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Two methods to obtain preferred efficiency for negative data (IS)

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Abstract. The original DEA models were applicable only to technologies characterized by positive inputs/outputs. We consider the interval scale (IS) variables especially when the IS variable is a difference of two different variables (like sales etc.) have been used as inputs and/or outputs. We measure Preferred Efficiency (PE) in Data Envelopment Analysis (DEA) with negative data when these data derived from IS variables. The PE is an efficiency concept that takes into account the decision maker's (DM) preferences. We search the Most Preferred combination of inputs and outputs of Decision Making Units (DMUs) which are efficient in DEA. Also, we approximate indifference contour of the unknown Preferred Function (PF) at Most Preference Solution (MPS) with supporting hyper plane on PPS at MPS. We propose a way to obtain this the supporting hyper plane and also assume this the hyper plane is tangent on the indifference contour of PF. We use from the radial DEA problems with Variable Returns to Scale (VRS) (BCC models) at the combination orientation (both outputs are maximized and inputs are minimized). Also, We decompose each IS variable into two Ratio Scale (RS) variables and then utilizing from a compromise solution approach generate Common Weights (CW) for the decomposed input/output variables. In other to, we will introduce an MOLP model which its objective functions are input/output variables subject to the defining constraints of production possibility set (PPS) of DEA models. Lastly, the procedure and the resulting PE scores are applicable to solving practical problems by the mentioned model.

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1. Introduction

The basic DEA was considered an advantage of DEA that no preference information is needed. But it is possible to incorporate into the analysis the DM's judgments. To incorporate the DM's preferences into efficiency analysis, developed by Halme et al. (1998), Korhonen et al (2002), Joro et al (2003), on the interpretation of PE by Korhonen et al (2005) and also the improve estimate of PE by Zohrehbandian (2011). We deal with the negative data which derived often from observations of variables measured on the IS. In many applications from DEA, the IS variables like profit and changes in different variables (like sales, loans etc.) have been used as inputs and/or outputs. Data on the IS does not allow division (since the zero point is not defined and only distances can be

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calculated, Halme et al (2002)). The approach of Halme et al.'s measuring PE of each DMU as a distance to an approximated indifference contour of a DM's VF at MPS. The different ways exist for obtain a MPS. A simple way for introduction a MPS is to first compute the technical efficiency of the unit after decomposition the IS variables, and then to make the choice from the set of efficient units. If the number of efficient units is large, the DM may need some to pick his/her MPS from this set. We approximate of the indifference contour of the unknown PF at MPS by the supporting hyper plane and then calculating the PE scores for each DMU in the selected direction by comparing the inefficient units to units having the same value as the MPS. We use from the dual the proposed radial models for the IS data by Halme et al. (which the proposed procedure by them maintain the applicability of the radial model after the decomposing IS variables) for introduce supporting hyper plane which approximate the indifference contour at MPS. The PE scores are calculated for each DMU, in output direction without solving any linear programming problems, comparing the inefficient units having the same value as the MPS. The proposed method in this paper doesn't worse from the method of Halme et al. and dependence to supporting hyper plane. The rest of this paper is organized as follows. In section 2 we review the IS data and PE analysis. Our estimations to produce a measure of PE scores is discussed in sections 3. Numerical example is presented in section 4 and finally, section 5 draws the conclusive remarks.

2. Basic Concept

The Negative data values were observed frequently. We encountered have been with the variables with negative observations that a result of a deduction of two Ratio Scale (RS) variables. Pastor (1994) lists the following examples of variables in the DEA literature with negative values: rate of growth of gross domestic product per capita, profit and taxes (profit = income - cost). We suggest that the original IS variable should be replaced by the two RS variables. However, even in the case when the values of the variable happen to be positive in the data we strongly suggest the approach among other things for the quite obvious reason that division on the interval scale is not allowed. We explain that who decomposing the IS variables into the RS variables as follows. Assume t inputs among the total of$m$, and s outputs among the total of$p$, have been measured on the IS. Replace each by two RS variables whose difference is the original variable. The new input matrix $X \in R_+^{(m+s)\times n}$ contains first the t new RS input variables originating from the IS input variable (minuends). Next come the s RS variables that originate from the IS output variable (subtrahends). As we arrange the new output variables originating from the IS input variables first (the subtrahends in the difference that corresponds to the IS input variable) and next the new output variables corresponding to the original IS outputs (minuends) for the output matrix$Y \in R_+^{(p+t)\times n}$. The coefficients of the new RS variables are set equal in the dual formulation. Each resulting new constraint in the dual creates a new variable, denoted here byπ, in the primal. The radial combined BCC dual model after decomposing the IS variables into one input and output each is as following:

