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Irrigation Water Resource Planning Optimization Model: The Case of Wine Grape Farming in Dodoma, Tanzania

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Abstract. Optimum cropping pattern in vineyard irrigated farming is one of the vital tasks for obtaining the best irrigation water reserves of the command. In this article the linear programming model was developed for optimal use of water and land resources. The model was tested by the data from Chinangali irrigated farmland with 120 cultivated hectares found in Dodoma, Tanzania. The results show that, the savings of 16 470.40 m^3 of water per annum will be observed if the planting of 14.18 hectares of Chardonnay, 27.97 hectares of Cabernet sauvignon, 56.14 hectares of Riesling and 21.39 hectares of Chenin blanc. Thus, it was recommended that 1 173 359.60 m^3 of water should be released to the irrigated farmland per annum for the best irrigation planning versus the 1 189 830 m^3 of water supplied currently per annum.

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1. Introduction

Water is an essential resource for the plant growth. It plays a vital role in most biological processes and transports nutrients from the soil to the plant through the roots. When the soil moisture in the root is inadequate to meet crop water requirements then, crop yield decrease below their potential values and the crop is said to be under water stress. In areas of limited water resources, where annual precipitation is not sufficient to meet crop water requirements, irrigation is needed to provide the soil root zone with adequate moisture to avoid physiological water stress in the crop and to achieve acceptable yield [1]. Land and water are the most important natural resources thus, the efficient use of these two resources is necessary for sustainable agricultural production. The scarcity of water in many parts of the world leads to the need of optimizing water usage from the field of irrigation system by adopting effective and efficient water management [2]. Under limited water conditions,

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the availability of water limits the acreage and type of crops that should be planted. Obtaining the optimal acreage of the crops can be done by formalizing the problem and then applying a suitable developed mathematical optimization model to it [3]. The aim is to use water resource efficiently to strike the balance between water availability and demand for it [4]. For that reason, mathematical optimization models are most widely accepted in the field of irrigation system planning and management.

Dodoma is a dry region with erratic rainfall of at most 570 mm per annum on average over the three quotas of its area. Most farmers in Dodoma therefore depend on irrigation and the major source of water in the region is underground wells which supply irrigation water to the farmers. Initially the water supply in the region was mainly surface water. This was due to insufficient technology to harvest the ground water reserves. In 1929 the Imagi reservoir was built to supply water in Dodoma urban from Imagi stream. The Imagi dam dried up in 1943 not because of its size but due to the increased demand triggered by the expansion of settlement, cultivation, deforestation, overgrazing and inadequate rainfall. Msalato reservoir was built in 1944 as a substitute of the Image reservoir. Another reservoir was constructed in south-west of Dodoma at the area known as Mkonze [5]. It was stated that the demand of water in the region was increased very fast in the fifties. For instance, from 1950 to 1951 the demand of water increased by 16% [6]. This was a sign that surface source of water in the semi-arid region of Dodoma will not meet the everlasting water supply of the region [7].

1.1. *Crop and irrigation water requirements*

Irrigation water requirements means the amount of water that must be supplied through the irrigation system to guarantee full crop water requirements were received by the crop. If irrigation is the only source of water supply for the plant, the irrigation requirement will always be greater than the crop water requirement to allow for inefficiencies in the irrigation system. If the crop receives some of its water from other sources (rainfall, water stored in the ground, underground, etc.), then the irrigation water requirement can be considerably less than the crop water requirement [8].

