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Analysis of a Discrete-Time Impatient Customer Queue with Bernoulli-Schedule Vacation Interruption

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Abstract. This paper investigates a discrete-time impatient customer queue with Bernoulli-schedule vacation interruption. The vacation times and the service times during regular busy period and during working vacation period are assumed to follow geometric distribution. We obtain the steady-state probabilities at arbitrary and outside observer's observation epochs using recursive technique. Cost analysis is carried out using particle swarm optimization. Computational experiences with a variety of numerical results are discussed.

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1. Introduction

The interest in discrete-time queues has experienced spectacular growth with the arrival of the digital technologies. A fundamental motive for studying discrete-time queues is that they became more appropriate than their continuous-time counterparts for analyzing computer and telecommunication systems. Performance modeling of queueing systems with impatient customers has attracted many researchers owing to their wide applications in real life congestion problems. Balking and reneging are two such impatient phenomena in queues; as a consequence, customers either decide not to join the queue or depart after joining the queue without getting service due to impatience. The lost revenues due to balking and reneging in various

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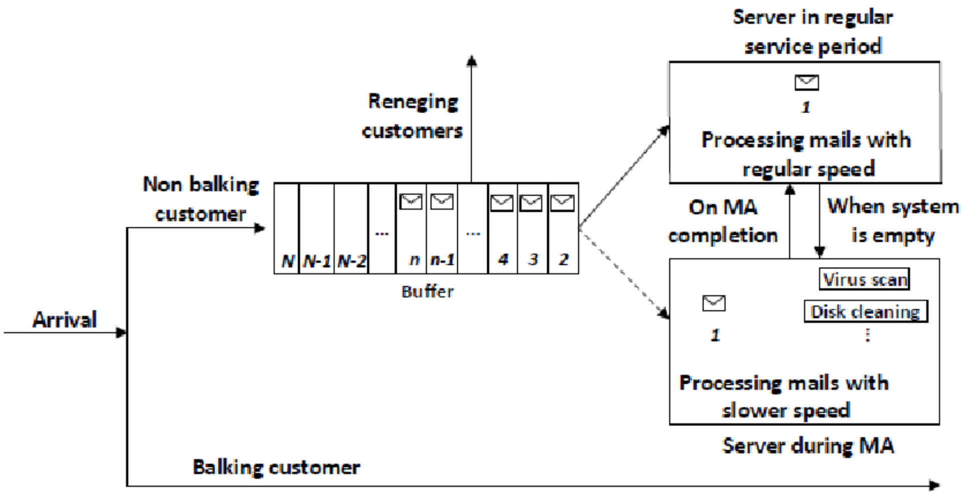

Figure 1. Inbound email contact center

industries can be enormous. A discrete-time single server queue with balking has been examined by [3]. [1] analyzed a discrete-time multi-server queue with balking.

Queueing systems with working vacation (WV) have been studied extensively due to their wide applications in several areas including computer and communication systems, manufacturing and production systems. In WV queues, it is assumed that the server remains active during the vacation period and serves the customers generally at a slower rate. At the end of a WV, if the queue is non-empty, a regular service period begins; otherwise, the server takes another WV. This policy is called multiple working vacation (MWV). It was introduced by Servi and Finn [5] in an $M/M/1$ queueing system. [6] considered a $Geo/Geo/1$ queue with MWV. [7] analyzed an $M/M/1/N$ queueing system with balking, reneging and WV. A discrete-time WV queue with balking and reneging has been studied in [2].

At a service completion instant during WV period, if there are customers in the system, the server can interrupt the vacation. This is known as WV with vacation interruption (VI). Under the Bernoulli-schedule VI, the server may continue the vacation with probability q or interrupt the vacation and resume regular service period with probability $1-q$, if there are customers in the system. [9] first studied an $M/M/1$ queue with VI under the Bernoulli rule. An impatient customer queue with Bernoulli-schedule VI has been analyzed in [8].

