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Two-Warehouse Inventory Model for Deteriorating Items with Time Dependent Demand and Partial Backlogging Under Inflation

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Abstract. This paper deals with a two-warehouse inventory model for deteriorating items with time dependent demand and partial backlogging under inflation. It is assumed that deterioration of items follows two-parameter Weibull distribution and demand rate varies exponentially with time. Shortages are allowed and partial backlogging depends on waiting time of next replenishment. A numerical example is provided to illustrate the considered model. Further, sensitivity analysis has also been made to show the behavior of the present model.

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1. Introduction

Deterioration of items is one of the key factors whose impact on inventory control problems cannot be ignored. Deterioration is defined as decay, spoilage pilferage and loss of utility that results in decreasing usefulness from original one. Food items, pharmaceuticals, fashionable goods are few examples of the items in which appreciable deterioration can take place during the normal storage period and consequently this loss must be taken into account while analyzing the model. A considerable amount of literature is available regarding inventory models for deteriorating items. Goswami and Chaudhuri [10] presented an EOQ model for deteriorating items considering linear trend in demand. Pal et al. [27] developed an inventory model for deteriorating items with stock dependent demand. Wee [44] studied a deterministic lot-size inventory model for deteriorating items with shortages and a declining market. Mandal and Maiti [22] considered inventory model for deteriorating items with stock dependent demand and shortages. Gupta and Aggarwal [11] analyzed an inventory model assuming time dependent deterioration rate. Sana and Chaudhuri [32] proposed an economic production lot size inventory model for deteriorating items with shortages and variable production rate assuming demand varying linearly with time. Ghosh and Chaudhuri [8] discussed an order level inventory model with shortages assuming deterioration following Weibull distribution and demand varying

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quadratically with time. Deng et al. [5] presented a note on the inventory models for deteriorating items with ramp type demand. He et al. [14] analyzed inventory models for deteriorating items considering the demand rate as a function of the on-hand inventory. Dye and Hsieh [7] discussed an optimal replenishment policy for deteriorating items with effective investment in preservation technology. Recently, Chauhan and Singh [2] developed an inventory model for deteriorating items to reflect the real situation of market for time dependent demand. Inventory model for deteriorating items with stock dependent demand has been analyzed by Tyagi et al. [43] taking variable holding cost and Goel et al. [9]considering shortages. Singh et al. [38] studied an EPQ model for non- instantaneous deteriorating items with demand dependent production rate. They discussed two cases: for the first case, lifetime of produced item was greater than the production stopping time point and for the second case, the lifetime of produced item was less than the production stopping time point.

Most of the inventory models unrealistically assume that during stock-out either all demand is backlogged or all is lost. In reality, often some customers are ready to wait until replenishment, especially if the waiting time is to be short, while others are more impatient and go elsewhere. Keeping this in view, Wee [45] proposed inventory models for deteriorating items with partial backlogging. Papachristos and Skouri [28] presented a partially backlogged inventory model for deteriorating items assuming that the backlogging rate decreases exponentially as the waiting time increases. Skouri and Papachristos [40] analyzed an inventory model for deteriorating items with time varying demand, linear replenishment cost and partial backlogging depending upon time. Zhou et al. [51] presented an inventory model considering time dependent demand and partial backlogging. Skouri et al. [41] developed an inventory model with ramp type demand and partial backlogging of the unsatisfied demand for deteriorating items. Sharma and Singh [35] analyzed an inventory model with partial backlogging for deteriorating items assuming stock and selling price dependent demand rate in fuzzy environment. Mishra et al. [23] discussed an inventory model for time dependent deterioration, demand and holing cost with partial backlogging. Taleizadeh and Pentico [42] developed an EOQ model with all units discount and partial backlogging at constant rate. Dutta and Kumar [3] presented an inventory model assuming time varying demand and holding cost with partial backlogging. Sangal et al. [33] discussed an inventory model with partial backlogging under the effect of fuzzy environment. Rastogi et al. [30] analyzed an inventory model for deteriorating items with price dependent demand, partial backlogging, credit limit policy and cash discount having time varying holding cost. Singh et al. [38] presented a production inventory model for deteriorating items with time dependent demand rate and demand dependent production rate assuming that shortages are allowed and partially backlogged.

The classical inventory models usually assume that the available warehouse has unlimited capacity. But in practice, the capacity of any warehouse is limited. In many practical situations, there exist many factors like temporary price discounts making retailers buy more goods than the capacity of own warehouse (OW). In this case, retailers will either rent other warehouses or construct new warehouses. However, from economical point of view, it is usually opted to rent one or more other warehouses known as rented warehouses (RW). The idea of two-warehouse inventory systems was proposed by Hartely [13] but gained wide popularity in last decade of 20th century. Pakkala and Achary [26] studied the two-warehouse inventory model for deteriorating items with finite replenishment rate and shortages. Zhou [50] presented a two-warehouse model for deteriorating items with time-varying demand and shortages during the finite planning horizon. Lee and Ying [19] developed an optimal inventory policy for two-warehouse inventory model for deteriorating items with time dependent demand. Zhou and Yang [52] analyzed a two-warehouse inventory model with stock dependent demand. Lee [19] discussed an inventory model for two-warehouse for deteriorating items under FIFO dispatching policy. Rong et al. [31] considered a two-warehouse inventory model for deteriorating items with partially and fully backlogged shortages in fuzzy environment. Lee and Hsu [20] presented the two-warehouse production model for deteriorating items with time dependent demand. Panda et al. [25] developed the two-warehouse inventory models with price and stock dependent demand for the single vendor and multiple retailers. Maity [21] studied two-warehouse production inventory problem under fuzzy inequality constraints. Dem and Singh [4] discussed two-warehouse inventory systems for EPQ model with quality consideration. Further, Sharma et al. [36] considered the production model for different demands in a two-warehouse inventory system. Recently, Ranjan and Uthayakumar [29] presented a two-warehouse inventory model for deteriorating items with permissible delay considering demand increasing exponentially with time. Kumar et al. [17] developed two-warehouse inventory model for non-instantaneous deteriorating items with stock-dependent demand to determined optimal replenishment policy.

