

## *F*-Closedness in bitopological spaces

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**Abstract.** The purpose of this paper is to introduce the concept of pairwise *F*-closedness in bitopological spaces. This space contains both of pairwise strong compactness and pairwise *S*-closedness and contained in pairwise quasi *H*-closedness. The characteristics and relationships concerning this new class of spaces with other corresponding types are established. Moreover, several of its basic and important properties are discussed.

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## 1. Introduction

The notion of bitopological spaces was initiated by Kelly [10] in 1963. This concept helps the authors to generalize the most results related to the topological spaces, which are known before. Among these results is the construction of several types of compact and closed spaces depending on the meaning of a pairwise-open cover in bitopological spaces due to Flether et al. [6]. So, in 1982, Mashhour et al., [17] gave the pairwise *S*-closedness as a generalization of the *S*-closed due to Thompson [25]. One year later, Noiri et al. [20] presented the spaces: pairwise quasi *H*-closed and pairwise nearly-compact. While in 1991, Kheder and Noiri [12] defined the pairwise almost co-compact space. Several properties of all previous notions have been studied by previous research workers and

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others. Therefore, the main goal of this paper, is to introduce the concept of pairwise F-closed space by utilizing the pairwise feebly-open cover. This new type of bitopological spaces is considered as a generalization for the F-closedness that were given by Chae and Lee [4] in 1987, via the feebly-open sets due to Maheshwari and Tapi [16].

## 2. Preliminaries

Throughout the present work, we will consider the bitopological space  $(X, \tau_1, \tau_2)$  which consists of a nonempty set  $X$  with two arbitrary topologies  $\tau_1$  and  $\tau_2$  on it, without any separation properties and whenever such properties are needed. These will be explicitly assumed. For any subset  $A$  of a topological space  $(X, \tau_1, \tau_2)$  the closure and the interior of  $A$  with respect to  $\tau_i$  are denoted by  $\tau_i - Cl(A)$  and  $\tau_i - Int(A)$ , respectively,  $i = 1, 2$ . However,  $\tau_i - Cl(A)$  (resp.  $\tau_i - Int(A)$ ) will be denoted by  $Cl_i(A)$  (resp.  $Int_i(A)$ ). For the simplicity, if the meaning is explicit.

**Definition 2.1** . Any subset  $A$  of  $X$  in a bitopological space  $(X, \tau_i, \tau_j)$  where  $i, j = 1, 2$  and  $i \neq j$  is said to be:

- (a)  $(\tau_i, \tau_j)$ - semi-open [12] (briefly  $(i, j)$ -semi-open) if there exists  $U \in \tau_i$ , such that  $U \subset A \subset Cl_j(U)$ , equivalently  $A \subset Cl_j(Int_i(A))$ ,
- (b)  $(\tau_i, \tau_j)$ - preopen [6] (briefly  $(i, j)$ -preopen) if there exists  $U \subset \tau_i$  such that  $A \subset U \subset Cl_j(U)$ , equivalently  $A \subset Int_i(Cl_j(A))$ ,
- (c)  $(\tau_i, \tau_j)$ - $\alpha$ -open [11] (briefly  $(i, j)$ - $\alpha$ -open) if there exists  $U \subset \tau_i$ , such that  $U \subset A \subset Int_i(Cl_j(U))$ , equivalently  $A \subset Int_i(Cl_j(Int_i(A)))$ ,
- (d)  $(\tau_i, \tau_j)$ -regular open (briefly  $(i, j)$ -regular open) if  $A = Int_i(Cl_j(A))$ .