This paper focusses on a discrete-time balking and reneging queue with Bernoulli-schedule VI. The model has potential applications in practical systems, for example an inbound email contact center wherein the potential customers (users in different offices, research centers, etc.) transmit emails across the network through an office automation system such as LAN. A picture depicting the email handling scheme is shown in Figure 1. Many communication networks are organized with complete synchronization, as a result of which channel requests, grants and data transmissions and receptions all proceed in predetermined fixed time intervals. As an email contact center is one such communication network, email sending, preprocessing and processing of requests are done in discrete slots. The emails received are processed immediately when the server is idle. If the server is busy, the received emails are placed in a queue. When the server is busy, there is a probability that requests may be terminated by users before arriving at the email server and if an email is not processed within a certain duration it is lost. To keep the functioning of the email server well and efficient, maintenance activities (MAs) such as virus scans, disk cleaning, etc., can be done when the server is idle. MAs consume some system resources and reduce the processing speed, but the server can still process emails waiting in the queue (represented by dotted line in Figure 1) at a lower speed. After processing an email during the MAs, if there are emails waiting to be processed, the server may interrupt or continue the MAs with some probability. On the other hand, when the MAs are completed and emails are waiting in the queue, they are processed with regular speed. Moreover, the number of users connected to the automation system server in different offices do not exceed a limited number. In this scenario, the number of users, the requests terminated by users, lost emails, the email server and the MAs correspond to finite buffer, balking, reneging, server and WV with Bernoulli-schedule VI, respectively.

This paper presents the analysis of a discrete-time balking and reneging queue with Bernoulli-schedule vacation interruption. The inter-arrival times of customers and service times are assumed to be independent and geometrically distributed. The service times during a WV period and vacation times are also assumed to be geometrically distributed. The arriving customers may decide either to balk with a certain probability or renege according to geometric distribution. The steady-state probabilities are obtained through recursive technique which is easy to implement. Various performance measures of the model such as the expected queue length, average balking rate, average reneging rate, etc., are presented. Further, a cost model is formulated to determine the optimum service rate during regular busy period using particle swarm optimization. Finally, the parameter effect on the performance measures of the model is demonstrated through some numerical results.

The rest of the paper is organized as follows. Section 2 presents the description of the model. The steady-state probabilities at arbitrary and outside observer's observation epochs are obtained in Section 3. Section 4 presents various performance measures and cost model. Numerical results in the form of a table and graphs are discussed in Section 5 followed by conclusions in Section 6.

2. Model Description

We consider a finite buffer discrete-time single server queue with balking, reneging and Bernoulli-schedule vacation interruption under the early arrival system (EAS). Assume that the time axis is slotted into intervals of equal length with the length of a slot being unity and it is marked as 0,1,2,..., t,... The potential arrivals occur in $(t, t+)$ and a potential departure takes place in $(t-, t)$. We assume that the capacity of the system is N.

The inter-arrival times A of customers are independent and geometrically distributed with probability mass function (p.m.f.) $a_n = Pr(A = n) = \bar{\lambda}^{n-1}\lambda, n \geq 1, 0 < \lambda < 1$, where for any real number $x \in [0, 1], \bar{x} = 1 - x$. On arrival, if a customer finds i other customers in the system then he either decides to join the queue with a probability b_i or balks with a probability $\bar{b}_i = 1 - b_i$. Furthermore, we assume that $0 < b_{i+1} \leq b_i \leq 1, 1 \leq i \leq N-1, b_0 = 1$ and $b_N = 0$. After joining the queue each customer will wait a certain length of time, say T, for service to begin before they get impatient and leave the queue without receiving service. This time T is assumed to follow geometric distribution with parameter α. The average reneging rate of a customer is given by $(i-1)\alpha, 1 \leq i \leq N$.

The server is allowed to take WV whenever the system becomes empty. During WV the server renders service at a different rate. At the instants of a service completion during WV, the server may continue the vacation with probability q or interrupt the vacation and resume regular service period with probability $\bar{q}$ (if there is at least one customer in the queue). On the other hand, on return from a WV if the system is non-empty, it switches to regular busy period; otherwise another WV commences. The regular service times, service times during WV and vacation times are assumed to be independent and geometrically distributed with parameters μ, η and ϕ, respectively. The customers are served according to first-come first-served (FCFS) service discipline.

