

A new approach to solve fuzzy system of linear equations by Homotopy perturbation method

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Abstract. In this paper, we present an efficient numerical algorithm for solving fuzzy systems of linear equations based on homotopy perturbation method. The method is discussed in detail and illustrated by solving some numerical examples.

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1. Introduction

Systems of linear equations are important for studying and solving a large proportion of the problems in many topics in applied mathematics. Usually, in many applications some of the parameters in our problems are represented by fuzzy number rather than crisp, and hence it is important to develop mathematical models and numerical procedures that would appropriately treat general fuzzy linear systems and solve them.

Friedman et al. [8] proposed a general model for solving an $n \times n$ fuzzy linear system, whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector, by the embedding approach. Allahviranloo [2, 3] used the Adomian and Jacobi iterative methods to approximate the unique solution of a fuzzy system of

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linear equations. Asady et al. [5], who merely discussed the full row rank system, used the same method to solve the $m \times n$ fuzzy linear system for $m \leq n$. Zheng and Wang [18, 19] discussed the solution of the general $m \times n$ consistent and inconsistent fuzzy linear system. Allahviranloo et al. [4] used the homotopy perturbation to solve fuzzy linear systems. Saberi Najafi et al. [17] proposed homotopy perturbation method and compared it with Adomians decomposition method for solving fuzzy linear systems. Indeed, they studied the homotopy analysis method (HAM). They used the convergence control parameter, just like what is proposed by Liao in HAM [15].

In recent years, the homotopy perturbation method (HPM), first proposed by He [10–12], has successfully been applied to solve many types of linear and nonlinear functional equations. This method, which is a combination of homotopy, in topology, and classic perturbation techniques, provides a convenient way to obtain analytic or approximate solutions to a wide variety of problems arising in different fields, see [1, 4, 6, 14] and the references there in.

In this paper, we investigate an $n \times n$ fuzzy linear system whose coefficients matrix is crisp and right-hand side column is a fuzzy number vector. We first replace the original $n \times n$ fuzzy linear system by a $(2n) \times (2n)$ crisp function linear system. Then, we apply the homotopy perturbation method for solving the new system.

In section 2, We introduce some basic definitions and results on fuzzy linear systems. In section 3, We use homotopy perturbation method for solving an $n \times n$ fuzzy system of linear equations. Numerical examples are given in section 4, and conclusions in section 5.

2. Preliminaries

We represent an arbitrary fuzzy number in parametric form [9, 16] by an ordered pair of functions $(\underline{u}(r), \bar{u}(r))$, $0 \leq r \leq 1$, which satisfy the following requirements:

- 1) $\underline{u}(r)$ is a left-continuous non-decreasing function over $[0, 1]$,
- 2) $\bar{u}(r)$ is a left-continuous non-increasing function over $[0, 1]$,
- 3) $\underline{u}(r) \leq \bar{u}(r)$, $0 \leq r \leq 1$.

A crisp number α is simply represented by $\underline{u}(r) = \bar{u}(r)$, $0 \leq r \leq 1$. By appropriate definitions the fuzzy numbers space $\{\underline{u}(r), \bar{u}(r)\}$ becomes a convex cone, E^1 . With the use of the extension principle [7], the addition and the scalar multiplication of fuzzy numbers are defined by

$$(u + v)(x) = \sup_{x=s+t} \min\{u(s), v(t)\},$$

$$(ku)(x) = u(x/k); k \neq 0,$$

for $u, v \in E^1$, $k \in \mathbb{R}$. Equivalently, for arbitrary $u = (\underline{u}, \bar{u})$, $v = (\underline{v}, \bar{v})$ and $k \in \mathbb{R}$, we may define the addition and the scalar multiplication as

$$(\underline{u} + \underline{v})(r) = \underline{u}(r) + \underline{v}(r),$$

$$(\overline{u + v})(r) = \bar{u}(r) + \bar{v}(r),$$

$$(\underline{ku})(r) = k\underline{u}(r), \quad (\overline{ku})(r) = k\overline{u}(r), \quad k \geq 0,$$

$$(\underline{ku})(r) = k\overline{u}(r), \quad (\overline{ku})(r) = k\underline{u}(r), \quad k < 0.$$

Definition 2.1. The $n \times n$ linear system

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &= y_1, \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &= y_2, \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \dots + a_{nn}x_n &= y_n, \end{aligned} \tag{1}$$

is called a fuzzy system of linear equations (FSL) where the coefficient matrix $A = (a_{ij})$ is a crisp $n \times n$ matrix and $y_i, i = 1, 2, \dots, n$ are fuzzy numbers.

