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Synchronization of fractional-order LU system with new parameters using the feedback control technique

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Abstract. In this paper, a feedback control method is employed for synchronization between two identical chaotic fractional order LU system (FOLUS) with the new parameters. We have shown that the convergence rate of synchronization error. Therefore, use encryption and its analysis for the chaotic FOLUS. In addition, we show that the method used here is better than other existing algorithm.

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1. Introduction

In recent years the application of dynamic systems in various sciences including, elecrtomagnetic waves, engineering-biology, dielectric polarization, electrode-electrolyte polarization [10, 11, 13, 15–18, 24, 27, 32], viscoelastic systems, is rapidly increasing. Chaotic characteristics were found to exist in many fractional-order systems whose orders are less than three, such as: Lu [24] presented a chaotic dynamics of the fractional-order LU system and its synchronization. Wang et al. [35] applied chaotic synchronization and secure communication based on descriptor observer. Wang [33] proposed projective synchronization of hyperchaotic LU system and Liu system. Ghosh et al. [9] applied projective synchronization of new hyperchaotic system with fully unknown parameters. Yang [36]

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proposed adaptive synchronization of LU hyperchaotic system with uncertain parameters based on single-input controller. Liu et al. [23] presented nonlinear state-observer control for projective synchronization of a fractional-order hyperchaotic system. Li et al. [21] applied anti-synchronization and intermitten anti-synchronization of two identical hyperchaotic Chua systems via impulsive control. Singh [30] presented single input sliding mode control for hyperchaotic LU system with parameter uncertainty. Sudheer et al. [31] proposed adaptive modified function projective synchronization between hyperchaotic Lorenz system and hyperchaotic LU system with uncertain parameters. Yu et al. [38] have investigated synchronization of unified chaotic systems with uncertain parameters based on the CLF. Zhang et al. [40] have studied dual projective synchronization between integer-order and fractional-order chaotic systems. Hegazi et al. [12] applied on chaos control and synchronization of the commensurate fractional-order Liu system. Yuan et al. [39] presented generalized projective synchronization of a class of hyperchaotic systems based on state observer. Li [19] have investigated tracking control and generalized projective synchronization of a class of hyperchaotic system with unknown parameter and disturbance. Chen et al. [5] applied projective and lag synchronization of a novel hyperchaotic via impulsive control. Wang et al. [34] have studied synchronization of hyperchaotic systems via linear control. Rafikov et al. [28] presented on control and synchronization in chaotic and hyperchaotic systems via linear feedback control. Gao et al. [7] proposed a new fractional-order hyperchaotic system and its modified projective synchronization. Zhou et al. [42] have studied adaptive control and synchronization of a new modified hyperchaotic LU system with uncertain parameters. Sadaoui et al. [29] applied predictive feedback control and synchronization of hyperchaotic systems. Ge et al. [8] have investigated chaos in a fractional-order modified Duffing system. Huang et al. [14] applied chaos and hyperchaos in fractional-order cellular neural network. Ahmad [1] proposed hyperchaos in fractional-order nonlinear systems. Chen et al. [4] have studied cluster synchronization in fractional-order complex dynamical. Zhang et al. [41] proposed stabilization of fractional-order chaotic system via a single state adaptive feedback controller. Li et al. [22] applied a novel equilibrium fractional-order chaotic system and its complete synchronization by circuit implementation. Yin et al. [37] have studied control of a novel class of fractional-order chaotic systems via adaptive sliding mode control approach. Bandyopadhyay et al. [2] applied stabilization and control of fractional-order systems a sliding mode approach. Changpin et al. [20] have studied remarks on fractional derivatives. Matignon et al. [25] proposed stability results for fractional differential equation with applications to control processing. Camacho et al. [3] have investigated Lyapunov functions for fractional-order system.

In this paper, the synchronization between the fractional-order LU systems are investigated using nonlinear feedback control technique in the presence of new parametric and different initial conditions. By using the Lyapunov stability, sufficient condition for achieving synchronization of the chaotic fractional-order LU system via feedback control is derived. In the following we showed the convergence error using the results of numerical simulation and drawing figures, which shows the effectiveness of the method.

