

Effects of Magnetic Field on the Red Cell on Nutritional Transport in Capillary-Tissue Exchange System

Nirmala P. Ratchagar^a and Vijaya Kumar R^b

^a*Department of Mathematics, Annamalai University, Annamalainagar, Chidambaram,
Tamilnadu-608002, India*

^b*Mathematics Section, Faculty of Engineering and Technology, Annamalai University,
Annamalainagar, Chidambaram, Tamilnadu-608002, India*

Abstract. A mathematical model for nutritional transport in capillary tissues exchange system in the presence of magnetic field has been studied. In this case, the cell is deformed. Due to concentration gradients, the dissolved nutrient in substrate diffuses into surrounding tissue. The analytical method is based on perturbation technique while the numerical simulation is based on finite difference scheme. Results concerning the concentration of dissolved nutrients, diffusive flux, normal component of velocity and skin friction coefficient, indicate that the presence of magnetic field influences the flow field considerably.

Received:20 June 2013, Revised:22 August 2013, Accepted:14 September 2013.

Keywords: Capillary-Tissue Exchange, Magnetic field, Slip velocity, Nutrients, Diffusive Flux.

Index to information contained in this paper

- 1 Introduction
- 2 Mathematical Formulation
- 3 Method of Solution
- 4 Results and Discussions
- 5 Conclusion
- 6 Appendices

1. Introduction

The most physiologically important function of blood circulation through capillaries is to supply nutrients to every living cell of the organism and also to remove various waste products from every cell. Nutrient, dissolved in plasma, enter the tissue from capillary wall. The material is transported by convection and diffusion in the capillary, whereas in the tissue the material is transported through diffusion only as the convection velocity in the tissue is small. Blood can be regarded as a magnetic fluid because the red blood cell(RBC) contain the hemoglobin molecule

* Corresponding author. Email: rathirath_viji@yahoo.co.in.

a form of iron oxide. So when we apply a magnetic field on the blood the only components of blood response to these field are erythrocytes. Pauling and Coryell are the first who reported that the erythrocytes orient with their disk plane parallel to the magnetic field [13]. Knut Aukland was found that other cell of blood, except the erythrocytes, like platelets, also orient with the applied magnetic field [11].

In order to investigate the flow of a biomagnetic fluid under the action of an applied magnetic field Haik [10] and Tzirtzilakis [21, 22] developed the mathematical model of Biomagnetic fluid dynamics. Rudraiah [15] have investigated the effect of slip and electrically conducting viscous fluid in a horizontal channel bounded on both sides by porous substrates of finite thickness. Two, three region flow models have also been developed Gupta [8]. Secomb [16–18], Tandon [19, 20] and Bali [1] developed Mathematical models for flow in narrow capillaries with diameter less than $8\mu m$ in which red cell travel as single file almost filling the lumen. Bali [2] also studied the nutritional transport through the plasma, in between the cell and capillary wall, into the tissue. Bharadwaj [5] analyzed the squeezing flow of red blood cell through a very narrow capillary enclosed by a tissue. Deepti Seth [7] concerned with the formulation of a simple mathematical model for the transport of oxygen from the surrounding retinal tissue.

In this paper, our aim is to study the deformation of cell on nutrition transport in capillary-tissue exchange in the presence of magnetic field. The momentum equation is solved analytically for velocity distribution. A first order perturbation method satisfying the slip-velocity at the porous surface is used. Knowing the average velocity field, we solve the diffusion equation numerically by employing a finite difference scheme.

2. Mathematical Formulation

The viscous, steady, two dimensional, incompressible, axis symmetrical, laminar biomagnetic fluid (blood) flow is considered taking place between two parallel flat plates(channel). We assume that the length of the channel version of a idealized krogh capillary-tissue exchange system is large compared to the width so that the end effects are negligible. A uniform magnetic field, H_0 is applied in the y-direction and the magnetic Reynolds number is assumed to be sufficiently small so that the perturbations to the magnetic field may be neglected.

