International Journal of
Mathematical Modelling & Computations
Vol. 06, No. 03, Summer 2016, 243- 252

Application of the Bellman and Zadeh's principle for identifying the fuzzy decision in a network with intermediate storage

Mohammad Rahimian [a], Esmaiel Keshavarz [b] and Hamid Hassasi [c,*]

[a] *Department of Mathematics, Islamic Azad University, Masjed-Soleiman Branch, Masjed-Soleiman, Iran*;
[b] *Department of Mathematics, Islamic Azad University, Sirjan Branch, Sirjan, Iran*;
[c] *Faculty of Management Sciences, Islamic Azad University, Central Tehran Branch, Tehran, Iran.*

Abstract. In most of the real-life applications we deal with the problem of transporting some special fruits, as banana, which has particular production and distribution processes. In this paper we restrict our attention to formulating and solving a new bi-criterion problem on a network in which in addition to minimizing the traversing costs, admissibility of the quality level of fruits is a main objective. However, the fruits are possibly stored at some intermediate node for practical purposes. We call the new model the best shipping pattern problem with intermediate storage. Here, it is assumed that both arc costs and times are crisp numbers. The main contribution of this model is an actual interpretation of the given fuzzy trapezoidal number, as the quality of delivered commodities. Since the presented problem has a fuzzy structure, the Bellman and Zadeh's max-min criterion can be used to treat it as a crisp single-objective problem, which is easily solvable. An illustrative example is solved, to explain the presented details.

Received: 16 March 2016, Revised: 5 May 2016, Accepted: 1 June 2016.

Keywords: Multiple criterion optimization; Best shipping pattern; Fuzzy number; Bellman-Zadeh's principle

AMS Subject Classification: 90B06; 90B50; 90C70; 90C90

Index to information contained in this paper

*Corresponding author. Email: Hamidhassasi@yahoo.com

http://www.ijm2c.ir

1. Introduction

Many real optimization problems contained in every facet of industrial planning, production and distribution can be modelled by network flows (see [1], for example). However, in general, network flow models, like any mathematical programming model, can be just projections of reality into mathematics involving simplifications and fuzzy data. Various modifications of mathematical programming tools, such as fuzzy (see [8]) and multi-criterion optimization (see, e.g., [11]), are considered to provide the decision maker, who bases his/her decisions on such models, with a larger variety of alternatives.

Since the fuzzy set theory was proposed by Zadeh in 1965 ([13]), we have been able to handle vague or fuzzy data to real world applications. The concept of fuzzy set theory has been found extensive applications in various fields. Originally, Bellman and Zadeh [2] described the role of fuzzy sets in decision processes. A few years after, the original statements of Bellman and Zadeh provide main motivation for the use of fuzzy set theory in several research studies, such as Zimmermann [15] who handled fuzzy linear programming with multiple objectives by assuming a continuous membership function. Soon after, his fuzzy model was developed into fuzzy multiobjective optimization models ([5, 9]). In particular, several fuzzy network flow problems with multiple objective functions were later proposed in the literature (For a more detailed discussion of such problems, the reader is referred to, for instance, ([4]-[14]) and references therein).

In this note, we deal with the following problem which arises quite naturally in the real world and so we will restrict our attention to formulating a new network flow model and then, try to solve it based on the Bellman and Zadeh's principle ([2]): Suppose that we wish to transport some special commodities, as banana, from one city to another city through a given road network. These fruits are picked unripe and continuation of the ripening process is done until reaching consumers. As different fruits need different times for ripening, and their quality and taste are changed over time and the ripening process be done during the shipping pattern from origin to destination, so besides the total traversing cost, the quality of delivered fruits is important. On the other hand, sometimes, we need to store the fruits at some intermediate node for improving the quality, which has additional storage costs. Therefore, we deal with a bi-objective problem, which is to determine a path from source to sink, so that its transportation cost is minimized and the quality of the transported fruits is maximized, as much as possible. In other words, the problem concentrates on two criteria: total transportation cost and the total traversing time, which must be close to a given time interval, as far as possible. We call it *the best shipping pattern problem with intermediate storage*. It is assumed that both the traversing and the storage costs of arcs in the network are crisp values, and a fuzzy trapezoidal number ([8]) is used to describe the dependence of the quality on both traversing and storage times. As will be noted in the next sections, this fuzzy number, as the quality of delivered commodities, is the main contribution of the new model. On the other hand, it is our motivation to use the Bellman-Zadeh's principle of decision making in the fuzzy environment to be able to solve the new model for identifying the fuzzy decision as the best path of a given road network discussed earlier. To this end, we shall first fuzzify the cost objective function and then use the Bellman-Zadeh's max-min criterion ([2]), to reformulate the problem as a mixed integer linear programming problem.