$$
\begin{aligned}
& min \quad \sum_{i=1}^{m+s} v_i x_{io} - \sum_{r=1}^{p+t} \mu_r y_{ro} + u \qquad s.t. \\
& \sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u \geq 0, \ \ j = 1, \dots, n \\
& \sum_{r=1}^{p+t} \mu_r y_{ro} + \sum_{i=1}^{m+s} v_i x_{io} = 1 \\
& \mu_r - v_i = 0, \ \ i = r = 1, \dots, t+s \\
& \mu_r, v_i \geq 0, \forall r, \forall i
\end{aligned} \tag{1}
$$

The model (1) is the dual of the following radial combined BCC primal model.

$$\begin{array}{ll} max & \sigma \\ s.t. & \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - \pi_r \geq y_{ro},\ \ r = 1, \dots, t+s \\ & \sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} \geq y_{ro},\ \ r = t+s+1, \dots, p+t \\ & \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} - \pi_i \leq x_{io}, i = 1, \dots, t+s \\ & \sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} \leq x_{io},\ \ i = t+s+1, \dots, m+s \\ & \sum_{j=1}^{n} \lambda_j = 1;\ \ \lambda_j \geq 0, j = 1, \dots, n \end{array} \qquad (2)$$

Naturally apart from the above model, input or output oriented models can be considered. If we set $x_{io}, i = 1, \dots, m+s,$ to zero in (2) we get the output oriented formulation. The input oriented model is derived analogously. Note that efficient units remain efficient after the decomposition. The increase of variables in DEA means also in this case that inefficient units may become efficient and in fact only the scores of the inefficient units change (for the more explain see the paper of Halme et al. (1998), Dealing with interval scale data in DEA). The purpose of PE Analysis (PEA) is to assess the efficiency of each unit in relation to the indifference contour of DM's PF passing through the MPS. This assess could be done easily, if we explicitly knew the PF. The idea of PEA is to incorporate the DM's preference information regarding a desirable combination of inputs and outputs into the analysis. The MPS is a (virtual or existing) DMU on the efficient frontier with the most desirable values of inputs and outputs. In practice, the PF is unknown and we cannot characterize the indifference curve precisely but we have to approximate it. Halme et al. (1999) assumed that the DM's (unknown) PF $v(u),\ u = (y, -x)^T$ is pseudo concave, and strictly increasing in u (i.e. strictly increasing in y and strictly decreasing in$x$) and with a maximal value$v(u^*)$, $u^* = (y^*, -x^*)^T \in PPS$, at MPS$u^*$. In the following models point (unit) $g = (g^y, g^x)^T \in PPS$ is preferred inefficient with respect to any strictly increasing pseudo concave PF $v(u),\ u = (y, -x)^T$ with a maximum at pointu^*, if the optimum value Z^* of the following problem is strictly positive:

$$\begin{array}{ll} max & \sigma + \varepsilon(1^T s^+ + 1^T s^-) \\ s.t. & Y\lambda - \sigma w^y - s^+ = g^y \\ & X\lambda + \sigma w^x + s^- = g^x;\ \ F\lambda + \eta = d, \\ & \lambda_j \geq 0,\ \ if\ \lambda_j^* = 0, j = 1, \cdots, n; \\ & \eta_j \geq 0,\ \ if\ \eta_j^* = 0,\ \ j = 1, \cdots, k \end{array} \qquad (3)$$

Where λ^* and η^* correspond to the MPS: $y^* = Y\lambda^*$, $x^* = X\lambda^*$.

$$\begin{array}{ll} min & v^T g^x - \mu^T g^y + \rho^T d \\ s.t. & -\mu^T y_j + v^T x_j + \rho^T F_j = 0, j \in \{j | \lambda_j^* > 0, j = 1, \cdots, n\} \\ & -\mu^T y_j + v^T x_j + \rho^T F_j \geq 0, j \in \{j | \lambda_j^* = 0, j = 1, \cdots, n\} \\ & \mu^T w^y + v^T w^x = 1;\ \ \mu, v \geq \varepsilon 1, \\ & \rho_j = 0,\ \ if\ \ \eta_j^* = 0,\ \ j = 1, \cdots, k \\ & \rho_j \geq 0,\ \ if\ \ \eta_j^* > 0,\ \ j = 1, \cdots, k. \end{array} \qquad (4)$$

The only difference compared with a standard primal DEA model is that some variables of λ, η are allowed to have negative values. This simple modification of the DEA model makes it possible to take into account value judgments in the form of the MPS.