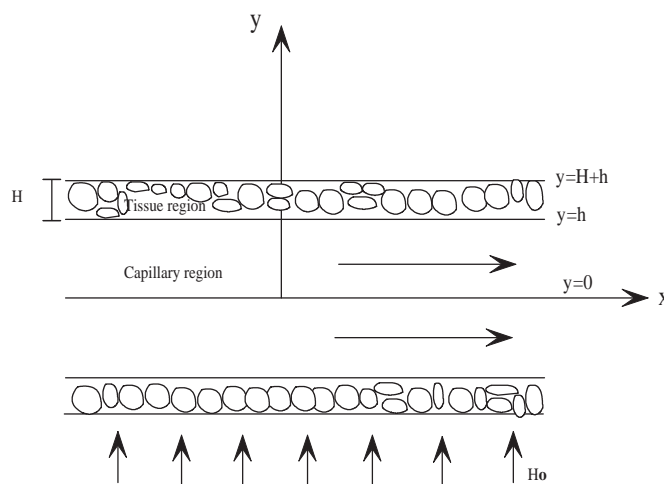


Figure 1. Physical configuration.

Flow region is divided into two sub-regions:

- (i) Fluid film region
- (ii) Porous tissue region.

Blood is considered as a homogeneous, electrically conducting biomagnetic fluid and Newtonian behavior is assumed. The force action on the erythrocytes when entering the magnetic field are shown in figure 1.

2.1 Governing Equations

2.1.1 Fluid Film Region

The governing equation of motion for the flow of plasma in between the cell and capillary tissue wall are given [6]

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} \right) - \frac{\mu^2 \sigma_0 H_0^2}{\rho} u' \quad (1)$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial y'} + \nu \left(\frac{\partial^2 v'}{\partial x'^2} + \frac{\partial^2 v'}{\partial y'^2} \right) \quad (2)$$

but due to small leakage in to the porous wall we retain the continuity equation as:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (3)$$

The Boundary conditions are

$$u' = -\frac{\sqrt{k}}{\alpha} \frac{\partial u'}{\partial y'} \quad \text{at } y' = \pm h \quad (4)$$

$$\frac{\partial u'}{\partial y'} = 0 \quad \text{at } y' = 0 \quad (5)$$

$$v' = 0 \quad \text{at } y' = 0 \quad (6)$$

$$v' = v_w \quad \text{at } y' = \pm h \quad (7)$$

where, u and v are the x and y components of the velocity, p is the pressure, σ_0 is the electrical conductivity, ρ is the density of the fluid, v_w is the vertical velocity in the porous layer, k is the permeability of the medium, α is the slip parameter, μ is the kinematic viscosity and $2h$ is the width of the channel Eq. (4) is the Beavers and Joseph(BJ)slip condition [3] at the lower and upper permeable surface.

To solve the Eqs (1)-(3), we use the following non dimensional quantities

$$y = \frac{y'}{h}, \phi = \frac{1}{\alpha \sigma}, \sigma = \frac{h}{\sqrt{k}}$$

For incompressible flows, the velocity vector V can be expressed in terms of a vector potential ψ as

$$V = \nabla \times \psi$$

This solution technique was first formulated by Berman [4].

By introducing the dimensionless stream function $\psi = \psi(x, y)$ defined by the expressions

$$u = \frac{\partial \psi}{\partial y'} = -\frac{1}{h} \frac{\partial \psi}{\partial y} \quad (8)$$

$$v = -\frac{\partial \psi}{\partial x'} \quad (9)$$

Now, we choose an appropriate form for the stream function as

$$\psi = [hu_0 - v_w x']f(y) \quad (10)$$

Substituting Eq.(10) into Eqs.(8)and(9), the velocity components are

$$u = [u_0 - v_w \frac{x'}{h}]f'(y) \quad (11)$$

$$v = v_w f(y) \quad (12)$$

By eliminating the pressure p from the first two and substituting (11)and (12) in (1)and (2), we obtained

$$\frac{\partial}{\partial y} [Re_w (f'^2 - f f'') + f'''(y) - M^2 f'] = 0 \quad (13)$$

Integrating (13) once, we get

$$[Re_w (f'^2 - f f'') + f'''(y) - M^2 f'] = K \quad (14)$$

where, K is the constant of integration, $Re_w = \frac{v_w h}{\gamma}$ is the Reynolds number, $M = \mu H_0 h \sqrt{\frac{\sigma_0}{\rho \gamma}}$ is the Hartmann number.

