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Research article

Dynamic analysis of guyed towers using direct time integration method by Newmark-β model

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Abstract

Direct time integration method is based on creating dynamic equation in selected time intervals. It is presented to solve equations of motion describing the dynamic response of the structural linear and nonlinear multi degree of freedom systems. In this research, dynamic analyses of guyed cables with small sagging have been studied. Nonlinear systems in the guyed cables with multi degree of freedoms are involved creating matrix equation. In these systems, response and derivations of response- time have been found. Evaluation of the forced vibration response for nonlinear guy cable with multi degree of freedom needs to solve second order differential equations in the general coordination system. Therefore solving the equation may be obtained using integration methods of time domain. Newmark-β model is one of the direct time integration methods to solve second order differential equations of motion using the difference formulation. After normalization of motion equations for the second central difference, this model changes to explicit extrapolation to dynamic response. Because of having no domain loss error and has a small error in the periodicity, it can be used as one of the best formulation methods of guy cables under dynamic loads.

Keywords: Dynamic analysis, Guy cable, Direct time integration, Motion equations

1- Introduction

Cables structures are nonlinear structures which experience different types of loading. Regarding growing importance of cables structures application, understanding their properties are very important, so there is a need for more information on cable behavior [1]. Many of cables are unstable geometrically and change the geometric shape in the load direction but they have no stiffness from the beginning the analysis, so special methods are needed to analysis [2]. Anyway, because of nonlinearity of cable behavior there is a mismatch between loading, stiffness and inner forces. In this case, it is possible to solve motion equation of guyed cables using integration methods of time integrity [3]. Time integrity method is frequently employed to compute numerical solutions of differential equations. This method is limited to linear systems but can be developed in linear systems. During the last decades, different time integrity methods have been used in structural systems. Each method have had different accuracy, stability and computational costs which according to displacement, speed and acceleration equations in step by step algorithm have different relations and can be obtained the dynamic response of the structure. The Newmark method is an effective method for numerical time integration in dynamic problems. Newmark-β method generalizes numerical integration method using two parameters, β and γ, which define numerical damping and controller of acceleration in the range of time step, respectively [4].Using this method, differential equation can be solved and using finite element analysis, it can be modeled dynamic systems.

2- Dynamic analysis of guyed cables underweight loading

Nonlinear structural systems which are multi degree of freedom systems involve equilibrium and dynamic matrix equations. In these equations, nonlinearity depends on time and time dependent variables. In nonlinear structures, the evaluation of the forced vibration response in multi degree of freedom requires solving quadratic differential equations in the general coordinate system. In this case, equation solving can be obtained by time domain integration methods.

2-1- Development of governing equations of motion

2-1-1- Main hypothesis

The following assumptions are used to simplify the derivations of equations of motion equations of cable element [5]:

(1) Cable material is elastic with linear strain (Lagrangian nonlinear strain)
(2) Cables are long and pre-tensioned with axial stiffness (bending and torsional stiffness are negligible). Due to increase of the length, the stability of the structure reduces [6].
(3) Along the x axis the tension in cable varies (x is the only considered independent variable).

2-1-2- Motion equations

Consider the case of single span inclined cable subjected to three dimensional partial differential equations in a parabolic form. Fig. 1 shows the inclined dynamic cable and its coordination. As shown in this figure, the left cable end (O) is considered as the origin of the coordinate system and the cable is initially placed in the x-z plane. According to the X axis, the cable has a chord length, L_{ch}, and inclination angle, θ. Following equation presents for force equilibrium in local direction, x'.

$$\frac{\partial}{\partial \bar{s}}\left(\bar{T}\frac{\partial \bar{x}}{\partial \bar{s}}\right)+(\tilde{q}_{x'}+\tilde{q}_{dx'})dx' = m.dx'\frac{\partial^2 u'}{\partial t^2}+c_{x'}.dx'\frac{\partial u'}{\partial t}+k_{x'}.dx'u' \quad (1)$$