3. Presenting our two Methods

3.1. *By supporting hyper plane (method 1)*

We purpose by the supporting hyper plane on PPS in MPS approximate the indifference contour of unknown PF. By dual of the following model obtain weights of output/input variables as the normal vector of the supporting hyper plane.

$$\begin{aligned} max \quad & \sigma + \varepsilon\left(\textstyle\sum_{i=1}^{m+s} s_i^- + \sum_{r=1}^{p+t} s_r^+\right) \\ s.t. \quad & \textstyle\sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - \pi_r - s_r^+ = y_{ro}, \ r = 1,\dots,t+s \\ & \textstyle\sum_{j=1}^{n} y_{rj}\lambda_j - \sigma y_{ro} - s_r^+ = y_{ro}, \ r = t+s+1,\dots,p+t \\ & \textstyle\sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} - \pi_i + s_i^- = x_{io}, \ i = 1,\dots,t+s \\ & \textstyle\sum_{j=1}^{n} x_{ij}\lambda_j + \sigma x_{io} + s_i^- = x_{io}, \ i = t+s+1,\dots,m+s \\ & \textstyle\sum_{j=1}^{n} \lambda_j = 1 \\ & s_i^-, s_r^+ \geq \varepsilon, \forall i, r; \ \ \varepsilon > 0; \lambda_j \geq 0, \ \text{If} \lambda_j^* = 0, j = 1,\dots,n. \end{aligned} \tag{5}$$

Where λ^* is corresponds to the MPS after the decomposing IS variables. The dual of the above model is as following:

$$\begin{aligned} min \quad & \textstyle\sum_{i=1}^{m+s} v_i x_{io} - \sum_{r=1}^{p+t} \mu_r y_{ro} + u \\ s.t. \quad & \textstyle\sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u = 0, \ j \in \{j|\lambda_j^* > 0, j = 1,\dots,n\} \\ & \textstyle\sum_{i=1}^{m+s} v_i x_{ij} - \sum_{r=1}^{p+t} \mu_r y_{rj} + u \geq 0, \ j \in \{j|\lambda_j^* = 0, j = 1,\dots,n\} \\ & \textstyle\sum_{r=1}^{p+t} \mu_r y_{ro} + \sum_{i=1}^{m+s} v_i x_{io} = 1; \ \mu_r - v_i = 0, \quad i = r = 1,\dots,t+s \\ & \mu_r, v_i \geq \varepsilon, \ \forall r, \forall i; \ \varepsilon > 0. \end{aligned} \tag{6}$$

The obtained hyperplane from the model (6) is tangent on PPS at DMU_j that $j \in \{j|\lambda_j^* > 0, j = 1,\dots,n\}$ and in fact these are the reference DMUs of MPS. Since the MPS is efficient, so set on the efficient frontier and usually the set $\{j|\lambda_j^* > 0, j = 1,\dots,n\}$ is including only from MPS. ‹Therefore, this hyper plane passes through MPS. The first, we obtain MPS. For its finding to compute the technical efficiency of each DMU (after the decomposing IS variables) and pick out MPS among the efficient DMUs by aid DM. We want to approximate the value θ such that$(x_j, y_j) + \theta(W^x, W^y) = (\bar{x}, \bar{y})$, where $(\bar{x}, \bar{y})$ is the projected point of DMU_j on the indifference contour PF at MPS which we utilizing the supporting hyper plane at MPS instead of it. We use from the model (6) and suppose that (v^*, μ^*, u^*) is its optimal solution. So the equation of the supporting hyper plane of PF at MPS is as $v^{*T}x - \mu^{*T}y + u^* = 0$. Hence, we have: $v^{*t}(x_j + \theta W^x) - \mu^{*t}(y_j + \theta W^y) + u^* = 0$. In other words, $\theta = -\frac{v^{*t}x_j - \mu^{*t}y_j + u^*}{v^{*t}W^* - \mu^{*t}W^y}$. We purpose obtain PE scores only in output orientation. So as to, we use from the output oriented direction$(W^x, W^y) = (0, y_j)$, that thus we have: $\theta = \frac{v^{*t}x_j + u^*}{\mu^{*t}y_j} - 1$. Note that we consider the case when both the two new variable in decomposing the IS variable into two ratio scale variables as objectives and don't consider the case when one of the new variables is non-discretionary by character.