Also, a subset  $A$  of  $(X, \tau_1, \tau_2)$  is said to be pairwise semi-open if it is  $(1, 2)$ -semi-open and  $(2, 1)$ -semi-open. While, pairwise preopen sets and pairwise  $\alpha$ -open sets are similarly defined. The complement of an  $(i, j)$ -semi-open (resp.  $(i, j)$ -preopen,  $(i, j)$ - $\alpha$ -open) set is said to be  $(i, j)$ -semi-closed (resp.  $(i, j)$ -preclosed,  $(i, j)$ - $\alpha$ -closed). A subset  $A$  is said to be pairwise semi-closed if it is  $(1, 2)$  semi-closed and  $(2, 1)$  semi-closed. Pairwise preclosedness and pairwise  $\alpha$ -closedness are similarly defined.

The intersection of all  $(i, j)$ -semi-closed sets of a space  $(X, \tau_1, \tau_2)$  containing  $A$  is called the  $(\tau_i, \tau_j)$ -semiclosure of  $A$  and is denoted by  $(\tau_i, \tau_j)$ - $sCl(A)$  (briefly  $(i, j)$ - $sCl(A)$ ).

The collection of  $(i, j)$ -regular-open,  $(i, j)$ -semi-open,  $(i, j)$ -preopen, and  $(i, j)$ - $\alpha$ -open of a bitopological space  $(X, \tau_1, \tau_2)$  will be denoted by  $(i, j)$ -RO(X),  $(i, j)$ -SO(X),  $(i, j)$ -PO(X) and  $(i, j)$ - $\alpha$ O(X), respectively, for  $i \neq j$  and  $i, j = 1, 2$ .

**Definition 2.2** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $(\tau_i, \tau_j)$ -feebly-open [7] (briefly  $(i, j)$ -feebly-open) if there exists a  $\tau_i$ -open set  $U$  such that  $U \subset A \subset (i, j) - scl(A)$ , for  $i \neq j$  and  $i, j = 1, 2$ .

A subset  $A$  of a bitopological space  $(X, \tau_i, \tau_j)$  is pairwise feebly-open if it is  $(1, 2)$ -feebly open and  $(2, 1)$ -feebly open.

A cover  $\mu$  of a bitopological space  $(X, \tau_1, \tau_2)$  is called pairwise open cover [4, 5, 10] (resp. pairwise semi-open [14], pairwise preopen [17], pairwise co-open [10]) if  $\mu \subset \tau_1 \cup \tau_2$  (resp.  $\mu \subseteq SO(X, \tau_1) \cup SO(X, \tau_2)$ ,  $\mu \subseteq PO(X, \tau_1) \cup PO(X, \tau_2)$ ,  $\mu \subseteq CO(X, \tau_1) \cup CO(X, \tau_2)$ ) and  $\mu$  contains at least one nonempty member of  $\tau_1$  (resp.  $SO(X, \tau_1)$ ,  $PO(X, \tau_1)$ ,  $CO(X, \tau_1)$ ) and one nonempty number of  $\tau_2$  (resp.  $SO(X, \tau_2)$ ,  $PO(X, \tau_2)$ ,  $CO(X, \tau_2)$ ). If every pairwise open cover of  $(X, \tau_1, \tau_2)$  has a finite subcover then, the space is called pairwise compact.

**Definition 2.3** see 20A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise strongly

compact if every pairwise preopen cover has a finite subcover.

**Definition 2.4** A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise nearly compact [17] if for every pairwise open cover  $\{V_\lambda : \lambda \in \nabla\}$  of  $X$ , there exist a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Int_i(Cl_j(V_\lambda)) : \lambda \in \nabla_\circ\}$ . for  $i, j = 1, 2$  and  $i \neq j$ .

**Definition 2.5** A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise quasi H-closed [2] (resp. pairwise almost co-compact [10], pairwise S-closed [14]) if for every pairwise open cover (resp. pairwise co-open cover, pairwise semi-open cover)  $\{V_\lambda : \lambda \in \nabla\}$  of  $X$ , there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Cl_j(V_\lambda) : \lambda \in \nabla_\circ\}$ .

**Definition 2.6** see 18A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise extremally disconnected if  $Cl_j(U)$  is  $\tau_i$ -open in  $X$  for each  $\tau_i$ -open set  $U$  in  $X$ .