The net irrigation water requirement does not comprise losses that are arising in the process of applying the water. The net irrigation water requirement plus losses constitute to the gross irrigation water requirement. It is important to understand that the estimation of crop water requirements is the first stage in the estimation of irrigation water requirements of a given cropping programme. The irrigation water requirement is one of the principal parameters for the planning, design and operation of irrigation and water resources systems. It is a parameter of prime importance in formulating the policy for optimal allocation of water resources as well as in decision-making in the day-to-day operation and management of irrigation systems. Hence estimating irrigation water requirements accurately is important for water project planning and management [9]. Inaccurate calculation of irrigation water requirements can result in significant system failures and the waste of valuable water resources. It may induce water logging, salinity, or nutrient leaching from the soil as a result of insufficient regulation of the soil moisture regime in the root zone. It may result in irrigation network or storage reservoir capacities that are insufficient to allow efficient water use and a reduction in irrigated area. Overestimating the amount of irrigation water required at peak demand could result in higher development expenses [10].

1.2. *Mathematical Optimization Modeling*

In a water-scarce environment, a mathematical optimization model can also be employed to solve the problem of cropping pattern. This will allow a farmer to determine which crops to grow on the available acreage and how much irrigation water to use in order to maximize his agricultural output. Linear programming is an optimization technique which under this study was used to allocate the limited water resource and rationalize the multiple characteristics of allocation problem. The LP is easy to apply to the problem of cultivation planning using several readily available program packages [11]. For the given problem, we formulate a mathematical description called a mathematical model to represent the situation. The model consists of the following components.

- Decision variables: The decision variables of the problem are represented using symbols such as $X_1, X_2, X_3, \ldots\ldots. X_n$. These variables represent unknown quantities which may be the number of items to produce or the amounts of money to invest in.
- Objective function: The objective function of the problem is expressed as a mathematical expression in the decision variables.
- Constraints: The limitations or requirements of the problem are expressed as inequalities or equations in the decision variables.
- Non-negativity restrictions: The restrictions on the decision variables, as in the physical problems, negative values of decision variables have no valid interpretation.

The irrigation water requirement for the four grapevine varieties in this study was calculated using a modified FAO Penman's approach, which is now suggested as the standard method for defining and computing reference evapotranspiration [11][12]. The method is widely accepted because it covers all parameters and provides more satisfactory results when measured data on temperature, humidity, wind, and sunshine duration are available, as opposed to other methods that require local or regional calibration before being used for irrigation project planning and design [13].The mathematical model used in this study was modified to a farmland in the Chinangali grape irrigation scheme in Tanzania's Dodoma area using linear programming methodology. The goal of the study was to determine the best water allocation to the planting areas where each of the four grapevine varieties, Cabernet sauvignon, Chardonnay, Chenin blanc, and Riesling, should be planted in order to minimize irrigation water use in vineyard agricultural production.

According to [14], linear programming models are good tools for supporting initial or periodic planning of irrigated agricultural businesses, but they require technical coefficients that can be calculated using computer simulation models. The ideal approach included considerable constraints on monthly water availability, labor, acreage, and productivity. In terms of water resource optimization, it was confirmed that small decreases in the maximum total net present value can result in significant reductions in irrigation requirements.

On data from a planned reservoir on the Havrias River in Northern Greece, [15] developed a Non-Linear Programming optimization model with an incorporated soil water balance to determine the optimal reservoir release policies, irrigation allocation to several crops, and the optimal cropping pattern in irrigated agriculture. The findings of this model were compared to those of a model in which cultivated areas were the only decision variable. The model had a lot of potential as a decision-making tool for irrigated area cropping patterns and irrigation scheduling in agricultural output.

Linear Programming models are effective instruments for supporting initial or periodic planning of irrigated agricultural businesses, according to the examined publications. The Linear Programming model was recognized as having tremendous potential to be used as a decision support tool for cropping patterns in an irrigated area and irrigation scheduling [15]. The findings in the reviewed articles have the drawback of relying on approximated and literature-based data, which is subjective and requires local or regional calibration.