Let $x_i = (\underline{x}_i(r), \overline{x}_i(r))$ and $y_i = (\underline{y}_i(r), \overline{y}_i(r)), i = 1, 2, \dots, n$ be fuzzy numbers. Then FSL (1) can be represented in the form of the following function linear system:

$$\begin{cases} \sum_{j=1}^n a_{ij}x_j = \sum_{j=1}^n a_{ij}\underline{x}_j = \underline{y}_i, \\ \sum_{j=1}^n \overline{a_{ij}x_j} = \sum_{j=1}^n \overline{a_{ij}\overline{x}_j} = \overline{y}_i, \end{cases} \quad i = 1, 2, \dots, n. \tag{2}$$

In particular, if $a_{ij} \geq 0, 1 \leq j \leq n$, for some i , then

$$\sum_{j=1}^n a_{ij}\underline{x}_j = \underline{y}_i, \quad \sum_{j=1}^n a_{ij}\overline{x}_j = \overline{y}_i. \tag{3}$$

Definition 2.2. A fuzzy number vector (x_1, x_2, \dots, x_n) given by

$$x_i = (\underline{x}_i(r), \overline{x}_i(r)), \quad 1 \leq i \leq n, \quad 0 \leq r \leq 1$$

is called a solution of the fuzzy system if it satisfies (2).

From (2) we have two crisp $(n) \times (n)$ linear systems for all i that can be extended to a $(2n) \times (2n)$ crisp linear system as follows:

$$SX = Y \longrightarrow \begin{pmatrix} S_1 \geq 0 & S_2 \leq 0 \\ S_2 \leq 0 & S_1 \geq 0 \end{pmatrix} \begin{pmatrix} X \\ \overline{X} \end{pmatrix} = \begin{pmatrix} Y \\ \overline{Y} \end{pmatrix}, \tag{4}$$

where S_1 and S_2 are $n \times n$ matrices and

$$\underline{X} = \begin{pmatrix} \underline{x}_1(r) \\ \vdots \\ \underline{x}_n(r) \end{pmatrix}, \quad \overline{X} = \begin{pmatrix} \overline{x}_1(r) \\ \vdots \\ \overline{x}_n(r) \end{pmatrix}, \quad \underline{Y} = \begin{pmatrix} \underline{y}_1(r) \\ \vdots \\ \underline{y}_n(r) \end{pmatrix}, \quad \overline{Y} = \begin{pmatrix} \overline{y}_1(r) \\ \vdots \\ \overline{y}_n(r) \end{pmatrix}.$$

Thus, FSL (1) is extended to a crisp (4) where $A = S_1 + S_2$. Eq. (4) can be written as

follows:

$$\begin{cases} S_1 \underline{X} + S_2 \overline{X} = \underline{Y}, \\ S_2 \underline{X} + S_1 \overline{X} = \overline{Y}. \end{cases} \quad (5)$$

Definition 2.3. Let $X = \{(\underline{x}_i(r), \overline{x}_i(r)), i = 1, 2, \dots, n\}$ denote the unique solution of $SX = Y$. The fuzzy number vector $U = \{(\underline{u}_i(r), \overline{u}_i(r)), i = 1, 2, \dots, n\}$ defined by

$$\underline{u}_i(r) = \min\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1)\},$$

$$\overline{u}_i(r) = \max\{\underline{x}_i(r), \overline{x}_i(r), \underline{x}_i(1), \overline{x}_i(1)\},$$

is called the fuzzy solution of $SX = Y$. If $(\underline{x}_i(r), \overline{x}_i(r)), 1 \leq i \leq n$ are all fuzzy numbers, then $\underline{u}_i(r) = \underline{x}_i(r), \overline{u}_i(r) = \overline{x}_i(r), 1 \leq i \leq n$, and U is called a strong fuzzy solution; otherwise, U is a weak fuzzy solution.

