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Progress and retrogression of costs Profitability by global Malmquist index

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Abstract

Data Envelopment Analysis (DEA) is a nonparametric method for measuring and does not need to have and calculate the production function, which is often difficult to calculate. In this article, we evaluate the units under investigation in terms of cost and production efficiency in several time periods and the progress (Improvements) or regression of each unit for this purpose, we use the method based on solving linear programing models using Malmquist productivity index. Data envelopment analysis is a non-parametric method for measuring the performance of decision-making units. Finally, by designing and solving a numerical example, we emphasize and test the applicability of the material presented in this article.

Keywords: Data envelopment Analysis (DEA), Progress, Improvements, Regressions, Cost efficiency, Profitability, Malmquist Global Index.

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1. Introduction

The concept of cost efficiency is that Measuring the ability of a DMU to produce outputs at the lowest input cost, Farrell began in 1957. He defined cost efficiency as the minimum cost to actual cost [6].

Lee and Johnson proposed in 2014 a short-term capacity planning approach [9]. Rostamy and Ahangari (2016) investigated earnings inefficiency with interval data [1]. Kazutoshi Ando and his colleagues (2016) measured the minimum inefficiency gap in data envelopment analysis [8]. Rostamy and his colleagues start working on inefficiency in 2019 . In 2015, Feng He and his colleagues examined the stability and sensitivity radius for uncertain boundaries [7]. Allahyar and Rostamy reviewed the performance analysis for negative data and performance-based scalability in 2015 very interesting work on economic efficiency was done fuzzily. In 2019, Rahmani and many other colleagues conducted activities and research with fuzzy data [13,14].

In 2019, Peykani and Mohammadi proposed a domain-oriented size model of the feasibility of ranking warehouses in the presence of negative and non-linear data [13,14].

In 2019 Akhlaghi and Rostamy used integer linear programming to determine the most efficient decision-making unit. Another key issue in data envelopment analysis is the Malmquist Productivity Index. The Malmquist Index was first introduced in 1953 as a qualitative index in the analysis of consumer inputs by Stan Malmquist and became well-known among researchers.

Malmquist Index has some drawbacks despite being widely used, including when linear programming techniques. For the calculation of the index, the invariance occurs which is a fundamental problem in the data structure. And sufficient data were obtained to resolve the problem, [15]. Pastor and Lovell showed that the source of the problem is the properties of adjacent step technologies in the index building [5].

However, each of the Malmquist index does not solve all three problems. Then a new index built from all data at all stages and periods of all manufacturers, its technology was presented as the Global Malmquist that has a circular feature and a It measures the magnitude of productivity change. The Malmquist Global Index can solve all three problems by specifying a fixed boundary. In 2012, Tohidi et al. used the Malmquist Global in cost estimation. [16] Emrooznejad and Yang used global Malmquist to investigate carbon dioxide emissions. [4] Tohidi and Razavyan used Malmquist Global Index in 2012 to calculate the profit of decision-making units. Not all articles present and research on the cost productivity of economic units were reviewed.

2. Basic Model in Data Envelopment Analysis

A nonparametric method for measuring the performance of decision-making units (DMUs) is Data Envelopment Analysis (DEA). Charnes, Cooper, and Rhodes provided the first linear models for this measurement (CCR model) [3].

The original CCR model only used fixed-rate (CRS) technology, developed by Banker et al.

If n is the decision unit (DMU_j, $j=1,...,n$) that uses m to produce the output S and the vectors $x_j=\left(x_{1j},...,x_{mj}\right)$ and $y_j=\left(y_{1j},...,y_{sj}\right)$ are the inputs and outputs, respectively. Therefore, $y_j \neq 0, x_j \neq 0$, $y_j \geq 0, x_j \geq 0$.

Input-based technical performance for DMU_o is listed below.

$$\theta^* = \min \theta$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j \le \theta x_o$$
$$\sum_{j=1}^{n} \lambda_j y_j \ge y_o \qquad (1)$$
$$\lambda_j \ge 0$$

2.1. Cost efficiency

Consider the problem of minimizing the cost of a company with inputs to produce outputs and cost vector $C\left(C \ne 0, C \ge 0\right)$. We obtain the minimum total cost of producing the output vector y_o.

$$\min C^t x$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j \le x$$
$$\sum_{j=1}^{n} \lambda_j y_j \ge y_o \qquad (2)$$
$$x \le x_o$$
$$\lambda_j \ge 0, x \ge 0$$

Cost efficiency is the ratio of the minimum cost to the real cost.

$$CE_o = \frac{C^t x^*}{C^t x_o}, \quad 0 \le CE_i \le 1 \qquad (3)$$

2.2. Calculating cost productivity improvement

Following the implementation of the CCR economic performance model, we seek to determine whether different decision makers have improved, regressed or unchanged in terms of their performance over different time periods. Accordingly, using the Malmquist Index, the performance improvement or regression of the units is analyzed. Malmquist index measurement requires calculation of distance functions.

To solve these functions, we can use the linear programming method of comprehensive data analysis. In this regard, for each decision-making unit, according to the Malmquist Index between two time periods t and t + 1, four distance functions must be calculated, which in turn requires solving four linear programming problems. Assuming constant-scale returns (assumed by Fare, Graskov, Norris, and Zhang in their analysis) [12].

