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Proposing a Stochastic Model for the Continuous Spread of Coronavirus Dynamics in Nigeria

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Abstract. The emergence of corona virus (COVID-19) has create a great public concern as the outbreak is still ongoing and government are taking actions such as holiday extension, travel restriction, temporary closure of public work place, borders, schools, quarantine/isolation, social distancing and so on. To mitigate the spread, we proposed and analyzed a stochastic model for the continue spread of corona virus (COVID-19) dynamics considering the impact of vaccination among susceptible, exposed and quarantine cases. The difference and differential-difference equations for the dynamics of (COVID-19) were derived and simulated with available parameters using stochastic simulation (Gillespie Algorithm). The study adopts Continues–Time Birth and Death stochastic process and the probability distribution describing the dynamics of coronavirus was derived and simulated which shows an exponential decay over time. As the time increase, the probability of coronavirus incidents decline to zero. We therefore conclude that vaccination has an impact of 20% among susceptible, exposed and quarantine cases.

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Keywords: Corona virus; Birth-Death process; Vaccination; Stochastic simulation; Differential-Difference equations.

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1. Introduction

The novel coronavirus (COVID-19) was originated in the city of Wuhan, China which is the capital of Hubei province on December 31, 2019. The first case was detected after developing the pneumonia without a clear cause and for which the vaccine or treatments were not available and effective [4]. The virus can be transmitted from human to human [2]. It takes 2 to 10 days for the appearance of the symptoms and those symptoms include coughing, difficulties in breathing, and fever. It has been reported that the virus might be originated from bat [10] and the transmission of the virus might originate to a seafood market [6,3]. In recent, the features and clinical findings of the infection have been reported [3,11]. Currently, the spread is still ongoing and has claimed 2663 lives, along with 77,658 confirmed cases and 2824 suspected cases in China as of 24 Febuary, 2020 [7]. The coronavirus cases not only spread in China but to some other part of the globe such as Europe, North America, Asia specific and Africa. According to a recent report by CNN, America recorded the highest cases of coronavirus.

In Nigeria, according to a report by center for disease control (NCDC), we have 184

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confirmed cases, 20 recovery and 2 die [8]. Therefore, there is urgent need to develop a stochastic model for the spread of such deathly disease so as study the dynamics of transmission of the virus and capture the impact of vaccination among susceptible, exposed and quarantine individual in a population. In this work, we proposed a stochastic model for the spread of coronavirus (COVID-19) cases in Nigeria considering the impact of vaccination.

Literature Review

In this work, we review the recent work of [1], [5] and [9] on mathematical modeling of novel coronavirus dynamics.

2. Methodology

We proposed a stochastic model for the spread of corona virus dynamics in Nigeria considering the impact of vaccination among susceptible, exposed and quarantine case. In this work, the dynamics of coronavirus (COVID-19) is considered as stochastic continuous-time birth and death process. Satisfying $\{X(t):t\}$, where $X(t)$ represent a stochastic process and the model was formulated via the schematic or model diagram below:

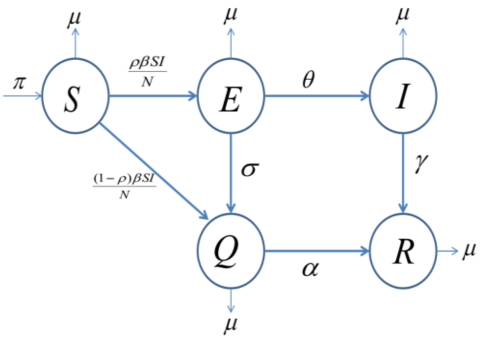

Figure 1. Model diagram.

From the model above, we first derived our various transitions and probabilities, let $X = (X_1 = S, X_2 = E, X_3 = Q, X_4 = I, X_5 = R)$ be a set of random variables representing Susceptible, Exposed, Quarantine, Infectious and Recovery respectively. Let the transition intensities and probability be represented in the table below:

To describe the probability of an event taking place due to transitions are obtained by set of difference-equation known as Kolmogorov- equations. For continues time birth and death stochastic process, we have

$$P_i(t+\Delta t) = P_i(t)\left[1 - i(\mu+\lambda)\Delta t + o(\Delta t)\right] + P_{i-1}(t)\left[(i-1)\lambda\Delta t + o(\Delta t)\right] + P_{i+1}(t)\left[(i+1)\mu\Delta t + o(\Delta t)\right] \tag{1}$$

Rearranging our equation (1), subtract $P_i(t)$ and divide Δt from both side, taking limit as $\Delta t \to 0$, we have

$$\frac{dP_i(t)}{dt} = (i-1)\lambda P_i(t) + (i+1)\mu P_{i+1}(t) - i(\mu+\lambda)P_i(t) \tag{2}$$

Table 1. Transition intensities and rates.