The rest of this paper is organized as follows: Section 2 contains the fundamental definitions, lemma, theorem and properties of fractional calculations. In Section 3, we introduce the chaotic FOLUS and its analysis. Section 4 analyzes the fractional-order LU system with new parameters and the different initial conditions. Also, we discuss the synchronization for commensurate order the chaotic FOLUS based on the feedback control technique. In Section 5, using the numerical simulation and the draw of the diagrams, we have investigated the results of this paper. Concluding remarks are presented in Section 6.

2. Preliminaries

In this section, we review some fundamental definitions of fractional calculus. Also, we present some useful stability theorems and properties of fractional-order dynamical systems.

Definition 2.1 [2] The *qth* order Caputo derivative of fractional for the function $G(t)$ is defined as follows:

$$^{c}D_t^q G(t) = D^{-(m-q)} D^{(m)} G(t) = \frac{1}{\Gamma(m-q)} \int_0^t (t-\zeta)^{m-q-1} G^{(m)}(\zeta) d\zeta, \tag{1}$$

where $m-1 < q \leqslant m, m \in N, q \in R^+, \Gamma(q) = \int_0^\infty t^{q-1} e^{-t} dt$.

Some properties of fractional order differential equations introduce in the following:

- The linear characteristic of the Caputo fractional-order derivatives is as follow:

$$^{c}D_t^q [c_1 G_1(t) + c_2 G_2(t)] = c_1^c D_t^q G_1(t) + c_2^c D_t^q G_2(t), \tag{2}$$

  where c_1, c_2 are constants and G_1, G_2 are functions of t [22].
- The Caputo fractional order derivative of a constant function is as follow:

$$^{c}D_t^q G(t) = 0, \tag{3}$$

  where $0 < q \leqslant 1$ [22].

Lemma 2.2 [3] Assume that $G(t) \in R$ is a continuously differentiable function. Then we have

$$\frac{1}{2}(^{c}D_t^q G^2(t)) \leqslant G(t)^c D_t^q G(t), \tag{4}$$

where $0 < q < 1$.

Theorem 2.3 [25] Autonomous system $D^q x = Ax, x(0) = x_0$ is asymptotically stable if $|arg(\lambda(A))| > \frac{q\pi}{2}$, where $0 < q < 1$ and $\lambda(A)$ represents the eigenvalues of matrix A. Also, this system is stable if and only if $|arg(\lambda(A))| \geqslant \frac{q\pi}{2}$, or those critical eigenvalues that satisfy $|arg(\lambda(A))| = \frac{q\pi}{2}$, have geometric multiplicity of one.

3. System description

We consider the following fractional-order LU system [6]:

$$\begin{cases} D^{q_1} x = \alpha(y(t) - x(t)), \\ D^{q_2} y = -x(t)z(t) + \gamma y(t), \\ D^{q_3} z = x(t)y(t) - \beta z(t), \end{cases} \tag{5}$$

where $0 < q_i \leqslant 1 (i = 1, 2, 3)$, are derivatives orders, and α, β, γ are system parameters. the system (5) has three equilibrium points $E_1 = (0, 0, 0)$, $E_2 = (\sqrt{\beta\gamma}, \sqrt{\beta\gamma}, c)$ and

$E_3 = (-\sqrt{\beta\gamma}, -\sqrt{\beta\gamma}, c)$. The jacobian matrix for equilibria $\bar{E} = (\bar{x}, \bar{y}, \bar{z})$ is defined as

$$J = \begin{pmatrix} -\alpha & \alpha & 0 \\ -\bar{z} & \gamma & -\bar{x} \\ \bar{y} & \bar{x} & -\beta \end{pmatrix}. \tag{6}$$

This approach is based on the definition of Caputo. Numerical approximation of qth derivative at the points mh, $(m = 1, 2, ...)$ has the following form [26]:

$$(m - \frac{L_n}{L}) D^q_{t_m} f(t) \simeq h^{-q} \sum_{i=0}^{m} (-1)^{(i)} \frac{q!}{q!(q-i)!} f(t_{m-1}), \tag{7}$$

where $(-1)^{(i)} \dfrac{q!}{q!(q-i)!}$ are binomial coefficients $c_i^q, (i = 0, 1, 2, ...)$, L_n is the memory space, and $t_m = mh$ is the time step of calculation.