3.2. *By common weights (method 2)*

We can with attention on the model (2) reform the model (3) for the IS variables as follows. It notes that, in the model (4) we put$(g^x, g^y) = (x_0, y_0)$.

$$max \quad \phi$$

$$
\begin{aligned}
s.t.\quad & \sum_{j=1}^{n} y_{rj}\lambda_j - \phi y_{ro} \geq y_{ro},\ r = 1,\dots,s-p \qquad (*)\\
& \sum_{j=1}^{n} y_{rj}\lambda_j - \phi y_{ro} - \pi_r \geq y_{ro},\ r = s-p+1,\dots,s+t \qquad (*)\\
& \sum_{j=1}^{n} x_{ij}\lambda_j \leq x_{io},\ i = 1,\dots,m-t \qquad (7)\\
& \sum_{j=1}^{n} x_{ij}\lambda_j - \pi_i \leq x_{io},\ i = m-t+1,\dots,m+p\\
& \sum_{j=1}^{n} \lambda_j = 1;\ \lambda_j \geq 0,\quad \hat{\lambda}_j = 0, j = 1,\dots,n.
\end{aligned}
$$

We can adjust the constraints (*) in the model (7) as $\sum_{j=1}^{n} y_{rj}\lambda_j \geq (1+\phi)y_{ro}$, $\sum_{j=1}^{n} y_{rj}\lambda_j - \pi_r \geq (1+\phi)y_{ro}$ .By conversion $\phi' = \phi + 1$ and again utilizing from the variable ϕ instead of ϕ' will obtain the following model.

$$
\begin{aligned}
\max\quad & \phi\\
s.t.\quad & \sum_{j=1}^{n} y_{rj}\lambda_j \geq \phi y_{ro},\ r = 1,\dots,s-p\\
& \sum_{j=1}^{n} y_{rj}\lambda_j - \pi_r \geq \phi y_{ro},\ r = s-p+1,\dots,s+t\\
& \sum_{j=1}^{n} x_{ij}\lambda_j \leq x_{io},\ i = 1,\dots,m-t \qquad (8)\\
& \sum_{j=1}^{n} x_{ij}\lambda_j - \pi_i \leq x_{io},\ i = m-t+1,\dots,m+p\\
& \sum_{j=1}^{n} \lambda_j = 1,\quad \lambda_j \geq 0,\ \hat{\lambda}_j = 0, j = 1,\dots,n
\end{aligned}
$$

Where $\hat{\lambda}$ is corresponds to the MPS ($\hat{y} = Y\hat{\lambda},\ \hat{x} = X\hat{\lambda}$). The dual of the model (8) is as following:

$$
\begin{aligned}
\min\quad & v^T x_o + u\\
s.t.\quad & -\mu^T y_j + v^T x_j + u = 0, j \in \{j|\hat{\lambda}_j > 0, j = 1,\cdots,n\}\\
& -\mu^T y_j + v^T x_j + u \geq 0, j \in \{j|\hat{\lambda}_j = 0, j = 1,\cdots,n\} \qquad (9)\\
& \mu_r - v_i = 0,\quad r = s-p+1,\dots,s+t,\ i = r + (m+p) - (s+t)\\
& \mu^T y_o = 1;\ \mu, v \geq 0.
\end{aligned}
$$

Where μ_r and v_i are the weights to be applied to the outputs and inputs, respectively. The optimum solution of the above problem, say $(-v^*, \mu^*, u^*)$ is associated to the normal vector of a supporting hyper plane that constants the PPS in only one of the half spaces and pass among $DMU_j s$ that $j \in \{j|\hat{\lambda}_j > 0, j = 1,\cdots,n\}$ and the MPS. Our aim is introducing an MOLP for finding CW which by an its efficient solution we obtain a the tangent hyper plane at MPS for approximate the indifference contour of the unknown PF. Firstly, we is introduced the following model (corresponding toDMU_o).