Theorem 2.1. The matrix S is nonsingular if and only if the matrices $A = S_1 + S_2$ and $S_1 - S_2$ are both nonsingular. See [8].

Theorem 2.2. Let S be nonsingular. Then the unique solution X of Eq. (4) is always a fuzzy vector for arbitrary vector Y , if S^{-1} is nonnegative. See [8].

Theorem 2.3. The matrix A with $a_{ii} > 0$ in Eq. (1) is strictly diagonally dominant if and only if matrix S is strictly diagonally dominant. See [3].

3. HPM for solving a FSL

Consider the system of linear equations, $SX = Y$, where S is an invertible matrix of size $2n$, $X \in \mathbb{R}^{2n}$ is an $2n$ -vector and $Y \in \mathbb{R}^{2n}$ is the right-hand-side vector. Nonsingularity of S implies that the system has a unique solution. Our aim is to obtain this solution using homotopy idea. Suppose $v^{(\circ)}$ is an initial guess of the solution, using an auxiliary matrix M (probably related to S), we construct a convex combination as follows:

$$\mathcal{S}(X; q) = (1 - q)M(X - v^{(\circ)}) + q(SX - Y). \quad (6)$$

This is a homotopy between $v^{(\circ)}$ and X (the exact solution). Enforcing the homotopy to be zero we obtain the following homotopy equation

$$(1 - q)M(X - v^{(\circ)}) + q(SX - Y) = 0. \quad (7)$$

Thus for every $q \in [0, 1]$, we have a system of linear equations whose solution is dependent upon q . When $q = 1$ the system is equivalent to $M(X - v^{(\circ)}) = 0$, if M is an invertible matrix then this system leads to the obvious solution $v^{(\circ)}$. In the case where $q = 1$, system (7) will be equivalent to $SX = Y$, i.e. the original system under study. We rewrite the homotopy equation in the form

$$M(X - v^{(\circ)}) + q\{(S - M)X + Mv^{(\circ)} - Y\} = 0. \quad (8)$$

Now, if we accept that the solution to the homotopy equation could be represented as an infinite series in the form

$$X = X^{(0)} + X^{(1)}q + X^{(2)}q^2 + X^{(3)}q^3 + \dots, \tag{9}$$

then substituting the series in (8), we would have

$$M(X^{(0)} + X^{(1)}q + X^{(2)}q^2 + \dots - v^{(0)}) + q\{(S - M)(X^{(0)} + X^{(1)}q + \dots) + Mv^{(0)} - Y\} = 0. \tag{10}$$

The above equation holds for every $q \in [0, 1]$, so considering equal powers of q , we would have

$$\begin{aligned} q^0 : & \quad M(X^{(0)} - v^{(0)}) = 0, \\ q^1 : & \quad MX^{(1)} + (S - M)X^{(0)} + Mv^{(0)} - Y = 0, \\ q^2 : & \quad MX^{(2)} + (S - M)X^{(1)} = 0, \\ & \quad \vdots \\ q^n : & \quad MX^{(n)} + (S - M)X^{(n-1)} = 0, \end{aligned}$$

which results in:

$$\begin{aligned} M(X^{(0)} - v^{(0)}) &= 0, \\ MX^{(1)} &= Y - SX^{(0)}, \\ MX^{(n)} &= (M - S)X^{(n-1)}, \quad n \geq 2. \end{aligned} \tag{11}$$

If we choose "M" to be an invertible matrix, each of the above systems have a unique solution and these solutions, from a theoretical point of view, would be:

$$\begin{aligned} X^{(0)} &= v^{(0)}, \\ X^{(1)} &= M^{-1}(Y - SX^{(0)}), \\ X^{(n)} &= (I - M^{-1}S)X^{(n-1)}, \quad n \geq 2. \end{aligned} \tag{12}$$

By this recursive relation, every $X^{(n)}$ could be expressed in terms of the initial guess,

$$\begin{aligned} X^{(0)} &= v^{(0)}, \\ X^{(n)} &= (I - M^{-1}S)^{(n-1)}M^{-1}(Y - Sv^{(0)}), \quad n \geq 2. \end{aligned} \tag{13}$$

However computing the inverse matrix M^{-1} is not recommended here, since it is an ill-conditioned (and costly) problem. For obtaining solutions (13), we try to solve equations (11), which are equivalent to our system. If M is a matrix which has a easy-computable inverse or it is triangular, then solving equations (11) may reduce to just doing a backward or forward substitution or even much simpler ones.