The rest of the paper is organized as follows: In the next Section, some basic concepts on fuzzy sets are reviewed. We describe and formulate a new best shipping pattern problem with intermediate storage, as a bi-objective optimization model in third Section. Subsequently, this new problem is reformulated as a crisp single criterion mixed integer problem, based on the fuzzy max-min criterion. The fourth Section of the paper tries to explain the details of the proposed approach by solving a numerical example. Finally, the last section of the paper submits our concluding observations and further directions.

2. Preliminary

In this section, some basic definitions from fuzzy set theory are reviewed ([8]).

Definition 2.1 If X is a collection of objects denoted generically by x, then a *fuzzy* set $\tilde{A}$ in X is a set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\}$$

$\mu_{\tilde{A}}(x)$ is called the *membership function* or *grade of membership* of x in $\tilde{A}$ that maps X to the unit interval $[0,1]$. The *support* of a fuzzy set $\tilde{A}$ is the crisp set of all $x \in X$ such that $\mu_{\tilde{A}} > 0$, and is denote by $supp(\tilde{A})$.

Definition 2.2 A fuzzy subset $\tilde{A}$ of $\mathbb{R}$ with membership mapping $\mu : \mathbb{R} \to [0,1]$ is called *fuzzy number* if its support is an interval $[a,b]$ and there exist real numbers s,t with $a \leqslant s \leqslant t \leqslant b$ and such that:

(1) $\mu(x) = 1 \;\; with \; s \leqslant x \leqslant t$
(2) $\mu(x) \leqslant \mu(y) \;\; with \; a \leqslant x \leqslant y \leqslant s$
(3) $\mu(x) \geqslant \mu(y) \;\; with \; t \leqslant x \leqslant y \leqslant b$
(4) $\mu(x)$ is upper semi-continuous.

3. A new problem

Let $G = (N, A)$ be a directed network defined by a set N of n nodes and a set A of m directed arcs ([1]). Each arc $(i,j) \in A$ has an associated cost c_{ij} and time t_{ij}, as crisp numbers, that denote the cost and time per unit flow on that arc, respectively. Network G has two distinguished nodes O and D, called Origin and Destination, respectively. A directed path P from node i to node j in G is an alternating sequence of nodes and arcs started at i and terminated in j. To simplify the notation, we represent a path only by its nodes. Every path has its own transit cost and time; the transit cost and time of a directed path is defined as the sum of costs and times of its constituent arcs, respectively. As mentioned earlier, we wish to transport some especial commodities, which their quality changes over time, from the origin to the destination. For practical purposes, the fruits are possibly stored at some intermediate node k with storage time of τ_k (note that τ_k is a variable which must be determined) and $\bar{c_k}$ as the cost of storage per unit time, which are also crisp numbers. To tackle this issue, we use a fuzzy trapezoidal number ([8]) $\tilde{T} = \langle l, u, v, L \rangle$, as defined below, to illustrate the quality of a commodity based on the total transit time.

$$\mu(T_P) = \begin{cases} \frac{T_P - l}{u - l}, & l \leqslant T_P \leqslant u; \\ 1, & u \leqslant T_P \leqslant v; \\ \frac{L - T_P}{L - v}, & v \leqslant T_P \leqslant L; \\ 0, & Otherwise. \end{cases} \tag{1}$$