For their calculation, we have

$$c_0^{(q)} = 1, \quad c_i^{(q)} = (1 - \frac{1+q}{i}) c_{i-1}^{(q)}. \tag{8}$$

Then, general numerical solution of the fractional differential equation is as follow [26]:

$$\begin{cases} x(t_m) = (a(y(t_{m-1}) - x(t_{m-1})))h^{q_1} - \sum_{i=0}^{m} c_i^{q_1} x(t_{m-i}), \\ y(t_m) = (-x(t_m)z(t_{m-1}) + \gamma y(t_{m-1})))h^{q_2} - \sum_{i=0}^{m} c_i^{q_2} y(t_{m-i}), \\ z(t_m) = (x(t_m)y(t_m) - \beta z(t_{m-1}))h^{q_3} - \sum_{i=0}^{m} c_i^{q_3} z(t_{m-i}), \end{cases} \tag{9}$$

where $c_i^q, (i = 0, 1, 2, ...)$ binomial coefficients, and T_{sim} is the simulation time, $m = 1, 2, ..., N$ for $N = \lceil \frac{T_{sim}}{h} \rceil$ and $(x(0), y(0), z(0))$ is the initial conditions. We consider new parameters $\alpha = 20$, $\beta = 14$, $\gamma = 15$ and the initial conditions $(x(0), y(0), z(0)) = (3, 2, 1), (x_1(0), y_1(0), z_1(0)) = (3.5, 2.5, 1.5)$ of the system (9). For the equilibrium $E_1 = (0, 0, 0)$ we obtain the following eigenvalues of the jacobian matrix (6): $\lambda_1 = -1, \lambda_2 = 15, \lambda_3 = -20$. For the equilibrium $E_2 = (3.87298, 3.87298, 15)$ we get $\lambda_1 = -9.999996, \lambda_{2,3} = 1.999998 \pm 7.483309$. For the equilibrium $E_3 = (-3.87298, -3.87298, 15)$ we have $\lambda_1 = -9.999996, \lambda_{2,3} = 1.999998 \pm 7.483309$. By Theorem 2.3, we can determine a minimal commensurate order for the system (5) $q > 0.8337$. Figure 1 shows that the lowest value of $q_i (i = 1, 2, 3)$ for which the system remains chaotic is commensurate order $q_1 = q_2 = q_3 = 0.86$ of the chaotic FOLUS (5). Consider the new parameters as a $\alpha = 20, \beta = 1, \gamma = 15$ and the different initial condition$(x(0), y(0), z(0)) = (3, 2, 1)$.

4. Synchronization of two identical chaotic fractional-order LU systems

The fractional-order LU system [6] is considered as the master system

$$\begin{cases} D^{q_1} x = \alpha(y - x), \\ D^{q_2} y = -xz + \gamma y, \\ D^{q_3} z = xy - \beta z, \end{cases} \tag{10}$$