Transitions	Event	Rates
$(\Delta X)_1 = (1,-1,0,0,0)$	$S \to E$	$\lambda_{12} X_1 X_2$
$(\Delta X)_2 = (1,0,-1,0,0)$	$S \to Q$	$\lambda_{13} X_1 X_3$
$(\Delta X)_3 = (0,1,-1,0,0)$	$E \to Q$	$\lambda_{23} X_2$
$(\Delta X)_4 = (0,1,0,-1,0)$	$E \to I$	$\lambda_{24} X_2$
$(\Delta X)_5 = (0,0,1,0,-1)$	$Q \to R$	$\lambda_{35} X_3$
$(\Delta X)_6 = (0,0,0,1,-1)$	$I \to R$	$\lambda_{45} X_4$
$(\Delta X)_7 = (1,0,0,0,0)$	$\to S$	π
$(\Delta X)_8 = (-1,0,0,0,0)$	μS	μX_1
$(\Delta X)_9 = (0,-1,0,0,0)$	μE	μX_2
$(\Delta X)_{10} = (0,0,-1,0,0)$	μQ	μX_3
$(\Delta X)_{11} = (0,0,0,-1,0)$	μI	μX_4
$(\Delta X)_{12} = (0,0,0,0,-1)$	μR	μX_5

Where $\lambda_{12} = \rho\beta/N$, $\lambda_{13} = (1-\rho)\beta/N$, $\lambda_{23} = \sigma,\ \lambda_{24} = \theta,\ \lambda_{35} = \alpha,\ \lambda_{45} = \gamma$.

Table 2. Parameters of the model.

Descriptions	Value	Source
$\pi =$ Recruitment rate	100	Assumed
$\mu =$ Natural mortality rate	0.02	Khan (2020)
$\beta =$ Effective contact rate	0.05	Khan (2020)
$\theta =$ Proportion of asymptomatic infection	0.1243	Khan (2020)
$\gamma =$ Recovery rate of infectious cases	0.09871	Khan (2020)
$\alpha =$ Recovery rate of quarantine cases	0.05	Assumed
$\rho =$ Proportion of exposed cases	0.02	Assumed
$\sigma =$ Quarantine rate	0.20	Assumed
$v =$ Vaccination parameter	$[0.2-0.8]$	Assumed

Equation (2) is a system of difference differential equations known as Kolmogorov forward differential equations and we shall derive our Kolmogorov equations for the dynamics of coronavirus (COVID-19) in our model using equation (1)

$$P_{X_1}(t+\Delta t) = P_{X_1}(t)\left[1 - X_1(\mu+\pi)\Delta t + o(\Delta t)\right] + P_{X_1-1}(t)\left[(X_1-1)\lambda_{12}\Delta t + o(\Delta t)\right] + P_{X_1+1}(t)\left[(X_1+1)\mu\Delta t + o(\Delta t)\right] \tag{3}$$

$$P_{X_2}(t+\Delta t) = P_{X_2}(t)\left[1 - X_2(\mu+\pi)\Delta t + o(\Delta t)\right] + P_{X_2-1}(t)\left[(X_2-1)\lambda_{13}\Delta t + o(\Delta t)\right] + P_{X_2+1}(t)\left[(X_2+1)\mu\Delta t + o(\Delta t)\right] \tag{4}$$

$$P_{X_3}\left(t+\Delta t\right)=P_{X_3}(t)\left[1-X_3\left(\mu+\lambda_{12}\right)\Delta t+o\left(\Delta t\right)\right]+P_{X_3-1}(t)\left[\left(X_3-1\right)\lambda_{23}\Delta t+o\left(\Delta t\right)\right]$$
$$+P_{X_3+1}(t)\left[\left(X_3+1\right)\mu\Delta t+o\left(\Delta t\right)\right] \qquad (5)$$