3. Analysis of the Model

In this section, we present the analytic analysis of the model. At steady-state, let $\pi_{i,0}(0 \le i \le N)$ represent the probability of i customers in the system and the server in WV and $\pi_{i,1}(1 \le i \le N)$ be the probability of i customers in the system and the server in regular busy period. To obtain the system length distribution at steady-state, we first develop the following system of difference equations:

$$
\begin{aligned}
\pi_{0,0} &= \left(\bar{\lambda} + \lambda\eta\right)\pi_{0,0} + \mathbf{s}_1(\eta)\pi_{1,0} + \mathbf{t}_2(\eta)\pi_{2,0} + \mathbf{s}_1(\mu)\pi_{1,1} + \mathbf{t_2}(\mu)\pi_{2,1}, &(1)\\
\pi_{1,0} &= \bar{\phi}\mathbf{u}_1(\eta)\pi_{1,0} + \bar{\phi}\lambda\bar{\eta}\pi_{0,0} + \bar{\phi}\mathbf{w}_2(\eta)\pi_{2,0} + \bar{\phi}\mathbf{f}_3(\eta)\pi_{3,0}, &(2)\\
\pi_{i,0} &= \bar{\phi}\mathbf{u}_i(\eta)\pi_{i,0} + \bar{\phi}\mathbf{m}_{i-1}(\eta)\pi_{i-1,0} + \bar{\phi}\mathbf{w}_{i+1}(\eta)\pi_{i+1,0} + \bar{\phi}\mathbf{f}_{i+2}(\eta)\pi_{i+2,0},\\
&\quad 2 \le i \le N-2, &(3)\\
\pi_{N-1,0} &= \bar{\phi}\mathbf{u}_{N-1}(\eta)\pi_{N-1,0} + \bar{\phi}\mathbf{m}_{N-2}(\eta)\pi_{N-2,0} + \bar{\phi}\mathbf{w}_N(\eta)\pi_{N,0}, &(4)\\
\pi_{N,0} &= \bar{\phi}\mathbf{u}_N(\eta)\pi_{N,0} + \bar{\phi}\mathbf{m}_{N-1}(\eta)\pi_{N-1,0}, &(5)\\
\pi_{1,1} &= \mathbf{r}_1(\mu)\pi_{1,1} + \mathbf{s}_2(\mu)\pi_{2,1} + \mathbf{t}_3(\mu)\pi_{3,1} + \phi\mathbf{r}_1(\eta)\pi_{1,0} + \phi\lambda\bar{\eta}\pi_{0,0} + \phi\mathbf{s}_2(\eta)\pi_{2,0}\\
&\quad +\phi\mathbf{t}_3(\eta)\pi_{3,0} + \bar{\phi}\mathbf{v}_1(\eta)\pi_{1,0} + \bar{\phi}\mathbf{z}_2(\eta)\pi_{2,0} + \bar{\phi}\mathbf{g}_3(\eta)\pi_{3,0},, &(6)\\
\pi_{i,1} &= \mathbf{r}_i(\mu)\pi_{i,1} + \mathbf{s}_{i+1}(\mu)\pi_{i+1,1} + \mathbf{m}_{i-1}(\mu)\pi_{i-1,1} + \mathbf{t}_{i+2}(\mu)\pi_{i+2,1} + \phi\mathbf{r}_i(\eta)\pi_{i,0}\\
&\quad +\phi\mathbf{m}_{i-1}(\eta)\pi_{i-1,0} + \phi\mathbf{s}_{i+1}(\eta)\pi_{i+1,0} + \phi\mathbf{t}_{i+2}(\eta)\pi_{i+2,0} + \bar{\phi}\mathbf{v}_i(\eta)\pi_{i,0}\\
&\quad +\bar{\phi}\mathbf{z}_{i+1}(\eta)\pi_{i+1,0} + \bar{\phi}\mathbf{g}_{i+2}(\eta)\pi_{i+2,0}, 2 \le i \le N-2, &(7)\\
\pi_{N-1,1} &= \mathbf{r}_{N-1}(\mu)\pi_{N-1,1} + \mathbf{s}_N(\mu)\pi_{N,1} + \mathbf{m}_{N-2}(\mu)\pi_{N-2,1} + \phi\mathbf{r}_{N-1}(\eta)\pi_{N-1,0}\\
&\quad +\phi\mathbf{m}_{N-2}(\eta)\pi_{N-2,0} + \phi\mathbf{s}_N(\eta)\pi_{N,0} + \bar{\phi}\mathbf{v}_{N-1}(\eta)\pi_{N-1,0} + \bar{\phi}\mathbf{z}_N(\eta)\pi_{N,0}, &(8)\\
\pi_{N,1} &= \mathbf{r}_N(\mu)\pi_{N,1} + \mathbf{m}_{N-1}(\mu)\pi_{N-1,1} + \phi\mathbf{r}_N(\eta)\pi_{N,0} + \phi\mathbf{m}_{N-1}(\eta)\pi_{N-1,0}, &(9)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{u}_i(x) &= \bar{\lambda}\bar{x}\overline{(i-1)\alpha} + \lambda\bar{b}_i\bar{x}\overline{(i-1)\alpha} + \lambda b_i q x\overline{(i-1)\alpha} + \lambda b_i\bar{x}(i-1)\alpha,\ i=1,\ldots,N,\\
\mathbf{v}_i(x) &= \lambda b_i\bar{q}x\overline{(i-1)\alpha},\ i=1,\ldots,N,\\
\mathbf{r}_i(x) &= \mathbf{u}_i(x)+\mathbf{v}_i(x),\ i=1,\ldots,N,\\
\mathbf{w}_i(x) &= \bar{\lambda}qx\overline{(i-1)\alpha} + \lambda\bar{b}_i qx\overline{(i-1)\alpha} + \bar{\lambda}\bar{x}(i-1)\alpha + \lambda\bar{b}_i\bar{x}(i-1)\alpha + \lambda b_i qx(i-1)\alpha,\\
&\quad i=1,\ldots,N-1,\\
\mathbf{z}_i(x) &= \bar{\lambda}\bar{q}x\overline{(i-1)\alpha} + \lambda\bar{b}_i\bar{q}x\overline{(i-1)\alpha} + \lambda b_i\bar{q}x(i-1)\alpha, i=1,\ldots,N-1,\\
\mathbf{s}_i(x) &= \mathbf{w}_i(x)+\mathbf{z}_i(x),\ i=1\ldots N-1,\\
\mathbf{f}_i(x) &= \lambda\bar{b}_i qx(i-1)\alpha + \bar{\lambda}qx(i-1)\alpha, i=3,\ldots,N,\\
\mathbf{g}_i(x) &= \lambda\bar{b}_i\bar{q}x(i-1)\alpha + \bar{\lambda}\bar{q}x(i-1)\alpha, i=3,\ldots,N,\\
\mathbf{t}_i(x) &= \mathbf{f}_i(x)+\mathbf{g}_i(x),\ i=3\ldots N,\\
\mathbf{m}_i(x) &= \lambda b_i\bar{x}\overline{(i-1)\alpha}, i=1,\ldots,N-1.
\end{aligned}
$$

