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Generalized hyperstability of the cubic functional equation in ultrametric spaces

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Abstract. In this paper, we present the generalized hyperstability results of cubic functional equation in ultrametric Banach spaces using the fixed point method.

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1. Introduction and preliminaries

The starting point of studying the stability of functional equations seems to be the famous talk of Ulam [20] in 1940, in which he discussed a number of important unsolved problems. Among those was the question concerning the stability of group homomorphisms.

Given a group G_1, a metric group G_2 with the metric $d(.,.)$ and a positive number ϵ, does there exists a $\delta > 0$ such that if $f : G_1 \to G_2$ satisfies $d(f(x.y), f(x)f(y)) \leqslant \delta$ for all $x, y \in G_1$, then a homomorphism $\phi : G_1 \to G_2$ exists with $d(f(x), \phi(x)) \leqslant \epsilon$, for all $x \in G_1$.

The first partial answer to Ulam question was given by Hyers [15] in the case of Cauchy equation in Banach spaces. Later, the result of Hyers was significantly generalized by Rassias [19] and Găvruţa [13]. Since then, the stability problems of several functional equations have been extensively investigated.

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We say a functional equation is hyperstable if any function f satisfying the equation approximately (in some sense) must be actually solutions to it. It seems that the first hyperstability result was published in [5] and concerned the ring homomorphisms. However, the term hyperstability has been used for the first time in [17]. Quite often the hyperstability is confused with superstability which admits bounded functions. The next definition more precisely describes the notion of hyperstability (B^A to mean "the family of all functions mapping from a nonempty set A into a nonempty set B").

Definition 1.1 Let X be a nonempty set, (Y, d) be a metric space, $\varepsilon \in \mathbb{R}_0^{X^n}$ and $\mathcal{F}_1$, $\mathcal{F}_2$ be operators mapping from a nonempty set $\mathcal{D} \subset Y^X$ into Y^{X^n}. We say that the operator equation

$$\mathcal{F}_1\varphi(x_1, \ldots, x_n) = \mathcal{F}_2\varphi(x_1, \ldots, x_n), \quad (x_1, \ldots, x_n \in X) \tag{1}$$

is ε-hyperstable provided that every $\varphi_0 \in \mathcal{D}$ which satisfies

$$d\left(\mathcal{F}_1\varphi_0(x_1, \ldots, x_n), \mathcal{F}_2\varphi_0(x_1, \ldots, x_n)\right) \leqslant \varepsilon(x_1, \ldots, x_n), \quad (x_1, \ldots, x_n \in X)$$

fulfills the equation (1).

For information concerning the notion of hyperstability we refer to the survey paper [12]. Numerous papers on this subject have been published and we refer to [1–4, 6–8, 14, 17, 18].

Throughout this paper, $\mathbb{N}$ stands for the set of all positive integers, $\mathbb{N}_0 := \mathbb{N} \cup \{0\}$, N_{m_0} the set of integers $\geqslant m_0$, $\mathbb{R}_+ := [0, \infty)$ and we use the notation X_0 for the set $X \setminus \{0\}$.

Let us recall (see, for instance, [16]) some basic definitions and facts concerning non-Archimedean normed spaces.

Definition 1.2 By a non-Archimedean field we mean a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot| : \mathbb{K} \to [0, \infty)$ such that for all $r, s \in \mathbb{K}$, the following conditions hold:

(1) $|r| = 0$ if and only if $r = 0$,
(2) $|rs| = |r||s|$,
(3) $|r + s| \leqslant \max\{|r|, |s|\}$.

The pair $(\mathbb{K}, |.|)$ is called a valued field.

In any non-Archimedean field we have $|1| = |-1| = 1$ and $|n| \leqslant 1$ for $n \in \mathbb{N}_0$. In any field $\mathbb{K}$ the function $|\cdot| : \mathbb{K} \to \mathbb{R}_+$ given by

$$|x| := \begin{cases} 0, x = 0, \\ 1, x \neq 0, \end{cases}$$

is a valuation which is called trivial, but the most important examples of non-Archimedean fields are p-adic numbers which have gained the interest of physicists for their research in some problems coming from quantum physics, p-adic strings and super strings.