$$P_{X\,4}\left(t+\Delta t\right)=P_{X_4}(t)\left[1-X_4\left(\mu+\lambda_{12}\right)\Delta t+o\left(\Delta t\right)\right]+P_{X_4-1}(t)\left[\left(X_4-1\right)\lambda_{24}\Delta t+o\left(\Delta t\right)\right]$$
$$+P_{X_4+1}(t)\left[\left(X_4+1\right)\mu\Delta t+o\left(\Delta t\right)\right] \qquad (6)$$

$$P_{X_5}\left(t+\Delta t\right)=P_{X_5}(t)\left[1-X_5\left(\mu+\lambda_{35}\right)\Delta t+o\left(\Delta t\right)\right]+P_{X_5-1}(t)\left[\left(X_5-1\right)\lambda_{45}\Delta t+o\left(\Delta t\right)\right]$$
$$+P_{X_5+1}(t)\left[\left(X_5+1\right)\mu\Delta t+o\left(\Delta t\right)\right] \qquad (7)$$

Equations (3) to (7) can be differentiated using equation (2) and obtained the sets of difference differential equations for the dynamics of coronavirus in our model known as Kolmogorov forward differential equations as below:

$$\frac{dP_{X_1}(t)}{dt}=\left(X_1-1\right)\lambda_{12}P_{X_1-1}(t)+\left(X_1+1\right)\mu P_{X_1+1}(t)-X_1\left(\mu+\pi\right)P_{X_1}(t) \qquad (1)$$

$$\frac{dP_{X_2}(t)}{dt}=\left(X_2-1\right)\lambda_{13}P_{X_2-1}(t)+\left(X_2+1\right)\mu P_{X_2+1}(t)-X_2\left(\mu+\pi\right)P_{X_2}(t) \qquad (2)$$

$$\frac{dP_{X_3}(t)}{dt}=\left(X_3-1\right)\lambda_{23}P_{X_3-1}(t)+\left(X_3+1\right)\mu P_{X_3+1}(t)-X_3\left(\mu+\lambda_{12}\right)P_{X_3}(t) \qquad (3)$$

$$\frac{dP_{X_4}(t)}{dt}=\left(X_4-1\right)\lambda_{24}P_{X_4-1}(t)+\left(X_4+1\right)\mu P_{X_4+1}(t)-X_4\left(\mu+\lambda_{12}\right)P_{X_4}(t) \qquad (4)$$

$$\frac{dP_{X_5}(t)}{dt}=\left(X_5-1\right)\lambda_{45}P_{X_5-1}(t)+\left(X_5+1\right)\mu P_{X_5+1}(t)-X_5\left(\mu+\lambda_{35}\right)P_{X_5}(t) \qquad (5)$$

To obtain the probability distribution that describes the dynamics of coronavirus (COVID-19) in a population over time, we use equation (8) to (12). Generally, using probability generating function, we multiply equation (3) by z^k, where z is a dummy variable in the range $0\le z\le 1$, and add over k as $G\left(z,t\right)=\sum_{k=0}^{\infty}z^kP_k(t)$, we have the Lagrange partial differential equation as

$$\frac{\partial G}{\partial t}=-\lambda z\frac{\partial G}{\partial z}-\mu z\frac{\partial G}{\partial z}+\lambda z^2\frac{\partial G}{\partial t}+\mu\frac{\partial G}{\partial z}=\frac{\partial G}{\partial z}\left(\lambda z-\mu\right)\left(z-1\right) \qquad (13)$$

Therefore, probability generation function (pgf) is a solution of equation (13); and similarly, $M\left(\theta,t\right)=\sum_{k=0}^{\infty}e^{\theta k}P_k(t)$, we have

$$\frac{\partial M}{\partial t}=\frac{\partial M}{\partial t}\left[(\mu e^{-\theta}-\mu)+(\lambda e^{-\theta}-\lambda)\right] \qquad (14)$$