2. Mathematical model formulation

The farm used as a case study has an area of 120 hectares of land. The total water available for irrigation based on grapevine crop characteristics for the whole command area is 1 189 830 m^3 per annum as calculated from the formula by [16]. Cabernet sauvignon, Chardonnay, Chenin blanc, and Riesling are the most common grape varietals planted by grape producers in the region. The least planting areas guarantee the availability of all grapevine varieties on the market, while the maximum planting areas guarantee that a farmer will not face storage or selling issues if harvests exceed storage capacity or if demand exceeds supply, causing the selling price to decline [17]. Under this study the minimum planting area is assumed to be 10 hectares because in wine processing the wines that are produced have special aroma and taste, and it is used to blend with other wines so as to give the desired quality and aroma. So it is necessary to have all four grapevine varieties in good proportion during wine making process.

It is assumed that the crops are not to be rain fed but rely solely on irrigation. We want to find out the optimum cropping areas with corresponding water resource allocations for each area planted with grapevine varieties to minimize irrigation water volume. In order to formulate the mathematical model the necessary data including the crop water requirements for the grapevine variety considered, the yield rates per unit area of land, the cost of production per unit area, local price data and water charges per cubic meters.

2.1 *The Objective function*

With the goal of preserving groundwater use in Dodoma grape agriculture, a linear programming model was developed to identify the ideal cropping areas for decreasing crop water demand and achieving large groundwater savings. In farm management research, the linear programming model is a very powerful technique that can efficiently handle a large number of linear constraints and variables (activities) at the same time [18].

The decision variables for this study are the amounts of land areas in hectares which should be planted with each grapevine variety denoted by $(A_i \; for \; i = 1,2,3,4)$ where;

- A_1 is the amount of area to be planted with Cabernet sauvignon
- A_2 is the amount of area to be planted with Chardonnay
- A_3 is the amount of area to be planted with Chenin blanc
- A_4 is the amount of area to be planted with Riesling

The objective function is to minimize the total water usage (TWU) such that:

$$Min\ (TWU) = \sum_{i=1}^{n} CWR_i * A_i \; for \; n = 4$$

where;

- A_i is the cultivated area of grapevine variety i in (ha)
- CWR_i is a crop water requirement per hectare of land for grape variety i in (m^3)
- TWU is the total water used for the veneyards planted with grape variety variety i in (m^3)

2.2. *Constraints*

- **Constraint 1:** Total irrigated area shall be less than or equal to cultivable area.

$$\sum_{i=1}^{n} A_i \leq TA \; for \; n = 4$$

where;

- A_i is the cultivated area of grapevine variety i in (ha)
- TA is the total area available for a cultivation in hactares i.e variety $\leq 120\ ha$

- **Constraint 2:** Total water required by all crops shall be less than or equal to total available water in that year.

$$\sum_{i=1}^{n} CWR_i * A_i \leq TWA\ for\ n = 4$$

where;

- A_i is the cultivated area of grapevine variety i in (ha)
- CWR_i is a crop water requirement per hectare of land for grape variety i in (m^3)
- TWA is the total water available for available supplied by undergraound wells for in (m^3)

The volume of water available is equal to the total of irrigation water requirement for each crop multiplied by the number of days the crop is irrigated multiplied by the area where the crop is grown. The formula by Adeyemo *et al.*, (2008) below is used.

$$TWA = \sum_{i=1}^{n} (GIWR_i * D_i * A_i\ \ for\ n = 4$$

where;

- $GIWR_i$ is gross irrigation water requirement per day for grapevine variety i
- D_i is the duration of irrigation for grapevine variety i in days

The total area (A) is equal to the sum of all areas of land where each of the grapevine variety is grown.

- **Constraint 3:** Under this study it is assumed that each area, (A_i) must be greater than or equal to 10ha. With this constraint, it is certain that all the crops will be grown in at least ten hectares of land to make sure that every grapevine variety is present in the market because during wine making some grapevine varieties are used to blend the wine of other varieties to give good test and aroma. Such that:

$$A_1, A_2, A_3, A_4 \geq 10\ ha$$

- **Constraint 4:** The sum of net returns (Profit) for the four grapevine varieties should be greater or equal to the total net return $(TNRA)$ available for the whole command area calculated from total revenue and total expenditure. Such that:

$$\sum_{i=1}^{n} (NR_i * A_i) \geq TNRA\ \ for\ n = 4$$

where;