The effects of inflation and the time value of money cannot be ignored in determining inventory policies. Buzacott [1] was the first researcher to include the concept of inflation in inventory modelling. He developed a minimum cost model for a single item inventory with inflation. In the last few years, many researchers developed inventory models for single warehouse and two-warehouses considering effects of inflation and time value of money. Sarker and Pan [34] assumed a finite replenishment model and analyzed the effects of inflation and time-value of money on order quantity when shortages are allowed. Hariga [12] analyzed the effects of inflation and time-value of money on an inventory model with shortages and time-dependent demand. Moon and Lee [24] discussed an EOQ model with the effects of inflation and time value of money. The effect of inflation in two-warehouse inventory models was first investigated by Yang [49] for deteriorating items with constant demand and shortages. Wee et al. [46] developed two-warehouse inventory model for deteriorating items assuming constant demand rate with constant partial backlogging under effect of inflation. Yang [47] discussed two- warehouse inventory models for deteriorating items considering partial backlogging and effect of inflation. Dey et al. [7] analyzed two-warehouse inventory problems with inflation. Jaggi and Verma [15] developed a two-warehouse inventory model for deteriorating items with linear trend in demand under the inflationary conditions considering constant deterioration rate for both warehouses. Singh et al. [37] discussed a production model with selling price dependent demand and partial backlogging under inflation. Yang and Chang [48] discussed two-warehouse inventory model for deteriorating items assuming partial backlogging and permissible delay in payment under inflation. Khurana [16] developed the two-warehouse inventory model for deteriorating items assuming time dependent demand under inflation. Singh et al. [39] analyzed inventory model for deteriorating items with multivariate demands in different phases, partial backlogging and inflation.

This paper analyzes a two-warehouse inventory model for deteriorating items with time varying demand and partial backlogging under inflation. The rates of deterioration in both warehouses are different and follow a two-parameter Weibull distribution. The demand has been assumed to be exponential function of time. Shortages have been assumed to be exponential function of time and are partially backlogged. A numerical example has been considered to illustrate the model. Further, the effect of various parameters such as deterioration parameters, inflation parameter, backlogging parameter and capacity of own warehouse on present value of total cost per unit time has been investigated. Convexity of the present value of total cost per unit time has been revealed graphically.

2. Assumptions and notations

To develop the mathematical model of the inventory system considered herein, the following assumptions have been made:

1. The inventory system involves only one item.
2. Deterioration of the items follows a two-parameter Weibull distribution.
3. Deterioration occurs as soon as items are received into inventory.
4. There is no replacement or repair of deteriorating items during the period under consideration.
5. Own warehouse has fixed capacity of W units, while rented warehouse has unlimited capacity.
6. The holding cost in RW is higher than that in OW.
7. Lead-time is zero and initial inventory level is zero.
8. The replenishment rate is infinite.
9. Demand rate is known and is equal to ae^{bt}, where a and b ($a > b$) are constants.
10. Shortages are allowed and backlogging rate is $e^{-\delta t}$. The backlogging parameter δ is positive constant and $0 < \delta << 1$.
11. Inflation is considered.
12. T_1 is time for holding inventory in RW and $T_1 + T_2$ is time for holding inventory in OW. Also T_3 is time when shortages occur in OW.
13. $I_i(t_i)$ is inventory level in OW at time t_i, $0 \le t_i \le T_i$, $i = 1,2,3$ and $I_r(t_1)$ is inventory level in RW at time t_1, $0 \le t_1 \le T_1$.

In addition, the following notations have been used throughout the paper,

W	Capacity of OW
I_r	Maximum inventory level in RW
α, β	Deterioration parameter for OW
g, h	Deterioration parameter for RW
r	Inflation parameter
c	Purchasing cost per item
c_o	Ordering cost per order
c_{oh}	Holding cost per item per unit time in OW
c_{rh}	Holding cost per item per unit time in RW

3. Network system

The OW inventory system has been shown in Figure 1 and can be divided into three phases depicted by T_1 to T_3. Figure 2 shows the RW inventory system. First W units of items are stored in the OW and then rest are dispatched to the RW. Therefore RW is utilized only after OW is full, but stocks in RW are consumed first. Stock in the RW depletes due to demand and deterioration until it reaches zero at $t = T_1$. During that time, the inventory in OW decreases due to deterioration only. The stock in OW depletes due to the combined effect of demand and deterioration during time T_2. Both warehouses are empty during the shortage time T_3 and part of the shortage is backlogged in the next replenishment.

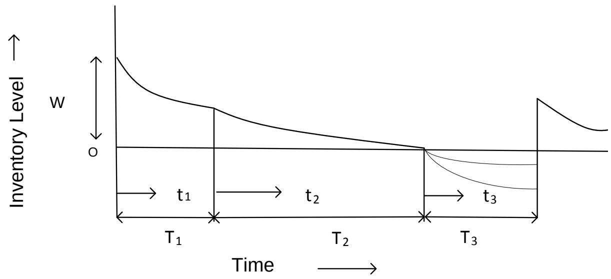

Figure 1. The OW inventory system.

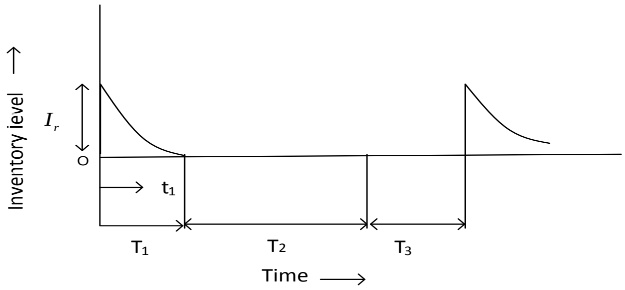

Figure 2. The RW inventory system.