Four issues will be addressed and solved for cost efficiency.

$$\min \ Cx$$
$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_j^K \le x$$
$$\sum_{j=1}^{n} \lambda_j y_j^K \ge y_o^L \qquad (4)$$
$$x \le x_o^L$$
$$\lambda_j \ge 0, x \ge 0$$

(Unit in time L – border in time K)

$$K = t, t+1 \qquad L = t, t+1$$

Cost effectiveness:

$$CE_{oL}^{K} = \frac{C^K x^*}{C^L x_o^L} \qquad (5)$$

We can calculate the Malmquist Index for the cost efficiency of DMU_o using the following equation.

$$CMPI_o = \sqrt{\frac{CE_{o(t+1)}^{t}}{CE_{o(t)}^{t}} \cdot \frac{CE_{o(t+1)}^{t+1}}{CE_{o(t)}^{t+1}}} \qquad (6)$$

- $CMPI_o > 1$ Indicates the improvement in cost efficiency of DMU0 from time period t to time period t + 1.
- $CMPI_o < 1$ Indicates the recovery of DMU0 cost efficiency from time period t to time period t + 1.

- $CMPI_o = 1$ Indicates the unchanged cost efficiency of DMU0 from time period t to time period t + 1.

2.3. Calculating cost productivity improvement and regression with the Malmquist Global Index

If the units are evaluated over time periods $t = 1, \ldots, T$, in the Malmquist Global Method, we examine all units in one place and the cost efficiency model for DMU_o at time k is given below. Be. In this section, each DMU is compared to itself over different time periods, and it has progressed in two periods, such as p and q.

$$
\begin{aligned}
&\min C^K x \\
&s.t. \quad \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j x_j^t \le x \\
&\qquad \sum_{t=1}^{T}\sum_{j=1}^{n} \lambda_j y_j^t \ge y_o^K \\
&\qquad x \le x_o^K \\
&\qquad \lambda_j \ge 0, x \ge 0, \qquad K = 1, \ldots, T
\end{aligned} \tag{7}
$$

And the cost efficiency comes from the following relationship:

$$CE_o^K = \frac{C^K x^*}{C^K x_o^K} \tag{8}$$

If we consider the productivity of DMU_o at time p compare to time q, we consider the solution of the model K = p, q and two problems arise and are solved. The Malmquist Global Index is written as follows:

$$GCMPI_q^p = \frac{CE_o^p}{CE_o^q} \tag{9}$$

- If $GCMPI_q^p > 1$ Let the unit at time p have progressed to q
- $GCMPI_q^p < 1$ If the unit at p has regressed to q.
- $GCMPI_q^p = 1$ Let unit be unchanged at p relative to q.

3. Numerical examples

For example, we consider 10 decision-making units with three inputs and two outputs over the three time periods t_1, t_2, t_3 which are listed in Table 1. Table 2 shows the cost, income, and profit efficiencies for all units over the three time periods.

Tables 3,4 &5 show the proposed models for calculate the improvement and regression of units with the Malmquist index from periods t_1 to t_1, t_2 to t_3 and t_2 to t_3.

Table 1: The inputs and outputs data in three-time period for ten DMUs

	DMU_1			DMU_2			DMU_3			DMU_4			DMU_5			DMU_6			DMU_7			DMU_8			DMU_9			DMU_{10}		
	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3	t_1	t_2	t_3
I_1	5	3	5	4	4	6	2	3	7	1	6	5	4	5	3	4	3	3	4	3	6	2	4	6	3	7	4	5	6	1
I_2	4	3	6	5	7	3	4	3	2	4	5	4	6	4	3	6	6	7	1	4	5	4	6	5	3	4	6	5	2	7
I_3	5	6	4	7	6	5	6	3	5	6	3	7	5	4	6	5	7	4	7	3	2	3	3	2	6	3	1	4	5	4
O_1	4	4	7	6	5	3	5	6	4	5	6	5	4	3	7	7	5	6	5	4	3	6	1	4	3	5	4	3	6	8
O_2	6	5	4	7	6	3	5	4	6	3	7	6	4	5	3	8	7	5	4	6	4	3	5	7	6	4	6	7	5	4

Table 2: Cost Efficiency in three-time period for ten DMUs

DMU	Cost		
	t_1	t_2	t_3
1	0/750	0/618	0/673
2	0/760	0/632	0/335
3	0/765	1	0/579
4	0/660	0/912	0/571
5	0/531	0/608	0/693
6	1	0/708	0/772
7	0/593	1	0/523
8	1	0/678	0/275
9	0/828	0/570	1
10	0/928	0/647	1

Table 3: Cost productivity improvement and regression with the Malmquist Index (t_1 to t_2)

DMU	CMPIO
1	0/98
2	0/84
3	0/08
4	0/31
5	1/11
6	0/28
7	0/76
8	4/88
9	8/86
10	1/10

Table 4: Cost productivity improvement and regression with the Malmquist Index (t_2 to t_3)

DMU	CMPIO
1	32/48
2	2/92

3	0/89
4	8/10
5	0/28
6	3/38
7	0/41
8	0/88
9	1/33
10	1/86

4. Conclusion

Calculating the Malmquist Productivity Index is a very useful and applicable system for determining the progress and regress of a system and helps managers in future decisions and policies of the system and leads to improved productivity. The Global Malmquist Productivity Index helps managers track the progress and remediation of a system or organization over two time periods, across time periods, and improve decision making and policy making. And better evaluate the efficiency of a unit.

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