Validation of the Conceptual Model for the Virtual Mathematics Curriculum in Primary Education: A Case Study of the Educational System in Tehran

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Abstract

This research aimed to validate the conceptual model of the virtual math curriculum for primary education. The research method was descriptive-survey type, including exploratory and confirmatory factor analysis. The statistical population consisted of primary school teachers from grades one to six in Tehran's education system. The sample size was determined using a multi-stage random cluster sampling method, resulting in 112 selected teachers. Data were collected using a researcher-developed questionnaire with 57 items on a 5-point Likert scale, designed to evaluate the internal validity of the model. For data analysis, descriptive statistics including mean, standard deviation, skewness, and kurtosis were used, and for inferential statistics, structural equation modeling and confirmatory factor analysis were conducted using Smart PLS software. The results of the analyses indicated that the proposed model had a good fit, and all conceptual relationships were significant based on the confirmatory factor analysis. Finally, the model was refined through focus group sessions with curriculum specialists in mathematics and education experts. The final virtual math curriculum model includes 48 components, divided into 11 components in the objectives dimension, 10 components in the content dimension, 15 components in the teaching-learning methods dimension, and 12 components in the evaluation methods dimension.

Key Words: Validation, Mathematics Curriculum, Virtual Education, Elementary Education.

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Introduction

In the digital age, virtual education has become one of the primary tools for knowledge transfer and learning worldwide. This transformation accelerated significantly with the outbreak of the COVID-19 pandemic in 2019 (March 2020), pushing educational systems toward extensive use of digital technologies. The development of new tools and technologies and the growing need for remote learning have created numerous opportunities. These opportunities have contributed to the creation of new learning environments that leverage pedagogical knowledge in virtual education, focusing on how to effectively integrate educational elements with technological capabilities. This process also enables the formation of blended learning (Hajizadeh et al., 2021).

In this context, mathematics education at the primary level requires special attention due to its complex and conceptual nature. Concepts such as numbers, geometric shapes, and mathematical operations demand deep understanding and continuous practice, which has traditionally been a significant part of face-to-face instruction (Bottle, translated by Bakhshali-Zadeh, 2019). The development of virtual mathematics classrooms must not only utilize multimedia content and materials to enhance the processing and deeper learning of mathematics in children but also address and mitigate issues related to the teaching and learning of mathematical concepts that were present in face-to-face settings. Integrating various curricular elements with information technology capabilities in a virtual learning environment can endow these elements with unique features that are not typically present in traditional face-to-face curricula (Abbasi et al., 2020).

Mathematics is considered one of the most important subjects in the educational curricula of many countries, although it is often perceived as one of the most challenging by students and even adults. Many students experience frustration with mathematics during their school years and frequently express dissatisfaction with the amount of time dedicated to teaching this subject. Nevertheless, numerous studies underscore the significance of mathematics in everyday adult life and affirm the necessity of teaching mathematics at the primary level. These studies highlight mathematics as a powerful tool for enhancing communication and reasoning skills, which not only strengthens the foundation of science in children but also contributes to their success and competence in adulthood (Dawachi, 2015).

The transition of mathematics education to the virtual space presents new challenges. One of the most significant challenges is the mismatch between existing educational content and the needs of students in a virtual environment. Curricula designed traditionally for face-to-face instruction cannot maintain the same effectiveness in a virtual setting. Additionally, the lack of face-to-face interactions, the inability of teachers to directly monitor students' progress, and technical issues are among the factors that can impact the quality of education. Furthermore, the ability of primary school students to use digital technologies directly affects their success in learning. Weaknesses in the equipment and tools of virtual systems, teachers' lack of skills in using teaching tools in a virtual environment (Sadeghi, 2021), parents' lack of training in supporting their children in virtual education, challenges related to students staying at home, difficulties in simulating a virtual mathematics classroom compared to a physical one, and issues in motivating students towards self-regulation, self-discipline, and responsibility, among other problems (Azadi et al., 2016), have led to virtual mathematics classes in primary education lacking the necessary quality and effectiveness. In many cases, teachers have resorted to merely explaining mathematical concepts and conducting a few exercises, bypassing hands-on activities and the stages of conceptual understanding of mathematics.

In virtual education, teachers and students must adapt to a new reality, where educational activities are pursued remotely (Khori, 2022). However, given the conceptual nature of mathematics and the need for deep understanding, it is unrealistic to expect students to learn mathematical concepts and processes by simply sitting, listening, and looking at textbook pages. A mathematics teacher must develop new plans, designs, and models for virtual teaching, adapting their teaching methods to modern tools and software. This requires both knowledge and awareness in order to foster the highest levels of interaction, creativity, and engagement, thereby facilitating the transmission of mathematical concepts and enhancing student learning through the small, monotonous, and potentially tiresome lens of virtual education (Hamzehloo & Rahimi, 2020). In recent years, with the expansion of virtual classes, extensive research has been conducted on the problems and challenges of online mathematics classes. Researchers have worked to offer various solutions and have made efforts to raise awareness among teachers, students, and their parents, taking significant steps toward improving the quality of these classes. We have witnessed considerable progress in the teaching and learning processes of these virtual classes, alongside a wave of interest and enthusiasm for change and reform in curricula, particularly in the mathematics curriculum.

For instance, Naqdi, in his research, pointed out that after the COVID-19 pandemic, teachers and educators realized that many mathematical concepts and topics could not be effectively taught using traditional methods. Therefore, today, the application of educational technology and diversifying the teaching process by teachers has become unavoidable for more effective teaching. In this regard, teachers must be familiar with modern technologies and use manipulatives and hands-on activities to enhance mathematics instruction in virtual environments (Naqdi, 2023). Ashnagar, in his research, acknowledged that the current implementation of virtual mathematics education, given the available resources, does not yield optimal results. However, by utilizing more advanced software and updated equipment, these challenges can be improved. Learners have recognized the effectiveness of mathematics instruction in virtual classrooms when up-to-date electronic content creation programs are used. It is recommended that training in the latest educational software be prioritized in teachers' professional development programs (Ashnagar et al., 2023).

Virtual mathematics education must aim to strengthen conceptual and practical learning of mathematics. As we witness technological advancements over time, we must strive to effectively integrate these technologies into the teaching and learning process. Today, we see many teachers searching for ways to increase the efficiency of virtual education. While at first glance, teaching in a virtual space may seem easy, this is not the case for all subjects. One of the most challenging aspects of online teaching is virtual mathematics instruction (Saeidi, 2020). Mathematics teachers should remember that just as repetition, concept reinforcement, practice, problem-solving, and assessments are important in in-person classes, these elements must also be emphasized in virtual classrooms. Achieving this requires the proper use of suitable tools and technologies to effectively deliver the course material (Mahdi et al., 2022).

For schools and teachers to succeed in implementing virtual mathematics education, precise and thoughtful planning is essential. As the primary facilitators of virtual classes, teachers must focus on the key components of the mathematics curriculum. By utilizing the opportunities and resources available in the virtual environment, along with technological tools, they can deliver effective instruction that meets students' learning needs (Ghaznavi et al., 2018). Additionally, leveraging learning management systems (LMS) in virtual classes can help achieve educational objectives, such as accessing content, communicating with classmates and the teacher, submitting assignments, and participating in the assessment process. Despite extensive research in the field of virtual education, a specialized and systematic review of conceptual models for elementary mathematics curricula, particularly within the context of Iran's education system, has received limited attention. This is noteworthy because a precise understanding and evaluation of these conceptual models can significantly enhance the quality of virtual education.

Given the above considerations, it is essential for curriculum planners, specialists, and educators to adopt a suitable model for the virtual mathematics curriculum at the elementary level. This model must be designed to address the needs of both students and teachers in the virtual learning environment. The researchers of this study, through a thorough examination of relevant studies, articles, and research on virtual mathematics curricula, as well as by incorporating the insights of educational experts, university professors specializing in mathematics education, and authors of elementary mathematics textbooks, have proposed the following model:

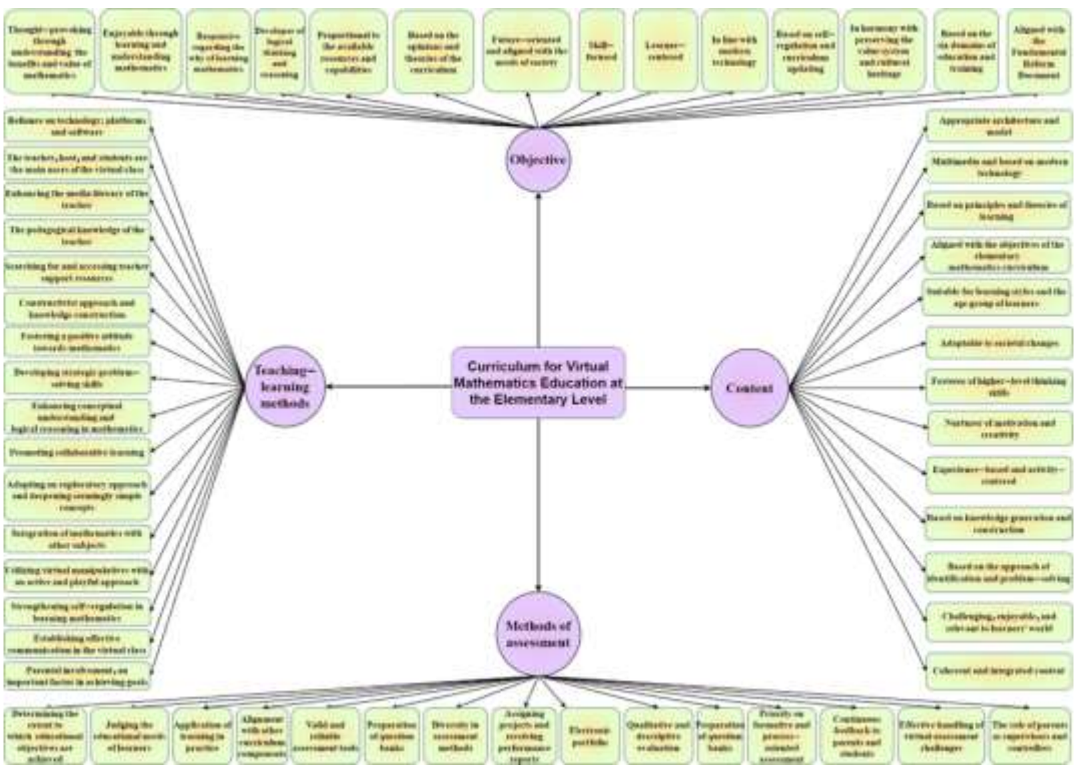


Figure 1. Proposed Curriculum Model for Virtual Mathematics Education in Primary School

Based on the proposed model, the curriculum for virtual mathematics education at the primary level, like any other curriculum, consists of four main components: objectives, content, teaching methods, and evaluation methods (Silver et al., 1997; Nichols & Nichols, 1989; Fathi Vajargah, 2007; Mehrmohammadi, 1988). Each of these components has its own specific indicators and markers, which play a crucial role in the design and implementation of the curriculum.

In alignment with the Fundamental Transformation Document, based on the six domains of education and training, consistent with maintaining the value system and cultural heritage, grounded in self-regulation and curriculum updating, in line with current technology, learner-centered, skill-oriented, future-oriented, and responsive to community needs, and supported by curriculum planning theories, it should be tailored to available resources and capabilities. The goals include fostering logical thinking and reasoning, answering the question of why learning mathematics is necessary, making learning enjoyable through mathematical understanding, and encouraging reflection on the benefits and values of mathematics.

Key indicators of the "objectives" include a suitable architectural and model design, multimedia-based and technology-driven, based on learning principles and theories, aligned with the goals of the primary school mathematics curriculum, adaptable to learning styles and age groups, responsive to societal changes, promoting higher-order thinking skills, encouraging motivation and creativity, experience-based and activity-oriented, focused on knowledge production and construction, problem-solving-oriented, challenging, enjoyable, relevant to learners' world, and presenting coherent and integrated content.

Important indicators for the "content" involve reliance on technology; platforms and software; teachers, hosts, and students as primary users in virtual classrooms; enhancing teachers' media literacy and pedagogical knowledge; searching for and accessing support resources for teachers; adopting a constructivist approach and knowledge construction; fostering a positive attitude towards mathematics; developing strategic problem-solving abilities; enhancing conceptual understanding and logical reasoning in mathematics; promoting collaborative learning; employing exploratory approaches and deepening seemingly simple concepts; integrating mathematics with other subjects; utilizing virtual tools with a focus on activity and play; strengthening self-directed learning in mathematics; establishing effective communication in virtual classrooms; and involving parents as a critical factor in achieving goals.

Indicators for the "teaching-learning methods" include determining the extent of achieving educational objectives, assessing learners' educational needs, applying learned knowledge in practice, aligning with other elements of the curriculum, utilizing valid and reliable assessment tools, preparing a question bank, diversifying assessment methods, providing projects and receiving performance reports, using electronic portfolios, qualitative and descriptive assessments, prioritizing process-oriented and formative assessments, providing continuous feedback to parents and students, effectively addressing challenges in virtual assessment, and recognizing the role of parents as monitors and controllers.

Research question

Although the proposed model is potentially credible, its validity needs to be assessed in practice from the perspective of various stakeholders. Therefore, the research question is: "Does the proposed model for the virtual mathematics curriculum in primary education possess the necessary credibility?"

Background of Research

Selected articles and texts are generally based on three significant areas: the role of technology in the curriculum of virtual mathematics education, teachers and methods of teaching and learning mathematics in virtual mathematics classrooms, and students in virtual mathematics classrooms with a focus on the curriculum of virtual primary mathematics education. These were examined with a focus on features such as objectives, content, teaching methods, and assessment techniques of the curriculum.

Table 1. Areas of Study in the Reviewed Articles with a Focus on the Curriculum of Virtual Mathematics Education for Primary Education

Main Research Topic	Number of Related Articles	Areas of Study in Related Research
Mathematics Virtual Curriculum for Elementary School	٨ domestic articles	a) Technology and its crucial role in the virtual mathematics curriculum
	٦ international articles	
	١٥ domestic articles	b) Teachers and teaching-learning methods in virtual mathematics classes
	١١ international articles	
	١١ domestic articles	c) Students in virtual mathematics classes and related challenges
	٧ international articles	

In his study titled "Investigating the Role of Information Technology in Primary Mathematics Education," Bozorgian (2024) emphasizes that in the era of technology, numerous tools have been designed to facilitate learning. These tools are particularly important in primary mathematics education, and the success of teachers in virtual teaching depends on their familiarity with technology. The use of technology not only makes mathematics instruction more engaging but also increases student motivation. Students can use computer technologies to learn at a pace that suits their needs and abilities.

In his research "Examining the Role of Technology in Mathematics Teaching and Learning," Ramazani (2024) demonstrates that with the advent of modern technologies and information technology's influence in various aspects of life, using these technologies as tools to facilitate learning across all subjects, especially mathematics, is unavoidable.

In the study "The Role of Educational Technologies in Transforming Mathematics Teaching and Learning in Primary Education," Rahmani Yadollah and colleagues (2024) acknowledge that using visual and multimedia representation of concepts, creating interactive experiences, and developing flexibility in time and place through educational technologies can improve the mathematics learning process and increase student motivation. Proper and effective use of educational technologies can enhance the quality of mathematics education in primary schools.

In their study "Investigating the Role and Importance of Technology in Teaching Mathematics to Primary Students," Mohammadi and colleagues (2023) have concluded that technology is a key factor in teachers' success in virtual teaching. They emphasize the need for enhancing teachers' skills to improve the quality of education and believe that the use of technology makes the learning process more engaging and increases student motivation.

Hamidi et al. (2021), in their study "The Role of Virtual Media and Educational Tools in Primary Students' Learning," conclude that information technologies and media, by creating interactive environments and sharing opportunities, have transformed traditional teaching methods and contributed to individual and social progress.

Valiollahi et al. (2021), in their research "Examining Strategies for Teaching Primary Mathematics Using Technology," emphasize that the use of technology in mathematics education increases students' interest and deeper understanding of concepts. This approach enhances problem-solving skills, logical thinking, and fosters collaboration, critical thinking, and creativity among students.

Amirahmadi (2021) in his study titled "Examining the Impact of Interactive Multimedia Programs on Enhancing Creativity and Basic Mathematics Skills" demonstrates that using graphics and animations in virtual learning increases students' motivation to learn mathematics, especially when familiar cartoon characters are used.

Hamidi et al. (2020), in their research "Utilizing Media in Virtual Mathematics Education," highlight the importance of technology during the COVID-19 pandemic and stress the need to equip teachers and students with modern virtual teaching methods. They also introduced several software applications for teaching mathematics in the virtual space.

In the study by Hadi Zadeh (2024) titled "Comparing the Impact of Face-to-Face, Virtual (Shad), and Hybrid (Shad and Face-to-Face) Teaching Methods on Fourth-Grade Students' Mathematics Learning," the results indicate that the hybrid method, i.e., virtual (Shad) - face-to-face, has been successful in improving mathematics scores. Therefore, combining face-to-face and virtual teaching methods can provide a more effective teaching and learning process.

Gholami (2022), in another study titled "Types of Virtual Education and Mathematics Assessment During the Pandemic," notes that teachers in virtual education must achieve the objectives outlined in the mathematics curriculum. They can employ various methods in the virtual space to teach mathematics and have also suggested strategies to improve the quality of instruction.

Gholami-Pour Sholoumi (2022), in the research "Examining Issues and Challenges of the Mathematics Curriculum During the Pandemic," identifies key challenges in virtual classrooms, such as conceptual understanding of mathematics, effective teacher-student communication, hands-on activities, monitoring assignments, and assessment.

Salehi et al. (2022), in their study "Examining Experiences of Mathematics Education in the Virtual Space After the Pandemic," suggest that to adapt mathematics education to modern virtual principles, teachers should design and deliver learning activities creatively and effectively using appropriate tools and media.

Nasseri et al. (2022) in their research "Knowledge Required for Virtual Teaching of Third-Grade Primary School Teachers" found that from teachers' perspectives, assessment is the most crucial component of virtual mathematics teaching. Interaction with students, information technology, and teaching methods are also among the next priorities.