To interpret the role of $\mu(T_P)$, we remind the example of transporting especial

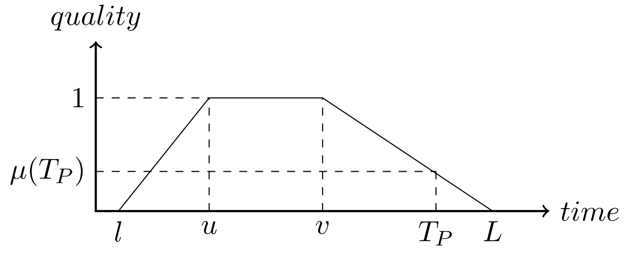

Figure 1.: Membership function of $\tilde{T}$

fruits as banana. These types of fruits are picked unripe, and the continuation of the ripening process is done until reaching consumers. The quality of transported fruits changes continually with time, this quality is monotonically increasing from l to u, is constant, as the highest level, between u and v, and is monotonically decreasing from v to L.

As shown in 1 and Figure 1, if the total time of a path P is $T_P = \sum_{(i,j)\in P} t_{ij} + \sum_{k\in P} \tau_k$, then $\mu(T_P)$, as the membership degree of T_P, shows the quality of the delivered commodity, sent through P. Therefore, in order to maximizing the quality of delivered commodity, T_P must be close to the interval $[u, v]$, as far as possible. On the other hand, minimizing the total traversing cost is an important objective, which should be included in the analysis of the model. Using these notations we can now formulate the following *best shipping pattern problem with intermediate storage*:

$$\begin{aligned}
&\min \quad C(x,\tau) = \sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n} \bar{c}_k \tau_k \\
&\max \quad \mu(T(x,\tau)) = \mu\Big(\sum_{(i,j)\in A} t_{ij}x_{ij} + \sum_{k=1}^{n} \tau_k\Big) \\
&\text{s.t.} \\
&\qquad \sum_{(i,j)\in A} x_{ij} + \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i = O; \\ 0, & i \neq O, D;\ i \in N \\ -1, & i = D. \end{cases} \\
&\qquad 0 \leqslant \tau_k \leqslant M \sum_{k=1}^{n} x_{ik} \\
&\qquad x_{ij} \in \{0,1\}, \qquad\qquad \forall i,j \in N.
\end{aligned} \tag{2}$$

where, the binary variable x_{ij} is one if arc (i, j) belongs to the selected path and is zero otherwise. For some sufficiently large constant M, the second constraint of Model 2 implies that if node k does not belong to the selected path, then its storage time is equal to zero, i.e., $\tau_k = 0$. For the sake of convenience, let us define the set X as the set of feasible solutions to the problem 2, so we can define a feasible solution $(x, \tau) \in X$ as the solution $(\ldots,\ x_{ij}, \ldots,\ \ldots,\ \tau_k, \ldots)$.

4. Problem solving

The presented problem in the previous section is a bi-objective optimization model; for these types of models, instead of the optimal solution, the concept of efficient solution is defined ([5, 11]). Following definitions introduced this concept.

Definition 4.1 Let $(x^*, \tau^*) \in X$ be a feasible solution of the model 2. (x^*, τ^*) is called *weakly efficient*, if there is no other feasible solution $(x, \tau) \in X$ such that $C(x, \tau) < C(x^*, \tau^*)$ and $\mu(T(x, \tau)) > \mu(T(x^*, \tau^*))$.

Definition 4.2 Let $(x^*, \tau^*) \in X$ be a feasible solution of the model 2. (x^*, τ^*) is called *efficient*, if there is no other feasible solution $(x, \tau) \in X$ such that $C(x, \tau) \leqslant C(x^*, \tau^*)$ and $\mu(T(x, \tau)) \geqslant \mu(T(x^*, \tau^*))$, with at least one strict inequality. If (x^*, τ^*) is efficient, then the vector $(C(x^*, \tau^*), T(x^*, \tau^*))$ is called *non-dominated* point.