The steady-state probabilities $\pi_{i,j}, j=0,1; j\le i\le N$ can be obtained by solving the system of equations (2) to (9) recursively as below.
Define $\xi_N=1$. From (5), we get $\pi_{N-1,0}$ in terms of $\pi_{N,0}$ as

$$\pi_{N-1,0}=\xi_{N-1}\pi_{N,0}, \tag{10}$$

where $\xi_{N-1}=\dfrac{1-\bar{\phi}\mathbf{u}_N(\eta)}{\bar{\phi}\mathbf{m}_{N-1}(\eta)}$.
Using (10) in (4), we obtain

$$\pi_{N-2,0}=\xi_{N-2}\pi_{N,0}, \tag{11}$$

where $\xi_{N-2}=\left(\dfrac{1-\bar{\phi}\mathbf{u}_{N-1}(\eta)}{\bar{\phi}\mathbf{m}_{N-2}(\eta)}\right)\xi_{N-1}-\left(\dfrac{\mathbf{w}_N(\eta)}{\mathbf{m}_{N-2}(\eta)}\right)\xi_N$.
The probabilities $\pi_{i,0}, 1\le i\le N-3$ are obtained from (3) as

$$\pi_{i,0}=\xi_i\pi_{N,0},\ i=N-3,\ldots,1, \tag{12}$$

where $\xi_i=\left(\dfrac{1-\bar{\phi}\mathbf{u}_{i+1}(\eta)}{\bar{\phi}\mathbf{m}_i(\eta)}\right)\xi_{i+1}-\left(\dfrac{\mathbf{f}_{i+3}(\eta)}{\mathbf{m}_i(\eta)}\right)\xi_{i+3}-\left(\dfrac{\mathbf{w}_{i+2}(\eta)}{\mathbf{m}_i(\eta)}\right)\xi_{i+2}$.
Equation (2) yields $\pi_{0,0}$ as