Gorgij et al. (2022), in their study "Addressing Weaknesses in Virtual Mathematics Education for Sixth-Grade Students," found that using colleagues' experiences, localizing content, employing flipped classroom methods, enhancing media literacy for both teachers and students, and applying practical mathematics in virtual primary mathematics classrooms can reduce problems associated with non-face-to-face mathematics classes and eliminate students' weaknesses in mathematics.

Mohammad Reza Dorostkar Siyani et al. (2022) in their article "Examining the Lived Experiences of Sixth-Grade Mathematics Teachers in Isfahan with Virtual Education During the 2020-2021 Academic Year" address the importance of sharing teachers' experiences in virtual education. The study's results indicate that teachers primarily used lecture and question-and-answer methods in the virtual format. The researchers categorized the positive and negative aspects of virtual education into three domains: educational, temporal, and economic. To address the existing limitations in virtual education, recommendations such as conducting training workshops for teacher empowerment, using electronic portfolios, and developing specific regulations for virtual environments have been proposed.

The results of the research by Mohsenzadeh (2021) in the article "Challenges in Teaching Primary Mathematics in Virtual Spaces" show that teachers can present mathematical topics to students in an engaging manner using hands-on activities and creative methods, which can lead to a deeper understanding of mathematical concepts in virtual classrooms.

Shamani et al. (2021), in their study "Examining the Familiarity and Application of Modern Teaching Methods in Virtual Mathematics Education," found that although teachers have a high awareness of modern teaching methods, their use in virtual education is at a moderate level due to existing obstacles.

In the study by Babazadeh et al. (2020) titled "Improving Virtual Mathematics Education for Primary School," the researcher introduces the site mathsframe.co.uk as a platform for virtual mathematics education and attempts to turn the challenges of virtual mathematics instruction into an opportunity for higher quality education.

According to the research by Khodabakhshi et al. (2020) titled "Providing a Primary Mathematics Curriculum Model with a Mobile Learning Approach," the proposed model includes three organizing themes: Learner, Organizer (comprising: learning planning, improving culture, attitudes, and goals), and Enrichment (comprising: enrichment of learning, performance and execution of learning, teacher competencies, tools and methods, interaction of individuals involved in education, and assessment). Based on the results, it appears that the proposed model could be effective in advancing educational goals in the field of mathematics curriculum with a mobile learning approach.

Saeidi's (2020) research, titled "Online Teaching and Virtual Mathematics Education," addresses the specific challenges in virtual mathematics education. The results of this study indicate that teaching mathematics in online classes is one of the most complex issues, especially due to the conceptual nature of the subject. It is not expected that students can learn mathematical concepts and processes merely by observing book pages. Therefore, mathematics teachers need to adapt their teaching methods using up-to-date tools and software and create conditions that facilitate easier concept transfer and learning for students through innovative virtual teaching.

Fathi et al. (2019) in their research "Comparing Traditional Education and Educational Software in Mathematics and Science Learning Levels" emphasize that the focus of education has shifted towards active learning. The use of educational technology in schools has helped increase student motivation and interaction with teachers, leading to improved mathematics learning.

In their 2019 study titled "The Effectiveness of Humor-Based Electronic Content on Mathematics Learning," Rostami-Nejad and colleagues demonstrated that integrating humor with electronic content has a profound impact on both learning and retention in mathematics. This approach not only enhances students' interest in mathematics but also fosters their creativity.

Chenari and colleagues (2023), in their research "Addressing Students' Comprehension Issues in Virtual Learning (Case Study: Mathematics)," pointed out that one of the significant challenges students face in virtual math instruction is the difficulty in understanding mathematical concepts. They suggest strategies such as parental involvement and collaborative learning in virtual classrooms to assist children in overcoming these learning difficulties.

In their 2023 study, "The Impact of the VAK Learning Method on Attitude and Academic Performance in Elementary Students' Virtual Mathematics Education During the COVID-19 Pandemic," Bardia and colleagues found that the VAK strategy, due to its use of visual, auditory, and kinesthetic content, as well as its attention to different learning styles, led to an improvement in students' attitudes towards mathematics and their academic performance.

Ashnagar and colleagues (2023), in their study titled "Investigating the Challenges of Virtual Assessment in Elementary Mathematics Education: A Case Study of Sarbaz City," argued that using up-to-date software and equipment can enhance the evaluation process in virtual math instruction. Learners acknowledged the effectiveness of virtual mathematics education through modern electronic content creation tools.

In their 2022 study titled "The Effectiveness of Virtual Mathematics Education on Creative Thinking and Academic Self-Efficacy in Female Students," Soltani and colleagues demonstrated that virtual math education has a positive impact on both creative thinking and academic self-efficacy among students. Therefore, it is crucial for policymakers and educational planners to emphasize the use of information technology by designing various software tools for teaching elementary mathematics.

In their 2022 research "Designing and Validating a Competency Model for Elementary Students' Visual-Spatial Abilities in Virtual Mathematics Education," Mohammadi and colleagues categorized visual-spatial competencies into two groups: "student-related competencies" and "teacher-related competencies." Student-related competencies include cognitive skills, mental performance, learning capacity, mental space construction, pattern recognition, visual memory, and simulation ability. Teacher-related competencies encompass content preparation, teaching skills, and instructional design.

In their 2022 study, Saedmocheshi and colleagues examined the impact of virtual education through the "Shad" program on the academic progress of fourth-grade students. They found that virtual education, compared to traditional methods, significantly enhances learning and academic achievement.

Fakhari and colleagues (2021), in their research "Enhancing Students' Learning in Solving Mathematical Problems through Online Teaching," highlighted several benefits of online education, including improved problem comprehension, increased mathematical knowledge, boosted self-confidence, and encouragement of group activities and participation in virtual learning.

Naseri's 2021 study titled "Examining the Quality of Learning and Teaching Mathematics in the Shad Program Using Animation from the Perspective of Second-Level Elementary Students in Tehran" revealed that using animation improves the understanding of mathematical concepts. It also increases students' interest in learning and following mathematical topics.

In their 2021 research, "Comparing the Academic Performance of Sixth-Grade Students in Mathematics with Virtual Teaching During the COVID-19 Pandemic," Kianfar and colleagues showed that the average academic performance in virtual education surpassed that of traditional in-person teaching. Based on these findings, they suggested increasing the use of virtual education in future school years, improving infrastructure, enhancing the quality of the Shad program and virtual classes, and offering training courses for teachers on virtual instruction.

Niayee and colleagues (2021), in their study "The Effectiveness of Flipped Teaching on Math Anxiety and Math Performance in Fifth-Grade Students in Marand," demonstrated that flipped teaching facilitates deep learning, increases motivation, and makes classes more engaging, ultimately boosting students' confidence in learning mathematics.

Gholami and colleagues (2020), in their research "The Effectiveness of Virtual Learning on Mathematics Education in Elementary School," emphasized that virtual education during the COVID-19 pandemic had a significant positive impact on improving knowledge, understanding, and application of mathematics among elementary school students.

In her 2024 article, "Implicit Learning in a Virtual Mathematics Classroom: A Case Study," Carmen addresses the challenges of online learning in mathematics, considering cognitive theory and the need for interaction between the mind, body, and environment. She argues that the limitations of physical activity in virtual education can negatively impact the effective learning of mathematical concepts.

In the study "Virtual Schools in America" (2023), Molnar and colleagues highlight that the COVID-19 pandemic pushed many teachers to adopt new educational technologies. This shift resulted in greater flexibility and part-time opportunities for teachers, while parents reported improvements in the quality of teaching and learning in virtual schools.

In 2022, in their research "A Virtual Reality-Based Mathematics Education System for Elementary Schools," Xie and colleagues demonstrated that virtual reality (VR) technology offers significant benefits for math education. These systems, by simulating mathematical operations and principles, aid in deeper comprehension of concepts and allow students to pursue independent, self-directed learning. Additionally, VR can boost students' enthusiasm and creativity, fostering a greater interest in the mathematics curriculum.

The findings of Figg and colleagues (2020) in their article "Virtualizing Mathematics" indicate that the COVID-19 pandemic presented teachers with new challenges in virtual math instruction. To enhance meaningful learning, significant portions of math curriculum planning need to be dedicated to virtual environments, with efforts to cultivate positive attitudes towards teaching math online. Collaborative activities in virtual classrooms, the formation of learning groups for teachers, and familiarizing them with digital tools and supportive technologies are crucial to improving the quality of instruction and fostering a better understanding of mathematical concepts.

Azid and colleagues (2020), in their study "The Impact of Using Web Tools on the Mathematics Achievement of Elementary School Students," highlighted the use of web-based tools and educational software in virtual learning environments and their role in enhancing students' learning and achieving higher-level educational goals. According to their findings, this method of instruction can motivate students and encourage them to complete their mathematical tasks and assignments. They emphasized that teachers must receive continuous training to improve their ability to teach effectively in virtual classrooms.

Liu and colleagues (2018), in their study "The Application of Virtual Reality for Elementary Mathematics Classrooms," demonstrated that virtual reality (VR) opens new horizons in mathematics education. This technology allows students to interact with classmates, utilize diverse learning opportunities, overcome time and space constraints, and improve teaching quality. VR also helps enhance the understanding of abstract concepts and enriches the overall educational experience.