The boundary conditions (4)-(7) in non-dimensional form are:

$$f'(y) = -\phi f''(y) \text{ at } y = 1 \quad (15)$$

$$f''(y) = 0 \text{ at } y = 0 \quad (16)$$

$$f(y) = 0 \text{ at } y = 0 \quad (17)$$

$$f(y) = 1 \text{ at } y = 1 \quad (18)$$

2.2 Nutritional Transport

2.2.1 Porous Tissue Region

Under the admissible assumptions, the approximate diffusion equation in porous matrix is given [14]

$$\bar{u}'_n \frac{\partial C'}{\partial x'} + \bar{v}'_n \frac{\partial C'}{\partial y'} = D' \left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2} \right) + m' \quad (19)$$

where, C' the concentration of dissolved nutrients in the tissue region, D' is the diffusion coefficient, \bar{u}'_n and \bar{v}'_n is the average velocity and m' is the rate of production of the nutrient within the tissue.

we assume that the longitudinal diffusion is much less than the transverse, that is $\frac{\partial^2 C'}{\partial x'^2} \ll \frac{\partial^2 C'}{\partial y'^2}$. Thus (19) can be rewritten as

$$\bar{u}'_n \frac{\partial C'}{\partial x'} + \bar{v}'_n \frac{\partial C'}{\partial y'} = D' \frac{\partial^2 C'}{\partial y'^2} + m' \quad (20)$$

The boundary conditions are:

$$C' = c_0 \text{ at } x' = 0 \quad (21)$$

$$C' = \eta c_0 \text{ at } y' = h \quad (22)$$

$$\frac{\partial C'}{\partial y'} = 0 \text{ at } y' = h + H' \quad (23)$$

Now we introduce the following non-dimensional quantities:

$$U = \frac{u'}{u_0}, V = \frac{v'}{v_w}, C = \frac{C'}{c_0}, \bar{U}_n = \frac{\bar{u}'_n}{u_0}, \bar{V}_n = \frac{\bar{v}'_n}{v_0}, H = \frac{H'}{h},$$

$$y = \frac{y'}{h}, x = \frac{v_w x'}{\bar{u}_0 h} = \frac{4Re_w x}{Re h}, Pe = \frac{v_w h}{D}, N = \frac{m' h}{c_0 v_0}, \alpha_0 = \frac{1}{Pe}$$

where, H is the thickness of Porous matrix, v_0 is the fluid velocity, c_0 is the uniform concentration of the nutrient in the capillary, η is the partition coefficient [11]. Using the non-dimensional scheme, the Eq.(20) is transformed as given below,

$$\bar{U}_n \frac{\partial C}{\partial x} + \bar{V}_n \frac{\partial C}{\partial y} = \alpha_0 \frac{\partial^2 C}{\partial y^2} + N \quad (24)$$

and the Boundary conditions are

$$C = 1 \quad \text{at } x = 0 \quad (25)$$

$$C = \eta \quad \text{at } y = 1 \quad (26)$$

$$\frac{\partial C}{\partial y} = 0 \quad \text{at } y = 1 + H \quad (27)$$

3. Method of Solution

3.1 Velocity distribution in capillary region

Solving the Eq.(14) using perturbation technique. we let

$$f(y) = f_0(y) + Re_w f_1(y) + O(Re_w^2), \quad (28)$$

and

$$K = K_0 + Re_w K_1 + O(Re_w^2) \quad (29)$$

where, f_n 's and K_n 's are independent of Re_w and Re_w is the perturbation parameter.

Substituting Eqs.(28)and (29) in Eq.(14) and collecting of like powers of Re_w , we have the following sets of equations.

Zeroth order equations

$$f_0''' - M^2 f_0' = K_0 \quad (30)$$

The corresponding boundary conditions:

$$f_0(y) = f_0''(y) = 0 \quad \text{at } y = 0 \quad (31)$$

$$f_0'(y) + \phi f_0''(y) = 0, f_0(y) = 0 \quad \text{at } y = 1 \quad (32)$$

The Solution is

$$f_0 = A_1 + A_2 e^{-My} + A_3 e^{My} - \frac{yK_0}{M^2} \quad (33)$$

Using the boundary conditions (31)and (32), we obtained

$$f_0 = \beta_1 \sinh My + \beta_2 y \quad (34)$$

and

$$K_0 = -M^2 \beta_2 \quad (35)$$

First order equations:

$$f_1''' - M^2 f_1' = K_1 - f_0'^2 + f_0 f_0'' \quad (36)$$

The corresponding boundary conditions are:

$$f_1(y) = f_1''(y) = 0 \quad \text{at} \quad y = 0 \quad (37)$$

$$f_1'(y) + \phi f_1''(y) = 0, f_1(y) = 0 \quad \text{at} \quad y = 1 \quad (38)$$

The Solution is

$$f_1 = B_1 + B_2 e^{-My} + B_3 e^{My} - \frac{y(K_1 - b_2^2 - b_1^2 M^2)}{M^2} \quad (39)$$