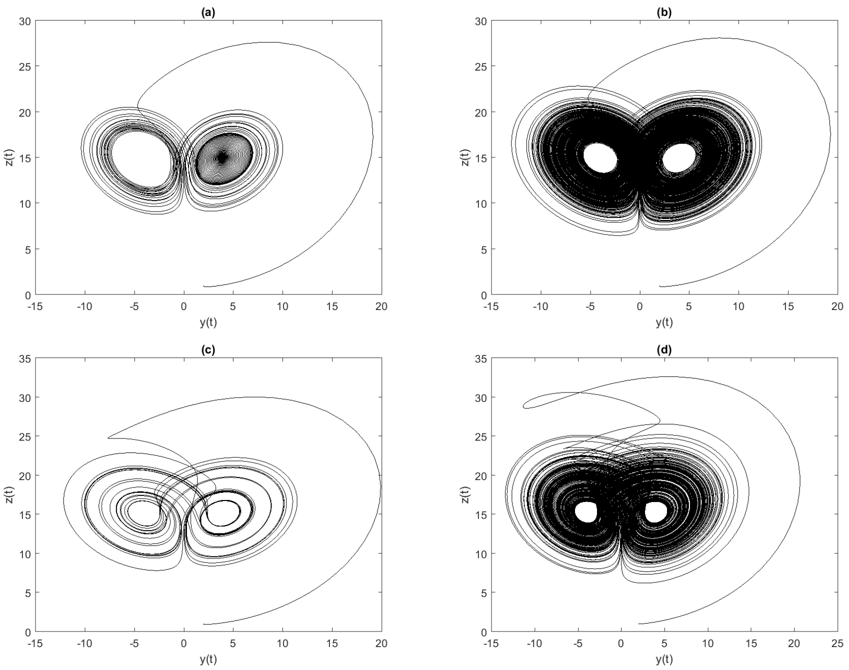

Figure 1. The phase portrait of fractional order LU system (5) for the commensurate orders at $q_1 = q_2 = q_3 = q$. (a)q=0.85, (b)q=0.86, (c)q=0.95, (d)q=1.

and also, the slave system as

$$\begin{cases} D^{q_1}x_1 = \alpha(y_1 - x_1) + u_1(t), \\ D^{q_2}y_1 = -x_1y_1 + \gamma y_1 + u_2(t), \\ D^{q_3}z_1 = x_1y_1 - \beta z_1 + u_3(t), \end{cases} \tag{11}$$

where $u_1(t), u_2(t)$ and, $u_3(t)$ are the controllers. Now, we find the controllers $u_1(t)$, $u_2(t)$ and $u_3(t)$ to regulate (synchronize) the states x_1, y_1 and z_1 of the system (11) to the states x, y and z of the system (10) with desired constant values α, β and .

Define a synchronization error between the master system (10) and slave system (11) as follows:

$$\begin{cases} e_1(t) = x_1(t) - x(t), \\ e_2(t) = y_1(t) - y(t), \\ e_3(t) = z_1(t) - z(t). \end{cases} \tag{12}$$

We obtain the error dynamics system by subtracting master system (10) of the slave system (11) as

$$\begin{cases} {}^cD_t^{q_1}e_1(t) = \alpha e_2 - \alpha e_1 + u_1(t), \\ {}^cD_t^{q_2}e_2(t) = -x_1e_3 - ze_1 + \gamma e_2 + u_2(t), \\ {}^cD_t^{q_3}e_3(t) = -x_1e_2 + ye_1 - \beta e_3 + u_3(t), \end{cases} \tag{13}$$

and we consider the feedback control method

$$\begin{cases} u_1(t) = -k_1e_1 - \alpha e_2, \\ u_2(t) = x_1e_3 + ze_1 - k_2e_2, \\ u_3(t) = -x_1e_2 - ye_1 - k_3e_3, \end{cases} \tag{14}$$

where k_1, k_2, k_3 are gain constants. By substituting of the system (14) in (13), we obtain the system error as follows:

$$\begin{cases} {}^cD_t^{q_1} e_1(t) = -\alpha e_1 - k_1 e_1, \\ {}^cD_t^{q_2} e_2(t) = \gamma e_2 - k_2 e_2, \\ {}^cD_t^{q_3} e_3(t) = -\beta e_3 - k_3 e_3. \end{cases} \tag{15}$$

The jacobian matrix of the system (15) is obtained as follows:

$$J = \begin{pmatrix} -\alpha - k_1 & 0 & 0 \\ 0 & \gamma - k_2 & 0 \\ 0 & 0 & -\beta - k_3 \end{pmatrix}, \tag{16}$$

when $\alpha = 20, \beta = 1, \gamma = 15$ and $k_1 = k_2 = k_3$ the eigenvalues $\lambda_1 = -30, \lambda_2 = 5, \lambda_3 = -11$, which satisfies $|arg(\lambda_i)| > \frac{q\pi}{2}, (i = 1, 2, 3)$ for $0 \leqslant q \leqslant 1$. Thus the synchronization error system converges to zero as $t \longrightarrow \infty$, and therefore, synchronization between the master system (10) and the slave system (11) is achieved.