$$\pi_{0,0}=\xi_0\pi_{N,0}, \tag{13}$$

where $\xi_0=\left(\dfrac{1-\bar{\phi}\mathbf{u}_1(\eta)}{\bar{\phi}\lambda\bar{\eta}}\right)\xi_1-\left(\dfrac{\mathbf{f}_3(\eta)}{\lambda\bar{\eta}}\right)\xi_3-\left(\dfrac{\mathbf{w}_2(\eta)}{\lambda\bar{\eta}}\right)\xi_2$.
Define $\zeta_N=1$ and $\gamma_N=0$. From (9), we get $\pi_{N-1,1}$ in terms of $\pi_{N,1}$ and $\pi_{N,0}$ as

$$\pi_{N-1,1}=\zeta_{N-1}\pi_{N,1}+\gamma_{N-1}\pi_{N,0}, \tag{14}$$

where $\zeta_{N-1}=\dfrac{1-\mathbf{r}_N(\mu)}{\mathbf{m}_{N-1}(\mu)}$ and $\gamma_{N-1}=-\dfrac{\phi}{\mathbf{m}_{N-1}(\mu)}\left(\mathbf{m}_{N-1}(\eta)\xi_{N-1}+\mathbf{r}_N(\eta)\right)$.

Using (14) in (8) yields $\pi_{N-2,1}$ as

$$\pi_{N-2,1} = \zeta_{N-2}\pi_{N,1} + \gamma_{N-2}\pi_{N,0}, \tag{15}$$

where $\zeta_{N-2} = \left(\frac{1-\mathbf{r}_{N-1}(\mu)}{\mathbf{m}_{N-2}(\mu)}\right)\zeta_{N-1} - \left(\frac{\mathbf{s}_N(\mu)}{\mathbf{m}_{N-2}(\mu)}\right)$ and
$\gamma_{N-2} = \left(\frac{1-\mathbf{r}_{N-1}(\mu)}{\mathbf{m}_{N-2}(\mu)}\right)\gamma_{N-1} - \left(\frac{\phi\mathbf{m}_{N-2}(\eta)}{\mathbf{m}_{N-2}(\mu)}\right)\xi_{N-2} - \left(\frac{\phi\mathbf{r}_{N-1}(\eta)+\bar{\phi}\mathbf{v}_{N-1}(\eta)}{\mathbf{m}_{N-2}(\mu)}\right)\xi_{N-1} - \left(\frac{\phi\mathbf{s}_N(\eta)+\bar{\phi}\mathbf{z}_N(\eta)}{\mathbf{m}_{N-2}(\mu)}\right)$.

From (7), we have

$$\pi_{i,1} = \zeta_i\pi_{N,1} + \gamma_i\pi_{N,0}, i = N-3,\dots,1, \tag{16}$$

where

$$\zeta_i = \left(\frac{1-\mathbf{r}_{i+1}(\mu)}{\mathbf{m}_i(\mu)}\right)\zeta_{i+1} - \left(\frac{\mathbf{s}_{i+2}(\mu)}{\mathbf{m}_i(\mu)}\right)\zeta_{i+2} - \left(\frac{\mathbf{t}_{i+3}(\mu)}{\mathbf{m}_i(\mu)}\right)\zeta_{i+3},$$
$$\gamma_i = \left(\frac{1-\mathbf{r}_{i+1}(\mu)}{\mathbf{m}_i(\mu)}\right)\gamma_{i+1} - \left(\frac{\mathbf{s}_{i+2}(\mu)}{\mathbf{m}_i(\mu)}\right)\gamma_{i+2} - \left(\frac{\mathbf{t}_{i+3}(\mu)}{\mathbf{m}_i(\mu)}\right)\gamma_{i+3} - \left(\frac{\phi\mathbf{r}_{i+1}(\eta)+\bar{\phi}\mathbf{v}_{i+1}(\eta)}{\mathbf{m}_i(\mu)}\right)\xi_{i+1}$$
$$- \left(\frac{\phi\mathbf{m}_i(\eta)}{\mathbf{m}_i(\mu)}\right)\xi_i - \left(\frac{\phi\mathbf{s}_{i+2}(\eta)+\bar{\phi}\mathbf{z}_{i+2}(\eta)}{\mathbf{m}_i(\mu)}\right)\xi_{i+2} - \left(\frac{\phi\mathbf{t}_{i+3}(\eta)+\bar{\phi}\mathbf{g}_{i+3}(\eta)}{\mathbf{m}_i(\mu)}\right)\xi_{i+3}.$$