Definition 1.3 Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean non-trivial valuation $|\cdot|$. A function $||\cdot||_* : X \to \mathbb{R}$ is a non-Archimedean norm valuation if it satisfies the following conditions:

(1) $\|x\|_* = 0$ if and only if $x = 0$,
(2) $\|rx\|_* = |r| \, \|x\|_*$ $(r \in \mathbb{K}, x \in X)$,
(3) The strong triangle inequality (ultrametric); namely

$$\|x+y\|_* \leqslant \max\left\{\|x\|_*, \|y\|_*\right\} \text{ for all } x, y \in X.$$

Then $(X, \|\cdot\|_*)$ is called a non-Archimedean normed space or an ultrametric normed space.

Definition 1.4 Let $\{x_n\}$ be a sequence in a non-Archimedean normed space X.

(1) A sequence$\{x_n\}_{n=1}^{\infty}$ in a non-Archimedean space is a Cauchy sequence iff the sequence $\{x_{n+1} - x_n\}_{n=1}^{\infty}$ converges to zero;
(2) The sequence $\{x_n\}$ is said to be convergent if, there exists $x \in X$ such that, for any $\varepsilon > 0$, there is a positive integer N such that $\|x_n - x\|_* \leqslant \varepsilon$, for all $n \geqslant N$. Then the point $x \in X$ is called the limit of the sequence $\{x_n\}$, which is denoted by $\lim_{n\to\infty} x_n = x$;
(3) If every Cauchy sequence in X converges, then the non-Archimedean normed space X is called a non-Archimedean Banach space or an ultrametric Banach space.

Let X, Y be normed spaces. A function $f : X \to Y$ is Cubic provided it satisfies the functional equation

$$f(2x+y) + f(2x-y) = 2f(x+y) + 2f(x-y) + 12f(x) \quad \text{for all } \ x, y \in X, \qquad (2)$$

and we can say that $f : X \to Y$ is Cubic on X_0 if it satisfies (2) for all $x, y \in X_0$ such that $x+y \neq 0$, $x-y \neq 0$, $2x+y \neq 0$ and $2x-y \neq 0$.

In this paper, we present some hyperstability results for the equation (2) in ultrametric Banach spaces using the fixed point method derived from [4, 7, 9]. The obtained results generalize the existing ones in [2]. Before proceeding to the main results, we state Theorem 1.5 which is useful for our purpose. To present it, we introduce three following hypotheses:

(H1) X is a nonempty set, Y is an ultrametric Banach space over a non-Archimedean field, $f_1, ..., f_k : X \longrightarrow X$ and $L_1, ..., L_k : X \longrightarrow \mathbb{R}_+$ are given.
(H2) $\mathcal{T} : Y^X \longrightarrow Y^X$ is an operator satisfying the inequality

$$\left\|\mathcal{T}\xi(x) - \mathcal{T}\mu(x)\right\|_* \leqslant \max_{1\leqslant i\leqslant k}\left\{L_i(x)\left\|\xi\Big(f_i(x)\Big) - \mu\Big(f_i(x)\Big)\right\|_*\right\}, \ \xi, \mu \in Y^X, \ x \in X.$$

(H3) $\Lambda : \mathbb{R}_+^X \longrightarrow \mathbb{R}_+^X$ is a linear operator defined by

$$\Lambda\delta(x) := \max_{1\leqslant i\leqslant k}\left\{L_i(x)\delta\Big(f_i(x)\Big)\right\}, \ \delta \in \mathbb{R}_+^X, \ x \in X.$$

Thanks to a result due to Brzdęk and Ciepliñski [11, re2], we state an analogue of the fixed point theorem [10, Theorem 1] in ultrametric spaces. We use it to assert the existence of a unique fixed point of operator $\mathcal{T} : Y^X \longrightarrow Y^X$.

Theorem 1.5 Let hypotheses (H1)-(H3) be valid and functions $\varepsilon : X \longrightarrow \mathbb{R}_+$ and $\varphi : X \longrightarrow Y$ fulfill conditions $\|\mathcal{T}\varphi(x) - \varphi(x)\|_* \leqslant \varepsilon(x)$ and $\lim_{n\to\infty} \Lambda^n\varepsilon(x) = 0$ for $x \in X$. Then there exists a unique fixed point $\psi \in Y^X$ of $\mathcal{T}$ with $\|\varphi(x) - \psi(x)\|_* \leqslant \sup_{n\in\mathbb{N}_0} \Lambda^n\varepsilon(x)$ for $x \in X$. Moreover $\psi(x) := \lim_{n\to\infty} \mathcal{T}^n\varphi(x)$ for $x \in X$.