- NR_i is the net return per hectare obtained for cultivating grapevine variety i
- $TNRA$ is the total net return available for whole command area

- **Constraint 5:** The sum of costs for water used the four grapevine varieties on cultivated vineyards must be less or equal to the total cost of water available for the whole command area in that year.

$$\sum_{i=1}^{n} TCWR_i * A_i \leq TIWC\ for\ n = 4$$

$where;$

- $TCWR_i$ is the cost of water used to irrigate grapevine variety i
- $TIWC$ is the total irrigation water cost for whole command area

- **Constraint 6:** The sum of preparation costs per hectare for the four vineyards must be less or equal to the total cost of preparing whole command in that year

$$\sum_{i=1}^{n} PC_i * A_i \leq TPC_i \; for \; n = 4$$

where;

- $TCWR_i$ is the cost of preparing a vineyard planted with grapevine variety i
- $TIWC$ is the total preparation cost for whole command area

The FAO Penman-Monteith method was used for estimating the crop water requirement of each of the four grapevine varieties studied. The crop evapotranspiration was estimated by FAO Penman-Monteith equation [19]. Therefore, crop water requirement *(CWR)* for a given crop was defined as:

$$CWR_i = \sum_{i=1}^{n} (KC_i * ET_0 - P_e) \; mm, units \; for \; n = 4$$

where:

- KC_i is the crop coefficient for wine grape variety i during the growth stage *t* and where *T* is the final growth stage
- ET_0 is reference evapotranspiration rate
- P_e is effective rainfall

3. Results and discussions

The objective function of this study was to minimize water usage for irrigation at Chinangali grapevine farm in Dodoma region. The formulated mathematical model is solved by MATLAB to obtain the optimum amount of water used for the whole command area. Then we analyze to see if we have been able to save water by taking the difference between the actual and optimal irrigation water demands.

The MATLAB Optimization Toolbox (MathWorks, 2013) provides a function linprog (linprog.m), which implements the simplex algorithm to solve a linear programming problem. According to [20] 'linprog.m' is available for minimization problems and modified accordingly for maximization. Under this study it was used for minimization of objective function (i.e., irrigation water usage) and for analyzing the developed optimization model for various case scenarios.

A standard software package (MATLAB Optimization Toolbox) was used to solve the formulated linear programming model and generate results of the minimization process as given;

The area allocated(A_1) *for Cabernets Avignon in hectares is 27.97*
The area allocated (A_2) *for Chardonnay in hectares is 14.18*
The area allocated (A_3) *for Chenin blank in hectares is 21.39*
The area allocated (A_4) *for Resieling in hectares is 56.14*
The minimum total amount of water to be used in Cums is 1 173 359.60

The amount of water saved in Cums is 16470.40
The profit generated in TZS is 1118170132.79 more than the actual case.

In this case the computed irrigation water requirements are the function of the areas. The best cropping areas should be planted with grapevine varieties such as Chardonnay, Chenin blanc, Cabernet sauvignon, and Riesling are shown in Figure 1. The actual and ideal configurations for the crop planning model for irrigation water minimization are shown in Figure 2.

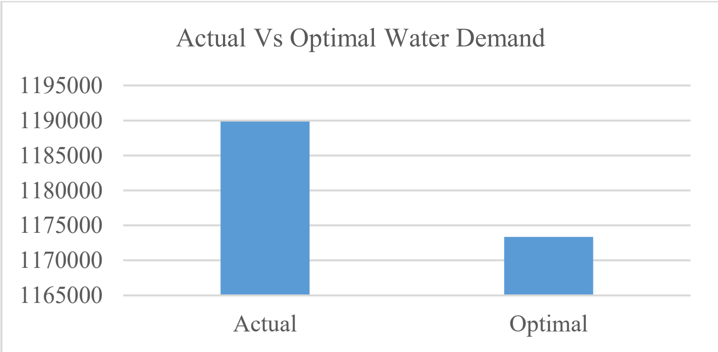

Figure 1. The actual and optimized irrigation water demands

This study offered the proposed areas of land in hectares for the four grapevine varieties modelled in the study area with the objective of minimizing irrigation water resource usage which is the main purpose of this study. Water allocation to cultivated vineyards might be increased, or alternative sources of water, such as rain water collecting, could be used to supplement irrigation under the Chinangali irrigation scheme.