**Proposition 2.7** If  $A$  is pairwise preopen of a space  $(X, \tau_1, \tau_2)$ , then

- (1)  $Cl_j(Int_i(Cl_j(A))) = Cl_j(A)$ .
- (2)  $(i, j) - sCl(A) = Int_i(Cl_j(A))$ ,  $i, j = 1, 2$  and  $i \neq j$ .

**Proof.** The first statement is obvious, while the second follows from the fact that,  $(i, j) - sCl(A) = A \cup Int_i(Cl_j(A))$ . ■

**Proposition 2.8** A subset  $A$  of  $(X, \tau_1, \tau_2)$  is pairwise feebly open if and only if it is pairwise  $\alpha$ -open.

**Proof.** Let  $A$  be pairwise feebly-open in  $(X, \tau_1, \tau_2)$ . There exists a  $U \in \tau_1 \cap \tau_2$  such that  $U \subset A \subset (i, j) - sCl(U)$ . But  $U \subset Int_i(A)$  and since  $(i, j) - sCl(U) = Int_i(Cl_j(U))$ . (see Proposition (2.7)) then we have  $A \subset Int_i(Cl_j(Int_i(A)))$  which means that  $A$  is pairwise  $\alpha$ -open in  $(X, \tau_1, \tau_2)$ . Conversely, let  $A$  be pairwise  $\alpha$ -open in  $(X, \tau_1, \tau_2)$ , then  $A \subset Int_i(Cl_j(Int_i(A)))$  and hence  $Int_i(A) \subset A \subset (i, j) - sCl(Int_i(A))$ . Therefore,  $A$  is pairwise feebly- open in  $(X, \tau_1, \tau_2)$ . ■

**Definition 2.9** A cover  $\mu$  of  $(X, \tau_1, \tau_2)$  is called a pairwise feebly-open cover if  $\mu \subset FO(X, \tau_1) \cup FO(X, \tau_2)$ ,  $\mu$  must contains at least one nonempty member of each of  $FO(X, \tau_1)$  and  $FO(X, \tau_2)$ .

**Proposition 2.10** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is an  $(i, j)$ - feebly-open set if and only if  $A$  is  $(i, j)$ -semi-open and  $(i, j)$ -preopen for  $i \neq j$  and  $i, j = 1, 2$ .

**Proof.** Follows by the equivalently in Theorem 5.1 of [14]. ■

We will also need the following definitions;

**Definition 2.11** (see [10]) A bitopological space  $(X, \tau_1, \tau_2)$  is called

- 1- pairwise regular if for each  $U \in \tau_1$  and  $x \in U$  there exists  $V \in \tau_1$  with  $x \in V \subseteq Cl_{\tau_2}(V) \subseteq U$  and for each  $U \in \tau_2$  and  $x \in U$  there exists  $V \in \tau_2$  with  $x \in V \subseteq Cl_{\tau_1}(V) \subseteq U$ .
- 2- pairwise normal if for a  $\tau_1$ -closed set  $A$  and a  $\tau_2$ -closed set  $B$  with  $A \cap B = \phi$  there exists a  $\tau_2$ -open set  $U$  and a  $\tau_1$ -open set  $V$  with  $A \subseteq U, B \subseteq V$  and  $U \cap V = \phi$ .

### 3. Pairwise F-closed Spaces

**Definition 3.1** A bitopological space  $(X, \tau_1, \tau_2)$  is said to be pairwise F-closed if for every pairwise feebly-open cover  $\{V_\lambda | \lambda \in \nabla\}$  of  $X$ , there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Cl_j(V_\lambda) | \lambda \in \nabla_\circ\}$ .