In the 2024 research by Weinhandl and colleagues, titled "Utilizing Student Personality Insights When Developing Digital Mathematics Learning Resources to Improve Teacher Education," it is noted that incorporating modern technologies in schools requires teachers to be prepared for new challenges, such as developing digital learning resources. Pre-service mathematics teachers, even without direct student contact, need to understand the unique characteristics of math students to effectively develop these resources during their training.

Yakubova and colleagues (2024), in their study "Virtual Learning and Mathematics Instruction for Students with Autism: The Impact of Video Modeling, Virtual Games, and Math Games," found that the simultaneous intervention of multiple components—educational clips, video modeling, virtual manipulations, digital math games, and guided exercises through these games—

helped students achieve 100% accuracy in acquiring, retaining, and generalizing all the skills taught in virtual math classes.

McCulloch and colleagues (2024), in their study "Exploring Mathematics with Technological Methods for Math Teachers," focused on how math teachers can implement high-quality and equitable mathematics instruction using dynamic, math-specific technologies. The authors aimed to expand the teaching landscape by incorporating these technologies, enabling students to learn through specialized mathematical tools that enhance their understanding.

Zydney and colleagues (2023), in their research "Learning from Design Failures: A Virtual Mathematics Teaching Program," examined the challenges faced in designing a virtual mathematics curriculum during the pandemic. They identified three key design failures: issues with flexible planning, difficulties encountered by students, and obstacles in collaborative communication tools. Despite these setbacks, the lessons learned have contributed to the improvement of future curriculum designs and the discovery of innovative ways to use advanced technologies for visualizing mathematical concepts in remote learning settings.

In their 2022 study, "The Use of Virtual Classroom Simulations in a Mathematics Methods Course to Develop Pre-Service Mathematics Teachers' Skills," Chi Lin demonstrated that the simulation of a virtual mathematics classroom significantly enhanced teachers' learning skills and their focus on these skills. Additionally, the qualitative data analysis revealed that participants had a positive perception of the virtual environment in terms of realism, enjoyment, and effectiveness.

Khalil (2022), in the study "Elementary School Mathematics Teachers' Beliefs about Teaching in Synchronous Virtual Classrooms: A Mixed-Methods Study," focused on the components of teaching efficacy, the philosophy of active learning, and mathematical progress. The study showed that in synchronous elementary mathematics classrooms, these components were of critical importance, and teachers believed the success of these classes was dependent on effectively addressing these factors.

Amedu et al. (2022), focusing on the topic "Teachers' Perception of Using Technology for Teaching Mathematics During the COVID-19 Virtual Learning Period," examined the impact of remote learning on mathematics education. In this study, teachers expressed concerns regarding difficulties in receiving feedback from students and their limited interaction in the virtual environment. They also highlighted challenges related to implementing curriculum-based assignments that require classroom interaction and discourse, as well as issues with the quality of educational resources and materials for remote teaching.

Kilinc et al. (2021), in their study titled "Elementary School Teachers’ Views on Teaching Mathematics During the Global COVID-19 Pandemic," explored the effects of virtual mathematics classrooms on elementary school teachers and provided recommendations for improving online education. These recommendations include providing technology training for teachers, students, and parents; organizing in-service training to enhance teachers' skills in remote education; and reviewing and expanding internet infrastructure to ensure educational equity. Additionally, they proposed programs to support disadvantaged students, prevent psychological issues resulting from quarantine, and offer remedial instruction for topics that require more repetition and practice.

Xie et al. (2021), in another study titled "Micro-Classes as an Educational Response to Elementary Mathematics During the COVID-19 Pandemic in China," demonstrated that online mathematics education has become a fundamental teaching method in response to the pandemic. This research shows that virtual mathematics instruction not only addresses students' psychological needs but is also highly accepted by students, creating opportunities to reduce educational inequities.

Akman et al. (2020), in their study titled "The Impact of Educational Virtual Reality on Elementary Students' Mathematics Achievement and Engagement," demonstrated that using technology for teaching mathematics positively affects academic achievement in the subject. Additionally, virtual reality has a beneficial impact on students' academic progress, and the use of multimedia applications and software significantly enhances student engagement in mathematics.

According to the findings of Mulenga et al. (2020) in their research "Is COVID-19 a Gateway to Digital Learning in Mathematics Education?", the COVID-19 pandemic and the shift of educational classes to the virtual space revealed that teachers with lower skill levels in using mobile technology and accepting social media for mathematics instruction were common. However, digital learning and virtual classrooms could effectively contribute to improving the quality of mathematics education.

In the study by Satsangi et al. (2023) titled "Virtual Manipulatives for Teaching Fraction Computation to Students with Mathematical Difficulties," it was shown that the increased use of digital technology has provided various strategies to support students in learning mathematics. One such technology is the use of virtual manipulatives to demonstrate mathematical concepts. This method has been particularly effective for children with learning disabilities in mathematics, greatly aiding in overcoming difficulties related to understanding fraction computation.

Bright et al. (2022), in their study "Examining the Impact of COVID-19 on Standardized Mathematics Test Scores in a Rural Ohio School," demonstrated that the national standardized test administered in the United States, which serves as a measure of student learning, revealed a significant decrease in students' mathematics scores compared to before the COVID-19 pandemic. This decline was attributed to the pandemic's disruption of various learning methods, including face-to-face instruction, blended learning, and reduced interaction levels. Consequently, the performance of students in virtual mathematics education environments was discussed.

The findings of Ukdem et al. (2022) in their study "Examining the Impact of Interventions Using Concrete and Virtual Manipulatives on Fifth-Grade Students' Understanding and Motivation" indicate that the use of manipulatives is effective in understanding mathematics, with minimal difference in efficacy between in-person and virtual classrooms. These manipulatives increased students' motivation and interest in learning mathematical concepts in both in-person and virtual settings.

Moliner et al. (2022), in their study "COVID-19 Restrictions and Their Impact on Student Achievement in Spain," found that the COVID-19 pandemic led to a decline in mathematics learning. Key challenges in virtual education during the pandemic included the shift to online classes, lack of support in virtual teaching, reduced interaction among students, teachers' difficulties in employing effective teaching methods, and decreased time allocated for mathematics classes.

Isnawan et al. (2022), in their study "Family Barriers During Remote Mathematics Learning in Indonesia: A Phenomenological Study," showed that deficiencies in teachers' explanations of mathematical concepts in virtual classrooms resulted in learning difficulties for students. Despite parents' crucial role in supporting their children in virtual classes, they were found to be inadequate in understanding and assisting with mathematics education. The study recommends adopting a blended learning approach that combines limited in-person and virtual instruction.

Darragh et al. (2022), in their study "Lessons from Quarantine: The Impact of Parental Involvement in Home Mathematics Learning During the COVID-19 Quarantine," explored the role of parents in home-based mathematics learning. The findings indicate that virtual mathematics assignments were stressful and unsuccessful for both parents and children. The study suggests that schools should listen to parental feedback regarding the quality and quantity of mathematics assignments and provide solutions to mitigate issues related to virtual learning.

Kalogeropoulos et al. (2021), in their research "Mathematics Learning from Home During the COVID-19 Pandemic," examined how elementary mathematics learning programs were planned and implemented and the associated challenges. The results showed that students had positive interactions with remote mathematics learning, but teachers expressed concerns about the content delivered and the reduction in student interaction.

Methodology

In this study, a descriptive-survey method was employed. The study population consisted of primary school teachers (grades 1 to 6) in Tehran's Department of Education. A multi-stage random cluster sampling method was used for sampling. Out of the 22 districts in Tehran, 4 districts were randomly selected. Within each district, 4 primary schools (2 boys' schools and 2 girls' schools) were randomly chosen. Assuming there were 2000 teachers in these 4 districts, the sample size was determined to be 322. To account for the possibility of non-returned or incomplete questionnaires, 330 questionnaires were distributed. Upon final review, 112 complete and valid questionnaires were used for the analysis. This equates to approximately 7 teachers per each of the 16 selected schools having completed the questionnaire. Structural equation modeling studies typically recommend a minimum sample size of 100-150 participants (Tabachnick & Fidell, 2013).

To collect data for the study, a researcher-developed questionnaire was used. This questionnaire, consisting of 57 items and utilizing a 5-point Likert scale (ranging from "Very Low" to "Very High"), was designed to gather data aimed at validating the internal structure of the model. At the beginning of the questionnaire, a general introduction to the research topic was provided, followed by an explanation of the study's objectives. Demographic information such as gender, years of service, academic degree, level of education, and field of study was collected.

The questionnaire was divided into four main sections: objectives, content, teaching-learning methods, and evaluation methods. The face validity of the questionnaire was confirmed by 5 experts, and after incorporating their feedback, the final version of the questionnaire was developed. The items were created in Google Forms, and the link to the questionnaire was shared with expert teachers via messaging apps, following their consent and a brief explanation of the study. Teachers were requested to complete the questionnaire using the provided link.