From (6), $\pi_{N,1}$ can be expressed in terms of $\pi_{N,0}$ as

$$\pi_{N,1} = k\pi_{N,0}, \tag{17}$$

where

$$k = \big(\mathbf{s}_2(\mu)\gamma_2 + \mathbf{t}_3(\mu)\gamma_3 + (\phi\mathbf{r}_1(\eta)+\bar{\phi}\mathbf{v}_1(\eta))\xi_1 + \phi\lambda\bar{\eta}\xi_0 + (\phi\mathbf{s}_2(\eta)+\bar{\phi}\mathbf{z}_2(\eta))\xi_2$$
$$+(\phi\mathbf{t}_3(\eta)+\bar{\phi}\mathbf{g}_3(\eta))\xi_3 - (1-\mathbf{r}_1(\mu))\gamma_1\big) / \left((1-\mathbf{r}_1(\mu))\zeta_1 - \mathbf{s}_2(\mu)\zeta_2 + \mathbf{t}_3(\mu)\zeta_3\right).$$

Using (17) in (14) to (16) yields $\pi_{i,1}(1 \le i \le N-1)$ in terms of $\pi_{N,0}$. Finally, the only unknown $\pi_{N,0}$ is obtained from the normalization condition $\sum_{i=0}^{N}\pi_{i,0} + \sum_{i=1}^{N}\pi_{i,1} = 1$ as

$$\pi_{N,0} = \left(\sum_{i=0}^{N}\xi_i + \sum_{i=1}^{N}(k\zeta_i + \gamma_i)\right)^{-1}.$$

3.1 *Outside Observer's Distribution*

In EAS, since an outside observer's observation epoch falls in a time interval after a potential arrival and before a potential departure, the probabilities $\pi_{i,j}^{o}(j = 0,1; j \le i \le N)$ that the outside observer finds i customers in the system and server in state

j are given by

$$\begin{aligned}
\pi^o_{0,0} &= \bar{\lambda}\pi_{0,0},\\
\pi^o_{i,0} &= (1-\lambda b_i)\pi_{i,0} + \lambda b_{i-1}\pi_{i-1,0},\ 1 \le i \le N-1,\\
\pi^o_{i,0} &= \pi_{N,0} + \lambda b_{N-1}\pi_{N-1,0},\\
\pi^o_{1,1} &= (1-\lambda b_1)\pi_{1,1},\\
\pi^o_{i,1} &= (1-\lambda b_i)\pi_{i,1} + \lambda b_{i-1}\pi_{i-1,1},\ 2 \le i \le N-1,\\
\pi^o_{N,1} &= \pi_{N,1} + \lambda b_{N-1}\pi_{N-1,1}.
\end{aligned}$$

This completes the evaluation of the steady-state probabilities at various epochs.

4. Performance Measures and Cost Model

Once the steady-state probabilities are obtained, one can evaluate various performance measures of the model. The average system length at an arbitrary epoch (L_s) and at an outside observer's observation epoch (L^o_s) are given by

$$L_s = \sum_{i=1}^{N} i(\pi_{i,0} + \pi_{i,1}); \quad L^o_s = \sum_{i=1}^{N} i(\pi^o_{i,0} + \pi^o_{i,1}).$$

The probability that the server is in working vacation (P_{wv}) and the probability of the server in regular busy period (P_b) are given by

$$P_{wv} = \sum_{i=0}^{N} \pi_{i,0}; \quad P_b = \sum_{i=1}^{N} \pi_{i,1}.$$

The average balking rate ($B.R.$), the average reneging rate ($R.R.$) and the average rate of loosing a customer ($L.R.$) are given as

$$B.R. = \sum_{i=1}^{N} \lambda \bar{b}_i(\pi_{i,0} + \pi_{i,1}); \quad R.R. = \sum_{i=1}^{N}(i-1)\alpha(\pi_{i,0} + \pi_{i,1}); \quad L.R. = B.R. + R.R.$$

We develop a total expected cost function with an objective to determine an optimum regular service rate (μ^*) and the optimum expected cost ($F(\mu^*)$). Let

$C_\mu \equiv$ cost per unit time during regular busy period,
$C_\eta \equiv$ cost per unit time during working vacation period,
$C_{ls} \equiv$ cost per unit time when a customer joins the queue and waits for service,
$C_{lr} \equiv$ cost per unit time when a customer balks or reneges.