The moment generating function is solution of equation (14). To solve our equation (13), we set up an auxiliary equation below

$$\frac{dt}{1}=\frac{dz}{\left(1-z\right)\left(\lambda z-\mu\right)}=\frac{dG}{0},\ \frac{dt}{1}=\frac{dz}{\left(1-z\right)\left(\lambda z-\mu\right)}$$

$$\left(\lambda-\mu\right)dt=\frac{dz}{\left(1-z\right)}+\frac{\lambda dz}{\left(\lambda z-\mu\right)},\lambda\neq\mu \tag{15}$$

Equation (15) integrates to give $k_1=\left((\lambda z-\mu)/(1-z)\right)e^{(\mu-\lambda)\theta}$ and $k_2=G$. To obtain the general solution is to set $k_2=\psi\left(k_1\right)$, where $\psi(.)$ is a function to be determined by the boundary condition, $N(0)=n$.

Thus, $G(z,t)=\psi\left(\left(\frac{\lambda z-\mu}{1-z}\right)e^{(\mu-\lambda)t}\right)$, $t=0$ and $z^n=\psi\left(\frac{\lambda z-\mu}{1-z}\right)$, by letting $\omega=\left(\frac{\lambda z-\mu}{1-z}\right)$, we have $\psi(\omega)=\left(\frac{\mu+\omega}{\lambda+\omega}\right)^n$. Hence,

$$G(z,t)=\left(\frac{\mu(1-z)+(\lambda z-\mu)e^{(\mu-\lambda)t}}{\lambda(1-z)+(\lambda z-\mu)e^{(\mu-\lambda)t}}\right)^n,\ \lambda\neq\mu. \tag{16}$$

The general probability solution can be obtained by expanding equation (16) as a power series in z.

For $n=1$, (first term)

$$P_j(t)=\left(\frac{\lambda}{\lambda+\mu\phi}\right)\left(1-\left(\frac{\lambda\phi}{\lambda+\mu\phi}\right)\right)\left(\frac{\lambda\phi}{\lambda+\mu\phi}\right)^{j-1},\ j\geq 1\ \&\ P_0(t)=\left(\frac{\mu\phi}{\lambda+\mu\phi}\right),\ j=0, \tag{17}$$

where $\phi=\left(\frac{\lambda}{\lambda-\mu}\right)\left(1-e^{(\mu-\lambda)t}\right)$ and $\lambda\neq\mu.$

For $n\geq 2,$

$$P_j(t)=\sum_{i=0}^{\min(n,j)}\binom{n}{j}\binom{n+j+i-1}{n-1}\alpha^{n-1}\beta^{j-1}\left(1-\beta-\alpha\right)^i,\ j\geq 1\ \&\ P_0(t)=\alpha^n,\ j=0, \tag{18}$$

where $\alpha=\frac{\mu\left(1-e^{(\mu-\lambda)t}\right)}{\left(\lambda-\mu e^{(\mu-\lambda)t}\right)}$ and $\beta=\frac{\lambda\left(1-e^{(\mu-\lambda)t}\right)}{\left(\lambda-\mu e^{(\mu-\lambda)t}\right)}$, $\lambda\neq\mu,\ \alpha,\beta>0.$

3. Numerical simulation of the model

We perform numerical simulations of our model using values obtained from Table 2. The sample paths resulting from our stochastic model were also shown. To validate the model and to determine which parameters are significant to the disease spread, we carried out sensitivity analysis. Lastly, we obtained the probability distribution describing the transition probabilities of our model.

4. Result and discussion

The simulation was conducted for the model in Figure 1 using data on coronavirus (COVID-19) cases obtained from the situation room of Nigeria Center for Disease Control (NCDC) and parameter values in Table 2 was used to plot the sample paths of the model using simulation. The plots of the model include the Susceptible, Exposed, Quarantine, Infectious and Recovery with and without vaccination.

Figure (2-4) shows individual in Susceptible, Exposed and Quarantine without vaccination via the initial conditions of the parameters in Table 2. It was observed that the population of Susceptible increases from 5th to 10th days as a result of interaction between infected cases of coronavirus to Exposed cases in a population and also the Quarantine cases rises because of the interactions between the infected. The susceptible begin to rise up to 5th day and decrease slowly at 10th day. Figure (7-9) shows individual in Susceptible, Exposed and Quarantine with vaccination which contradict the lack of vaccination cases. It was observed that both the population of Susceptible and exposed cases decreases at 20th day while that of quarantine cases maintain some stability from 1 to 50 day and decreases to 100th days as a result of vaccination impact. Figure (5-6) shows the plot of infected and recovery cases, the infected cases decline and the recovery cases increases due intervention given among susceptible, exposed and quarantine individuals. Figure 10 represent Confluent Hypergeometric probability distribution, which describes a probability distribution associated with coronavirus in our model using Kummer function. It is a simulation of equation (18) which shows an exponential decay over time. As the time increase, the probability coronavirus incidents decline to zero.