The basis of the questionnaire was the conceptual model presented for the virtual mathematics curriculum of the elementary level. According to this model, each of the indicators of the four components or essential elements of the virtual mathematics curriculum was translated into questions for validation. To assess the content validity of each question, content analysis by experts and the Content Validity Ratio (CVR) were employed. The calculated CVR values ranged from 0.6 to 1, indicating that the items were deemed essential by the experts and demonstrating the questionnaire's adequate validity. Table 2 shows the content validity ratio values.

Table 2. Content Validity Ratio (CVR) Values

Components	Items	CVR
Objective	Items 1 to 14	1
Content	Items 15 to 27	1
Method	Items 28 to 43	0/60

Assessment	Items 44 to 57	0/60

To assess the reliability of the questionnaire, Cronbach's alpha coefficient was used, yielding a value of 0.974, which indicates very good internal consistency of the questionnaire. In data analysis, descriptive statistical methods including mean, standard deviation, skewness, and kurtosis were employed. For inferential analysis, confirmatory factor analysis (CFA) was utilized. The statistical analyses were conducted using Smart PLS software.

Demographic Information of the Research Sample

Descriptive indicators related to the demographic information of the study group are detailed below.

Gender

Table (3) shows the frequency distribution of the gender of the sample members. According to this data, among the research participants, 82 are female (approximately 73%) and 30 are male (approximately 27%) (Chart 1).

Table 3. Distribution of Gender Frequency of Sample Members

Gender	Frequency	Percentage
woman	82	73
man	30	27
All	112	100

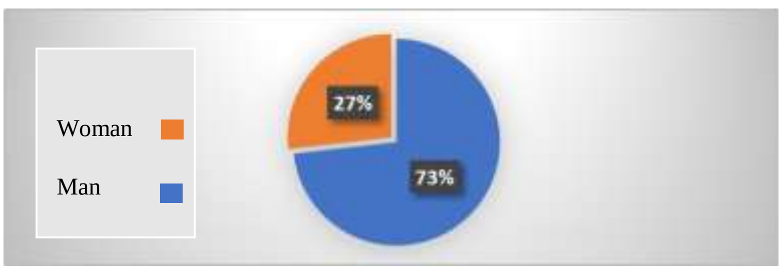


Chart 1. Frequency Distribution of Gender of Sample Members

Length of Service Frequency

Table (4) presents the frequency distribution of the length of service among the sample members. According to this data, among the research participants, 21 have less than 2 years of service (19%), 9 have 5 years of service (8%), 7 have 10 years of service (6%), 26 have 15 years of service (23%), and 49 have more than 15 years of service (44%) (Chart 2).

Table 4. Distribution of Service Tenure Frequency among Sample Members

Service Tenure	Frequency	ercentage
Less than 2 years	21	19%
5 years	9	8%
10 years	7	6%
15 years	26	23%
More than 15 years	49	44%
All	112	100

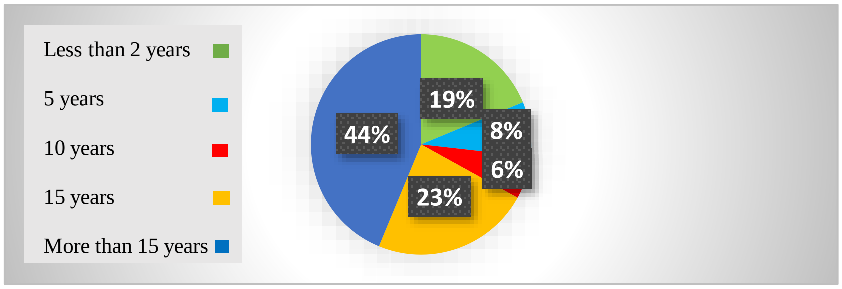


Chart 2. Frequency Distribution of Length of Service of Sample Members

Academic Degree

Table (5) presents the frequency distribution of academic degrees among the sample members. According to this data, among the research participants, 85 are teachers, 6 are instructors, 5 are assistant professors, 9 are associate professors, and 7 are full professors (Chart 3).

Table 5. Distribution of Academic Degree Frequency Among Sample Members

Academic Rank	Frequency	Percentage
Teacher	85	76
Instructor	6	5
Assistant Professor	5	4
Associate Professor	9	8
Professor	7	6
Total	112	100

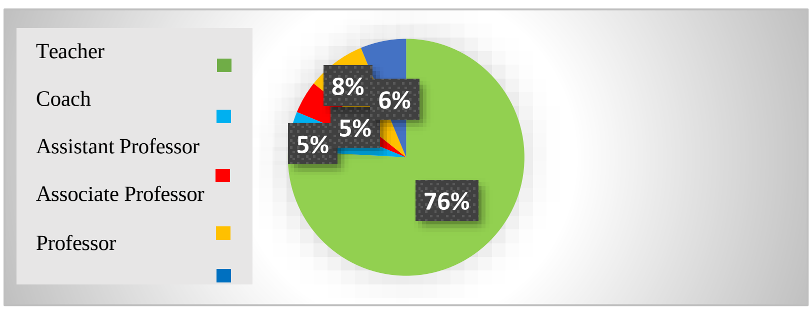


Chart 3. Frequency Distribution of Academic Degrees of Sample Members

Educational Level

Table (6) presents the frequency distribution of educational levels among the sample members. According to this data, among the research participants, 8 have a high school diploma, 21 have an associate degree, 42 have a bachelor's degree, 20 have a master's degree, and 21 have a doctoral degree (Chart 4).

Table 6. Distribution of Educational Level Frequency Among Sample Members

Education Level	Frequency	Percentage
High School Diploma	8	7%
Associate's Degree	21	19%
Bachelor's Degree	42	37%

Master's Degree	20	18%
Doctorate	21	19%
Total	112	100%

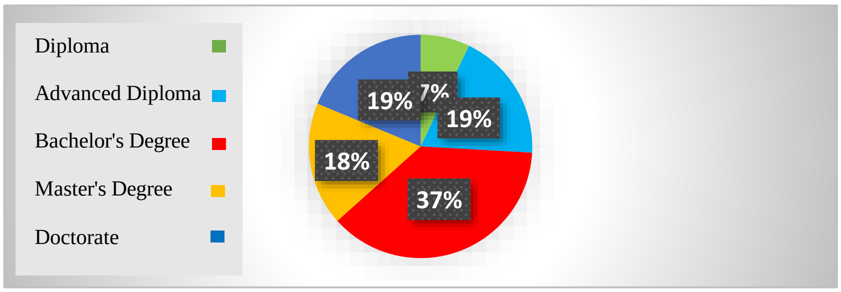


Chart 4. Frequency Distribution of Educational Levels of Sample Members

Research Findings

Based on the research question, "To what extent is the validity of the virtual mathematics curriculum model for elementary education reliable?", the process and results of validating the mentioned model are as follows:

Model Evaluation

Table (7) presents the statistical description of the scores related to the elements of the virtual mathematics curriculum model for elementary education, including skewness and kurtosis, along with the mean and standard deviation of the scores. Additionally, considering that the skewness and kurtosis values of the data are between +2 and -2, the data conforms to a normal distribution at the 0.05 significance level.

Table 7. Descriptive Statistics of Scores for the Elements of the Curriculum Model for Virtual Mathematics Education in the Primary Period

	Standard Deviation	Mean	Kurtosis	Skewness
Objectives	12.051	31.72	-0.877	-0.033
Content	11.803	29.72	-1.070	0.027
Teaching-Learning Methods	14.863	38.04	-1.106	-0.191
Evaluation	13.971	33.40	-1.352	-0.150

Factor Loadings and t-values

The factor loadings and t-values for each component of the virtual mathematics curriculum model for elementary education are presented in the table below:

Table 8. Factor Loadings and t-values for Items of Elements in the Virtual Mathematics Elementary Curriculum Model

Significance Level	t-value	Factor Loading	Items	Factor

0.01	10.317	0.656	1	Goals of the Curriculum
0.01	9.554	0.630	2	
0.01	9.529	0.625	3	
0.01	10.960	0.659	4	
0.01	9.931	0.641	5	
0.01	9.666	0.627	6	
0.01	7.996	0.590	7	
0.01	12.649	0.666	8	
0.01	9.098	0.631	9	
0.01	10.857	0.645	10	
0.01	23.925	0.778	11	
0.01	14.713	0.709	12	
0.01	9.357	0.595	13	
0.01	14.719	0.704	14	
0.01	10.670	0.640	15	Curriculum Content
0.01	7.379	0.559	16	
0.01	8.019	0.570	17	
0.01	12.398	0.680	18	
0.01	13.162	0.703	19	
0.01	15.124	0.711	20	
0.01	11.442	0.663	21	
0.01	19.382	0.748	22	
0.01	13.334	0.701	23	
0.01	13.886	0.669	24	
0.01	13.990	0.712	25	
0.01	22.333	0.783	26	
0.01	11.530	0.665	27	
0.01	10.856	0.667	28	Teaching-Learning Methods
0.01	9.722	0.623	29	
0.01	12.653	0.695	30	
0.01	15.292	0.728	31	
0.01	12.132	0.681	32	
0.01	22.642	0.787	33	
0.01	18.055	0.749	34	
0.01	18.024	0.741	35	
0.01	17.273	0.741	36	
0.01	9.856	0.622	37	
0.01	14.136	0.695	38	
0.01	10.136	0.632	39	
0.01	9.874	0.592	40	
0.01	14.379	0.707	41	
0.01	9.751	0.644	42	
0.01	11.666	0.683	43	
0.01	17/522	0.775	44	Assessment Methods
0.01	16/446	0.763	45	
0.01	16/034	0.753	46	
0.01	16/936	0.767	47	
0.01	10/832	0.680	48	
0.01	20/400	0.796	49	

0.01	11/856	0.684	50	
0.01	17/124	0.773	51	
0.01	10/954	0.683	52	
0.01	17/463	0.772	53	
0.01	25/878	0.824	54	
0.01	14/349	0.725	55	
0.01	23/284	0.804	56	
0.01	18/308	0.785	57	

Based on the results of the factor loadings analysis for the items related to the elements of the virtual mathematics curriculum model, including objectives, content, teaching-learning methods, and assessment strategies, and the t-statistics presented in Table (8), all factor loadings for each of these elements have t-values higher than 1.96 and are significant at the 0.05 level ($P<0.05$). In other words, the factor loadings of all items across these four dimensions of the virtual curriculum are statistically significant.