From solutions (13), we construct the series solution

$$\begin{aligned} X &= X^{(0)} + X^{(1)} + X^{(2)} + X^{(3)} + \dots, \\ &= v^{(0)} + \sum_{n=1}^{\infty} (I - M^{-1}S)^{(n-1)}M^{-1}(Y - Sv^{(0)}), \end{aligned} \tag{14}$$

where we have substitute $q = 1$. We see that the matrix $I - M^{-1}S$ plays a key role in the homotopy method. We will refer time by time to this matrix, whenever we are concerned with the convergence problem.

3.1 Discussion on convergence

The approximations (14), obtained by the homotopy method, wouldn't be worthwhile unless we get sure that the series is convergent, here we study the conditions under which the solutions (14) are convergent for every choice of the initial guess.

Theorem 3.1 (Neumann's Series) Let S be a square matrix of size $2n$, if there is a matrix norm, $\|\cdot\|$, for which $\|S\| < 1$, then $I - S$ is a nonsingular matrix and $\sum_{k=0}^{\infty} S^k = (I - S)^{-1}$.

Corollary 3.2 If we find a matrix norm such that $\|I - M^{-1}S\| < 1$, then the series $\sum_{n=0}^{\infty} (I - M^{-1}S)^n$ will converge to $S^{-1}M$.

Theorem 3.3 If the auxiliary matrix, M , is chosen in such a way that the spectral radius of $I - M^{-1}A$ is less than one, i.e. $\rho(I - M^{-1}S) < 1$, then the solution series obtained by homotopy method, for solving the system $SX = Y$, will converge to the exact solution.

Proof. If $\rho(I - M^{-1}S) < 1$, then there exists a matrix norm for which $\|S\| < 1$, so according to the above mentioned corollary we have $\sum_{n=0}^{\infty} (I - M^{-1}S)^n = S^{-1}M$, therefore for the solution X , obtained from (14), we have

$$\begin{aligned} SX &= S(v^{(\circ)} + \sum_{n=1}^{\infty} (I - M^{-1}S)^{(n-1)} M^{-1} (Y - Sv^{(\circ)})) \\ &= Sv^{(\circ)} + S(S^{-1}M)M^{-1}(Y - Sv^{(\circ)}) \\ &= 0. \end{aligned}$$

■

This is the case which is studied by Keramati in [13] With choosing M to be the identity matrix, the solution series, according to (14), would be of the form:

$$\begin{aligned} X &= X^{(\circ)} + X^{(1)} + X^{(2)} + X^{(3)} + \dots, \\ &= v^{(\circ)} + \sum_{n=1}^{\infty} (I - S)^{(n-1)} (Y - Sv^{(\circ)}). \end{aligned} \quad (15)$$

The convergence criteria of this special case would be $\rho(I - S) < 1$. In the case where $v^{(\circ)}$ is chosen to be the zero vector (the case which is studied by Keramati), if the coefficient matrix is strictly diagonally dominant then by rewriting the system, in a suitable form, convergence conditions could be satisfied.

Definition 3.1.1 A square matrix $S = [s_{ij}]$ is said to be strictly row diagonally dominant if for each i , $i = 1, 2, \dots, 2n$, we have $\sum_{j \neq i} |s_{ij}| < |s_{ii}|$.

Theorem 3.4 If S is SRDD and M is the lower (or upper) triangular (with diagonal) part of S , then the homotopy method, for solving the system $SX = Y$, is convergent.