**Theorem 3.2** The following are equivalent for a bitopological space  $(X, \tau_1, \tau_2)$ :

- (a)  $(X, \tau_1, \tau_2)$  is pairwise F-closed.
- (b) For any pairwise preopen cover  $\{V_\lambda | \lambda \in \nabla\}$  of  $X$ , there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Cl_j(V_\lambda) | \lambda \in \nabla_\circ\}$ .
- (c) For any pairwise preclosed family  $\{F_\lambda | \lambda \in \nabla\}$  of subsets of  $X$  satisfying  $\cap\{F_\lambda | \lambda \in \nabla\} = \phi$ , there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $\cap\{Int_i(F_\lambda) | \lambda \in \nabla_\circ\} = \phi$ .

**Proof.** (a) $\Rightarrow$ (b): Let  $\{V_\lambda | \lambda \in \nabla\}$  be a cover of  $X$  by pairwise preopen set of  $X$ . Since for each  $\{\lambda \in \nabla\}$ ,  $V_\lambda \subset Int_i(Cl_j(V_\lambda))$ , this means that  $\{Int_i(Cl_j(V_\lambda)) | \lambda \in \nabla\}$  is a pairwise open cover of  $X$ , and therefore it is pairwise feebly open. So, there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Cl_j(Int_i(Cl_j(V_\lambda))) | \lambda \in \nabla_\circ\}$ , by Proposition (2.7), we obtain  $X = \cup\{Cl_j(V_\lambda) | \lambda \in \nabla_\circ\}$ .

(b) $\Rightarrow$ (c): Let  $\{F_\lambda | \lambda \in \nabla\}$  be a pairwise preclosed family of subsets of  $X$  satisfying  $\cap\{F_\lambda | \lambda \in \nabla\} = \phi$ . Then  $\{X \setminus F_\lambda | \lambda \in \nabla\}$  is a pairwise preopen cover of  $X$ . There exists a finite subsets  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Cl_j(X \setminus F_\lambda) | \lambda \in \nabla_\circ\}$ , therefore, we obtain  $\cap\{F_\lambda | \lambda \in \nabla_\circ\} = \phi$ , for  $i \neq j$  and  $i, j = 1, 2$ .

(c) $\Rightarrow$ (a): Let  $\{U_\lambda | \lambda \in \nabla\}$  be a pairwise preopen cover of  $X$ . Then  $\cap\{X \setminus U_\lambda | \lambda \in \nabla\} = \phi$ . So,  $\cap\{Int_i(X \setminus U_\lambda) | \lambda \in \nabla_\circ\} = \phi$ . This shows that  $\cup\{Cl_j(U_\lambda) | \lambda \in \nabla_\circ\} = X$  and hence,  $(X, \tau_1, \tau_2)$  is pairwise F-closed. ■

**Theorem 3.3** Let  $(X, \tau_1, \tau_2)$  be pairwise extremally disconnected. Then it is pairwise F-closed if and only if it is pairwise S-closed.

**Proof.** Follows from Proposition 2.7 and the fact that; in any pairwise extremally disconnected space  $(X, \tau_1, \tau_2)$ , the two classes pairwise  $\alpha$ -sets and pairwise semi-open sets are coincides. ■

The following theorem shows that the pairwise F-closed spaces are properly contains the pairwise nearly compact spaces.

**Theorem 3.4** Every pairwise nearly compact space  $(X, \tau_1, \tau_2)$  is pairwise F-closed.

**Proof.** Let  $\{V | \lambda \in \nabla\}$  be a pairwise feebly-open cover of a pairwise nearly compact space  $(X, \tau_1, \tau_2)$ . Then  $X = \cup\{Int_i(Cl_j(Int_i(V_\lambda))) | \lambda \in \nabla\}$  because  $Int_i(V_\lambda) \subset V_\lambda \subset Int_i(Cl_j(Int_i(V_\lambda)))$  for each  $\lambda \in \nabla$ . Since  $(X, \tau_1, \tau_2)$  is pairwise nearly compact and  $Int_i(V_\lambda)$  is pairwise open in  $X$ , we have a finite subfamily  $\nabla_\circ$  of  $\nabla$  such that  $X = \cup\{Int_i(Cl_j(Int_i(V_\lambda))) | \lambda \in \nabla_\circ\}$ . Thus  $X = \cup\{Cl_j(V_\lambda) | \lambda \in \nabla_\circ\}$  and therefore  $(X, \tau_1, \tau_2)$  is pairwise F-closed. ■