Applying the chain rule of $\frac{\partial}{\partial \bar{s}}=\frac{\partial}{\partial x'}\cdot\frac{\partial x'}{\partial s}$ and substituting $\bar{x}'=x'+u'$, we get:

$$\frac{\partial}{\partial \bar{s}}\left(\bar{T}\frac{\partial \bar{x}}{\partial \bar{s}}(x'+u')\frac{\partial x'}{\partial \bar{s}}\right)dx'+(\tilde{q}_{x'}+\tilde{q}_{dx'})dx' = m.dx'\frac{\partial^2 u'}{\partial t^2}+c_{x'}.dx'\frac{\partial u'}{\partial t}+k_{x'}.dx'u' \quad (2)$$

Suppose that equation (2) is written as follow:

$$\frac{\partial}{\partial s}\left(\overline{H'}\left(1+\frac{\partial u'}{\partial x'}\right)\right)dx' + (\tilde{q}_{x'} + \tilde{q}_{dx'})dx' = m.dx'\frac{\partial^2 u'}{\partial t^2} + c_{x'}.dx'\frac{\partial u'}{\partial t} + k_{x'}.dx'u' \quad (3)$$

Simplifying, we have:

$$\overline{H}`\left(0+\frac{\partial^2 u'}{\partial x'}\right) + \frac{\partial\overline{H'}}{\partial x'}\left(1+\frac{\partial u'}{\partial x'}\right) + (\tilde{q}_{x'} + \tilde{q}_{dx'}) = m\left(\frac{\partial^2 u'}{\partial t^2}\right) + c_{x'}.\left(\frac{\partial u'}{\partial t}\right) + k_{x'}.(u') \quad (4)$$

Similarly, the following expressions can be obtained and equilibrium forces in direction of z' , y' :

$$\overline{H'}\left(0+\frac{\partial^2 v'}{\partial x'^2}\right) + \frac{\partial\overline{H'}}{\partial x'}\left(0+\frac{\partial v'}{\partial x'}\right) + (\tilde{q}_{y'} + 0) = m.\left(\frac{\partial^2 v'}{\partial t^2}\right) + c_{y'}\left(\frac{\partial v'}{\partial t}\right) + k_{y'}.(v') \quad (5)$$

$$\overline{H'}\left(\frac{\partial^2 z'}{\partial x'^2} + \frac{\partial^2 w'}{\partial x'^2}\right) + \frac{\partial\overline{H'}}{\partial x'}\left(\frac{\partial z'}{\partial x'} + \frac{\partial w'}{\partial x'}\right) + (\tilde{q}_{z'} + q_{dz'}) = m.\left(\frac{\partial^2 w'}{\partial x'^2}\right) + c_{z'}\left(\frac{\partial w'}{\partial t}\right) + k_{z'}.(w') \quad (6)$$

where:

T' : total cable tension (static+ dynamic)

$\bar{H}' = H'_o + h'_{(t)}$: Cable tension in x' direction (static+ dynamic)

m : cable mass in the cable length unit

$\tilde{q}_{x'}, \tilde{q}_{y'}, \tilde{q}_{z'}$: Applied dynamic force per unit cable length in the direction of x', y', z' and $C_{x'}, C_{y'}, C_{z'}$ Cable Damping

Component per unit of length in x', y', z' direction. $K_{x'}, K_{y'}, K_{z'}$ stiffness(external) per unit of lemgth in the direction of x', y', z'.

$u' = u'_{(x',t)}, v' = v'_{(x',t)}, w' = w'_{(x',t)}$ are displacement copmponents due to vibrations in x', y', z' directions.

2-2- Motion equation normalization

Every modeling problem has its own solution space related to dependent and independent variables. Accurate calculation of these variables is a fundamental step to mathematical model formulation [7]. The difference in variable spaces is applicable for analytical and computational analysis of both models. Where all values are dimensionless, the normalization transformation technique creates a linear scaling value for each of dependent and independent variables and transforms the equations based on governing relations. Any forcing functions for the problem are scaled in a similar manner [8].