Using the boundary conditions (37) and (38), we obtained

$$f_1 = \alpha_1 y + \alpha_2 \sinh My + \alpha_3 y \cosh My + \alpha_4 y^2 \sinh My \quad (40)$$

and

$$K_1 = \alpha_5 \quad (41)$$

Substituting Eqs.(34)and (40) in Eq.(28), Eqs.(35) and (41) in Eq.(29), are obtained a first-order perturbation solution as

$$f(y) = (\beta_1 \sinh My + \beta_2 y) + Re_w (\alpha_1 y + \alpha_2 \sinh My + \alpha_3 y \cosh My + \alpha_4 y^2 \sinh My) \quad (42)$$

and

$$K = -M^2 \beta_2 + \alpha_5 Re_w \quad (43)$$

where, $\alpha_n'^s$ and $\beta_n'^s$ are functions of ϕ and $A_1, A_2, A_3, B_1, B_2, B_3$ are the constants, given in the Appendix A.

Using the non-dimensional quantities and substituting (42) in (11) and (12), we obtain the velocity profile as

$$U = \left(1 - \frac{4Re_w x'}{Re h}\right) [(\beta_2 + M\beta_1 \cosh My) + Re_w \{\alpha_1 + (\alpha_2 M + \alpha_3) \cosh My + (\alpha_3 M + 2\alpha_4)y \sinh My + \alpha_4 M y^2 \cosh My\}] \quad (44)$$

and

$$V = (\beta_2 y + \beta_1 \sinh My) + Re_w (\alpha_1 y + \alpha_2 \sinh My + \alpha_3 y \cosh My + \alpha_4 y^2 \sinh My) \quad (45)$$

The normalized axial component of velocity obtained from (44) is

$$\frac{U}{\bar{U}} = (\beta_2 + M\beta_1 \cosh My) + Re_w \{ \alpha_1 + (\alpha_2 M + \alpha_3) \cosh My + (\alpha_3 M + 2\alpha_4) y \sinh My + \alpha_4 M y^2 \cosh My \} \quad (46)$$

where,

$$\bar{U} = \frac{1}{2} \int_{-1}^1 u(y) dy = 1 - x \quad (47)$$

3.2 skin friction

In non-dimensional form, the coefficient of skin friction C_f is defined as

$$\begin{aligned} C_f &= \frac{\tau_w}{\frac{1}{2}\rho v_0^2} = \frac{2}{Re} \frac{\partial U}{\partial y} \\ &= \frac{2}{Re} \left(1 - \frac{4Re_w x'}{Re h} \right) [M^2 \beta_1 \sinh My + Re_w ((\alpha_2 M + \alpha_3) M \sinh My \\ &\quad + (\alpha_3 M + 2\alpha_4)(yM \cosh My + \sinh My) + \alpha_4 M (y^2 M \sinh My + 2y \cosh My))] \end{aligned}$$

The coefficient of skin friction C_f at $y = 1$ is defined as

$$\begin{aligned} C_f &= \frac{2}{Re} \left(1 - \frac{4Re_w x'}{Re h} \right) [M^2 \beta_1 \sinh M + Re_w ((\alpha_2 M + \alpha_3) M \sinh M \\ &\quad + (\alpha_3 M + 2\alpha_4)(M \cosh M + \sinh M) + \alpha_4 M (M \sinh M + 2 \cosh M))] \end{aligned} \quad (48)$$

3.3 Concentration distribution in tissue region

Solving Equation (24) using finite difference implicit scheme in y with the average velocity field obtained from (44) and (45). First choose the grid, $i = 1$ at $y = 1, i = 2$

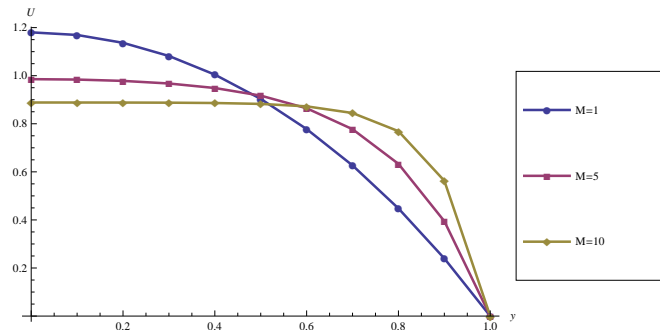


Figure 2. Axial velocity profiles for different values of magnetic field.

at $y = 1 + \Delta y$, $i = 2$ at $y = 1 + 2\Delta y$ and so on to $i = NI$ at $y = 1.25$. Similarly, $j = 1$ at $x = 0$, where Δy and Δx are increments in the y and x-direction, respectively.