**Remark 1** A pairwise strongly compact space is pairwise F-closed but not conversely.

**Example 3.5** Let  $X$  be the set of real numbers with two topologies  $\tau_1 = \{\phi, X, (-\infty, a]\}$  and  $\tau_2 = \{\phi, X, (a, \infty)\}$  for any point  $a \in X$ . Then  $(X, \tau_1, \tau_2)$  is pairwise F-closed but not pairwise strongly compact for  $\{\{x\} \text{ for every } x \in X, x \leq a\} \cup \{\{x\} : x \in X, x > a\}$  is pairwise preopen cover which has no finite subcover.

**Remark 2** A pairwise F-closed space is pairwise quasi H-closed but not conversely.

**Example 3.6** Let  $X$  be an infinite set,  $\tau_1$  be the discrete topology on  $X$  and let  $\tau_2$  be the excluding point topology with excluding point  $p \in X$ . One can deduce that  $(X, \tau_1, \tau_2)$  is pairwise quasi H-closed but not pairwise F-closed.

**Example 3.7** There exists a pairwise F-closed space  $(X, \tau_1, \tau_2)$  which is not pairwise compact. Let  $X$  be an infinite set,  $\tau_1$  be the discrete trophology and  $\tau_2 = \{\phi, V \subseteq X : p \in V, \text{ for a fixed point } p \in X\}$ . Then  $(X, \tau_1, \tau_2)$  is pairwise S-closed and consequently it

is pairwise  $S$ -closed but not pairwise compact since  $\mu = \{\{x\} : \text{for all } x \in X\}$  is pairwise open cover which has no finite subcover.

From Remark 5.2 of [12] and Definitions 2.3, 2.4, 2.5 and 3.1, we easily obtain the above diagram.

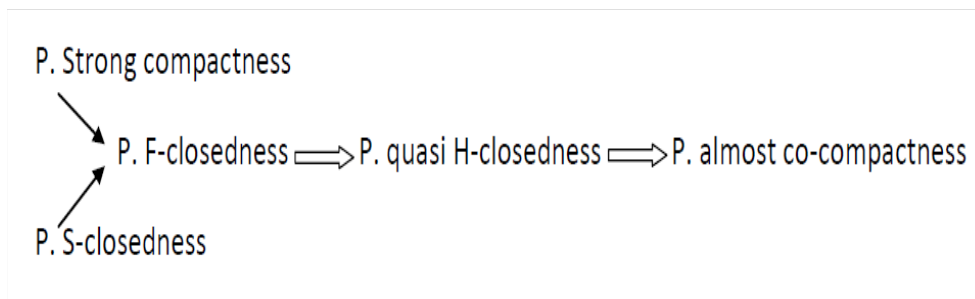


Figure 1. where "P" denotes the term " pairwise"

**Theorem 3.8**

A pairwise extremally disconnected pairwise F-closed space  $(X, \tau_1, \tau_2)$  is pairwise nearly-compact.

**Proof.** Let  $\{V_\lambda \mid \lambda \in \nabla\}$  be a pairwise open cover of  $X$ , and since  $(X, \tau_1, \tau_2)$  is pairwise extremally disconnected, then  $Cl_j(V_\lambda) = Int_i(Cl_j(V_\lambda))$  for each  $\lambda \in \nabla$ . Hence  $\{V_\lambda \mid \lambda \in \nabla\}$  is a pairwise feebly-open cover of  $X$ , there exists a finite subset  $\nabla_o$  of  $\nabla$  such that  $X = \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_o\} = \cup\{Int_i(Cl_j(V_\lambda)) \mid \lambda \in \nabla_o\}$ . This shows that  $(X, \tau_1, \tau_2)$  is pairwise nearly compact. ■

**Definition 3.9** 24A space  $(X, \tau_1, \tau_2)$  is called pairwise almost regular if for every  $x \in X$  and every  $i$ -open set  $V$  containing  $x$ , there exists an  $i$ -open set  $U$  such that  $x \in U \subset Cl_j(U) \subset Int_i(Cl_j(V))$ , for  $i \neq j$  and  $i, j = 1, 2$ .