$$
\begin{aligned}
\min\quad & v^T x_o - \phi_o^*(\mu^T y_o) + u\\
s.t.\quad & -\phi_j^*(\mu^T y_j) + v^T x_j + u = 0, j \in \{j|\hat{\lambda}_j > 0, j = 1,\cdots,n\}\\
& -\phi_j^*(\mu^T y_j) + v^T x_j + u \geq 0, j \in \{j|\hat{\lambda}_j = 0, j = 1,\cdots,n\} \qquad (10)\\
& \mu_r - v_i = 0,\quad r = s-p+1,\dots,s+t,\ i = r + (m+p) - (s+t)\\
& \sum_{i=1}^{m+p} v_i + \sum_{r=1}^{s+t} \mu_r = 1, (*);\quad \mu, v \geq 0.
\end{aligned}
$$

Where $\phi_j^*, j = 1,\dots,n$ is optimum value obtained from the model (7), when DMU_j is under consideration. It notes that the MPS is on the efficient frontier and also is the most preference solution, so, usually, we have $\{j|\hat{\lambda}_j > 0, j = 1,\dots,n\} = \{DMU_{MPS}\}$. Effect of the constraint (*) in the model (10) is to have normalized weights and uniqueness of optimal solution. We introduce the following MOLP problem for the identification of CW.

$$\begin{array}{ll} min & \{v^T x_1 - \phi_1^*(\mu^T y_1) + u, \cdots, v^T x_n - \phi_n^*(\mu^T y_n) + u\} \\ s.t. & \text{[Precisely the constraints of model (10)]} \end{array} \quad (11)$$

So as to solve the MOLP model (11) we use the compromise programming to generate a vector of deviation scores closest to the scores computed from model (10). Thus, a vector of zero scores is considered as a ideal solution. The mathematical programming is as below and called compromise solution with parameter$p$.

$$\begin{array}{ll} min & \left[\sum_{j=1}^{n} w_j \left(\left(v^T x_j - \phi_j^*(\mu^T y_j) + u\right) - 0\right)^p\right]^{\frac{1}{p}}, p \geq 1 \\ s.t. & \text{[Precisely the constraints of model (10)]} \end{array} \quad (12)$$

Here, we set$p = \infty$, and then the above model converts to the weights mini-max problem that its linear model is as follows.

$$\begin{array}{l} min \;\; f \\ s.t. \;\; w_j\left(v^T x_j - \phi_j^*(\mu^T y_j) + u\right) \leq f, j = 1, \dots, n \quad (13) \\ \text{[The remainder constraints are precisely the constraints of model (10)]} \end{array}$$

Solving model (13) gives us a CW and then we obtain the tangent hyper plan ${v^*}^T x - {\mu^*}^T y + u^* = 0$ at MPS which is as approximation of the indifference contour of (unknown) PF. We purpose to measure the value θ that$(x_j, y_j) + \theta(W^x, W^y) = (\bar{x}, \bar{y})$, where $(\bar{x}, \bar{y})$ is the projected point of DMU_j on the indifference contour PF at MPS. As mentioned already, since PF is unknown we can use the tangent hyper plane instead of its indifference contour. Hence, we must be have: ${v^*}^t(x_j + \theta W^x) - {\mu^*}^t(y_j + \theta W^y) + u^* = 0$. We want measure PE scores only in output orientation. So as to, we use from the output oriented direction $(W^x, W^y) = (0, y_j)$, that thus we have $\theta = \frac{{v^*}^t x_j + u^*}{{\mu^*}^t y_j} - 1$. Note that we consider the case when both the two new variable in decomposing the IS variable into two ratio scale variables as objectives and don't consider the case when one of the new variables is non-discretionary by character.

4. Numerical Example

In this section, we use the data recorded in table 1 to illustrate how the approach revised in this work perform. The VE of 14 units each consuming one input to produce two outputs in to be assessed. Some of the DMUs have negative data. In fact, this is a consequence of the IS output variable$O2(O2 = y - x)$.

Table1. Their input/output variable values.