OW inventory system can be represented by the following differential equations:

$$\frac{dI_1(t_1)}{dt_1} = -\alpha\,\beta\,t_1^{\beta-1} I_1\left(t_1\right), \qquad 0 \le t_1 \le T_1, \tag{1}$$

$$\frac{dI_2(t_2)}{dt_2} = -ae^{bt_2} - \alpha\,\beta\,t_2^{\beta-1} I_2\left(t_2\right), \qquad 0 \le t_2 \le T_2, \tag{2}$$

$$\frac{dI_3(t_3)}{dt_3} = -ae^{bt_3}e^{-\delta t_3} = -ae^{(b-\delta)t_3}, \qquad 0 \le t_3 \le T_3, \tag{3}$$

with boundary conditions $I_1(0) = W$, $I_1(T_1) = I_2(0)$ and $I_3(0) = 0$. The solution of differential equations (1), (2) and (3) leads to

$$I_1\left(t_1\right) = We^{-\alpha t_1^{\beta}}, \qquad 0 \le t_1 \le T_1, \tag{4}$$

$$I_2\left(t_2\right) = \frac{We^{-\alpha T_1^{\beta}} - a\int_0^{t_2} e^{bu+\alpha u^{\beta}}du}{e^{\alpha t_2^{\beta}}}, \qquad 0 \le t_2 \le T_2, \tag{5}$$

$$I_3\left(t_3\right) = \frac{a}{b-\delta}\left(1 - e^{(b-\delta)t_3}\right), \qquad 0 \le t_3 \le T_3. \tag{6}$$

The RW inventory system can be represented by the following differential equation:

$$\frac{dI_r(t_1)}{dt_1} = -ae^{bt_1} - ght_1^{h-1}I_r(t_1), \qquad 0 \le t_1 \le T_1, \tag{7}$$

with boundary condition $I_r(0) = I_r$. The solution of differential equation (7) provides

$$I_r(t_1) = \frac{I_r - a\int_0^{t_1} e^{bu+gu^h}du}{e^{gt_1^h}}, \qquad 0 \le t_1 \le T_1, \tag{8}$$

where, $I_r = a\int_0^{T_1} e^{bu+gu^h}du = a\left(T_1 + \frac{bT_1^2}{2} + \frac{gT_1^{h+1}}{h+1}\right).$

The present value of ordering cost

$$OC = c_o. \tag{9}$$

The present value of OW holding cost

$$HD_0 = c_{oh}\left[\int_0^{T_1} I_1(t_1)e^{-rt_1}dt_1 + \int_0^{T_2} I_2(t_2)e^{-r(T_1+t_2)}dt_2\right]$$

$$= c_{oh}\left[\begin{array}{l} W\left(T_1 - \frac{\alpha T_1^{\beta+1}}{\beta+1} - \frac{rT_1^2}{2} + \left(1-\alpha T_1^{\beta} - rT_1\right)T_2\right) + \frac{\left(-W - a + arT_1\right)T_2^2}{2} \\ + \left(ar - \frac{ab}{2} + \frac{abrT_1}{2}\right)\frac{T_2^3}{3} + \frac{abrT_2^4}{8} - \frac{W\alpha T_2^{\beta+1}}{(\beta+1)} + \frac{(a\alpha\beta + a\alpha rT_1)T_2^{\beta+2}}{(\beta+1)(\beta+2)} \\ + \frac{ab\alpha T_2^{\beta+3}}{2(\beta+3)} + \frac{\left(a\alpha^2 + a\alpha r\right)}{2(\beta+1)^2}T_2^{2\beta+2} \end{array}\right]. \tag{10}$$

The present value of OW shortage cost

$$SC = c_s\int_0^{T_3}\left[-I_3(t_3)\right]e^{-r(T_1+T_2+t_3)}dt_3 = c_s a\left[\frac{\left(1-rT_1-rT_2\right)T_3^2}{2} - \frac{rT_3^3}{3}\right]. \tag{11}$$

The present value of OW lost sale cost

$$LSC = c_l\int_0^{T_3}\left(1-e^{-\delta t_3}\right)ae^{bt_3}e^{-r(T_1+T_2+t_3)}dt_3 = c_l a\delta\left[\frac{\left(1-rT_1-rT_2\right)T_3^2}{2} - \frac{(r-b)T_3^3}{3}\right]. \tag{12}$$

The present value of OW item cost

$$IT_0 = c\left(W - I_3(T_3)e^{-r(T_1+T_2+T_3)}\right)$$

$$= c\left(W - \frac{a}{b-\lambda}\left(1-e^{(b-\delta)T_3}\right)e^{-r(T_1+T_2+T_3)}\right) = c\left[W + aT_3(1-r\left(T_1+T_2+T_3\right)\right]. \tag{13}$$

The present value of RW holding cost

$$HD_r = c_{rh}\int_0^{T_1} I_r(t_1)e^{-rt_1}dt_1$$

$$= c_{rh}\, a\left[\begin{array}{l} \dfrac{T_1^2}{2}+\dfrac{bT_1^3}{3}-\dfrac{rT_1^3}{6}-\dfrac{brT_1^4}{8}+\dfrac{ghT_1^{h+2}}{(h+1)(h+2)} \\ -\dfrac{(2bg+(h+1)gr)T_1^{h+3}}{2(h+1)(h+3)}-\dfrac{g^2}{2(h+1)}T_1^{2h+3} \end{array}\right]. \tag{14}$$

The present value of RW item cost

$$I\,T_r = c\left(a\int_0^{T_1} e^{bu+gu^h}\,du\right) = ca\left[T_1+\frac{bT_1^2}{2}+\frac{gT_1^{h+1}}{h+1}\right]. \tag{15}$$

The present value of total cost per unit time during the cycle

$$TUC(T_1,T_2,T_3) = [OC + HD_0 + SC + LSC + IT_0 + HD_r + IT_r]$$

$$= \frac{1}{T}\left[\begin{array}{l} c_o + c_{oh}\left[W\left(T_1-\dfrac{\alpha T_1^{\beta+1}}{\beta+1}-\dfrac{rT_1^2}{2}+\left(1-\alpha T_1^{\beta}-rT_1\right)T_2\right)+\dfrac{(-W-a+arT_1)T_2^2}{2}\right. \\ +\left(ar-\dfrac{ab}{2}+\dfrac{ab\,rT_1}{2}\right)\dfrac{T_2^3}{3}+\dfrac{abrT_2^4}{8}-\dfrac{W\alpha T_2^{\beta+1}}{(\beta+1)}+\dfrac{(a\alpha\beta+a\alpha rT_1)T_2^{\beta+2}}{(\beta+1)(\beta+2)} \\ \left.+\dfrac{ab\alpha T_2^{\beta+3}}{2(\beta+3)}+\dfrac{(a\alpha^2+a\alpha r)T_2^{2\beta+2}}{2(\beta+1)^2}\right]+c_s a\left[\dfrac{(1-rT_1-rT_2)T_3^2}{2}-\dfrac{rT_3^3}{3}\right] \\ +c_l a\delta\left[\dfrac{(1-rT_1-rT_2)T_3^2}{2}-\dfrac{(r-b)T_3^3}{3}\right]+c\left[W+aT_3\left(1-r\left(T_1+T_2+T_3\right)\right)\right] \\ +c_{rh}a\left[\dfrac{T_1^2}{2}+\dfrac{bT_1^3}{3}-\dfrac{rT_1^3}{6}-\dfrac{brT_1^4}{8}+\dfrac{ghT_1^{h+2}}{(h+1)(h+2)}-\dfrac{(2bg+(h+1)gr)T_1^{h+3}}{2(h+1)(h+3)}\right. \\ \left.-\dfrac{g^2}{2(h+1)}T_1^{2h+3}\right]+ca\left(T_1+\dfrac{bT_1^2}{2}+\dfrac{gT_1^{h+1}}{h+1}\right) \end{array}\right]. \tag{16}$$