2. Main results

In this section, we use Theorem 1.5 as a basic tool to prove the hyperstability results of the cubic functional equation in ultrametric Banach spaces. In the rest of the paper $\{\alpha_n\}_n$ is a sequence of real numbers such that $\lim_{n\to\infty}\alpha_n = 0$. Moreover, we always assume that the characteristic of $\mathbb{K}$ is not 2 (i.e., $2 \neq 0$).

Theorem 2.1 Let X be a real linear space and $(Y, \|\cdot\|_*)$ be an ultrametric Banach space. Assume that the $\varphi : X \times X \to [0, +\infty)$ be a function fulfils the conditions

$$\lim_{m\to\infty} \max_{i+j+k+l=n} \left\{\varphi\left(a_m^i b_m^j c_m^k d_m^l x, a_m^i b_m^j c_m^k d_m^l y\right)\right\} = 0 \tag{3}$$

and

$$\lim_{n\to\infty} \max_{i+j+k+l=n} \left\{\varphi\left(a_m^{i+1} b_m^j c_m^k d_m^l x, a_m^i b_m^j c_m^k d_m^l \alpha_m x\right)\right\} = 0 \tag{4}$$

for all $x, y \in X_0$ and for sufficiently large integers m, where $a_m = \frac{1+\alpha_m}{2}$, $b_m = \frac{1+3\alpha_m}{2}$,$c_m = \frac{1-\alpha_m}{2}$ and $d_m = 2\alpha_m + 1$. Assume that $f : X \to Y$ satisfies

$$\|f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 12f(x)\|_* \leqslant \varphi(x, y) \tag{5}$$

for all $x, y \in X_0$. Then f is cubic on X_0.

Proof. Replacing y by $\alpha_m x$ and x by $\frac{1+\alpha_m}{2}x$ in (5), where $\alpha_m \in \mathbb{R}$, we have

$$\left\|12f\left(\frac{1+\alpha_m}{2}x\right) + 2f\left(\frac{1+3\alpha_m}{2}x\right) + 2f\left(\frac{1-\alpha_m}{2}x\right) - f((2\alpha_m+1)x) - f(x)\right\|_* \leqslant \varphi(\frac{1+\alpha_m}{2}x, \alpha_m x),$$

which implies

$$\left\|12f(a_m x) + 2f(b_m x) + 2f(c_m x) - f(d_m x) - f(x)\right\|_* \leqslant \varphi(a_m x, \alpha_m x) \tag{6}$$

for all $x \in X_0$, where $a_m = \frac{\alpha_m+1}{2}$. Define operators $\mathcal{T}_m : Y^{X_0} \to Y^{X_0}$ and $\Lambda_m : \mathbb{R}_+^{X_0} \to \mathbb{R}_+^{X_0}$ by

$$\mathcal{T}_m\xi(x) := 12\xi(a_m x) + 2\xi(b_m x) + 2\xi(c_m x) - \xi(d_m x), \quad \xi \in Y^{X_0},\ x \in X_0,$$
$$\Lambda_m\delta(x) := \max\left\{\ \delta(a_m x)\ ,\ \delta(b_m x),\ \delta(c_m x)\ , \delta(d_m x)\right\} \quad \delta \in \mathbb{R}_+^{X_0},\ x \in X_0,$$

and write

$$\varepsilon_m(x) := \varphi(a_m x, \alpha_m x), \quad x \in X_0. \tag{7}$$

It is easily seen that Λ_m has the form described in (H3) with $k = 4$, $f_1(x) = a_m x$, $f_2(x) = b_m x$, $f_3(x) = c_m x$, $f_4(x) = d_m x$ and $L_1(x) = L_2(x) = L_3(x) = L_4(x) = 1$. Further, (6) can be written in the form $\|\mathcal{T}_m f(x) - f(x)\|_* \leqslant \varepsilon_m(x)$ for $x \in X_0$. Moreover, for every $\xi, \mu \in Y^{X_0}$, $x \in X_0$