Model Validation

To validate the virtual mathematics curriculum model for elementary education, two methods were employed:

a) Model validation using structural equation modeling (SEM)
b) Model validation using the focus group method

a) Model Validation Using Structural Equation Modeling (SEM):

To validate the virtual mathematics curriculum model for elementary education from the users' perspective, structural equation modeling (SEM) was applied using the partial least squares (PLS) method and the Smart PLS software.

To evaluate the model fit, three criteria were used: reliability, convergent validity, and discriminant validity. Reliability was assessed through three methods: factor loadings, Cronbach's alpha, and composite reliability.

The reliability coefficients and the values of the average variance extracted (AVE) criterion are shown in Table (8). All the constructs in the model exhibit high composite reliability, with values greater than 0.60, indicating strong internal consistency of the research data. Cronbach's alpha values are above 0.70, demonstrating acceptable reliability.

The average variance extracted (AVE) criterion, which represents the average shared variance between a construct and its indicators, is a key measure for evaluating the outer model. This criterion requires that a construct shares more variance with its indicators than with any other construct in the model. Convergent validity was examined using this criterion, with values greater than 0.50 indicating acceptable convergent validity.

Based on the results, the average variance extracted for all latent variables is greater than 0.50, confirming that the model has suitable convergent validity.

It is important to note that composite reliability is generally considered more accurate than Cronbach's alpha because composite reliability assesses the reliability of each indicator separately, resulting in a more reliable composite measure and providing more dependable results regarding the reliability of a measurement tool. In contrast, Cronbach's alpha assumes equal reliability for all indicators. Therefore, due to its robustness and wide application in structural equation modeling, composite reliability is the preferred choice for measuring reliability in assessment tools.

Table 9. AVE Values and Reliability Indices

Variables	Cronbach's Alpha (>0.7)	Composite Reliability (>0.6)	AVE (>0.5)
Goals	0.897	0.913	0.530
Content	0.903	0.918	0.565
Teaching-Learning Methods	0.925	0.935	0.589
Assessment Methods	0.942	0.949	0.574
Virtual Mathematics Curriculum	0.974	0.975	0.512

Discriminant Validity, the third criterion for evaluating the fit of measurement models, compares the correlation between the indicators of a construct and that construct itself with the correlation between those indicators and other constructs. According to the results in Table (10), the square root of the average shared variance of the latent variables in this study, which is placed in the diagonal cells of the matrix, is greater than the correlations between them, which are arranged in the lower and right-hand cells of the diagonal. This indicates that each construct in the research model has greater interaction with its own indicators than with other constructs. This demonstrates that the model has adequate discriminant validity and that the measurement models in the study are well-fitted.

Table 10. Fornell-Larcker Matrix for Examining Discriminant Validity

Objectives	1	2	3	4	5
Content	0.728				
Teaching-Learning Methods	0.560	0.751			
Assessment Methods	0.627	0.453	0.767		
Elementary Level Virtual Mathematics Curriculum	0.604	0.529	0.474	0.757	
Objectives	0.649	0.633	0.549	0.539	0.715

Based on the results of reliability, convergent validity, and discriminant validity, it is observed that the measurement models in the structural equation model of the research effectively measure the latent variables. Therefore, the structural model fit will be examined in the following section.

Structural Model

In this section, the relationships between the latent variables of the study are examined. The first criterion for assessing the fit of the structural model is the R^2 coefficients related to the endogenous (dependent) variables of the model. R^2 is a measure that indicates the effect of an exogenous variable on an endogenous variable, with values of 0.19, 0.33, and 0.67 representing weak, moderate, and strong effects, respectively. According to the results in Table 10, the R^2 values obtained indicate a suitable fit of the structural model of the research.

The second criterion for evaluating the fit of the structural model is the Q^2 values of the endogenous variables. This criterion specifies the predictive power of the model. Values of 0.02, 0.15, and 0.35 for this criterion represent weak, moderate, and strong predictive power, respectively. Based on the results in Table (11), the Q^2 values for all endogenous variables are greater than 0.15, indicating adequate predictive power of the model and confirming the good fit of the structural model of the research.

Table 11. Results of R^2 and Q^2 Coefficients

Variables:	$Q^۲$	$R^۲$
Objectives	0/296	0/721
Content	0/388	0/870
Teaching-Learning Methods	0410	0/900
Assessment	0/488	0/883

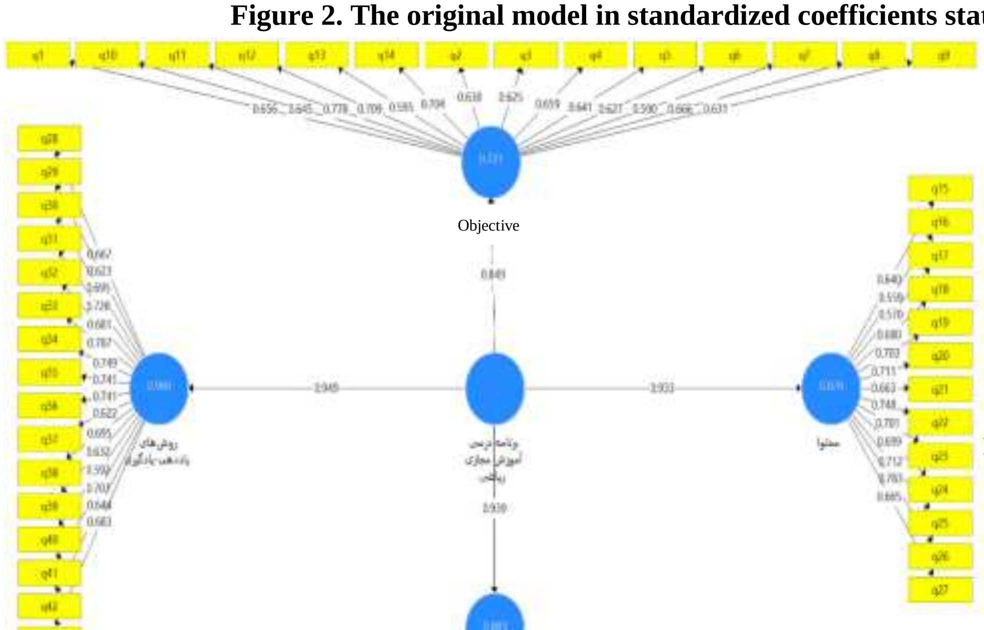
Overall Model Fit Evaluation

After examining the fit of the measurement models and the structural model, the overall structural equation model of the research was assessed using the Goodness of Fit (GOF) index. This index considers both the measurement and structural models and is used as a criterion for evaluating the overall performance of the model. It is equal to the geometric mean of the average shared variance and the average R^2 of the endogenous latent variables.

$$GOF = \sqrt{\overline{communalities} \times \overline{R^2}}$$

GOF values of 0.01, 0.25, and 0.36 are considered weak, moderate, and strong, respectively. Given that the GOF value obtained for the research model is 0.578, this indicates a good fit for the overall model of the research.