The present value of total cost per unit time *TUC* is a function of T_1, T_2 and T_3. The objective of the problem is to determine the values of T_1, T_2 and T_3, which minimize *TUC*. The necessary conditions for minimization of *TUC* are

$$\frac{\partial TUC}{\partial T_1}=0,\ \frac{\partial TUC}{\partial T_2}=0 \ \text{ and } \ \frac{\partial TUC}{\partial T_3}=0. \tag{17}$$

The sufficient condition of minimization of total cost per unit time is that *TUC* is convex function or

$$\frac{\partial^2 TUC}{\partial T_1^2}>0,\ \begin{vmatrix} \dfrac{\partial^2 TUC}{\partial T_1^2} & \dfrac{\partial^2 TUC}{\partial T_1\partial T_2} \\ \dfrac{\partial^2 TUC}{\partial T_2\partial T_1} & \dfrac{\partial^2 TUC}{\partial T_2^2} \end{vmatrix}>0 \ \text{ and } \begin{vmatrix} \dfrac{\partial^2 TUC}{\partial T_1^2} & \dfrac{\partial^2 TUC}{\partial T_1\partial T_2} & \dfrac{\partial^2 TUC}{\partial T_1\partial T_3} \\ \dfrac{\partial^2 TUC}{\partial T_2\partial T_1} & \dfrac{\partial^2 TUC}{\partial T_2^2} & \dfrac{\partial^2 TUC}{\partial T_2\partial T_3} \\ \dfrac{\partial^2 TUC}{\partial T_3\partial T_1} & \dfrac{\partial^2 TUC}{\partial T_3\partial T_2} & \dfrac{\partial^2 TUC}{\partial T_3^2} \end{vmatrix}>0. \tag{18}$$

The simultaneous equations (17) can be solved to obtain the values of T_1, T_2 and T_3. The optimal values of T_1, T_2 and T_3 are those values of T_1, T_2 and T_3, which satisfy equation (18) or for which *TUC* is a convex function. Substituting these optimal values in equation (16), the optimal present value of *TUC* is obtained.

4. Numerical Results

By using the methodology given in the preceding section an optimal replenishment policy has been derived to minimize the present value of total cost per unit time. The values of various parameters have been taken from literature in their appropriate units.

Example

The input data for parameters are taken as
a = 1000, b = 0.4, c_o = 100, c_{oh} = 3, c_{rh} = 4.8, c_s = 30, c_l = 18, c = 20, r = 0.08, W = 150, δ = 0.05, α = 0.08, β = 1.9, g = 0.04 and h = 0.9.

By using MATHEMATICA 8.0, the optimal values of T_1, T_2, T_3 and *TUC* have been obtained and are:
T_1^* = 2.12829, T_2^* = 4.51428, T_3^* = 6.07156 and TUC^* = 16106.7.

It is not possible to demonstrate the convexity of total cost per unit time by theoretical results because equation (18) is highly nonlinear function of decision variables. Therefore, the convexity of the total cost per unit time with respect to decision variables has been shown graphically in figures .3-5.

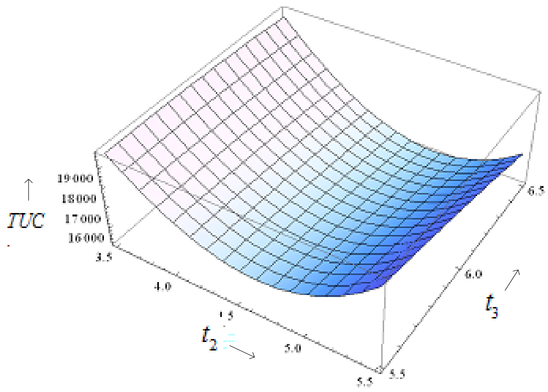

Figure 3. Convexity of total cost per unit time *TUC* for $T_1^* = 2.12829$

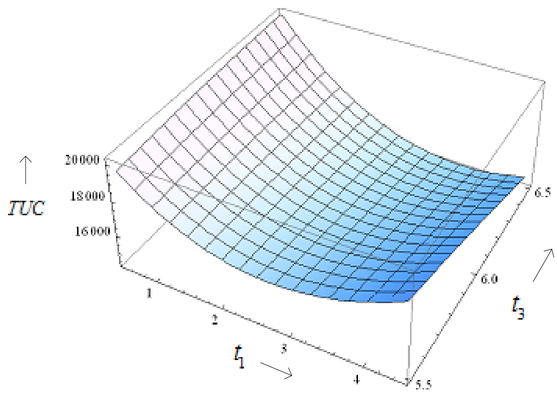

Figure 4. Convexity of total cost per unit time *TUC* for $T_2^* = 4.51428$.

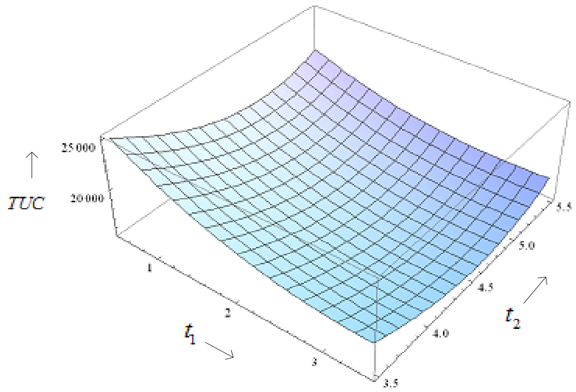

Figure 5. Convexity of total cost per unit time *TUC* for $T_3^* = 6.07156$.