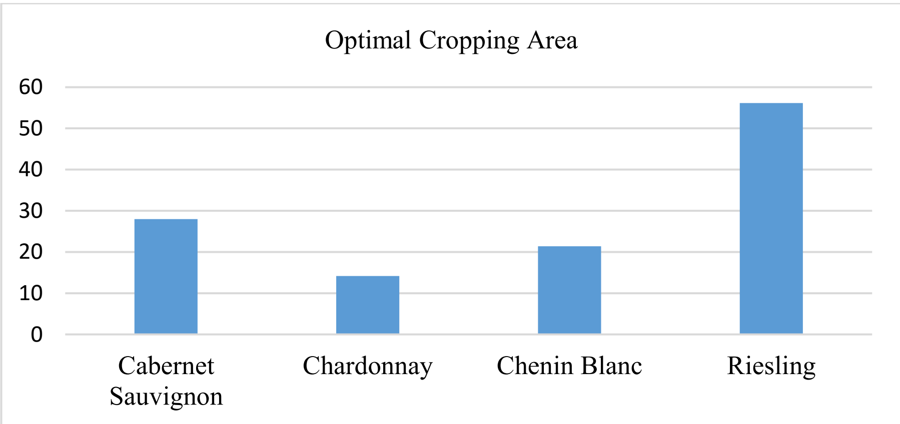

Figure2. The Optimal cropping areas for grapevine varieties.

Cropping pattern may not be dependent only on water resource availability it is also affected by other factors in an area being opened for irrigation. The factors fall into three categories namely: natural, social and economic. The natural factors are soil structure, climate and the amount of irrigation water available. The social factors are those relating to ways the farmers get organized. The economic factors include prices for farm products, marketability, the level of technology adopted, manpower needs, use of credit and the trends of industrialization in the region. The mathematical model formulated however is suitable enough for grape growers to make good decision in relation to water saving strategy from the grapevine farming in the study area.

Table 1 shows the optimal cropping areas per grapevine variety. Under this scenario the available irrigation release for the period of one year was used optimally in order to achieve minimum usage of water resource in irrigation of grapevines of the study area.

Table 1. The Optimal cropping areas with their corresponding grapevine varieties.

S/n	Grapevine variety	Cropping areas (ha)
1	Cabernet sauvignon	27.97
2	Chardonnay	14.18
3	Chenin blanc	21.39
4	Riesling	56.14

It was also found that the optimum irrigation water resource was less than the actual case from the whole grapevine farmland. It was then concluded that the actual irrigation water available in cubic meters was 1 189 830 and the optimal irrigation water in cubic meters was 1 173 359.60. Therefore, the amount of irrigation water saved in cubic meters was 16 470.40.

4. CONCLUSION

The mathematical MATLAB based optimization model was successfully analyzed for sustainable agricultural and water resources planning in irrigation for various scenarios and criteria. It gives water resources planners various options for different needs thereby enabling them to tradeoff between different optimal solutions. Upon analyzing various scenarios, the suitable criteria offer better alternatives to the planner. The proposed mathematical MATLAB based optimization model provided optimal cropping areas to the given grapevine varieties and operational policies to the grape growers to achieve desired result for sustainable water resources utilization. The irrigation water resources management can benefit from application of economic analysis and methodology of optimization. Modelling can test alternatives of water use in order to help policy makers, planners and grape growers to improve the use of water resources in vineyard irrigated agriculture under water shortage situations.

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