To find the unknown function of Equation (24) we use central difference for discretizing the second order and first order derivative and backward difference for the first order derivatives. Thus, with respect to (i,j) point the discretization is

$$\frac{\partial^2 C}{\partial y^2} = \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j}}{\Delta y^2}, \tag{49}$$

$$\frac{\partial C}{\partial y} = \frac{C_{i+1,j} - C_{i-1,j}}{2\Delta y}, \tag{50}$$

$$\frac{\partial C}{\partial x} = \frac{C_{i,j} - C_{i,j-1}}{\Delta x}, \tag{51}$$

Substituting Equations (49)-(51) in Equation (20) we get

$$A_i C_{i-1,j} + B_i C_{i,j} + E_i C_{i+1,j} = F_i, \tag{52}$$

where,

$$A_i = -\frac{V}{2\Delta y} - \frac{\alpha_0}{\Delta y^2}, \tag{53}$$

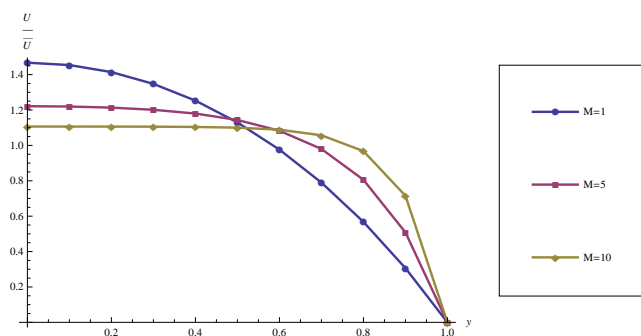


Figure 3. Axial velocity profiles for different values of ϕ .

$$B_i = \frac{U}{\Delta x} + 2\frac{\alpha_0}{\Delta y^2}, \quad (54)$$

$$E_i = \frac{V}{2\Delta y} - \frac{\alpha_0}{\Delta y^2}, \quad (55)$$

$$F_i = \frac{U}{\Delta x}C_{i,j-1}, \quad (56)$$

Using the Taylor series expansion for $i = 1$ and $i = NI$, the boundary conditions (26) and (27) become

$$C_{i,1} = \eta, \quad i = 1, 2, \dots, NI. \quad (57)$$

$$\left(\frac{U}{\Delta x} + \frac{2\alpha_0}{\Delta y^2}\right)C_{NI,j} - \frac{2\alpha_0}{\Delta y^2}C_{NI-1,j} = \frac{U}{\Delta x}C_{NI,j-1}. \quad (58)$$

Solving the above Equations (52)-(56), can be written in a way that constitutes a scalar tridiagonal system along each x grid line (j constant) and which are solved using the Thomas Algorithm [12]. It can be seen that this scheme is implicit, considering each j line constant, and therefore is called line by line implicit method (L.L.I.M.). The solution of the equation is achieved iteratively by solving, for all j lines, the arising tridiagonal systems until the unknown function at all the grid points of the computational domain has been evaluated up to an accuracy. It was found that convergence of the numerical of the solution is satisfactory for grid size 0.025 for y and 0.1 for x , solving methods are given in the Appendix B.

3.4 Diffusive flux

In non-dimensional form, Diffusive flux on the wall is given

$$D_f = -D\frac{\partial C}{\partial y} = -D\left\{\frac{C_{i+1,j} - C_{i,j}}{\Delta y}\right\} \quad (59)$$

4. Results and Discussions

Velocity profiles, skin friction coefficient, concentration distribution and diffusive flux of the flow field have been computed using MATHEMATICA 8.0. Figure 2 represent the variation of the velocity U with axial distance y for the fixed wall Reynolds number $Re_w = 0.1$. The curves are plotted for different values of the M with $\phi = 0$. Clearly when Hartmann number increases, the axial velocity decreases and profiles get flatter and approach those of plug flow.

Figure 3 shows that as slip coefficient ϕ increases, the wall shear decreases and velocity profiles flatten for $M = 1$. Figures 4 and 5 shows the normalized axial components of velocity profiles which are plotted against y for different valves of Hartmann number and slip coefficient respectively. It is similar to figure 2 and 3

which is independent of entrance Reynolds number and longitudinal position. Figure 6 depicts the coefficient skin friction C_f at $y = 1$ increases as Hartmann number increase.