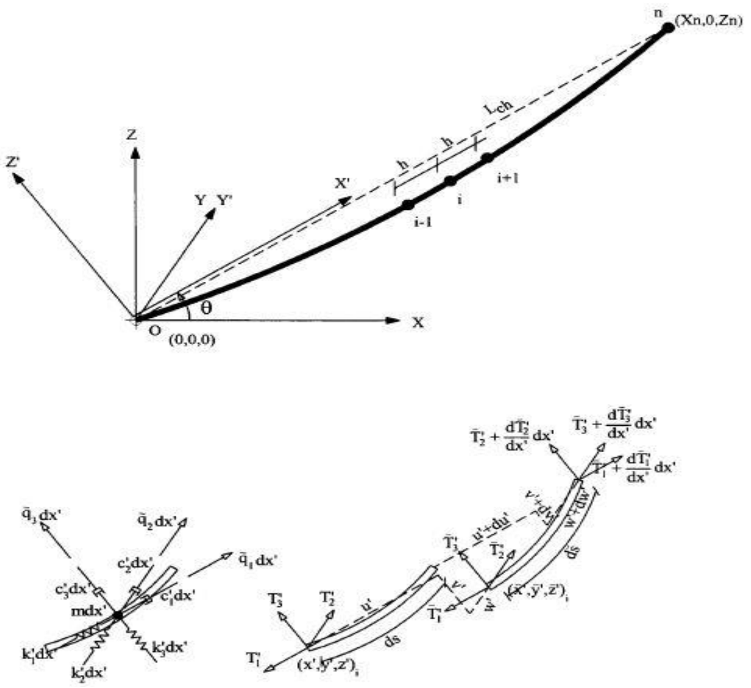

Fig. 1 3D cable model for dynamic analysis

2-2-1- Need for normalization

Math based optimization methods need to have optimum answer to start problem solution [9]. One of these methods is normalization. The importance of normalization is just as for linear systems and it can often provide solutions with independent parameter. Using normalization, the probability of round-off errors in numerical solution of equation can be reduced. The importance of normalization becomes clear when we have to solve a set of differential equations. In these cases, the variables are different in terms of numerical values, which are cause of problems. Sometimes these problems are related to selection of variable units [10]. Equation normalization has effect on the problem of variables creating and their derivatives, and if used in numerical methods of solving equations, it can show different values. Normalization provides important information about the problem and can often reduce the complexity of problem because of specifies the importance of each term governing equations.

2.2.2- Non dimensionalization

Using the normalization technique by applying a new set of non- dimensionalized variables can rewrite equations (4), (5) and (6) and provide non- dimensional equations for motion of single inclined cable.

$$\hat{H}\left(\frac{\partial^2 \hat{u}}{\partial \hat{x}^2}+0\right)+\frac{\partial \hat{H}}{\partial \hat{x}}.\left(1+\frac{\partial \hat{u}}{\partial \hat{x}}\right)+(\hat{\tilde{q}}_x+\hat{\tilde{q}}_{dx}) = \hat{m}.\left(\frac{\partial^2 \hat{u}}{\partial \hat{t}^2}\right)+\hat{c}_x.\left(\frac{\partial \hat{u}}{\partial \hat{t}}\right)+\hat{k}_y.\hat{u} \tag{7}$$

$$\hat{H}\left(\frac{\partial^2 \hat{v}}{\partial \hat{x}^2}+0\right)+\frac{\partial \hat{H}}{\partial \hat{x}}.\left(0+\frac{\partial \hat{u}}{\partial \hat{x}}\right)+\hat{\tilde{q}}_y = \hat{m}.\left(\frac{\partial^2 \hat{v}}{\partial \hat{t}^2}\right)+\hat{c}_y.\left(\frac{\partial \hat{v}}{\partial \hat{t}}\right).+\hat{k}_y.\hat{v} \tag{8}$$

$$\hat{H}\left(\frac{\partial^2 \hat{w}}{\partial \hat{x}^2}+\frac{\partial^2 \hat{z}}{\partial \hat{x}^2}\right)+\frac{\partial \hat{H}}{\partial \hat{x}}.\left(\frac{\partial \hat{z}}{\partial \hat{x}}+\frac{\partial \hat{w}}{\partial \hat{x}}\right)+(\hat{\hat{q}}_y+\hat{\hat{q}}_{dz})= \hat{m}.\left(\frac{\partial^2 \hat{w}}{\partial \hat{t}^2}\right)+\hat{c}_y.\left(\frac{\partial \hat{v}}{\partial \hat{t}}\right).+\hat{k}_y.\hat{w} \quad (9)$$

where: $\tilde{x},\tilde{y},\tilde{z}$ are normalized functions in x',y',z' directions.

$$\hat{x}=\left(\frac{1}{L_{ch}}\right)x' \quad \hat{y}=\left(\frac{1}{L_{ch}}\right)y' \quad \hat{z}=\left(\frac{1}{L_{ch}}\right)z' \quad (10\text{-}1)$$

$$\hat{t}=\left(\sqrt{\frac{EA}{mL_{ch}^2}}\right)t \quad (10\text{-}2)$$

$\tilde{t}$ and $\tilde{m}$ are normalized time and cable mass per unit length, respectively.