Proof. It suffices to prove $\rho(I - M^{-1}S) < 1$. We do the proof for the lower triangular case, the other case is similar. Let λ be an arbitrary eigenvalue of $I - M^{-1}S$ and X be the corresponding eigenvector, without loss of generality we can assume $\|X\|_{\infty} = 1$, therefore we have

$$(I - M^{-1}S)X = \lambda X \implies MX - SX = \lambda MX.$$

Since M is the lower triangular part of S , for every i , $1 \leq i \leq 2n$, we have:

$$\begin{aligned}
 - \sum_{j=i+1}^{2n} s_{ij} X_j &= \lambda \sum_{j=1}^i s_{ij} X_j \\
 \implies \lambda s_{ii} x_i &= -\lambda \sum_{j=1}^{i-1} s_{ij} X_j - \sum_{j=i+1}^{2n} s_{ij} X_j,
 \end{aligned}$$

now we can choose an index k such that

$$|X_k| = 1 \geq |X_j|, \quad 1 \leq j \leq 2n,$$

therefore:

$$|\lambda| |s_{kk}| \leq |\lambda| \sum_{j=1}^{k-1} |s_{kj}| + \sum_{j=k+1}^{2n} |s_{kj}|,$$

then diagonally dominance of S results in:

$$|\lambda| \leq \frac{\sum_{j=k+1}^{2n} |s_{kj}|}{|s_{kk}| - \sum_{j=1}^{k-1} |s_{kj}|} < 1.$$

Since λ was an arbitrary eigenvalue, this completes the proof. ■

Theorem 3.5 If S is SRDD and M is the lower (or upper) Hessenberg (with diagonal) part of S , then the homotopy method, for solving the system $SX = Y$, is convergent.

Proof. It suffices to prove $\rho(I - M^{-1}S) < 1$. We do the proof for the lower Hessenberg case, the other case is similar. Let λ be an arbitrary eigenvalue of $I - M^{-1}S$ and X be the corresponding eigenvector, without loss of generality we can assume $\|X\|_\infty = 1$, therefore we have

$$(I - M^{-1}S)X = \lambda X \implies MX - SX = \lambda MX.$$

Since M is the lower Hessenberg part of S , for every i , $1 \leq i \leq 2n$, we have:

$$\begin{aligned}
 - \sum_{j=i+2}^{2n} s_{ij} X_j &= \lambda \sum_{j=1}^{i+1} s_{ij} X_j \\
 \implies \lambda s_{ii} x_i &= -\lambda \sum_{j=1}^{i-1} s_{ij} X_j - \sum_{j=i+2}^{2n} s_{ij} X_j - s_{i,i+1},
 \end{aligned}$$

now we can choose an index k such that

$$|X_k| = 1 \geq |X_j|, \quad 1 \leq j \leq 2n,$$

therefore:

$$|\lambda| |s_{kk}| \leq |\lambda| \sum_{j=1}^{k-1} |s_{kj}| + \sum_{j=k+2}^{2n} |s_{kj}| + |s_{k,k+1}|,$$

then diagonally dominance of S results in:

$$|\lambda| \leq \frac{\sum_{j=k+2}^{2n} |s_{kj}| + |s_{k,k+1}|}{|s_{kk}| - \sum_{j=1}^{k-1} |s_{kj}|} < 1.$$

Since λ was an arbitrary eigenvalue, this completes the proof. ■

4. Numerical examples

The case proposed by Alahviranloo et al. [4] and Saberi Najafi et al. [17] is very simple and easy to use (identity matrix as auxiliary matrix), but it lacks the great flexibility we have in choosing the M . Even for the same examples, other choices of M may result in better approximations. We present two examples and their numerical results.

Example 4.1. Consider the 2×2 fuzzy linear system

$$\begin{cases} 2x_1 - x_2 = (r, 2 - r), \\ x_1 + 3x_2 = (4 + r, 7 - 2r). \end{cases}$$

The matrix of the system is strictly diagonally dominant. The extended 4×4 matrix S is

$$S = \begin{pmatrix} 2 & 0 & 0 & -1 \\ 1 & 3 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

