scheduling problem in a production company. *Verimlilik Dergisi* . 2022; (1): 110-119. DOI: http://doi.org/10.51551/verimlilik.819041

[23] Wiers VCS. A review of the applicability of OR and AI scheduling techniques in practice. *Omega*. 1997; 25(2): 145-153. DOI: http://doi.org/10.1016/S0305-0483(96)00050-3

[24] Zeydan M, Bostanc B, Oralhan B. A new hybrid decision making approach for housing suitability mapping of an urban area. *Mathematical Problems in Engineering*. 2018; 2018. DOI: http://doi.org/10.1155/2018/7038643

Mithat Zeydan
Department of Industrial Engineering
İstanbul Medeniyet University
İstanbul, Turkey
E-mail: mithat.zeydan@medeniyet.edu.tr

Murat Güngör
Department of Industrial Engineering
İstanbul Medeniyet University
İstanbul, Turkey
E-mail: murat.gungor@medeniyet.edu.tr

Burak Urazel
Department of Electrical and Electronics Engineering
Eskişehir Osmangazi University
Eskişehir, Turkey
E-mail: burazel@ogu.edu.tr