Figure 2. The original model in standardized coefficients state



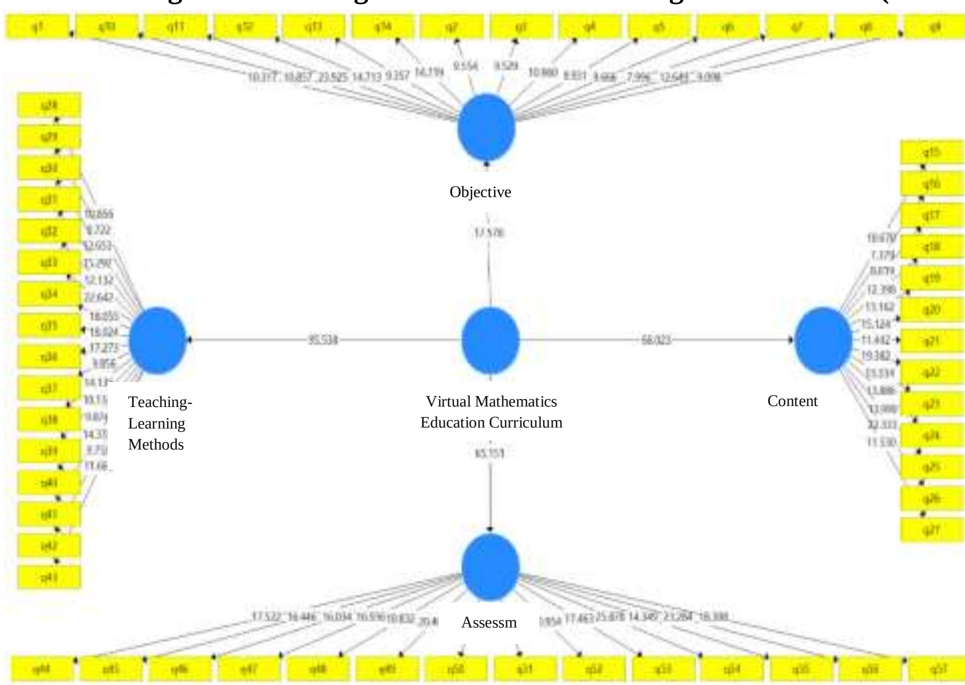
Teaching-
Learning
Methods

Virtual Mathematics
Education Curriculum

Content

Assessmen

Figure 3. The original model in terms of significance levels (t-values)



The numbers on the paths represent the t-values for each path. To assess the significance of the path coefficients, the t-value for each path needs to be greater than 1.96. In this analysis, the t-values for all paths are greater than 1.96, indicating that they are statistically significant at a 95% confidence level.

Table 12. Factor Loadings and t-values for Dimensions of the Virtual Mathematics Curriculum for Elementary Level

Path			**beta**	**t-value**	**level of significance**
The Virtual Mathematics Curriculum Model	→	Objectives	0/849	17.578	0.01

The Virtual Mathematics Curriculum Model	→	Content	0/933	66.023	0.01
The Virtual Mathematics Curriculum Model	→	Teaching-Learning Methods	0/949	95.538	0.01
The Virtual Mathematics Curriculum Model	→	Evaluation Approaches	0/939	65.151	0.01

In Table (12), the factor loadings and t-statistics for the dimensions of the virtual mathematics curriculum model for elementary education are presented. For the path coefficients to be considered significant, the t-statistic must be greater than 1.96. The results indicate that all factor loadings in the dimensions of objectives, content, teaching-learning methods, and assessment strategies have t-values greater than 1.96 and are significant at the 0.01 level ($P<0.01$).

b) Validation of the Model Using Focus Groups

After validating the virtual mathematics curriculum model using structural equation modeling, qualitative validation was also conducted through focus group sessions. These sessions included curriculum planning experts, authors of elementary mathematics textbooks, and experienced teachers. The purpose of the session was to provide a final assessment, improvement, and confirmation of the proposed model. Combining quantitative validation with structural equation modeling and qualitative validation through focus groups helped create a more comprehensive and accurate approach to evaluating the model, leveraging the strengths of each method in a complementary manner.

The focus group members provided their final opinions as follows:

- The model addresses all dimensions and components of the virtual mathematics curriculum for elementary education in a precise and comprehensive manner, covering all relevant aspects.
- In some dimensions of the model, certain components overlap in terms of meaning, content, and scope, and they could be merged without compromising the framework and aspects of the model. This would streamline the components and simplify the final model.
- Some components would benefit from a revision of the language used, including a review of writing and sentence structure.

The main modifications to the model regarding overlapping and merging of certain components were as follows:

- In the objectives dimension, the components "aligned with the Fundamental Transformation Document," "based on the six domains of education and training," and "aligned with the preservation of the value system and cultural heritage" were merged into one under the title "aligned with societal policy and value system." Additionally, the component "provoking through understanding the benefits and value of mathematics" was merged into "responsive to the reasons for learning mathematics."
- In the content dimension, the component "suitable architecture and pattern" was renamed "appropriate architecture and organization," and the components "coherent and integrated content" and "challenging, enjoyable, and related to learners' world" were merged into it. The component "based on principles and learning theories" was merged into "suitable for learning styles and learner age groups."

- In the teaching-learning methods dimension, the components "constructivist approach and knowledge construction" and "adopting an exploratory approach and deepening seemingly simple concepts" were merged under the title "diversity in teaching methods."
- In the evaluation methods dimension, the component "question bank preparation" was merged into "diversity in evaluation methods." Additionally, the component "role of parents as supervisors and controllers" was merged into "effective handling of virtual evaluation challenges."

  In conclusion, the virtual mathematics curriculum model for elementary education was finalized with the approval of 48 components across 4 dimensions: objectives, content, teaching-learning methods, and evaluation methods.

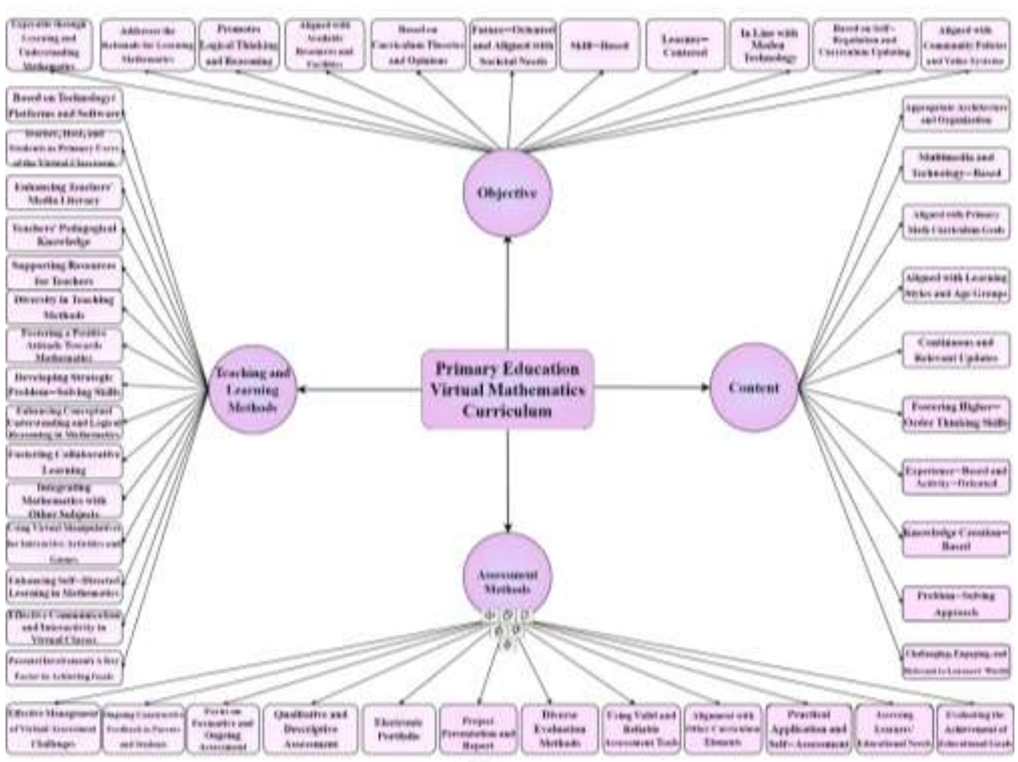


Figure 4. Diagram of the Final Model of the Virtual Mathematics Curriculum for Elementary Education

Discussion and Conclusion

In today's world, the development of technology and communication has made online education a powerful tool in enhancing the educational process. In particular, online mathematics education at the elementary level can serve as one of the main and important strategies in the teaching and learning process, offering mathematical concepts in a more engaging and impactful way (Bottle, translated by Bakhshali-zadeh, 2019). Online education allows learners to engage with mathematical concepts through digital tools and diverse educational resources. However, existing conditions reveal that due to various challenges and problems, the curriculum for online mathematics education can play a crucial and determining role in effectiveness and success. Therefore, the focus of this study was on identifying the features that should be considered in designing and developing an online mathematics curriculum for elementary students. Consequently, the researchers aimed to provide a conceptual model for the online mathematics curriculum and validate it for use in elementary education and schools.

Based on the findings of this study, if the **objectives** of the online mathematics curriculum align with societal policies and value systems, are based on self-regulation and curriculum updates, compatible with modern technology, learner-centered, skill-oriented, future-oriented, and based on community needs, and are grounded in curriculum theories, as well as responsive to the reasons for learning mathematics and enjoyable through the understanding of mathematics, it can be concluded that such a curriculum is capable of designing and implementing the mathematics learning process effectively, attractively, and efficiently. This approach not only helps improve students' academic performance but also develops their key skills for future success. The curriculum must always be updated and in line with scientific and technological advancements. According to McCulloch and colleagues (2023), Bozorgian (2023), and Valiollahi et al. (2021), the curriculum must keep pace with new changes and needs in mathematics and teaching methods, including incorporating new learning techniques, digital tools, and updating content to ensure that the curriculum remains relevant and effective. The curriculum should be designed to address the real needs of the community and available resources to meet educational and social needs and use resources optimally. According to Ramazani (2023), modern technologies and their influence in various aspects of life are indispensable as tools for facilitating learning in all subjects, especially mathematics. Designing a curriculum centered on learner needs contributes to the development of critical thinking skills, problem-solving, and analytical abilities, which are essential for students' future academic and professional success (Satsangi et al., 2023; Gholamipour Sholoumi, 2022). Utilizing valid curriculum theories for setting precise objectives and enhancing essential skills in mathematics is crucial. The curriculum should include activities and problems that encourage analytical thinking and provide students with opportunities to use logical reasoning to solve problems (Valiollahi et al., 2021). Additionally, understanding the personal and psychological characteristics of students is important for effective curriculum design (Weinhandl et al., 2024).