Figures 7 and 8 shows the concentration distribution in tissue region with longitudinal position x and transverse position y for different values of Hartmann number. In this region as Hartmann number increases, the concentration also increases. In this case, the cells of the tissue of the deeper region gets proper nutrition. Due to increase in Hartmann number, more fluids are enters into tissue so that more dissolved nutrients enter into the tissue along with fluid. Figure 9 represent the diffusive flux on the wall in tissue region at $y = 1$ with x . Diffusive flux on the wall increases for M value increases. Therefore in deeper region also get proper nutrition.

5. Conclusion

The biomagnetic (blood) fluid flow in a channel with axi-symmetric is studied. The present analysis deals with the deformation of the cell are useful for continuous blood flow through a capillary in the presence of magnetic field. When the blood flow is increased nutrients are distributed to the tissues much more effectively and quickly, so that the tissue gets more nutrients.

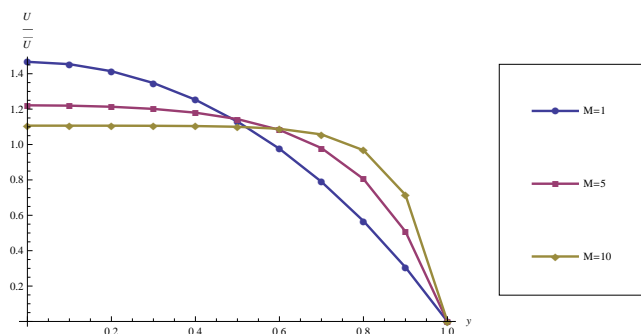


Figure 4. Velocity profiles for different values of magnetic field.

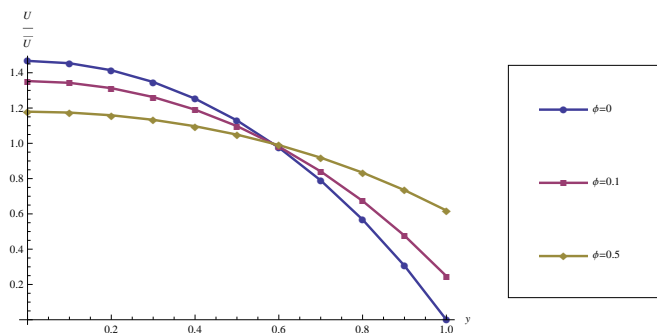


Figure 5. Velocity profiles for different values of ϕ .

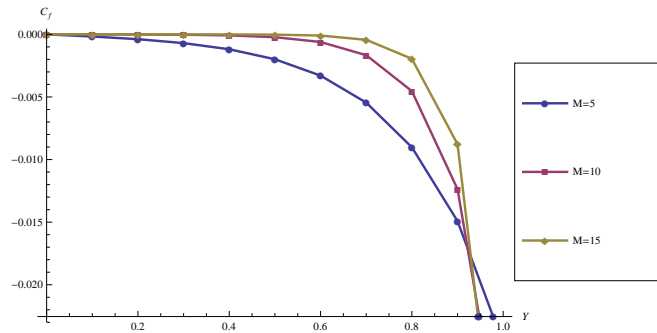


Figure 6. Skin friction coefficient of the upper plate for different values of magnetic field at $y=1$

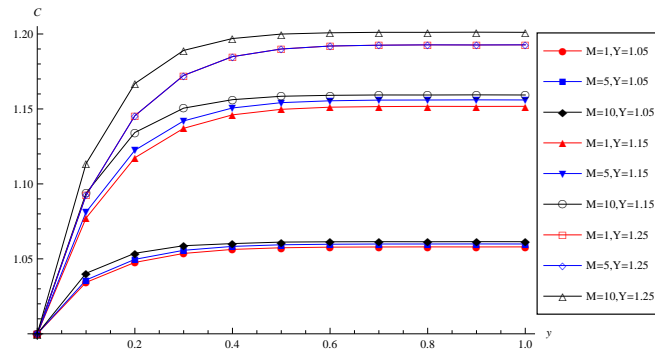


Figure 7. Concentration distribution in tissue region with longitudinal position y