5. Sensitivity analysis

The sensitivity analysis has been performed to study how the changes in parameters affect the optimal solution. The optimal values T_1^*, T_2^*, T_3^* and TUC^* for T_1, T_2, T_3 and TUC, respectively, have been obtained for example give above. The new optimal values T_1^0, T_2^0, T_3^0 and TUC^0 for T_1, T_2, T_3 and TUC, respectively, have been obtained by changing the values of one of model parameters, W, δ, r, α, β, g and h by $\pm$10% and $\pm$20%, at a time and keeping the remaining parameters unchanged. The percentage change in T_1, T_2, T_3 and TUC is given by $\left|\frac{T_1^o - T_1^*}{T_1^*}\right| \times 100$, $\left|\frac{T_2^o - T_2^*}{T_2^*}\right| \times 100$, $\left|\frac{T_3^o - T_3^*}{T_3^*}\right| \times 100$ and $\left|\frac{TUC^0 - TUC^*}{TUC^*}\right| \times 100$, respectively. The results of the sensitivity analysis have been presented in Tables 1 to 7.

Table 1. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when *W* varies.

W	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
120	2.12972	4.50458	6.08112	16153.2	0.0672	0.2149	0.1575	0.2887
135	2.12900	4.50943	6.07634	16130.1	0.0334	0.1074	0.0787	0.1453
165	2.12759	4.51912	6.06677	16083.3	0.0329	0.1072	0.0789	0.1453
180	2.12689	4.52397	6.06192	16057.9	0.0658	0.2147	0.1588	0.3030

Table 2. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when δ varies.

δ	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
0.040	2.04115	4.49902	6.00763	15859.6	4.0944	0.3380	1.0529	1.5341
0.045	2.08456	4.50663	6.03952	15982.9	2.0547	0.1695	0.5277	0.7686
0.055	2.17234	4.52197	6.10375	16231.1	2.0697	0.1703	0.5302	0.7723
0.060	2.21671	4.52971	6.13609	16356.0	4.1545	0.3418	1.0628	1.5478

Table 3. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when *r* varies.

r	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
0.064	3.41582	4.86137	8.11681	22884.4	60.4960	7.6887	33.6857	42.0800
0.072	2.69911	4.67100	6.95645	19063.8	26.8206	3.4716	14.5743	18.3594
0.088	1.66170	4.38282	5.37676	13736.3	21.9232	2.9121	11.4435	14.7169
0.096	1.27229	4.27092	4.81836	11780.8	40.2201	5.3909	20.6405	26.8578

Table 4. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when α varies.

α	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
0.064	1.91389	4.82395	5.96146	14648.2	10.0738	6.8598	1.8134	9.0552
0.072	2.02868	4.65848	6.02002	15428.3	4.6803	3.1943	0.8489	4.2119
0.088	2.21611	4.38684	6.11738	16705.0	4.1263	2.8230	0.7547	3.7146
0.096	2.29450	4.27293	6.15845	17238.6	7.8096	5.3464	1.4311	7.0275

Table 5. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when β varies.

β	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
1.52	1.33965	5.62494	5.69955	11682.3	37.0551	24.6033	6.1271	27.4693
1.71	1.79033	4.99874	5.90225	14157.7	15.8794	10.7317	2.7886	12.1006
2.09	2.39490	4.12802	6.20992	17677.5	12.5270	8.5564	2.2788	9.7525
2.28	2.61411	3.81314	6.32059	18963.3	22.8268	15.5316	4.1016	17.7355

Table 6. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when g varies.

g	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
0.032	2.13598	4.51216	6.06512	16079.2	0.3613	0.0470	0.1061	0.1707
0.036	2.13212	4.51322	6.06835	16093.0	0.1800	0.0235	0.0529	0.0851
0.044	2.12450	4.51532	6.07474	16120.4	0.1781	0.0230	0.0524	0.0851
0.048	2.12074	4.51635	6.07790	16134.0	0.3547	0.0459	0.1044	0.1695

Table 7. Percentage changes in T_1^*, T_2^*, T_3^* and TUC^* when h varies.

h	T_1^0	T_2^0	T_3^0	TUC^0	% change in T_1^*	% change in T_2^*	% change in T_3^*	% change in TUC^*
0.72	2.12884	4.51413	6.07111	16107.0	0.0258	0.0033	0.0074	0.0019
0.81	2.12857	4.51420	6.07133	16106.8	0.0132	0.0018	0.0038	0.0006
1.08	2.12802	4.51435	6.07179	16106.6	0.0127	0.0016	0.0038	0.0006
1.18	2.12774	4.51443	6.07202	16106.5	0.0258	0.0033	0.0076	0.0012

The main conclusions drawn from the sensitivity analysis are as follow:

(1) Table 1 presents the values of percentage changes in T_1^*, T_2^*, T_3^* and TUC^* with respect to capacity of own warehouse *W*. It is seen that T_1^*, T_2^*, T_3^* and TUC are very less sensitive to changes in *W*. The values of percentage changes in T_1^*, T_2^*, T_3^* and TUC^* with respect to inflation parameter δ have been presented in Table 2. It is revealed that T_1^* is fairly sensitive, while T_2^* is very less sensitive to changes in δ. Moreover, T_3^* and TUC^* are less sensitive to changes in δ. Table 3 reflects the values of percentage changes in T_1^*, T_2^*, T_3^* and TUC^* with respect to inflation parameter r. It is observed that T_1^*, T_3^* and TUC^* are highly sensitive, while T_2^* is fairly sensitive to changes in r.

(2) Tables 4 and 5 show the values of percentage changes in T_1^*, T_2^*, T_3^* and TUC^* with respect to deterioration parameters for own warehouses α and β, respectively. It is found that T_1^*, T_2^* and TUC^* are fairly sensitive, whereas, T_3^* is less sensitive to changes in α. In addition, T_1^*, T_2^* and TUC^* are highly sensitive, while T_3^* is fairly sensitive to changes in β. Tables 6 and 7 exhibit the values of percentage changes in T_1^*, T_2^*, T_3^* and TUC^* with respect to deterioration parameters for rented warehouse g and h, respectively. It is seen that T_1^*, T_2^*, T_3^* and TUC^* are very less sensitive to changes in g and h.