$\tilde{m}=1$

(10-3) $u'=u'_{(x',t)},v'=v'_{(x',t)},w'=w'_{(x',t)}$

are normalized displacement components due to vibration in the x',y',z' direction.

$$\hat{u}=\left(\frac{1}{L_{ch}}\right)u' \quad \hat{v}=\left(\frac{1}{L_{ch}}\right)v' \quad \hat{w}=\left(\frac{1}{L_{ch}}\right)w' \quad (10\text{-}4)$$

$\tilde{H}$ is cable tension component in the x' normalized direction (static-dynamic).

$$\hat{H}=\left(\frac{H'_o+h'_{(t)}}{EA}\right) \quad (10\text{-}5)$$

$h'_{(t)}$ is dynamic tension component in direction of x' .

$\hat{\hat{q}}_{dx}$, $\tilde{q}_{dz}$ are the normalized self-weight component per unit length,

$$\hat{\hat{q}}_{dx}=\left(\frac{L_{ch}}{EA}\right)\tilde{q}_{dx};\ \tilde{q}_{dz}=\left(\frac{L_{ch}}{EA}\right)\tilde{q}_{dz} \quad (10\text{-}6)$$

$\hat{\hat{q}}_x,\hat{\hat{q}}_y,\hat{\hat{q}}_z$ are normalized dynamic force in per unit length.

$$\hat{\hat{q}}_x=\left(\frac{L_{ch}}{EA}\right)\tilde{q}_x;\hat{\hat{q}}_y=\left(\frac{L_{ch}}{EA}\right)\tilde{q}_y,\hat{\hat{q}}_z=\left(\frac{L_{ch}}{EA}\right)\tilde{q}_z \quad (10\text{-}7)$$

$\widehat{c}_x,\widehat{c}_y,\widehat{c}_z$ are normalized damping component in per unit cable length.

$$\widehat{c}_x=\left(\frac{L_{ch}\sqrt{EA/m}}{EA}\right)c_{x'};\widehat{c}_y=\left(\frac{L_{ch}\sqrt{EA/m}}{EA}\right)c_{y'};\quad \widehat{c}_z=\left(\frac{L_{ch}\sqrt{EA/m}}{EA}\right)c_{z'}; \quad (10\text{-}8)$$

$\widehat{k}_x,\widehat{k}_y,\widehat{k}_z$ are hardness normalized component in per unit cable length.

$$\widehat{k}_x=\left(\frac{L_{ch}{}^2}{EA}\right)k_{x'};\widehat{k}_y=\left(\frac{L_{ch}{}^2}{EA}\right)k_{y'};\ \widehat{k}_z=\left(\frac{L_{ch}{}^2}{EA}\right)k_{z'}; \quad (10\text{-}9)$$

$\left(\frac{\partial \hat{u}}{\partial \hat{t}}\right),\left(\frac{\partial \hat{v}}{\partial \hat{t}}\right),\left(\frac{\partial \hat{w}}{\partial \hat{t}}\right)$ is normalized speed component and $\left(\frac{\partial \hat{u}}{\partial \hat{t}^2}\right),\left(\frac{\partial \hat{v}}{\partial \hat{t}^2}\right),\left(\frac{\partial \hat{w}}{\partial \hat{t}^2}\right)$ is acceleration normalized component regarding to vibration, respectively.

2-3- Direct time integration method

Unlike superposition methods, direct time integration technique is limited to linear systems but it can easily be expanded in nonlinear systems. This method can be used as much as possible for components with high frequency in a straightforward manner. Although equations of the direct time integration cannot be used as a black box. Indeed, the parameters of this method should be controlled correctly according to the necessary accuracy and stability according to the numerical damping control [11]. Also, each of them has different relationships relating displacement, velocity and acceleration in step by step algorithms to obtain structural dynamic response.