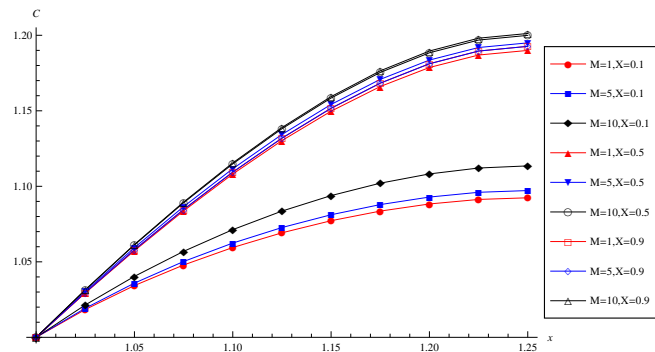


Figure 8. Concentration distribution in tissue region with transverse position x

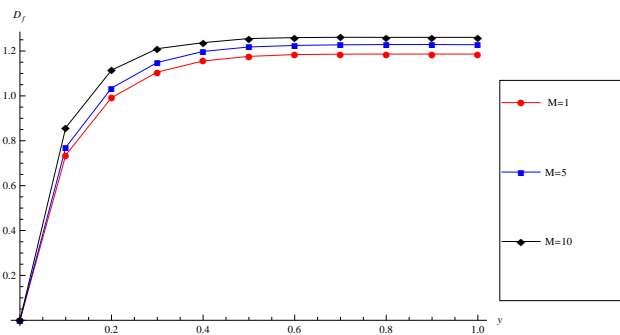


Figure 9. Diffusive flux on the wall for different values of magnetic field.

6. Appendices

6.1 Appendix 1

Constant defined in Eqs.(33), (39) and (42) are

$$A_1 = 0, B_1 = 0 \quad (60)$$

$$A_1 = \frac{K_0}{M^2(2M \cosh M + 2\phi M^2 \sinh M)}, \quad (61)$$

$$A_4 = -\frac{K_0}{M^2(2M \cosh M + 2\phi M^2 \sinh M)}, \quad (62)$$

$$B_2 = \frac{1}{2M \cosh M + 2m^2\phi \sinh M} \left[\frac{b_1^2 b_2 b_3}{4M} + \frac{7b_1 b_2 \cosh M}{4M} + \frac{5b_1 b_2 \sinh M}{4} \right. \\ \left. - \frac{b_1 b_2 M \cosh M}{4} + 3b_1 b_2 \phi \sinh M + \frac{3b_1 b_2 M \phi \cosh M}{4} - \frac{b_1 b_2 M^2 \phi \sinh M}{4} \right] \quad (63)$$

$$B_3 = -\frac{1}{2M \cosh M + 2m^2\phi \sinh M} \left[\frac{b_1^2 b_2 b_3}{4M} + \frac{7b_1 b_2 \cosh M}{4M} + \frac{5b_1 b_2 \sinh M}{4} \right. \\ \left. - \frac{b_1 b_2 M \cosh M}{4} + 3b_1 b_2 \phi \sinh M + \frac{3b_1 b_2 M \phi \cosh M}{4} - \frac{b_1 b_2 M^2 \phi \sinh M}{4} \right] \quad (64)$$

$$\alpha_1 = -\frac{\beta_1^2 \beta_2 \beta_3}{4M}, \quad (65)$$

$$\alpha_2 = \frac{\beta_1^2 \beta_2 \beta_4}{4}, \quad (66)$$

$$\alpha_3 = -\frac{7\beta_1 \beta_2}{4M}, \quad (67)$$

$$\alpha_4 = \frac{\beta_1 \beta_2}{4}, \quad (68)$$

$$\alpha_5 = \frac{\beta_2^2 + M^2 \beta_1^2 + \beta_1^2 \beta_2 \beta_3 M}{4}, \quad (69)$$

$$\beta_1 = -\frac{1}{M \cosh M - \sinh M + \phi M^2 \sinh M}, \quad (70)$$

$$\beta_2 = -\beta_1 (M \cosh M + \phi M^2 \sinh M) \quad (71)$$

$$\begin{aligned} \beta_3 = & 7M \cosh^2 M - 5M \sinh^2 M + 2\phi M^2 \sinh 2M - 7 \sinh M \cosh M \\ & - 12\phi M \sinh^2 M \end{aligned} \quad (72)$$

$$\beta_4 = (\phi M^2 - 12\phi - 6) \sinh M + (M - 3\phi M) \cosh M \quad (73)$$

6.2 Appendix 2

Solving the Eqs. (52)-(58) using Thomas algorithm for tridiagonal matrix

$$b_i = \frac{B_i - A_i E_{i-1}}{b_{i-1}} \quad \text{with } b_1 = B_1 \quad (74)$$

$$t_i = \frac{F_i - A_i t_{i-1}}{b_i} \quad \text{with } t_i = \frac{F_1}{B_1} \quad (75)$$