The variation of total cost per unit time *TUC* is analysed graphically in Figures 6-12. Graphs have been plotted for various values of parameters to determine the effect of these parameters on total cost per unit time.

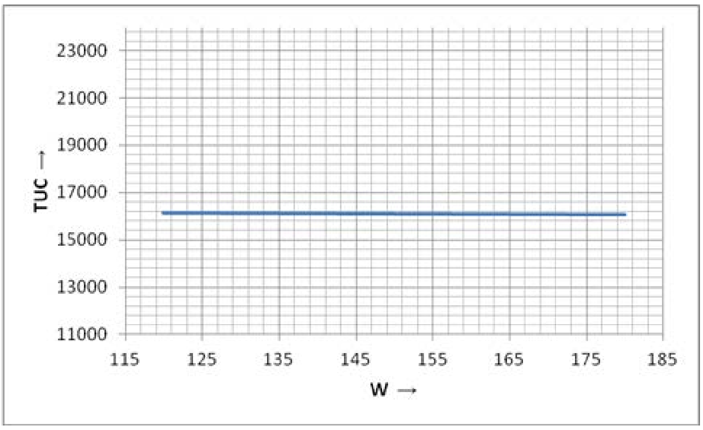

Figure 6. Variation of *TUC* w.r.t. capacity of own warehouse *W*.

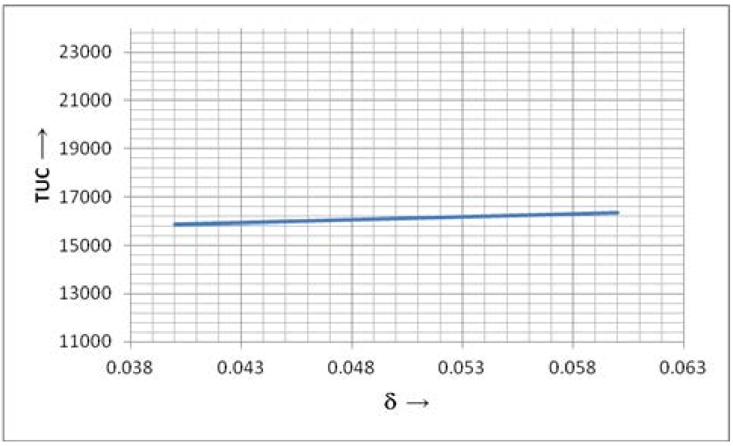

Figure 7. Variation of *TUC* w.r.t. change in backlogging parameter δ.

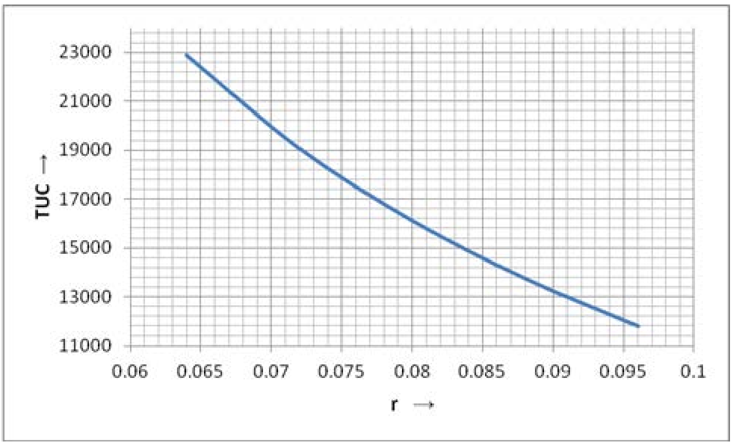

Figure 8. Variation of *TUC* w.r.t. change in inflation parameter r.

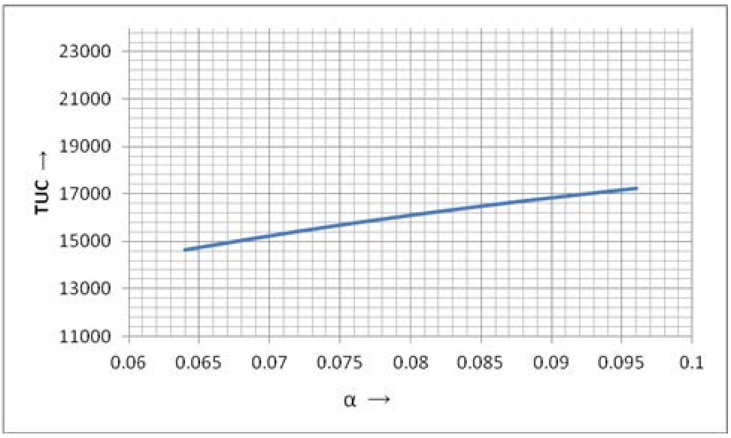

Figure 9. Variation of *TUC* w.r.t. change in deterioration parameter α for own warehouse.

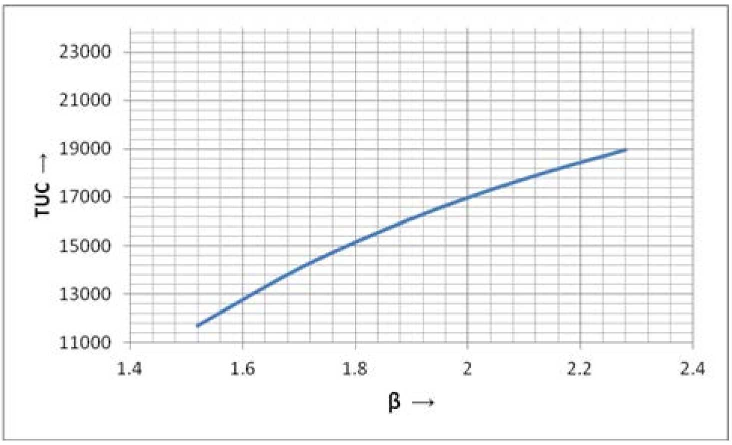

Figure 10. Variation of *TUC* w.r.t. change in deterioration parameter β for own warehouse.