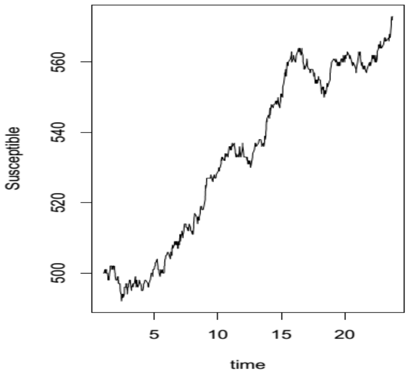

Figure 2. Plot showing sample of
non-vaccinated Susceptible individual

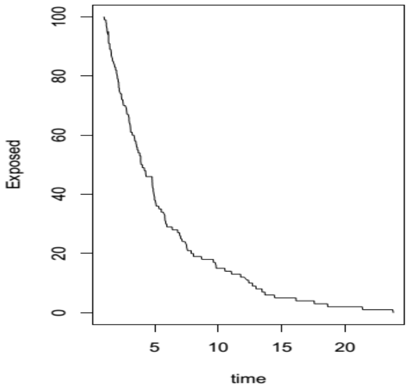

Figure 3. Plot showing sample of
non-vaccinated Exposed individual.

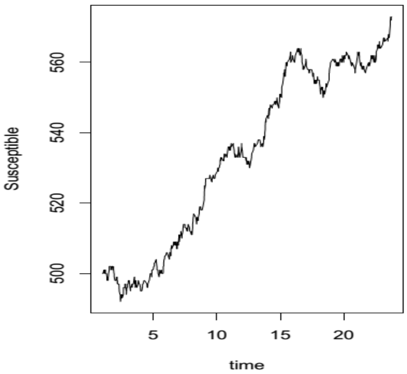

Figure 4. Plot showing sample of
non-vaccinated Susceptible individual

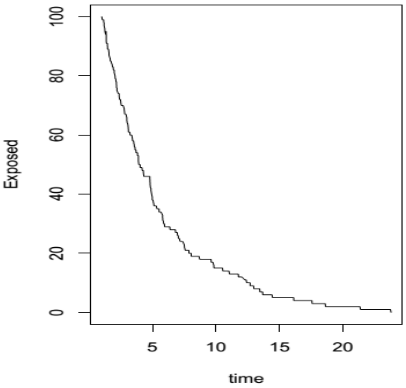

Figure 5. Plot showing sample of
non-vaccinated Exposed individual.

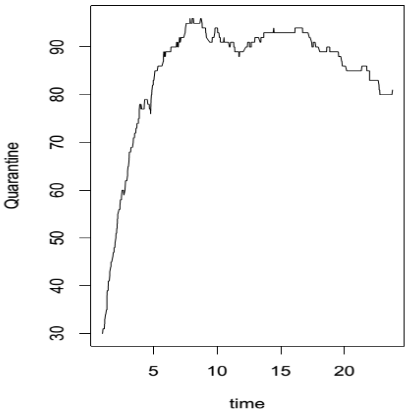

Figure 6. Plot showing sample of non Vaccinated quarantine.

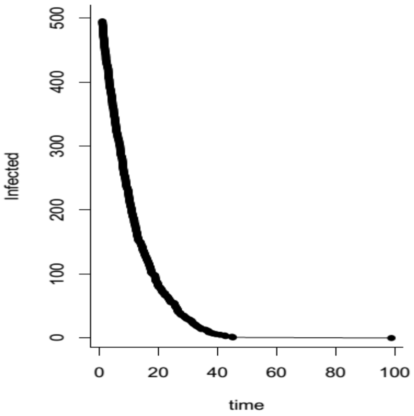

Figure 7. Plot showing sample of infected case.