We consider the Lyapunov function as follows

$$V(t) = \frac{1}{2} \sum_{i=1}^{3} e_i^2(t), \tag{17}$$

where by using Lemma 2.2, we get

$$D^{q_i} V(t) \leqslant \sum_{i=1}^{3} e_i(t) D^{q_i} e_i(t). \tag{18}$$

By substituting the values of $D_t^{q_i} e_i, i = 1, 2, 3$ from (15) in (18), we get

$$D^{q_i} V(t) \leqslant e_1(-\alpha e_1 - k_1 e_1) + e_2(\gamma e_2 - k_2 e_2) + e_3(-\beta e_3 - k_3 e_3). \tag{19}$$

Let $e_1^2 = e_2^2 = e_3^2$, $\alpha + \beta > \gamma$ and $k_1 = k_2 = k_3 = k$. Then we get $D^{q_i} V(t) \leqslant 0$.

The error system (15) is asymptotically stable and thus the synchronization error between the master system (5) and the slave system (11) is achieved.

5. Numerical simulation and results

The Adams-bashforth-Moulton method is used to solve the systems with time step size 0.005. For the numerical simulation, we choose the new parameters as $\alpha = 20, \beta = 1, \gamma = 15$ and the initial conditions of the master and slave systems are taken as $(x(0), y(0), z(0)) = (3, 2, 1), (x_1(0), y_1(0), z_1(0)) = (3.5, 2.5, 1.5)$ respectively. Thus the initial error are $(0.5, 0.5, 0.5)$. Figure 2(a) and Figure 3(a) show that the trajectories of the controller chaotic FOLUS with commensurate order $q = 0.85$ converge to the equilibrium points $E_1 = (3.87298, 3.87298, 15)$ and $E_2 = (-3.87298, -3.87298, 15)$ respectively. Figure 2(b) and Figure 3(b) are not stabilized to the equilibrium E_1 and E_2. In Figure 4 phase portraits shows synchronization with the commensurate order of FOLUS at $q_1 = q_2 = q_3 = q = 0.86$. In Figure 5 (a-c) show the synchronization of the states for the master system (10) and the slave system (11) at $q_1 = q_2 = q_3 = 0.9$ after applying

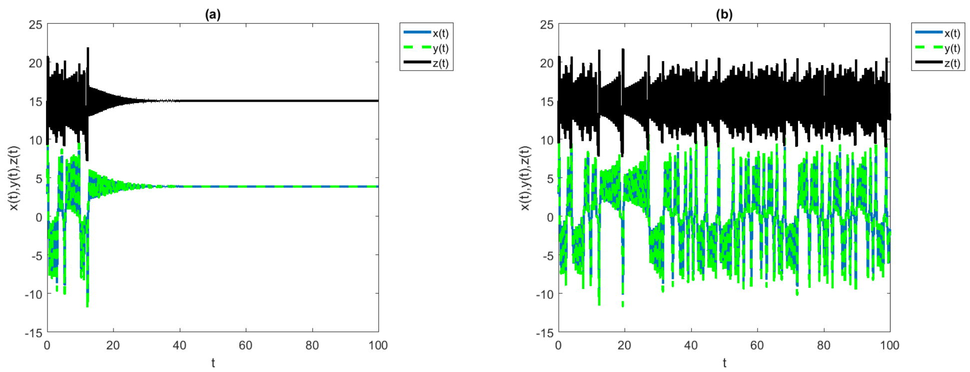

Figure 2. The trajectories of the fractional order system (11); (a) are stabilized to the equilibrium point $E_1 = (3.87298, 3.87298, 15)$ at $q = 0.85$; (b) are not stabilized to the equilibrium point $E_1 = (3.87298, 3.87298, 15)$ at $q = 0.86$