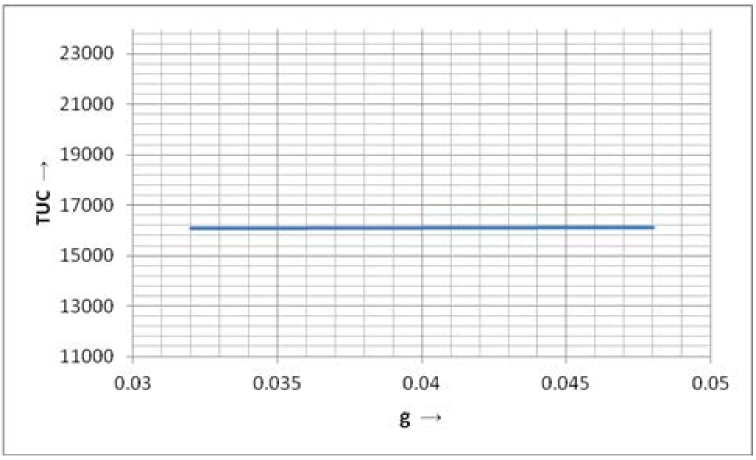

Figure 11. Variation of *TUC* w.r.t. change in deterioration parameter g for rented warehouse.

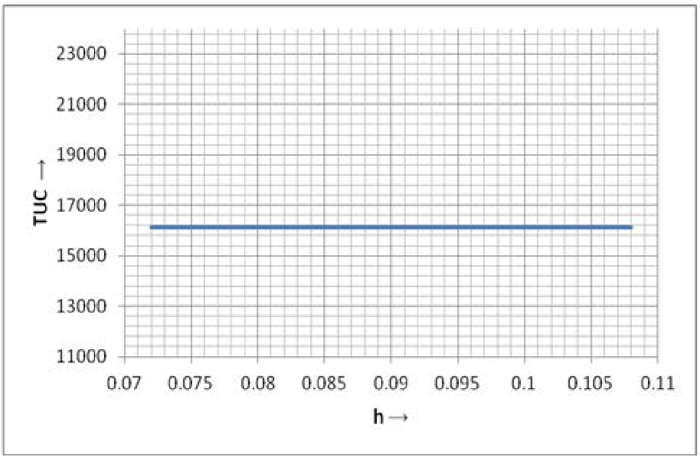

Figure 12. Variation of *TUC* w.r.t. change in deterioration parameter h for rented warehouse.

Figure 6 depicts the effect of capacity of own warehouse *W* on total cost per unit time *TUC* for *W* = 120, 135, 165 and 180. It is observed that *TUC* decreases with increasing values of *W*, but the effect of *W* is very-very low. Figure 7 shows the effect of backlogging parameter δ on total cost per unit time *TUC* for $\delta = 0.04$, 0.045, 0.05, 0.055 and 0.06. It is found that *TUC* increases fairly by increasing δ. The effect of inflation parameter *r* on total cost per unit time *TUC* for $r = 0.064$, 0.072, 0.088 and 0.096 has been depicted by Figure 8. It is seen that *TUC* decreases rapidly with increase in inflation parameter *r*.

Figures 9 and 10 present the graphs of *TUC* versus deterioration parameters of own warehouse α and β, respectively. It is observed that there a considerable increase in total cost per unit time *TUC* with increase of values of α and β. Figures 11 and 12 reflect the variation of total cost per unit time *TUC* with respect to deterioration parameters *g* and *h* for rented warehouse. It is seen that *TUC* increases with increase in *g*, while *TUC* decreases by increasing *h*, but the effect of increase in g is very-very low while that of h is almost negligible.

6. Conclusion

In this study, a two-warehouse inventory model for deteriorating items with time dependent demand and partial backlogging under inflation has been analyzed. The holding cost at rented warehouse is higher as compared to own warehouse. The rates of deterioration in both warehouses are different and follow a two- parameter Weibull distribution. A numerical example has been discussed and optimal values of T_1, T_2, T_3 and present value of total cost per unit time *TUC* have been obtained. Convexity of *TUC* has been shown graphically. Further, sensitivity analysis has also been performed. It has been observed that the present value of total cost per unit time *TUC* can be decreased by decreasing the values of backlogging parameter λ, deterioration parameters α and β of own warehouse and scale parameter of deterioration *g* of rented warehouse and by increasing capacity *W* of own warehouse, inflation parameter *r* and shape parameter of deterioration of rented warehouse. Further, r, α and β affect the present value of total cost per unit time reasonably, while W, λ and *g* do not affect much. The effect of shape parameter of deterioration *h* of RW is negligible. In future research, this model can be extended by considering fuzzy environment, trade credit and stock dependent demand.

References

[1] J. A. Buzacott, Economic order quantities with inflation, Operational Research Quarterly, **26 (3)** (1975) 553-558.

[2] A. Chauhan and A. P. Singh, Optimal replenishment and ordering policy for time dependent demand and deterioration with discounted cash flow analysis, International Journal of Mathematics in Operational Research, **6 (4)** (2014) 407-436.

[3] D. Dutta and P. Kumar, A partial backlogging inventory model for deteriorating items with time-varying demand and holding cost, International Journal of Mathematics in Operational Research, **7 (3)** (2015) 281-296.

[4] H. Dem and S. R. Singh, Two-warehouse production model with quality consideration, Procedia Engineering, **38** (2012) 242–3259.

[5] P. S. Deng, R. H. J. Lin and P. Chu, A note on the inventory models for deteriorating items with ramp type demand, European Journal of Operational Research, **178 (1)** (2007) 112-120.

[6] J. K. Dey, S. K. Mondal and M. Maiti, Two storage inventory problem with dynamic demand and interval valued lead-time over finite time horizon under inflation and time-value of money, European Journal of Operational Research, **185 (1)** (2008) 170-194.

[7] C. Y. Dye and T. P. Hsieh, An optimal replenishment policy for deteriorating items with effective investment in preservation technology, European Journal of Operational Research, **218 (1)** (2012) 106-112.

[8] S. K. Ghosh and K. S. Chaudhuri, An order-level inventory model for deteriorating items with Weibull distribution deterioration, time-quadratic demand and shortages, Advanced Modeling and Optimization, **6 (1)** (2004) 21-35.