$$\begin{aligned}
&\Big\|\mathcal{T}_m\xi(x) - \mathcal{T}_m\mu(x)\Big\|_* \\
&= \Big\|12\xi(a_mx) + 2\xi(b_mx) + 2\xi(c_mx) - \xi(d_mx) - 12\mu(a_mx) - 2\mu(b_mx) - 2\mu(c_mx) + \mu(d_mx)\Big\|_* \\
&\leqslant \max\Big\{\Big\|(\xi-\mu)(a_mx)\Big\|_*, \Big\|(\xi-\mu)(b_mx)\Big\|_*, \Big\|(\xi-\mu)(c_mx)\Big\|_*, \Big\|(\xi-\mu)(d_mx)\Big\|_*\Big\} \\
&= \max\Big\{\Big\|(\xi-\mu)\big(f_1(x)\big)\Big\|_*, \Big\|(\xi-\mu)\big(f_2(x)\big)\Big\|_*, \Big\|(\xi-\mu)\big(f_3(x)\big)\Big\|_*, \Big\|(\xi-\mu)\big(f_4(x)\big)\Big\|_*\Big\}.
\end{aligned}$$

So, (H2) is valid. By using the mathematical induction, we will show that for all $n \in \mathbb{N}_0$ and for each $x \in X_0$, we have

$$\Lambda_m^n \varepsilon_m(x) = \max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+1}b_m^jc_m^kd_m^lx, a_m^ib_m^jc_m^kd_m^l\alpha_mx\Big)\Big\}, \tag{8}$$

where $a_m = \frac{1+\alpha_m}{2}$. From (7), we obtain that (8) holds for $n = 0$. Next, we will assume that (8) holds for $n = k$, where $k \in \mathbb{N}$. Then we have

$$\begin{aligned}
\Lambda_m^{k+1}\varepsilon_m(x) = \Lambda_m\big(\Lambda_m^k\varepsilon_m(x)\big) &= \max\Big\{\Lambda_m^k\varepsilon_m(a_mx), \Lambda_m^k\varepsilon_m(b_mx), \Lambda_m^k\varepsilon_m(c_mx), \Lambda_m^k\varepsilon_m(d_mx)\Big\} \\
&= \max\Big\{\max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+2}b_m^jc_m^kd_m^lx, a_m^{i+1}b_m^jc_m^kd_m^l\alpha_mx\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+1}b_m^{j+1}c_m^kd_m^lx, a_m^ib_m^{j+1}c_m^kd_m^l\alpha_mx\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+1}b_m^jc_m^{k+1}d_m^lx, a_m^ib_m^jc_m^{k+1}d_m^l\alpha_mx\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+1}b_m^jc_m^kd_m^{l+1}x, a_m^ib_m^jc_m^kd_m^{l+1}\alpha_mx\Big)\Big\}\Big\} \\
&= \max_{i+j+k+l=n+1}\Big\{\varphi\Big(a_m^{i+1}b_m^jc_m^kd_m^lx, a_m^ib_m^jc_m^kd_m^l\alpha_mx\Big)\Big\}
\end{aligned}$$

This shows that (8) holds for $n = k+1$. Now, we can conclude that the equality (8) holds for all $n \in \mathbb{N}_0$. From (4) and (8), we obtain $\lim_{n\to\infty}\Lambda^n\varepsilon_m(x) = 0$ for all $x \in X_0$. Hence, according to Theorem 1.5, there exists a unique solution $C_m : X_0 \to Y$ of the equation

$$C_m(x) = 12C_m(a_mx) + 2C_m(b_mx) + 2C_m(c_mx) - C_m(d_mx) \tag{9}$$

such that

$$\|f(x) - C_m(x)\|_* \leqslant \sup_{n\in\mathbb{N}_0}\Big\{\max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^{i+1}b_m^jc_m^kd_m^lx, a_m^ib_m^jc_m^kd_m^l\alpha_mx\Big)\Big\}\Big\} \tag{10}$$

for all $x \in X_0$. Moreover, $C_m(x) := \lim_{n\to\infty}\mathcal{T}_m^nf(x)$ for all $x \in X_0$. Now we show that