The exact solutions from $X = S^{-1}Y$ are as follows:

$$\begin{aligned} x_1 &= (\underline{x}_1(r), \bar{x}_1(r)) = (0.91428557 + 0.22857143r, 1.51428570 - 0.37142857r), \\ x_2 &= (\underline{x}_2(r), \bar{x}_2(r)) = (1.02857143 + 0.25714285r, 1.82857143 - 0.54285714r). \end{aligned}$$

from [14] we approximate the solution vector \underline{U} and \bar{U} as follows

$$\underline{U} \approx \underline{U}_1 + \underline{U}_2 + \cdots + \underline{U}_n,$$

$$\bar{U} \approx \bar{U}_1 + \bar{U}_2 + \cdots + \bar{U}_n,$$

Table 1
The numerical results for Example 4.1. with present method

n	M	solution
2	Hessenberg	$x_1 = (0.91428557 + 0.22857143r, 1.51428570 - 0.37142857r)$ $x_2 = (1.02857143 + 0.25714285r, 1.82857143 - 0.54285714r)$
10	Triangular	$x_1 = (0.91428517 + 0.22857159r, 1.51428580 - 0.37142859r)$ $x_2 = (1.02857161 + 0.25714280r, 1.82857140 - 0.54285713r)$
15	Identity	$x_1 = (0.91428755 + 0.22857188r, 1.51428898 - 0.3714295r)$ $x_2 = (1.0285696 + 0.2571409r, 1.82857033 - 0.54285741r)$

Example 4.2. Consider the 3×3 fuzzy linear system

$$\begin{cases} 2x_1 + 0x_2 + x_3 = (r - 1, 1 - r), \\ 0x_1 + 2x_2 - x_3 = (r, 2 - r), \\ -x_1 + x_2 + 3x_3 = (2 + r, 3). \end{cases}$$

The matrix of the system is strictly diagonally dominant. The extended 6×6 matrix S is

$$S = \begin{pmatrix} 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & -1 \\ 0 & 1 & 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\begin{cases} x_1 = (\underline{x}_1(r), \bar{x}_1(r)) = (-0.8751 + 0.5625r, 0.3752 - 0.6877r), \\ x_2 = (\underline{x}_2(r), \bar{x}_2(r)) = (0.1248 + 0.6877r, 1.3752 - 0.5627r), \\ x_3 = (\underline{x}_3(r), \bar{x}_3(r)) = (0.7500 - 0.1250r, 0.2500 + 0.3750r). \end{cases}$$

Which implies that x_3 is not fuzzy number; therefore, the corresponding fuzzy solution is a weak fuzzy solution given by

$$\begin{cases} \hat{x}_1 = (-0.8751 + 0.5625r, 0.3752 - 0.6877r), \\ \hat{x}_2 = (0.1248 + 0.6877r, 1.3752 - 0.5627r), \\ \hat{x}_3 = (0.2500 + 0.3750r, 0.7500 - 0.1250r). \end{cases}$$

Table 2
The numerical results for Example 4.2. with present method

n	M	solution
7	Hessenberg	$x_1 = (-0.8751 + 0.5625r, 0.3752 - 0.6877r)$ $x_2 = (0.1248 + 0.6877r, 1.3752 - 0.5627r)$ $x_3 = (0.2500 + 0.3750r, 0.7500 - 0.1250r)$
10	Triangular	$x_1 = (-0.875057 + 0.562461r, 0.374991 - 0.687477r)$ $x_2 = (0.125008 + 0.687477r, 1.375057 - 0.562485r)$ $x_3 = (0.250006 + 0.374982r, 0.749997 - 0.124970r)$
15	Identity	$x_1 = (-0.8751 + 0.5624r, 0.3748 - 0.6875r)$ $x_2 = (0.1252 + 0.6875r, 1.3751 - 0.5624r)$ $x_3 = (0.2500 + 0.3748r, 0.7497 - 0.1248r)$

5. Conclusion

In this paper, we considered an $n \times n$ fuzzy system of linear equation (FSL). By the embedding method, the original system is converted in to a $(2n) \times (2n)$ crisp linear system. Then we apply the homotopy perturbation method (HPM) to approximate the unique solution of FSL. If the unique solution of $SX = Y$ is a strong or weak fuzzy number, then the approximate solution of iterative method would also be a strong or weak fuzzy number.

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