2-3-1- Classification 0f time integration methods

The time integration algorithms used in the structure dynamics can be classified under four broad categories: single step multi value, multi- step, multi- stage and predictor and corrector methods. Each method can be categorized as implicit or explicit. Either explicit or implicit integration can be used, depending on nature of the problem to be solved. Most of implicit methods can be absolute and stable and create large time steps. However compared to explicit method, the cost per time steps and storage requirements is more. Therefore it is necessary to have simulated equations system to provide more appropriate solutions (for example, more computational efforts per time step). On the other hand, most of explicit methods are stable and need small time steps for numerical stability [12]. It is compensated based on this fact that the cost of each time step and the storage requirements is relatively low and it is not necessary to solve the simulated equations (for example, less computation per time step). Of course, the time integration method is a critical choice that combines accuracy and efficiency. Regarding to the applications, choice of method is guided and its usage limitation has been specified. Usually, the implicit methods are effective for structural dynamic problems; whereas explicit methods are used for wave propagation problems. In a general way, multi- stage integration methods for the first order systems are expressed in this form:

$$u_{n+1} = \sum_{j=1}^{m} \alpha_j u_{n+1-j} - \Delta t \sum_{j=0}^{m} \beta_j \dot{u}_{n+1-j} \quad (11)$$

where: $\Delta t = t_{n+1} - t_n$ is time step and

$u_{n+1}^T = \left[\dot{q}_{n+1}^T \; q_{n+1}^T \right]$ is the state vector at t_{n+1} calculated from state vector at the n time from their derivatives and also u_{n+1} derivation.

If $\beta_0 = 0$, the integration plan in the equation (12) is absolute (implicit) because quality vector in t_{n+1} shows time derivative therefore the integration relations reform before solving equations. Solving method repeats in non- linear manner. If $\beta_0 = 0$, the time vector in $tn+1$can be provide directly using the presented results in previous time steps. This method is explicit.

Also, when for j>1 we consider $\alpha j=0$ and $\beta_j = 0$, equation (11) is according to one-step method and in t_{n+1}, the system shows previous manner in t_n.

2-3-2- Time integration using Newmark-β

The effective numerical algorithm has follow specifications:
1-uncoditional stability
2-controllable algorithmic damping
3-motion stability of structure
4-high accuracy
5-lacking historical affiliation

So, because of high accuracy and unconditional stability, the single step multi value method creates balance between numerical effective dispersion therefore it is a suitable algorithm for dynamic application of structure. For time integrity, Newmark method has necessary specifications for effective integrity and numerical stability and accuracy. This method is a single integrated formula [13]. The state vector of system $t_n + 1 = t_n + \Delta t$ is driven from manner vector in t in which development of Taylor sets are seen for displacements and velocities.

$$f(t_n + \Delta t) = f(t_n) + \Delta t.f'(t_n) + \frac{\Delta t^2}{2} f''(t_n) + ... + \frac{\Delta t^2}{S!} f^S(t_n) + R_S \quad (12\text{-}1)$$

Here R_S is expansion of S formula:

$$R_S = \frac{1}{S!}\int_{t_n}^{t_n+\Delta t} f^{(S+1)}(\tau)[t_n + \Delta t - \tau]^S d\tau \quad (12\text{-}2)$$

Equation (12-1) is used to calculate of speeds and displacements of system in t_{n+1}.

$$\dot{q}_{n+1} = \dot{q}_n + \int_{t_n}^{t_{n+1}} \ddot{q}(\tau) \quad (13\text{-}1)$$

$$q_{n+1} = q_n + \Delta t.\dot{q}_n + \int_{t_n}^{t_{n+1}} (t_{n+1} - \tau).\ddot{q}(\tau)d(\tau) \quad (13\text{-}2)$$