$$\begin{aligned}
&\|12\mathcal{T}_m^nf(x) + 2\mathcal{T}_m^nf(x+y) + 2\mathcal{T}_m^nf(x-y) - \mathcal{T}_m^nf(2x+y) - \mathcal{T}_m^nf(2x-y)\|_* \\
&\leqslant \max_{i+j+k+l=n}\Big\{\varphi\Big(a_m^ib_m^jc_m^kd_m^lx, a_m^ib_m^jc_m^kd_m^ly\Big)\Big\},
\end{aligned} \tag{11}$$

for every $x, y \in X_0$ such that $x+y \neq 0$, $x-y \neq 0$, $2x-y \neq 0$, $2x+y \neq 0$. Since the case $n=0$ is just (5), take $k \in \mathbb{N}$ and assume that (11 holds for n and every $x, y \in X_0$ such that $x+y \neq 0$, $x-y \neq 0$, $2x-y \neq 0$, $2x+y \neq 0$. Then

$$\begin{aligned}
&\Big\|12\mathcal{T}_m^{n+1}f(x) + 2\mathcal{T}_m^{k+1}f(x+y) + 2\mathcal{T}_m^{n+1}f(x-y) - \mathcal{T}_m^{n+1}f(2x+y) - \mathcal{T}_m^{n+1}f(2x-y)\Big\|_* \\
&= \big\|144\mathcal{T}_m^n f(a_m x) + 24\mathcal{T}_m^n f(b_m x) + 24\mathcal{T}_m^n f(c_m x) - 12\mathcal{T}_m^n f(d_m x) + 24\mathcal{T}_m^n f(a_m(x+y)) \\
&\quad + 4\mathcal{T}_m^n f(b_m(x+y) + 4\mathcal{T}_m^n f(c_m(x+y) - 2\mathcal{T}_m^n f(d_m(x+y)) + 24\mathcal{T}_m^n f(a_m(x-y)) \\
&\quad + 4\mathcal{T}_m^n f(b_m(x-y) + 4\mathcal{T}_m^n f(c_m(x-y) - 2\mathcal{T}_m^n f(d_m(x-y)) - 12\mathcal{T}_m^n f(a_m(2x+y)) \\
&\quad - 2\mathcal{T}_m^n f(b_m(2x+y) - 2\mathcal{T}_m^n f(c_m(2x+y) + \mathcal{T}_m^n f(d_m(2x+y)) - 12\mathcal{T}_m^n f(a_m(2x-y)) \\
&\quad - 2\mathcal{T}_m^n f(b_m(2x-y) - 2\mathcal{T}_m^n f(c_m(2x-y) + \mathcal{T}_m^n f(d_m(2x-y))\Big\|_* \\
&\leqslant \max\{\|12\mathcal{T}_m^n f(a_m x) + 2\mathcal{T}_m^n f(a_m(x+y)) + 2\mathcal{T}_m^n f(a_m(x-y)) - \mathcal{T}_m^n f(a_m(2x+y)) \\
&\quad - \mathcal{T}_m^n f(a_m(2x-y))\|_*, \|12\mathcal{T}_m^n f(b_m x) + 2\mathcal{T}_m^n f(b_m(x+y)) + 2\mathcal{T}_m^n f(b_m(x-y)) \\
&\quad - \mathcal{T}_m^n f(b_m(2x+y)) - \mathcal{T}_m^n f(b_m(2x-y))\|_*, \|12\mathcal{T}_m^n f(c_m x) + 2\mathcal{T}_m^n f(c_m(x+y)) \\
&\quad + 2\mathcal{T}_m^n f(c_m(x-y)) - \mathcal{T}_m^n f(c_m(2x+y)) - \mathcal{T}_m^n f(c_m(2x-y))\|_*, \|12\mathcal{T}_m^n f(d_m x) \\
&\quad + 2\mathcal{T}_m^n f(d_m(x+y)) + 2\mathcal{T}_m^n f(d_m(x-y)) - \mathcal{T}_m^n f(d_m(2x+y)) - \mathcal{T}_m^n f(d_m(2x-y))\|_*\} \\
&\leqslant \max\Big\{ \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i+1}b_m^j c_m^k d_m^l x, a_m^{i+1}b_m^j c_m^k d_m^l y\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i}b_m^{j+1} c_m^k d_m^l x, a_m^{i}b_m^{j+1} c_m^k d_m^l y\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i}b_m^{j} c_m^{k+1} d_m^l x, a_m^{i}b_m^{j} c_m^{k+1} d_m^l y\Big)\Big\}, \\
&\quad \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i}b_m^{j} c_m^{k} d_m^{l+1} x, a_m^{i}b_m^{j} c_m^{k} d_m^{l+1} y\Big)\Big\}, \\
&\leqslant \max_{i+j+k+l=n+1} \Big\{\varphi\Big(a_m^{i}b_m^{j} c_m^{k} d_m^{l} x, a_m^{i}b_m^{j} c_m^{k} d_m^{l} y\Big)\Big\}\Big\}
\end{aligned}$$