Using the definitions of each cost element listed above, the total expected cost function per unit time is given by

$$F(\mu) = C_\mu \mu + C_\eta \eta + C_{ls} L^o_s + C_{lr}\, L.R.$$

We employ particle swarm optimization (PSO) to solve the above optimization problem, as the computation of the derivatives of the total expected cost fucntion is a non-trivial task. For a detailed algorithm of PSO, one may refer [4].

Table 1. Optimum values for various values of η and α

	$\alpha \rightarrow$	0.05	0.07	0.1
$\eta = 0.1$	μ^*	0.77157	0.74010	0.69875
	$F(\mu^*)$	115.069	112.442	109.106
	L_s^*	1.40803	1.37378	1.33126
	L_s^{o*}	1.80238	1.76828	1.72593
	P_{wv}^*	0.58795	0.58511	0.58154
	P_b^*	0.41205	0.41489	0.41846
	$L.R.^*$	0.03994	0.05101	0.06594
$\eta = 0.3$	μ^*	0.77816	0.75500	0.72274
	$F(\mu^*)$	97.8809	96.6992	95.1007
	L_s^*	0.91357	0.90582	0.89606
	L_s^{o*}	1.30991	1.30220	1.29248
	P_{wv}^*	0.67398	0.67135	0.66761
	P_b^*	0.32603	0.32865	0.33239
	$L.R.^*$	0.02057	0.02667	0.03532

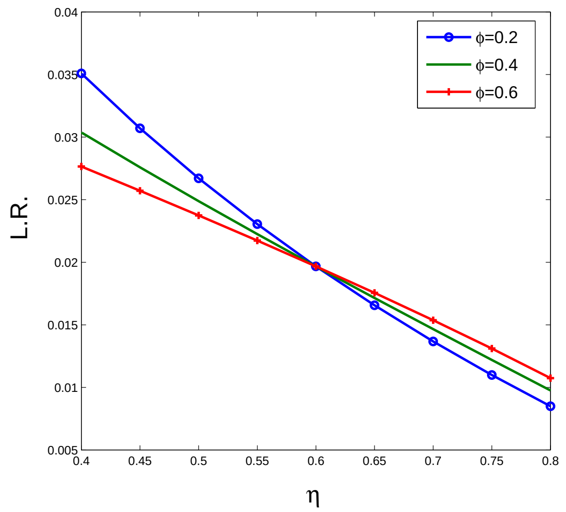

Figure 2. Effect of η on $L.R.$

5. Numerical Results

To demonstrate the applicability of the theoretical investigations made in the previous sections, we present some numerical results. The capacity of the system is fixed as $N = 10$. The balking function is taken as $b_i = 1 - (i/N^2), 1 \leq i \leq N-1$ and $b_0 = 1, b_N = 0$. The cost elements are taken to be $C_{ls} = 45, C_\mu = 40, C_\eta = 25$ and $C_{lr} = 15$. The parameters of the model are chosen to be $\lambda = 0.4, \mu = 0.6, \eta = 0.3, \phi = 0.2, q = 0.4$ and $\alpha = 0.1$, unless they are considered as variables or their values are mentioned in the respective table and graphs.

Table 1 presents the optimum values of μ, the minimum expected cost $F(\mu^*)$, using PSO, along with the corresponding performance measures $L_s^*, L_s^{o*}, P_{wv}^*, P_b^*$ and $L.R.^*$ for various values of η and α. From the table, we observe that except P_b^* and $L.R.^*$ all other optimum values decrease with the increase of α for fixed η. Further, for fixed α, μ^* and P_{wv}^* increase with the increase of η whereas other optimum values decrease with η.