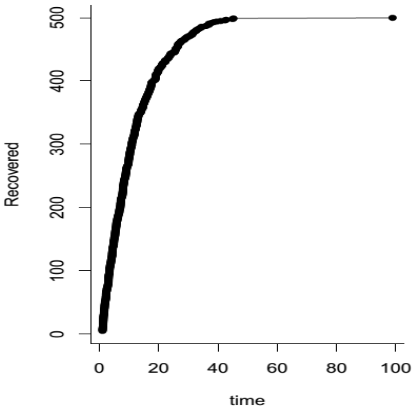

Figure 8. Plot showing sample path of recovery individuals.

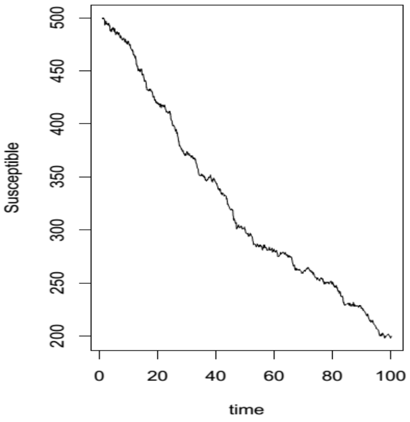

Figure 9. Plot showing sample path of Susceptible vaccinated individual $(v = 0.2)$.

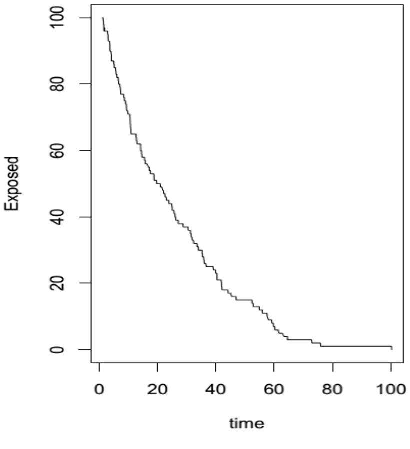

Figure 10. Plot showing sample path of Vaccinated Exposed $(v = 0.2)$.

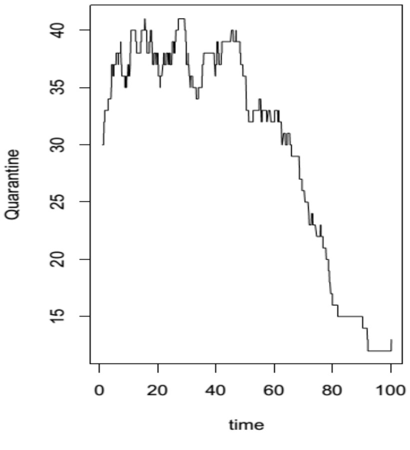

Figure 11. Plot showing sample path of Vaccinated Quarantine $(v = 0.2)$.

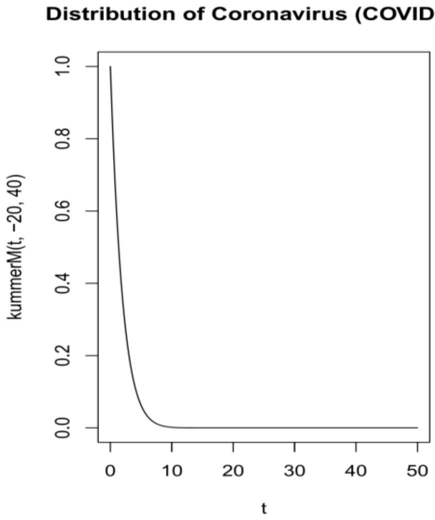

Figure 12. Confluent Hypergeometric distribution of COVID-19 model using Kummer function.

5. Conclusion

We proposed and analyzed a stochastic model for the continue spread of corona virus (COVID-19) dynamics considering the impact of vaccination among susceptible, exposed and quarantine cases. The sets of difference and difference-differential equations for the dynamics of (COVID-19) were derived and simulated with available parameters using stochastic simulation. The study adopt continues – time Birth and Death process and the probability distribution describing the transitions probabilities dynamics of coronavirus was obtained. We therefore, concluded that vaccination has 20% impact on susceptible, exposed and quarantine cases.

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