Kilinc et al. (2021) and Amirahmadi (2021) consider the use of technology and interactive multimedia programs as an effective alternative to traditional methods of teaching mathematics at the elementary level. These tools make learning more engaging and dynamic through animations and interactive activities, providing students with greater motivation for active participation and learning. Additionally, educational technology can tailor content to meet individual learning needs and pace, facilitating personalized learning.

In terms of content, effective and complete online mathematics curriculum content for elementary education should be well-architected and organized, multimedia-based, aligned with curriculum goals, suited to learning styles and age groups, regularly updated to match societal changes, foster higher-order thinking skills, stimulate motivation and creativity, experiential and activity-based, knowledge-creating, and problem-solving-oriented. This approach ensures that the content is designed to actively engage students and utilize modern educational methods. According to Figg and colleagues (2020) and Mohammadi et al. (2022), the use of digital tools and multimedia resources to promote visual and interactive learning should be prioritized to make the learning process more engaging and effective. Furthermore, activity design should enhance analytical and problem-solving skills, boosting cognitive and practical abilities of students. This aligns with the recommendations of Yakubova et al. (2024), Hamidi et al. (2020), and Valiollahi et al. (2021), who emphasize the creation of interactive content and a novel learning environment suitable for learning styles and age groups. For the development of higher-order thinking skills in students, content should challenge them intellectually and strengthen their critical, analytical, and creative thinking skills. According to Xie et al. (2022), Azid et al. (2020), and Babazadeh (2020), using real-life problems and situations that require analysis and creative solutions is crucial. These problems should be complex and multifaceted to encourage deep thinking. Designing questions that require critical thinking and cannot be answered with simple or definitive responses should compel students to evaluate and analyze.

Teaching and Learning Methods

In the section on teaching and learning methods, the curriculum should be technology-based, incorporating platforms and software. The teacher should act as the facilitator, while students are the primary users of the virtual classroom. It should enhance the teacher’s media literacy, pedagogical knowledge, and support resources. The curriculum should offer diverse teaching methods, foster a positive attitude towards mathematics, develop strategic problem-solving abilities, improve conceptual understanding and logical reasoning in mathematics, encourage collaborative learning, integrate mathematics with other subjects, use virtual manipulatives with activity and game-based approaches, strengthen self-regulation in mathematics learning, and involve parents as crucial partners in achieving educational goals. In this way, it can lead to an effective implementation of the online mathematics curriculum for elementary students.

In virtual mathematics classes, the teacher’s role as the facilitator and the students’ role as primary users can be highly effective. Chi Lin (2022), Khalil (2022), and Mohammadi et al. (2022) state that in this environment, teachers should make the learning experience engaging, interactive, and educational for students. Tools such as online whiteboards, concept maps, and mathematics education software can help make the class more active. These tools allow students to interact directly with the content. Live Q&A sessions for solving problems and providing additional explanations are also effective.

Using virtual manipulatives with activity and game-based approaches in the online mathematics curriculum means leveraging digital technologies and online tools to create engaging and interactive learning experiences. This approach can especially help in strengthening students' understanding and interest in mathematical concepts. Yakubova et al. (2024), Zeydini et al. (2023), Satsangi et al. (2023), Ukdem et al. (2022), Mohsenzadeh (2021), and Babazadeh et al. (2020) focus on using multimedia content and virtual manipulatives and games in mathematics classes. Virtual manipulatives include tools, software, and technologies available digitally that can enhance the learning process. Interactive and educational programs such as GeoGebra, Matific, or DreamBox offer various tools for learning mathematical concepts. Interactive games and simulations designed to teach mathematical concepts in a fun and engaging way. Designing activities where students can work interactively with mathematical concepts. Using games to teach mathematical concepts, such as puzzle games, matching games, and strategic games that can enhance computational and logical skills. These games can be played individually or in groups and help motivate and engage students. Hosting online challenges and competitions that encourage students to collaborate and compete, promoting active and practical learning of mathematical concepts. Games and interactive activities can significantly increase student motivation and encourage more active participation in the learning process.

According to findings by Weinhandl et al. (2024), Kilinc et al. (2021), Gorgij et al. (2022), and Kianfar et al. (2021), which emphasize enhancing teachers' media literacy, strengthening teachers' abilities to effectively use and leverage digital technologies and educational media in teaching is crucial. These abilities include selecting, analyzing, producing, and managing digital content to improve learning and teaching. Teachers need to become familiar with educational digital software, such as lesson design tools, mathematical simulation programs, and online interaction tools. Effective training in using these tools can improve teaching and interaction with students. Teachers should have the ability to create digital educational content, including making instructional videos, designing online activities, and producing interactive exercises and assessments. Additionally, teachers should be skilled in using Learning Management Systems (LMS) and online tools for conducting virtual classes, communicating with students, and tracking their academic progress.

Assessment Methods

In the section on assessment methods, determining the extent of achieving educational goals, evaluating learners' educational needs, applying learned material in practice, alignment with other curriculum elements, using valid and reliable assessment tools, diversity in assessment methods, providing projects and receiving performance reports, electronic portfolios, qualitative and descriptive assessments, prioritizing formative assessments and continuous evaluation, providing ongoing feedback to parents and students, and effectively addressing challenges in virtual assessment can contribute to an effective online mathematics curriculum for elementary students. In this context, determining the extent of achieving educational goals in assessment methods involves examining whether students have met the set objectives and the degree of their progress. The purpose of assessment is to provide feedback to students and teachers on learning progress to improve learning outcomes (Mosanabadi et al., 2022). According to Rahmani Valiollahi et al. (2023), this evaluation should help improve educational processes and enhance learning quality. Additionally, effectively addressing challenges in virtual assessment in the online mathematics curriculum for elementary education, as noted by Isnawan et al. (2022), Darragh et al. (2022), and Dorostkar Siani et al. (2022), involves identifying, analyzing, and managing specific problems and barriers that may arise in the assessment process in online and virtual educational environments. The goal is to use appropriate solutions to maintain the quality and accuracy of assessment and to enhance the teaching-learning process. Problems such as internet connectivity issues, lack of access to appropriate devices or required software, absence of face-to-face interactions and direct communication with the teacher, which may affect student motivation and understanding, difficulties in accurately and qualitatively assessing students' learning through digital tools, the potential for cheating or inaccuracies in submitting work and tests online, and differences in learning levels and educational needs of students that may be challenging to identify in a virtual environment.

In terms of assessment diversity in virtual mathematics classes, prioritizing formative assessment and continuous evaluation in the online mathematics curriculum for elementary education means focusing on methods of assessment that continuously and consistently track students' progress throughout the learning process. Research findings by Bright et al. (2022), Azid et al. (2020), Hadizadeh (2023), and Nasrati et al. (2022) indicate that these methods help teachers continuously analyze students' performance over time and make necessary improvements and adjustments based on these analyses. Using online self-assessment tests, periodic projects and assignments, and educational platforms with continuous assessment and immediate feedback capabilities are effective in this regard and contribute to creating a dynamic and flexible learning environment.

The findings indicate that different aspects of the online mathematics curriculum for elementary education are highly interconnected and have a deep relationship. Paying attention to the features of all four aspects of the model (goals, content, teaching methods, and assessment methods) is essential. However, empowering teachers in the aspects of teaching methods and assessment methods is particularly important, and it is recommended to consider appropriate planning and implementation to strengthen these two aspects.

Recommendations based on research findings

- One of the fundamental challenges in virtual education is the lack of adequate infrastructure. Research findings indicate that improving internet quality, equipping schools and students with appropriate digital tools can significantly impact the success of virtual education. Therefore, it is recommended that educational policymakers invest more in the development of technology infrastructure.
- Using content that aligns with the needs of elementary students in Tehran can significantly improve the quality of virtual education. It is recommended that curriculum developers and educational designers create and produce educational materials tailored to the local culture, needs, and conditions.
- Many teachers require additional training in the use of modern educational technologies. It is recommended that ongoing training programs be held for teachers so they can become familiar with the latest technologies and methods in virtual education and use them effectively.
- Based on the research findings, continuous assessment and feedback on the virtual education process play a crucial role in improving the performance of both students and teachers. It is recommended that dynamic, data-driven assessment systems be developed for virtual education to enable ongoing analysis and continuous improvement.
- Educational programs should be updated and enhanced to align with the needs and changes in society and technology. Continuous evaluation and feedback on the effectiveness of the curriculum model can contribute to its improvement and advancement.
- To enhance student motivation and engagement in virtual education, it is recommended that interactive and motivational methods, such as educational games and group activities, be incorporated into the curriculum.

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