Figure 2 displays the effect of η on the average rate of loosing a customer for

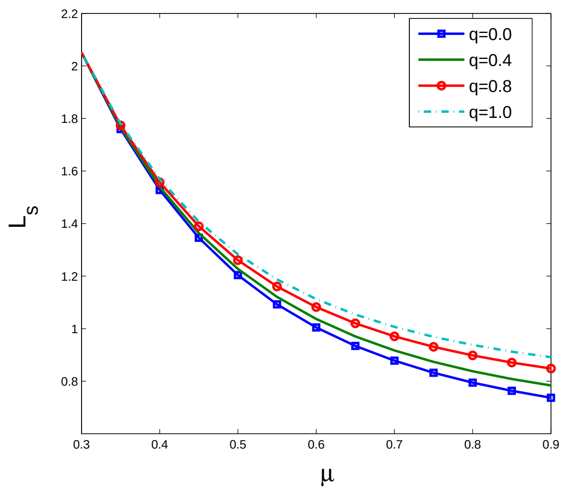

Figure 3. Impact of μ on L_s

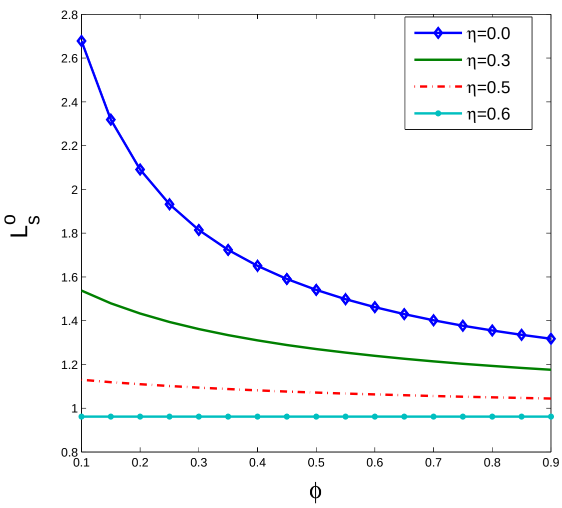

Figure 4. ϕ versus L_s^o for different η

different values ϕ. From the figure, one may observe that the average rate of loosing a customer decreases with the increase of η for fixed ϕ. Further, for $\eta < \mu$, $L.R.$ decreases with the increase of ϕ and for $\eta > \mu$, $L.R.$ increases with the increase of ϕ. Hence, a better performance of the model can be achieved by choosing the value of $\eta < \mu$.

The impact of μ on the average system length (L_s) is presented in Figure 3 for different q. For fixed q, L_s decreases with the increase of μ which is consistent with our intuition. On the other hand, for fixed μ, the average system length increases with q and L_s is higher in models without VI ($q = 1$) and least in VI models ($q = 0$) .

Figure 4 shows the effect of vacation rate (ϕ) on L_s^o for various values of η. For $\eta < \mu$, L_s^o decreases as ϕ increases. This is due to the fact that the larger the vacation rate, the shorter the vacation duration and the probability that the customer is served with regular service rate increases which leads to the decrease in L_s^o. One may also note that for $\eta = \mu = 0.6$, vacation rate (ϕ) has no effect on L_s^o. Further, for fixed ϕ, L_s^o is higher in models without WV (η=0.0).

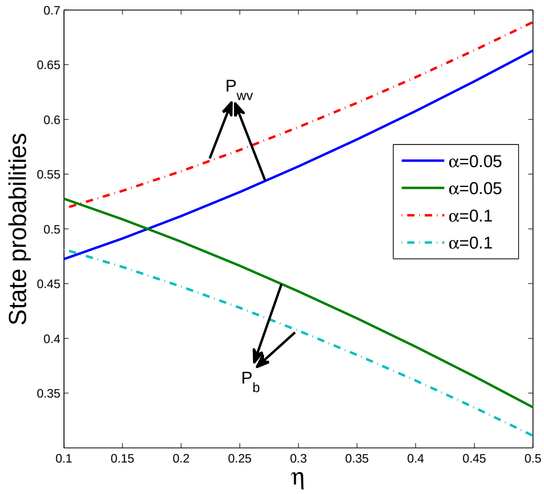

Figure 5. Effect of η on state probabilities

Figure 5 plots the impact of η on the state probabilities of the server for $\alpha = 0.05$ and 0.1. It is observed that as η increases, the probability that the server in working vacation (P_{wv}) increases whereas P_b decreases with η. Further, for fixed η, P_{wv} increases with α whereas P_b decreases.

6. Conclusions

In this paper, we have carried out the analysis of a discrete-time finite buffer queue with balking, reneging and Bernoulli-schedule vacation interruption for an early arrival system. We have obtained the steady-state probabilities at arbitrary and outside observer's observation epochs. Some important performance measures of the model such as average system length, average balking rate, average reneging rate, etc., are obtained. Cost analysis is carried out using particle swarm optimization. Computational experiences with a variety of numerical results are discussed to display the effect of the system parameters on the performance measures of the model. The present model can be generalized to a renewal input impatient customer queue with Bernoulli-schedule vacation interruption.

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