It should be noted that the efficient solution is not generally unique (see [11]), and from the geometrical point of view, efficient solutions are points of the feasible space, but non-dominated points, as the image of the efficient solutions, are located in the objectives space.

Now, we try to reformulate the problem 2 as a single objective optimization problem, and solve it to find an efficient solution. Toward this end, at first we fuzzify the cost objective function ([8]). Let $\tilde{C}$ denote the fuzzy form of the total traversing cost, we define the membership function of $\tilde{C}$ as a linear monotonically decreasing function that shown in 3 and Fig.2.

$$\pi(C(x,\tau)) = \pi(\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n} \bar{c_k}\tau_k) = \begin{cases} 1, & C(x,\tau) \leqslant Z_0; \\ \frac{Z_1 - C(x,\tau)}{Z_1 - Z_0}, & Z_0 < C(x,\tau) < Z_1; \\ 0, & Otherwise. \end{cases} \tag{3}$$

Where, Z_0 and Z_1, as constant numbers, are the lower and upper bounds of $\tilde{C}$, respectively. These bounds are subjectively dependent on the decision makers suggestion; nevertheless, we can use the following logical values, as alternative choices.

$$Z_0 = \min_{(x,\tau)\in X}(\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n} \bar{c_k}\tau_k) \quad \text{and} \quad Z_1 = \max_{(x,\tau)\in X}(\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n} \bar{c_k}\tau_k) \tag{4}$$

Using the above notations and the fuzzy max-min criterion ([5, 8]), suggested by

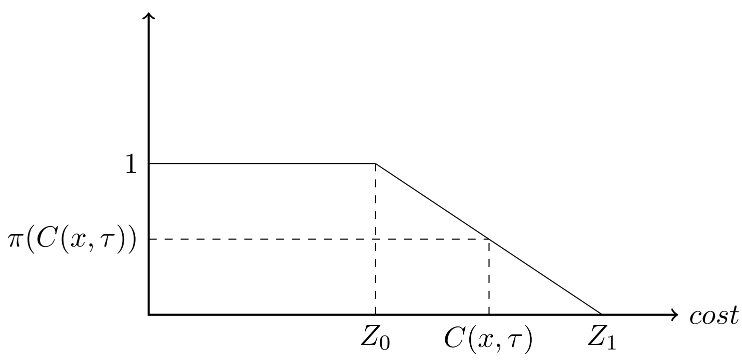

Figure 2.: Membership function of $\tilde{C}$

Bellman and Zadeh [2], we can modify the model 2 as the following model.

$$\max_{(x,\tau)\in X} \min \{\pi(C(x,\tau)), \mu(T(x,\tau))\} \tag{5}$$

or

$$
\begin{aligned}
&\max\ \min\left\{\pi\Big(\sum_{(i,j)\in A} c_{ij}x_{ij}+\sum_{k=1}^{n}\bar{c_k}\tau_k\Big),\mu\Big(T\Big(\sum_{(i,j)\in A} t_{ij}x_{ij}+\sum_{k=1}^{n}\tau_k\Big)\Big)\right\}\\
&\text{s.t.}\\
&\qquad \sum_{(i,j)\in A} x_{ij}+\sum_{(j,i)\in A} x_{ji}=\begin{cases}1, & i=O;\\ 0, & i\neq O,D;\ i\in N\\ -1, & i=D.\end{cases} \qquad . \qquad (6)\\
&\qquad 0\leqslant \tau_k\leqslant M\sum_{k=1}^{n} x_{ik}\\
&\qquad x_{ij}\in\{0,1\}, \qquad \forall i,j\in N.
\end{aligned}
$$