**Theorem 3.10**

Every pairwise almost regular pairwise quasi H-closed space  $(X, \tau_1, \tau_2)$  is pairwise F-closed.

**Proof.** It is known that an almost-regular space is quasi H-closed if and only if only it is nearly compact [23, Theorem (2.3)]. Similar result can be verified in bitopological spaces which is useful to complete the proof. ■

**Theorem 3.11** A pairwise almost regular pairwise F-closed space  $(X, \tau_1, \tau_2)$  is pairwise nearly-compact.

**Proof.** Follows directly by applying the meaning of pairwise almost-regularity on  $(X, \tau_1, \tau_2)$  which having the pairwise F-closedness property. ■

Since "the class of pairwise almost co-compactness of  $(X, \tau_1, \tau_2)$  is properly contains the class of pairwise H-closedness of the same bitopological space", then the following two theorems show under what conditions that the equivalently between these two spaces and the other corresponding previously types will be satisfied.

**Remark 3** In view of the above diagram and Theorems 5.3 and 5.4 of [12], we have the following theorem.

**Theorem 3.12** For a pairwise extremally disconnected space  $(X, \tau_1, \tau_2)$  the pairwise F-closedness, pairwise S-closedness, pairwise nearly-compactness, pairwise quasi H-closedness and pairwise almost co-compactness are equivalent.

**Theorem 3.13** If  $(X, \tau_1, \tau_2)$  is pairwise almost regular, then the following properties are equivalent: pairwise F-closedness, pairwise nearly compactness, pairwise quasi H-closedness and pairwise co-compactness.

Recall that a space  $(X, \tau)$  is called hyperconnected [7] if every nonempty open set is dense or, equivalently, if  $RC(X, \tau) = \{X, \emptyset\}$ .

Observation Let  $(X, \tau_1, \tau_2)$  be a bitopological space. If  $(X, \tau_1)$  (or  $(X, \tau_2)$ ) is hyperconnected then  $(X, \tau_1, \tau_2)$  is pairwise F-closed.

#### 4. Pairwise F-closed Subspaces

The purpose of this section is to investigate and study two new types of subspaces in a bitopological space  $(X, \tau_1, \tau_2)$ , which are pairwise F-closedness and pairwise quasi H-closedness subspaces relative to  $X$ .

**Definition 4.1** A subset  $A$  of  $(X, \tau_1, \tau_2)$  is said to be pairwise F-closed (resp. pairwise quasi H-closed [1]) relative to  $X$  if for every cover  $\{V_\lambda \mid \lambda \in \nabla\}$  of  $A$  by pairwise feebly-open (resp. pairwise open) sets of  $X$ , there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $A \subset \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_\circ\}$ .

It is clear that: "The pairwise quasi H-closed subspaces are properly contained in the pairwise F-closed subspaces". While the converse established throughout the following result.

**Theorem 4.2** A subset  $A$  of  $(X, \tau_1, \tau_2)$  is pairwise F-closed relative to  $X$  if it is pairwise quasi H-closed relative to  $X$ .