These equations are calculated integrity of equations (13) accompany by evaluation of the quadrant of the numerical circle. So, in the time interval $(t_n,\ t_{n+1}),\ q(t)$ is shown as $q_n,\ q_{m+1}$ function and substitution in (12-2) and these formula is provided:

$$\int_{t_n}^{t_{n+1}} \ddot{q}(\tau)d(\tau) = (1-\gamma).\Delta t.\ddot{q}_n + \gamma.\Delta t.\ddot{q}_{n+1} + r_n \quad (14\text{-}1)$$

$$\int_{t_n}^{t_{n+1}} (t_{n+1} - \tau).\ddot{q}(\tau)d(\tau) = \left(\frac{1}{2} - \beta\right).\Delta t^2.\ddot{q}_n + \beta.\Delta t^2.\ddot{q}_{n+1} + r'_n \quad (14\text{-}2)$$

and the errors are calculated as follow:

$$r_n = \left(\gamma - \frac{1}{2}\right).\Delta t^2.q^{(3)}(\tilde{\tau}) + o(\Delta t^3.q^{(4)}) \quad (14\text{-}3)$$

$$r'_n = \left(\beta - \frac{1}{6}\right).\Delta t^3.q^{(3)}(\tilde{\tau}) + o(\Delta t^4.q^{(4)}) \quad (14\text{-}4)$$

Substitution equations (13) in (14) lead to approximate formula for Newmark method:

$$(15\text{-}1)\ \dot{q}_{n+1} = \dot{q}_n + (1-\gamma).\Delta t.\ddot{q}_n + \gamma.\Delta t.\ddot{q}_{n+1}$$

$$(15\text{-}2)\ q_{n+1} = q_n + \Delta t.\dot{q}_n + \left(\frac{1}{2} - \beta\right).\Delta t^2.\ddot{q}_n + \beta.\Delta t^2.\ddot{q}_{n+1}$$

For that, β and γ are parameters related to a quarter of circle plane. By choosing $\gamma = \frac{1}{2}$ and $\beta = \frac{1}{6}$ a linear interpolation for accelerations: $\ddot{q}(\tau) = \ddot{q}n + (\tau - t_n).\frac{\ddot{q}_{n+1} - \ddot{q}_n}{\Delta t}$

in the system of with time interval (t_n, t_{n+1}) is provided and equation (15) is written as follow:

$$\dot{q}_{n+1} = \dot{q}_n + \frac{\Delta t}{2}(\ddot{q}_n + \ddot{q}_{n+1}) \quad (16\text{-}1)$$

$$q_{n+1} = q_n + \Delta t.\dot{q}_n + \frac{\Delta t^2}{2}.\left(\frac{2\ddot{q}_n + \ddot{q}_{n+1}}{3}\right) \quad (16\text{-}2)$$

In some methods, choosing $\gamma = ½$ and $\beta = ¼$ is provided by considering average of acceleration $\ddot{q}(\tau) = \frac{\ddot{q}_{n+1} + \ddot{q}_n}{2}$ in time interval (t_n, t_{n+1}) and equation (15) is written as follow:

$$\dot{q}_{n+1} = \dot{q}_n + \frac{\Delta t}{2}.(\ddot{q}_n + \ddot{q}_{n+1}) \quad (17\text{-}1)$$

$$q_{n+1} = q_n + \Delta t.\dot{q}_n + \frac{\Delta t^2}{2}.\left(\frac{\ddot{q}_n + \ddot{q}_{n+1}}{2}\right) \quad (17\text{-}2)$$

3- Conclusion

According to the equations provided by Newmark-β method, it is deduced that the cables with high initial tension (small sagging ratio) have hardness behavior in the lower parts (strain) so their dynamic tensions is small. Due to the relatively light weight of the structure, the fluctuation and change of the mass of cable have no effect on the displacement or tensile behavior of the cables. Also, the numerical stability in the non- linear oscillation problems of cable is decisive to the number of cable parts and the used time steps. Based on the selected time phase and the number of cable divisions in a problem, the stability index is determined .By using this index a stable and accurate solution can be provided. Although having a large number of divisions of N_{div} in terms of quality is better than having smaller Δt time interval Δt in terms of accuracy. The effect of stability index $\Delta t / Ttr$ on initial strain parameter or initial strain index of cable is in such way that when the cable hardness is created, the speed of the wave increases and the time of wave movement in the cable parts gets smaller. In this case, need to use smaller time steps. Also, due to definition of stability index for guyed cable $\Delta t / Ttr$ three functions including:

$\left(\frac{\tilde{q}_1.L_{ch}}{H_0}\right),\left(\frac{\tilde{q}_2.L_{ch}}{H_0}\right),\left(\frac{\tilde{q}_3.L_{ch}}{H_0}\right)$ are introduced as effective parameters in analysis of guyed cable.

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