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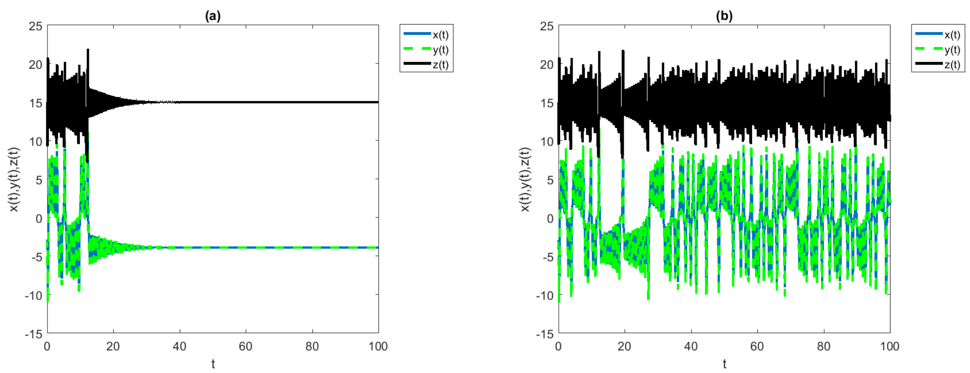

Figure 3. The trajectories of the fractional order system (11); (a) are stabilized to the equilibrium point $E_2 = (-3.87298, -3.87298, 15)$ at $q = 0.85$; (b) are not stabilized to the equilibrium point $E_2 = (-3.87298, -3.87298, 15)$ at $q = 0.86$.

the controller (14). The error of synchronization are shown in Figure 6. In Figure 6(a) shows the error converges to zero at approximate $t = 0.025$ for the commensurate order $q = 0.86$. Figure 6(b) shows the error converges to zero at approximate $t = 0.022$ for the commensurate order $q = 0.89$. Figure 6(c) shows the error converges to zero at approximate $t = 0.022$ for the commensurate order$q = 0.92$. Figure (6)(d) shows the error converges to zero at approximate $t = 0.02$ for the commensurate order $q = 95$. Figure 6(e) shows the error converges to zero at approximate $t = 0.018$ for the commensurate order $q_1 = q_2 = q_3 = 1$.

6. Conclusion

In this paper, we first discussed the sensitivity analysis of the FOLUS using the new parameters. Then, we investigated the synchronization, of the chaotic FOLUS with new parameters. In [24], for $q = 0.9$ in chaotic FOLUS the synchronization occurs at an approximate time $t = 5s$. In fact, using feedback control method, we showed that synchronization and synchronization error occurs at an approximate time $t = 0.018s$. We also analyze the stability of the system using Lyapunov function, it indicates the effectiveness of the method studied in this paper.