**Proof.** Suppose that  $A$  is pairwise quasi H-closed relative to  $X$  and let  $\{V_\lambda \mid \lambda \in \nabla\}$  be a cover of  $A$  by pairwise feebly-open sets of  $X$ . Then  $\{Int_i(Cl_j(Int_i(V_\lambda))) \mid \lambda \in \nabla\}$  is a cover of  $A$  by  $\tau_i$ -open sets of  $X$ . Hence there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $A \subset \cup\{Cl_j(Int_i(V_\lambda)) \mid \lambda \in \nabla_\circ\}$ . But by Proposition (2.10) we have  $V_\lambda$  is pairwise semi-open for each  $\lambda \in \nabla$  and so,  $Cl_j(Int_i(V_\lambda)) = Cl_j(V_\lambda)$ . Therefore,  $A \subset \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_\circ\}$ . ■

**Theorem 4.3** Let  $A$  be a pairwise preopen set of  $(X, \tau_1, \tau_2)$ . Then a subspace  $A$  is pairwise quasi H-closed if and only if  $A$  is pairwise quasi H-closed relative to  $X$ .

**Proof.** Assume that  $A$  is pairwise preopen in  $X$  and pairwise quasi H-closed relative to  $X$ . Let  $\{V_\lambda \mid \lambda \in \nabla\}$  be a cover of  $A$  by pairwise open sets of the subspace  $A$ . Then for  $\lambda \in \nabla$ , there exists a pairwise open set  $W_\lambda$  of  $X$  such that  $V_\lambda = W_\lambda \cap A$ . Since  $A$  is pairwise preopen in  $X$ , then  $V_\lambda \subset W_\lambda \cap Int_i(Cl_j(A)) = Int_i(W_\lambda \cap Cl_j(A)) \subset Int_i(Cl_j(W_\lambda \cap A)) = Int_i(Cl_j(V_\lambda))$ . Therefore,  $V_\lambda$  is pairwise preopen in  $X$  and  $\{Int_i(Cl_j(V_\lambda)) \mid \lambda \in \nabla\}$  is a cover of  $A$  by pairwise open sets of  $X$ . By Proposition (2.7) there exists a finite subset  $\nabla_\circ$  of  $\nabla$  such that  $A \subset \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_\circ\}$ . Therefore, we obtain  $A \subset \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_\circ\} = \cup\{Cl_A(V_\lambda) \mid \lambda \in \nabla_\circ\}$ , where  $Cl_A(V_\lambda)$  denotes the closure of  $V_\lambda$  in the subspace  $A$ . This shows that  $A$  is pairwise quasi H-closed. ■

We generalize the concept of feebly-irresolute functions due to Chae and Lee [3] in bitopological spaces as follows.

**Definition 4.4** A function  $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be pairwise feebly

irresolute if the inverse image of each  $(i, j)$ -feebly-open set of  $(Y, \sigma_1, \sigma_2)$  is  $(i, j)$ -feebly-open in  $(X, \tau_1, \tau_2)$  for  $i \neq j$ ;  $i, j = 1, 2$ .

**Theorem 4.5** Let  $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a pairwise feebly irresolute surjection and let  $A$  be pairwise F-closed relative to  $X$ , then  $f(A)$  is pairwise F-closed relative to  $Y$ .

**Proof.** Let  $\{V_\lambda \mid \lambda \in \nabla\}$  be a pairwise feebly-open cover of  $f(A)$ . Then  $\{f^{-1}(V_\lambda) \mid \lambda \in \nabla\}$  is a pairwise feebly-open cover of  $A$ . Since  $A$  is pairwise F-closed relative to  $X$ , then there exists a finite subset  $\nabla_o$  of  $\nabla$  such that  $A \subset \cup\{Cl_j(f^{-1}(V_\lambda)) \mid \lambda \in \nabla_o\}$ . Since  $f$  is pairwise feebly irresolute and surjective, then we have  $f(A) \subset \cup\{Cl_j(V_\lambda) \mid \lambda \in \nabla_o\}$  and this completes the proof. ■

**Corollary 4.6** Pairwise F-closedness is preserved under pairwise feebly irresolute surjection.

## 5. Concolusion

The theory of pairwise F-closedness is one of the most important subject in bitopological spaces. So, we introduced the characteristics and relationships concerned this Theorem with other corresponding types, several of its basic and important properties are defined.

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