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Transmission of Medical Images Based on Multi-mode Synchronization of Delayed Fractional-Order Coullete Chaotic Systems

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Abstract

In this study, a safety mechanism is used for the transmission of medical data with disturbance and unknown parameters and unknown time delays using a new communication method. A new synchronization method for Fractional Order Systems (FOCS) for encryption of medical images based on the Coullete system with unknown time delay is proposed. In the proposed method, the control laws are determined using the Lyapunov stability theorem such that the convergence of the synchronization error to zero is guaranteed. In this study, multiple state synchronization is performed in the presence of disturbance and unknown time delay. The control laws are determined using the Lyapunov function such that the synchronization and estimation errors converge to zero. For medical color images encryption, we use the fractional-order chaotic synchronization system alongside the chaos masking technique.

To test the efficiency of the proposed method in medical image transmission, various statistical parameters such as histogram, correlation, number of pixel change rate (NPCR), signal to peak noise ratio (PSNR), and information entropy are calculated. According to the values obtained for Entropy=7.8596, Correlation=0.9999, etc., the results show successfully encrypts the medical color images.

Keywords: Internet of Medical Things, Encryption Medical Images, Fractional-order chaotic system, Lyapunov stability.

1. INTRODUCTION

Digital communication is an essential tool used by a human, which is required in most daily works [1-3]. Also, digital communications are used for commerce in different industrial and medical contexts [4-

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5]. Therefore, information security in this context is critical. The digital communication system includes a transmitter, communication channel, and receiver. Information security in communication channels is of great importance. The particular importance of information security in digital communications has led to extensive studies in this domain, and scientists have presented various methods [6-9]. Encryption is one of the most significant contexts of information security in digital communications and includes multiple methods like chaos theory techniques [10-11]. The high efficiency of the chaos systems in information encryption has attracted the attention of many scientists. One of the most essential features of a chaotic system in information encryption is that these methods are highly sensitive to small changes in the initial conditions [12-14]. Synchronization methods are also used along with the chaos method. Synchronization of chaotic systems (SOCSs) was first presented by Peccora and Carrol. [15], and extensive studies have been done since then. The main purpose of using the synchronization technique is to adjust the parameters of the control system in chaotic systems [16]. In recent years, chaotic system synchronization has grown significantly, and the scientists of this context have tried to present methods to increase the information security of the communication channels [17-19]. In these studies, the primary goal is to create the control methods, aiming to synchronize and present the current chaotic systems to increase information security. Synchronization of FOCSs is one of the most recent contexts of information encryption in which extensive studies are being carried out [20-23]. Some of the essential fractional-order system synchronization methods for secure communication applications have been introduced in the following.

Zhu et al. [24] have designed an adaptive fuzzy control to synchronize the time-delayed chaotic system. To estimate the unknown parameters of uncertain chaotic systems, adaptive laws with a smooth projection operator are used. Also, the adaptive law is considered part of the Lyapunov function to stabilize the chaotic systems. Finally, .K.F. the closed-loop stability conditions are obtained by defining a proper Lyapunov function (L.K.F.) and using the linear matrix inequality theorem.

The internet of things (IoT) in medicine is one of the newest research and industrial fields and information security plays a vital role in them [25-26]. Medical data, including medical images, medical signals, etc., are recorded digitally, which has led to the transfer of this information between specialist physicians to facilitate the diagnosis and treatment of many patients [27-28]. One way to exchange information between medical specialists is through the Internet of Medical Things (IoMT) [29-30]. IoMT technology is one of the newest fields of medicine that aims to reduce patient costs and improve health care quality. IoMT is a mechanism that integrates physical objects, software, and hardware to interact. Communication technologies including Wi-Fi, Bluetooth, radio-frequency identification (RFID), and LAN are widely used in IoMT[31-32]. In the IoMT, keeping patient information confidential is one of the most important factors. So far, various methods have been proposed to increase data security in communication channels, and encryption is one of the most important of these methods [33-34].