By setting $\lambda = \min\left\{\pi\Big(\sum_{(i,j)\in A} c_{ij}x_{ij}+\sum_{k=1}^{n}\bar{c_k}\tau_k\Big),\mu\Big(T\Big(\sum_{(i,j)\in A} t_{ij}x_{ij}+\sum_{k=1}^{n}\tau_k\Big)\Big)\right\}$, problem 6 can be reformulated as:

$$
\begin{aligned}
&\max \quad \lambda\\
&\text{s.t.} \quad \lambda\leqslant \frac{Z_1 - C\sum_{(i,j)\in A} c_{ij}x_{ij}-\sum_{k=1}^{n}\bar{c_k}\tau_k}{Z_1-Z_0} && \text{(a)}\\
&\qquad \lambda\leqslant \frac{\sum_{(i,j)\in A} t_{ij}x_{ij}+\sum_{k=1}^{n}\tau_k - l}{u-l} && \text{(b)}\\
&\qquad \lambda\leqslant \frac{L-\sum_{(i,j)\in A} t_{ij}x_{ij}-\sum_{k=1}^{n}\tau_k}{L-v} && \text{(c)}\\
&\qquad \sum_{(i,j)\in A} x_{ij}+\sum_{(j,i)\in A} x_{ji}=\begin{cases}1, & i=O;\\ 0, & i\neq O,D;\ i\in N\\ -1, & i=D.\end{cases}\\
&\qquad 0\leqslant \tau_k\leqslant M\sum_{k=1}^{n} x_{ik}\\
&\qquad x_{ij}\in\{0,1\}, \qquad (i,j)\in A\\
&\qquad \lambda\geqslant 0 && (7)
\end{aligned}
$$

Constraint (7 (a)) is related to the membership function of cost and the constraints (7 (b)) and (7 (c)) are related to the membership function of time, which indicate lower bounds for the quality of delivered commodity. Each of these three constraints implies that $\lambda\leqslant 1$. It is easy to check that the model 7 can be simplified as follows:

$$
\begin{aligned}
\max \quad & \lambda \\
\text{s.t.} \quad & (Z_1 - Z_0)\lambda + \sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n} \bar{c_k}\tau_k \leqslant Z_1 && \text{(a)} \\
& (u-l)\lambda - \sum_{(i,j)\in A} t_{ij}x_{ij} - \sum_{k=1}^{n} \tau_k \leqslant -l && \text{(b)} \\
& (L-v)\lambda + \sum_{(i,j)\in A} t_{ij}x_{ij} + \sum_{k=1}^{n} \tau_k \leqslant L && \text{(c)} \\
& \sum_{(i,j)\in A} x_{ij} + \sum_{(j,i)\in A} x_{ji} = \begin{cases} 1, & i = O; \\ 0, & i \neq O, D;\ i \in N \\ -1, & i = D. \end{cases} \\
& 0 \leqslant \tau_k \leqslant M\sum_{k=1}^{n} x_{ik} \\
& x_{ij} \in \{0,1\}, \qquad (i,j) \in A \\
& \lambda \geqslant 0 && (8)
\end{aligned}
$$

It is obvious that Model 8 is a mixed integer linear programming problem and can be solved using the well-known methods [3]. Following theorem shows that the optimal solution of 8 is associated with an efficient solution of the bi-objective problem 2.

Theorem 4.1 If (λ^*, x^*, τ^*) is an optimal solution of the model 8, then (x^*, τ^*) is an efficient solution of the model 2.

Proof. Since the model 8 is a reformulated form of the model 5, we prove the theorem using the model 5. Now let (λ^*, x^*, τ^*) be an optimal solution of 8, so $\lambda^* = \min\{\pi(C(x^*, \tau^*)), \mu(T(x^*, \tau^*))\}$. If (x^*, τ^*) is not efficient, so based on Definition 4.2, there is another feasible solution as $(\bar{x}, \bar{\tau})$ such that, one of the following cases occurs:

(a) $C(\bar{x}, \bar{\tau}) < C(x^*, \tau^*)$ *and* $\mu(T(\bar{x}, \bar{\tau})) \geqslant \mu(T(x^*, \tau^*))$;
(b) $C(\bar{x}, \bar{\tau}) \leqslant C(x^*, \tau^*)$ *and* $\mu(T(\bar{x}, \bar{\tau})) > \mu(T(x^*, \tau^*))$.