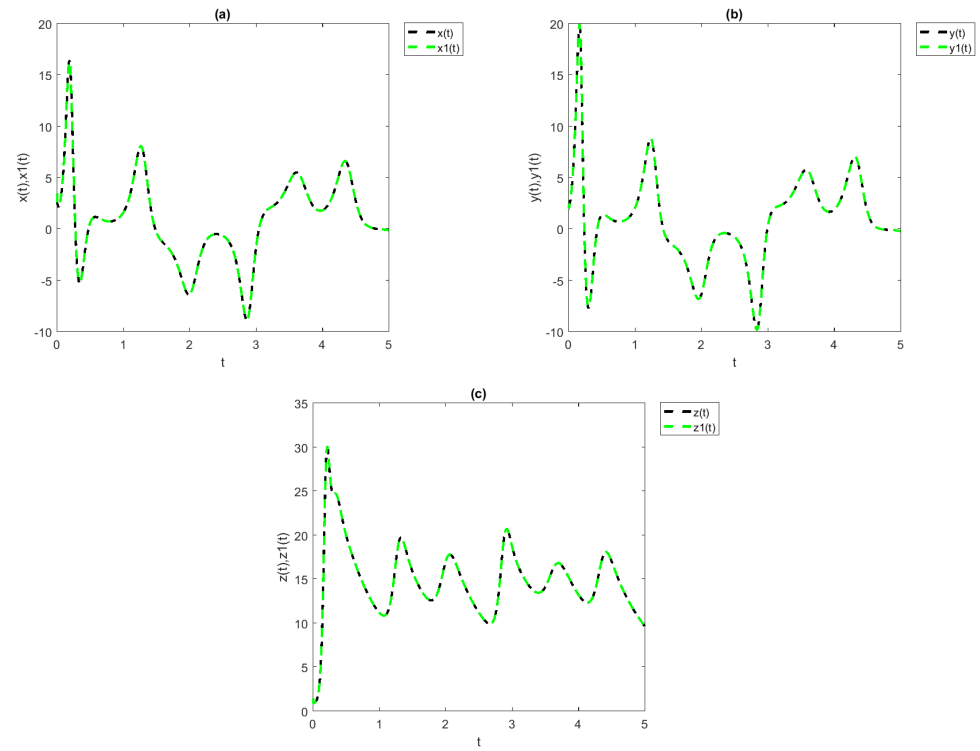

Figure 4. Depicts the phase portraits of synchronization of the master system (10) and the slave system (11).

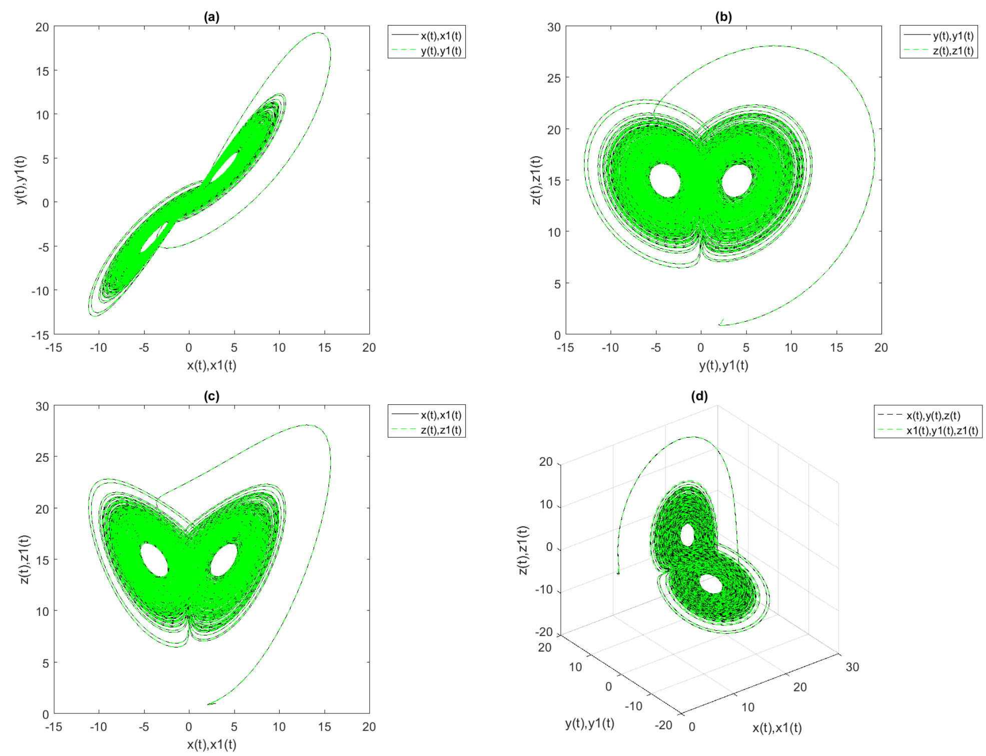

Figure 5. Synchronization for the commensurate orders, $q_1 = q_2 = q_3 = q = 0.86$ of the master system (10) and the slave system (11).