Compute the backward from $C_{NI,j}$ to $C_{1,j}$ as follows:

$$C_{i,j} = t_i - \frac{E_i C_{i+1}}{b_i} \quad C_{NI,j} = t_{NI} \quad (76)$$

References

- [1] Bali R, Mishra S and Dubey S, *A Mathematical model for red cell motion in narrow capillary surrounded by tissue*, Applied Mathematics and Computation, **196** (2008) 193–199.
- [2] Bali R, Mishra S and Mishra M, *Effects of Deformation of Red cell on Nutritional Transport in Capillary-Tissue Exchange System*, Applied Mathematics, **2** (2011) 1417–1423.
- [3] Beaners.G.S and Joseph D.D, *Boundary conditions at a naturally permeable wall*, J.Fluid Mech., **30** (1967) 197–208.
- [4] Berman A.S, *Laminar flow in channel with porous*, Journal of Applied Physics, **24** (1953) 1232–1235.
- [5] Bhardwaj R and Karmchari Nagar, *Swueezing flow of red blood cell through a very narrow capillary enclosed by a tissue*, Indian Journal of Biomechanics, **7** (2009) 147–150.
- [6] Chandrasekhar.B.C and Rudraiah N, *MHD flow through a channel of varying gap*, Indian J.Pure and Appl., **11** (1980) 1105–1123.
- [7] Deepti Seth, *An Analytical Solution for Diffusion and Nonlinear Uptake of Oxygen in the Retina*, International Journal of Mathematical Modelling and Computations, **2 (3)** (2012) 181–188.
- [8] Gupta B.B, *Analysis of human blood flow through narrow tubes*, *Ph.D. Thesis.*, IIT Delhi (1976)

- [9] Haik Y, Pai V and .Chen C.J, *Development of magnetic Device for cell Separation*, Journal of Magnetism Magn Mate., **194** (1999) 254–261.
- [10] Higashi T, Yamagishi A, Takeuchi T, Kawaguchi N, Sagawa S, Onishi S and Date M, *Orientation of blood cells in static magnetic field*, American Society of Hematology, **82** (4) (1993) 1328-1334.
- [11] Knut Aukland, Bruce F.Bower and Robert W.Berlines, *Measurement of local blood flow with hydrogen gas*, Circulation Research. , **14** (1964) 164–187.
- [12] Patankar S.V, *Numerical heat transfer and fluid flow*, Hemisphere Publication Corporation, Washington - New York - London. McGraw Hill Book Company, New York,(1980) 1. Aufl., 197 S., 76 Abb., geb., DM 71,90.
- [13] Pauling L and Coryell CD, *The magnetic properties and Structure of Hemoglobin, Oxyhemoglobin and Carbonmonoxy Hemoglobin,USA*, Proceedings of the National Academy of Science, **22** (1936) 210–216.
- [14] Ramana B and Sarojamma G, *Unsteady Convective Diffusion in a Herschel–Bulkley Fluid in a Conduit with Interphase Mass Transfer*, International Journal of Mathematical Modelling and Computations, **2** (3) (2012) 159 – 179.
- [15] Rudraiah N, Dulal Pal and Shivakumar PN, Effects of Slip and magnetic field on composite system, Fluid Dynamics Research, **4** (1988) 255–270.
- [16] Secomb T.W and Skalak R, *A Two Dimensional Model for the Capillary Flow of an Axisymmetric Cell*, Microvascular Research, **24** (1982) 194–203.
- [17] Secomb T.W, Skalak R, Ozakaya N and Gross J.F, *Flow of Axisymmetric Red Blood Cells in Narrow Capillaries*, Journal of Fluid Mechanics, **163** (1986) 405–423.
- [18] Secomb T.W, Hsu R and Pries A.R, *Motion of Red Blood Cell in A capillary with an Endothelial Surface Layer: Effect of Flow Velocity*, American Journal of Physiology: Heart and Circulatory Physiology, **28** (2001) H629–H636.
- [19] Tandon P.N, Mishra M and Chaurasia A, *A Model for Nutritional Transport in Capillary-Tissue Exchange System*, International Journal of Bio-Medical Computing, **37** (1994) 19–28.
- [20] Tandon P.N, Nirmala P, Tiwari M and Rana U.V., *Analysis of Nutritional Transport through a Capillary- Normal and Stenosed*, Computers and Mathematics with Applications, **22** (12) (1998) 3–13.
- [21] Tzitzilakis E.E and Loukopoulos V., *Biofluid flow in a channel under the action of a uniform localized magnetic field*, Comput Mech., **151** (2005) 360–374.
- [22] Tzitzilakis E.E., *A mathematical model for blood flow in magnetic field*, Physics of fluids., **17** (2005) 077103–077115.