for all $x, y \in X_0$ such that $x+y \neq 0$, $x-y \neq 0$, $2x-y \neq 0$ and $2x+y \neq 0$. Thus, by induction, we have shown that (11) holds for every $n \in \mathbb{N}_0$. Letting $n \to \infty$ in (11), we obtain that

$$C_m(2x+y) + C_m(2x-y) = 2C_m(x+y) + 2C_m(x-y) - 12C_m(x)$$

for all $x, y \in X_0$ such that $x+y \neq 0$, $x-y \neq 0$, $2x-y \neq 0$ and $2x+y \neq 0$. In this way, we obtain a sequence $\{C_m\}_{m \geqslant m_0}$ of cubic functions on X_0 such that

$$\|f(x) - C_m(x)\|_* \leqslant \sup_{n \in \mathbb{N}_0} \Big\{ \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i+1}b_m^j c_m^k d_m^l x, a_m^{i}b_m^j c_m^k d_m^l \alpha_m x\Big)\Big\}\Big\}, \quad x \in X_0$$

this implies that

$$\|f(x) - C_m(x)\|_* \leqslant \max_{i+j+k+l=n} \Big\{\varphi\Big(a_m^{i+1}b_m^j c_m^k d_m^l x, a_m^{i}b_m^j c_m^k d_m^l \alpha_m x\Big)\Big\}, \quad x \in X_0.$$

Letting $m \to \infty$ in (3), it follows that f is cubic on X_0. ■

The following corollaries are immediate consequences of Theorem 2.1.

Corollary 2.2 [2, Theorem 2.1] Let $(X, \|\cdot\|)$ and $(Y, \|\cdot\|_*)$ be normed space and ultrametric Banach space respectively, $c \geqslant 0$, $p, q \in \mathbb{R}$, $p+q < 0$ and $q < 0$. If $f : X \to Y$ satisfies

$$\|f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 12f(x)\|_* \leqslant c\, \|x\|^p\, \|y\|^q$$

for all $x, y \in X_0$. Then f is cubic on X_0.

Proof. The proof follows from Theorem 2.1 by taking $\varphi(x,y) = c\, \|x\|^p\, \|y\|^q$ for all $x, y \in X_0$. It is clear that φ satisfies the conditions (3) and (4). Then, we can choose $\alpha_m = m$, where $m \in \mathbb{N}$ to get the desired result. ■

By similar method we can prove the following corollary.

Corollary 2.3 [2, Theorem 2.2] Let $(X, \|\cdot\|)$ and $(Y, \|\cdot\|_*)$ be normed space and ultrametric Banach space respectively, $c \geqslant 0$, $p, q \in \mathbb{R}$, $p+q > 0$ and $q > 0$. If $f : X \to Y$ satisfies

$$\|f(2x+y) + f(2x-y) - 2f(x+y) - 2f(x-y) - 12f(x)\|_* \leqslant c\, \|x\|^p\, \|y\|^q$$

for all $x, y \in X_0$. Then f is cubic on X_0.

Proof. Putting $\varphi(x,y) = c\, \|x\|^p\, \|y\|^q$ for all $x, y \in X_0$ in Theorem 2.1, we get the the desired result when we choose $\alpha_m = \frac{-2}{m}$, where $m \in \mathbb{N}_3$. ■

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