[9] R. Goel, P. S. Singh and S. Sharma, Supply chain model with stock dependent demand, quadratic rate of deterioration with allowable shortage, International Journal of Mathematics in Operational Research, **7 (2)**

(2015) 156-177.
[10] A. Goswami and K. S. Chaudhuri, An EOQ model for deteriorating items with a linear trend in demand, Journal of the Operational Research Society, **42 (2)** (1991) 1105-1110.
[11] P. N. Gupta and R. N. Aggarwal, An order level inventory model with time dependent deterioration, Opsearch, **37 (4)** (2000) 351-359.
[12] M. A. Hariga, Effects of inflation and time-value of money on an inventory model with time-dependent demand rate and shortages, European Journal of Operational Research, **81 (3)** (1995) 512-520.
[13] R. V. Hartely, Operations Research—A Managerial Emphasis, Good Year Publishing Company, California, (1976) 315-317.
[14] Y. He, S. Y. Wang and K. K. Lai, An optimal production-inventory model for deteriorating items with multiple-market demand, European Journal of Operational Research, **203 (3)** (2010) 593–600.
[15] C. K. Jaggi and P. Verma, Two-warehouse inventory model for deteriorating items with linear trend in demand and shortages under inflationary conditions, International Journal of Procurement Management, **3 (1)** (2010) 54-71.
[16] D. Khurana, Two warehouse inventory model for deteriorating items with time dependent demand under inflation, International Journal of Computer Applications, **114 (7)** (2015) 975-987.
[17] S. Kumar, D. Chakrabarty and A. K. Malik, A two warehouse inventory model with stock-dependent demand and variable deterioration rate, International Journal of Future Revolution in computer Science and Communication Engineering, **3 (9)** (2017) 20-24.
[18] C. Lee and C. Ying, Optimal inventory policy for deteriorating items with two- warehouse and time dependent demands, Production Planning and Control, **11 (7)** (2000) 689-696.
[19] C. C. Lee, Two- warehouse inventory model with deterioration under FIFO dispatching policy, European Journal of Operational Research, **174 (2)** (2006) 861-873.
[20] C. C. Lee and S. L. Hsu, A two- warehouse production model for deteriorating inventory items with time–dependent demands, European Journal of Operational Research, **194 (3)** (2009) 700-710.
[21] K. Maity, Possibility and necessity representations of fuzzy inequality and its application to two warehouse production inventory problem, Applied Mathematical Modeling, **35 (3)** (2011) 1252-1263.
[22] M. Mandal and M. Maiti, Inventory model for damageable items with stock dependent demand and shortages, Opsearch, **34 (3)** (1997) 155-166.
[23] V. K. Mishra, L. S. Singh and R. Kumar, An inventory model for deteriorating items with time dependent demand and time varying holding cost under partial backlogging, Journal of Industrial Engineering International, **9 (4)** (2013) 1-5.
[24] I. Moon and S. Lee, The effects of inflation and time-value of money on an economic order quantity model with a random product life cycle, European Journal of Operational Research, **125 (3)** (2000) 588-601.
[25] D. Panda, M. K. Maiti and M. Maiti, Two warehouse inventory models for single vendor multiple retailers with price and stock dependent demand, Applied Mathematical Modelling, **34 (11)** (2010) 3571-3585.
[26] T.P.M. Pakkala and K.K. Achary, A deterministic inventory model for deteriorating items with two warehouses and finite replenishment rate, European Journal of Operational Research, **57 (1)** (1992) 71-76.
[27] S. Pal, A. Goswami and K. S. Chaudhuri, A deterministic inventory model for deteriorating items with stock dependent demand rate, International Journal of Production Economics, **32 (3)** (1993) 291-299.
[28] S. Papachristos and K. Skouri, An optimal replenishment policy for deteriorating item with time varying demand and partial exponential type-backlogging, Operations Research Letters, **27 (4)** (2000) 175-184.
[29] R. S. Ranjan and R. Uthayakumar, A two–warehouse inventory model for deteriorating items with permissible delay under exponentially increasing demand International Journal of Supply and Operation Management, **2 (1)** (2015) 662-682.
[30] M. Rastogi, S. R. Singh, P. Kushwaha and S. Tayal, A EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount, Uncertain Supply Chain Management, **5 (1)** (2017) 27-42.
[31] M. Rong, N. K. Mahapatra and M. Maiti, A two warehouse inventory model for a deteriorating item with partially/ fully backlogged shortage and fuzzy lead time, European Journal of Operational Research, **189 (1)** (2008) 59-75.
[32] S. Sana and K. S. Chaudhuri, On a volume flexible production policy for a deteriorating item with time-dependent demand and shortages, Advanced Modeling and Optimization, **6 (1)** (2004) 57–74.
[33] I. Sangal, A. Agarwal and S. Rani, A fuzzy environment inventory model with partial backlogging under learning effect, International Journal of Computer Applications, **137 (6)** (2016) 25-32.
[34] B. R. Sarker and H. Pan, Effects of inflation and time value of money on order quantity and allowable shortage, International Journal of Production Economics, **34 (1)** (1994) 65-72.
[35] S. Sharma and S. R. Singh, An inventory model for decaying items, considering multivariate consumption rate with partial backlogging, Indian Journal of Science and Technology, **6 (7)** (2013) 4870-4880.
[36] S. Sharma, S. R. Singh and H. Dem, Two-warehouse production policy for different demands under volume flexibility, International Journal of Industrial Engineering and Computations, **4 (4)** (2013) 599–609.
[37] S. Singh, R. Dube and S. R. Singh, Production model with selling price dependent demand and partial backlogging under inflation, International Journal of Mathematical Modelling and Computations, **1 (1)** (2011) 1-7.
[38] S. Singh, S. R. Singh and S. Sharma, A partially backlogged EPQ model with demand dependent production and non-instantaneous deterioration, International Journal of Mathematics in Operational Research, **10 (2)** (2017) 211-228.
[39] S. R. Singh, S. Sharma and S. Singh, Inventory model with multivariate demands in different phases with

customer returns and inflation, International Journal of Mathematics in Operational Research, **8 (4)** (2016) 477-489.

[40] K. Skouri and S. Papachristos, A continuous review inventory model with deteriorating items, time-varying demand, and linear replenishment cost, partially time varying backlogging, Applied Mathematical Modelling, **26 (5)** (2002) 603-617.

[41] K. Skouri, I. Konstantaras, S. Papachristos and I. Ganas, Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate, European Journal of Operational Research, **192 (1)** (2009). 79-92.