If case (a) is true, then

$$\pi(C(x^*, \tau^*)) = \tfrac{Z_1 - C(x^*, \tau^*)}{Z_1 - Z_0} < \pi(C(\bar{x}, \bar{\tau})) = \tfrac{Z_1 - C(\bar{x}, \bar{\tau})}{Z_1 - Z_0}$$

and so

$$\lambda^* = \min\{\pi(C(x^*, \tau^*)), \mu(T(x^*, \tau^*))\} \leqslant \min\{\pi(C(\bar{x}, \bar{\tau}), \mu(T(\bar{x}, \bar{\tau}))\}.$$

If $\lambda^* = \pi(C(x^*, \tau^*))$ then we will have $\lambda^* < \min\{\pi(C(\bar{x}, \bar{\tau}), \mu(T(\bar{x}, \bar{\tau}))\}$, which contradicts the optimality of λ^*. If $\lambda^* = \mu(T(x^*, \tau^*)) < \mu(T(\bar{x}, \bar{\tau}))$, again we have $\lambda^* < \min\{\pi(C(\bar{x}, \bar{\tau}), \mu(T(\bar{x}, \bar{\tau}))\}$, which is a contradiction. Finally, if $\lambda^* =$

$\mu(T(x^*,\tau^*)) = \mu(T(\bar{x},\bar{\tau}))$ then (x^*,τ^*) is a weakly efficient solution.

Case (b) can be deduced as a similar manner. □

5. Numerical example

In this section, in order to show how the proposed approach can be used to compute a best shipping pattern, we provide a numerical illustrative example. For the sake of exposition, we solve a small size problem and present the computational results (One can use $CPLEX$ solver of the $GAMS$ Software (available at $http://gams.com/$) to solve it).

Consider the bi-objective *best shipping pattern problem*, defined on a network having 8 nodes and 15 arcs, as depicted in Fig.3. We assume that nodes 1 and 8 are origin and destination nodes, respectively, and the given fuzzy trapezoidal number $\widetilde{T} = \langle l,u,v,L\rangle$ is $\langle 45, 59, 63, 70\rangle$. Also, by solving the problems of 4, it is easy to show that:

$$Z_0 = \min_{(x,\tau)\in X}\Big(\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n}\bar{c}_k\tau_k\Big) = 23$$

and

$$Z_1 = \max_{(x,\tau)\in X}\Big(\sum_{(i,j)\in A} c_{ij}x_{ij} + \sum_{k=1}^{n}\bar{c}_k\tau_k\Big) = 235$$

Using the $CPLEX$ solver of the $GAMS$ software, the optimal solution of the problem 8 is as follows:

$$\lambda^* = 0.745$$
$$x^*_{12} = x^*_{24} = x^*_{46} = x^*_{67} = x^*_{78} = 1$$
$$\tau^*_1 = \tau^*_7 = 9,\ \tau^*_2 = 2$$

and all the other variables equal to zero.

$$C(x^*,\tau^*) = 77, \quad T(x^*,\tau^*) = 56.$$

Clearly, $\mathbf{P = 1-2-4-6-7-8}$ is the best path and the quality level of the delivered commodity on this path is $\mu(T(x^*,\tau^*) = \frac{56-45}{59-45} = 0.786$. Also, the membership degree of traversing cost is $\pi(C(x^*,\tau^*)) = \frac{235-77}{235-23} = 0.745$.