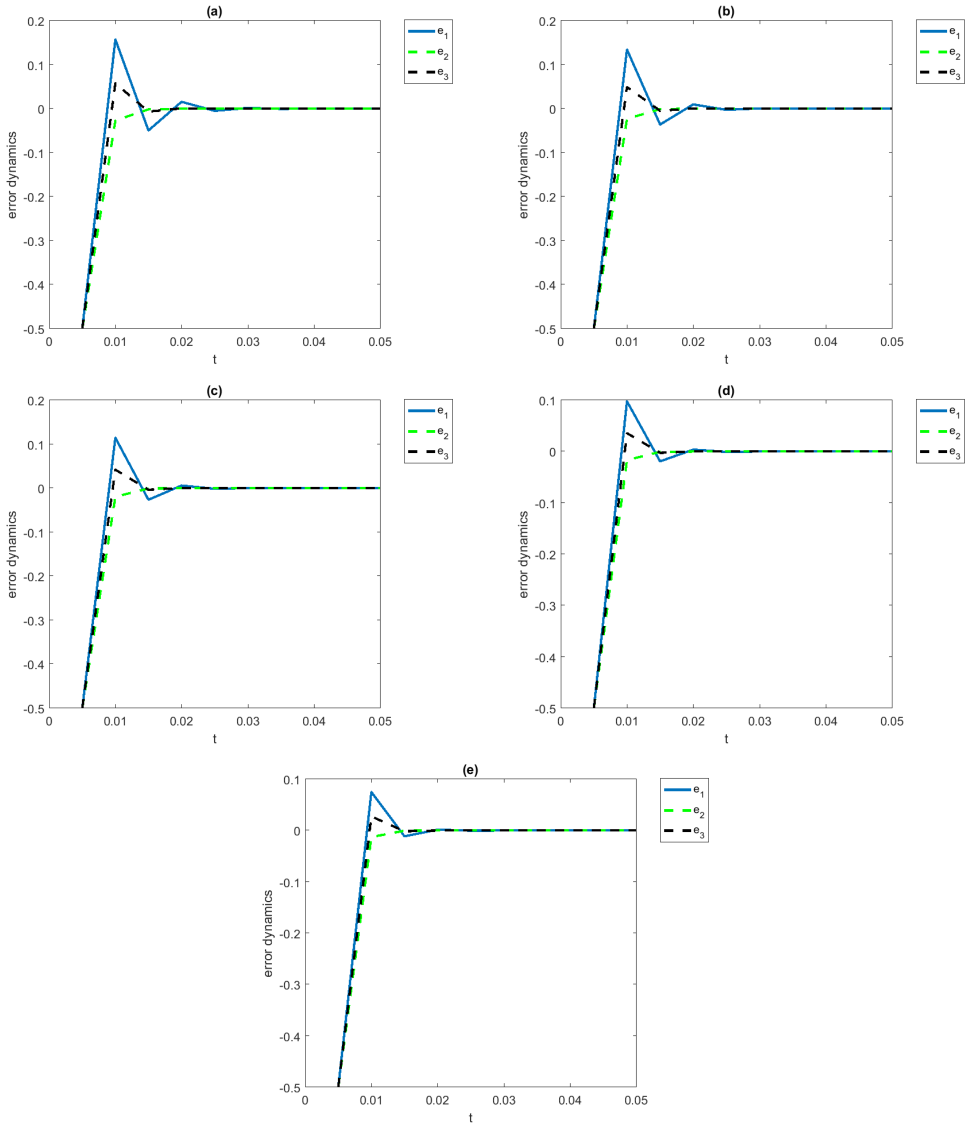

Figure 6. Synchronization error for the commensurate orders, (a) $q_1 = q_2 = q_3 = q = 0.86$, (b) $q = 0.89$, (c) $q = 0.92$, (d) $q = 95$, (e) $q = 1$.

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