$DMUs$	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**
$I1$	50	48	49	49	48	50	47	47	45	48	47	35	19	23
$O1$	58	48	45	35	34	25	25	25	16	15	14	13	4	4
$O2$ y x	-16 38 54	-17 32 49	-6 33 39	5 36 31	4 35 31	-12 19 31	3 31 28	-14 26 40	2 30 28	-4 31 35	1 21 20	1 19 18	3 7 4	-5 6 11

We need that decomposing the IS output variable $O2$ into two RS variables which $O2$ generated by difference two the RS values. For this example we have:$p = 2$,$m = 1$, $t =$

$0,\ s = 1$.

Table 2. The new variables values after decomposing$O2$.

$DMUs$	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**G**	**K**	**L**	**M**	**N**
x_1	54	49	39	31	31	31	28	40	28	35	20	18	4	11
x_2	50	48	49	49	48	50	47	47	45	48	47	35	19	23
y_1	38	32	33	36	35	19	31	26	30	31	21	19	7	6
y_2	58	48	45	35	34	25	25	25	16	15	14	13	4	4

Estimation 1: To compute the technical efficiency for each DMU with the new variables values after decomposing, utilizing from the data of table 2. Each units A, E, and M is efficient. We picking the unite A as MPS and$\lambda_A^* = 1$. Both variables y_1, x_1 are objectives. The variable x_1 can be viewed either as output or input. We considered that as input. For finding output/input weights, we use from the following model:

$$\begin{aligned} min \quad & 54v_1 + 50v_2 - 38\mu_1 - 58\mu_2 + u \\ s.t. \quad & 54v_1 + 50v_2 - 38\mu_1 - 58\mu_2 + u = 0 \\ & 49v_1 + 48v_2 - 32\mu_1 - 48\mu_2 + u \geq 0 \\ & \vdots \\ & 11v_1 + 23v_2 - 6\mu_1 - 4\mu_2 + u \geq 0 \\ & 54v_1 + 50v_2 + 38\mu_1 + 58\mu_2 = 1 \\ & -v_1 + \mu_1 = 0; \quad v_1, v_2, \mu_1, \mu_2 \geq \varepsilon, \varepsilon > 0. \end{aligned} \tag{17}$$

The obtained weights are $v_1^* = 0.0050,\ v_2^* = 0.0053,\ \mu_1^* = 0.0050,\ \mu_2^* = 0.0048,\ u^* = -0.0066$.

Estimation 2: The first, we must be choice a MPS. To calculate the technical efficiency for each DMU with the new variables values after decomposing, utilizing from the data of table 2. The units A, D, and M are efficient. We picking the unite M as MPS and$\lambda_M^* = 1$. The variable x_2 can be viewed either as output or input. We considered that as input. Solving the compromise model we obtain an efficient solution as follows, which this is a CW for output/input variables:$v_1^* = 0.1992, v_2^* = 0.3141,\ \mu_1^* = 0.1725,\ \mu_2^* = 0.3141$, $u^* = -2.1529$.

Table 3. Obtained PE scores.

$DMUs$	**The Halme et al.**	**Estimation1**	**Estimation2**
A	0.0000	0.0000	0.1288
B	0.0477	0.1086	0.2437
C	0.0084	0.01851	0.0953
D	0.0000	0.0000	0.1241
E	0.0010	0.0134	0.0171
F	0.2179	0.6434	0.7066
G	0.0441	0.1724	0.1391
H	0.1303	0.5298	0.5845
I	0.0310	0.3747	0.2810

J	0.0574	o.5977	o.4932
K	0.1934	0.6398	0.4973
L	0.0665	0.3271	0.2756
M	0.0000	0.0000	0.0000
N	0.1586	1.2435	1.2855

5. Conclusion

The main contribution of this paper is introducing the particular ways to estimate preferred efficiency by the supporting hyper plane at MPS which we assume that this is tangent hyperplane of (unknown) preferred function at MPS and by common weights. In other words, by solving a dual model for the obtained data of decomposing of the IS variables found weights for input/output variables. Then the PE scores, produce by simple calculations. We can use from methods that utilizing the original IS variables without decomposing data and the most preference weights for obtain PE. We can use CCR models instead of BCC models, in which case the PPS will change, and certainly the supporting hyper plane and thus the measure of the PE will change. Perhaps using the optimal weights of each decision maker unit instead of the common weights will take us away from the desired PE for each unit, but the decision maker's preferences can help to achieve real performance. Finally, this process can be used to obtain cost efficiency, etc., provided that the cost function is unknown.

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