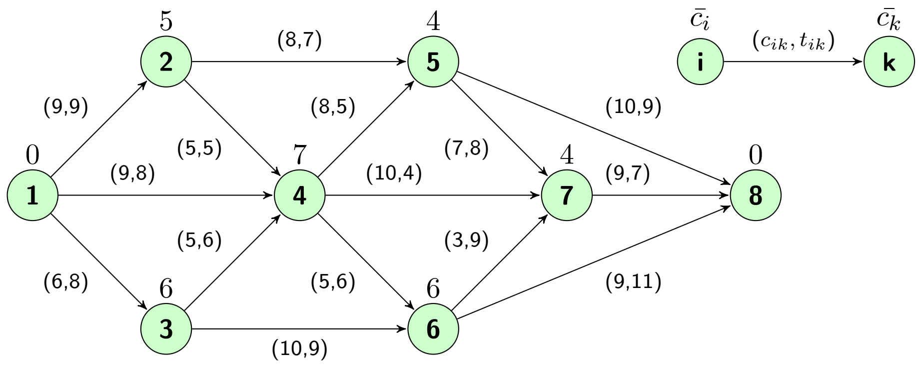

Figure 3.: Network of the numerical example

6. Conclusion

In this paper a best shipping pattern problem with intermediate storage, on a network with crisp arc costs, crisp arc times and the total traversing time as a fuzzy trapezoidal number, is formulated and solved. The main contribution of this model is an actual interpretation of the given fuzzy trapezoidal number, as the quality of delivered commodities. Since the presented problem has a fuzzy structure, the Bellman and Zadeh's max-min criterion, has been used to formulate the problem as a crisp single objective model, which is easily solvable.

Authors believe that the extension of the proposed model for the networks, with fuzzy arc costs and times, will be an important future research issue, and also the ability to present a labeling technique will be an important future research issue.

Acknowledgment

This study is supported under research project entitled "A searching best path problem with imprecise total time" by Islamic Azad University, Masjed Soleiman Branch. The first author is grateful for this financial support.

References

[1] R. K. Ahuja, T. L. Magnanti, J. B. Orlin, *Network Flows*, Prentice-Hall, Englewood Cliffs, NJ, (1993).
[2] R. E. Bellman, L. A. Zadeh, *Decision-making in a fuzzy environment*, Manage. Sci. **17B** (1970), 141–164.
[3] D. Bertsimas, J. N. Tsitsiklis, *Introduction to Linear Optimization*, Athena Scientific, Belmont, Massachusetts (1997).
[4] A. K. Bit, M. P. Biswal & S. S. Alam, *An additive fuzzy programming model for multi objective transportation problem*, Fuzzy Set Syst. **57** (1993) 313-319.
[5] C. Kahraman, *Fuzzy Multi-Criteria Decision Making: Theory and Applications with Recent Developments*, Springer, (2008).
[6] M. Kaur, A. Kumar, *Optimal compromise solution of multi-objective minimal cost flow problems in fuzzy environment*, Appl. Math. Model. **37** (2013) 1677-1693.

[7] E. Keshavarz, E. Khorram, *A fuzzy bi-criteria transportation problem*, Comput. Ind. Eng. **61** (2011) 947-957.
[8] G. J. Klir, Bo Yuan, *Fuzzy sets and fuzzy logic (Theory and applications)*, Prentice Hall, (1995).
[9] Y.-Y. Lai and C.-L. Hwang, *Fuzzy Multiple objective Decision Making: Methods and Applications*, Springer-Verlag, Berlin, (1996).
[12] S. Okada, T. Soper, *A shortest path problem on a network with fuzzy arc lengths*, Fuzzy Set Syst. **109** (2000) 129–140.
[11] R. E. Steuer, *Multiple Criteria Optimization: Theory, Computation, and Application*, John Wiley & Sons, New York, (1986).
[12] P. Tapkan, L. Ozbakr, A. Baykasoglu, *Solving fuzzy multiple objective generalized assignment problems directly via bees algorithm and fuzzy ranking*, Expert Syst. Appl. **40** (2013) 892-898.
[13] L. A. Zadeh, *Fuzzy sets*, Inform. Cont. **8** (1965), 338-353.
[14] F. A. Zammori, M. Braglia, M. Frosolini, *A fuzzy multi-criteria approach for critical path definition*, INT. J. Proj. Manage. **27** (2009) 278-291.
[15] H. J. Zimmermann, *Fuzzy programming and linear programming with several objective functions*, Fuzzy Set Syst. **1** (1978) 45-55.