a. The proposed method presents the synchronization of multiple state chaotic systems. Multiple chaotic systems (slave) with a chaotic system (master) are synchronized in this method. Then, the control laws are determined using the Lyapunov stability theorem such that the convergence of the synchronization error to zero is guaranteed. Next, multiple mode synchronization with disturbance and time delay is presented. The control laws are determined using the Lyapunov function such that the synchronization and estimation errors tend to zero. Subsequently, the presented method is applied to fractional order Coullete systems and encrypted using a new chaotic masking of different color images (benchmark and medical images). The efficiency of the method is evaluated through Histogram Analysis, Correlation Analysis, NPCR, unified average changing Intensity (UACI), PSNR, and Information Entropy. In conclusion, the performance of the presented method is desired compared to other studies. The proposed methods can be used in IoMT to increase the security of medical data such as medical images and signals. The experiment results show that this paper's proposed medical encryption method helps increase IoMT security and performance.

b. In section 2, fractional-order chaotic systems are introduced and then multi-state synchronization for fractional-order systems with disturbance and delay is explained. In the following section 2, Coullete chaotic system and its synchronization are introduced. In section 3, encryption based on new chaotic masking is stated. In section 4, simulation on standard benchmark images and medical color images have been done for two different values of derivative order and two different series of delays.

2. PROBLEM FORMULATION

In this section, the proposed scheme of the paper is stated. First, fractional-order systems are presented. Subsequently, SOCS of multiple transmission fractional order is described. Finally, multiple circular synchronizations are discussed. In both cases, adaptive rules and controllers are designed using the adaptive control method. Two examples are given to demonstrate the efficiency and performance of the proposed method.

2.1. Basic Definitions

2.1.1. Fractional-Order Derivative

Due to simple implementation and high performance, several numerical definitions for solving fractional differential equations have been proposed [35]. In this paper, the definition of Caputo is utilized, the fractional derivative of which is as follows [35]:

$$D^q f(x) = I^{m-q} h^{(m)}(x), \quad q > 0 \tag{1}$$

where $h^{(m)}$ represents the derivative of mth order of h (x), m = [q] is the first integer that is less than q, and Riemann-Lewil integral operator with order q of function g (x) is described as follows [38]:

$$I^q g(x) = \frac{1}{\Gamma(q)}\int_0^x (x-t)^{q-1}\, g(t)dt, q > 0 \quad (2)$$

where$\Gamma(q)$, is the gamma function. D^qoperator is called Caputo fractional operator of q order. Stability analysis of fractional-order systems by Lyapunov's direct method and determining the necessary and sufficient conditions guaranteeing stability with Mittag-Leffler concept [36] and stability analysis based on convex Lyapunov functions [40] for nonlinear systems are demonstrated.

Lemma 1[36]: Suppose $h(t) \in R$ is a continuous and derivable function. Then for $t \geq t_0$ we have:

$$D^q h^2(t) \leq 2h(t) \cdot D^q h(t) \quad (3)$$

Lemma 2 [36]: Suppose $h(t) \in R^n$ is a continuous and derivable function. Then for $t \geq t_0$ we have:

$$D^q h^T(t)h(t) \leq 2h^T(t) \cdot D^q h(t) \quad (4)$$

Theorem 1[39]: Suppose x = 0 is the equilibrium point of the fractional-order system (5), and its definition domain includes the origin. Suppose $V(t.x(t))$ is a continuous, derivable and Lipschitz function is relative to x such that:

$$D^q x(t) = f(x.t) \quad (5)$$

$$a_1\|x\|^a \leq V(t.x(t)) \leq a_2\|x\|^{ab} \quad (6)$$

$$D^q V(t.x(t)) \leq -a_3\|x\|^{ab} \quad (7)$$

Where $0 < q < 1$ and $a_1.a_2.a_3.a.b$ are arbitrary and positive constants. Then x = 0 is stable in the sense of Mittag-Leffler.

Definition 1: The continuous function $p: [0,\infty) \to [0,\infty)$ belongs to the class-K if it is strictly increasing and $p(0) = 0$.

Theorem 2[37]: Suppose x = 0 is the equilibrium point of the fractional-order system (5), where $f(x.t)$satisfies the Lipshitz condition with constant l>0 and $q \in (0,1)$. If the relations (9) are established for the Lyapunov function $V(t.x(t))$ and exist the class-K functions δ_i:

$$\delta_1(\|x\|) \leq V(t.x(t)) \leq \delta_2(\|x\|) \quad (8)$$

$$D^q V(t.x(t)) \leq -\delta_3(\|x\|) \quad (9)$$

Then the equation (5) is asymptotically stable in the sense of Mittag-Leffler [38].

Theorem 3 [39]: For the fractional-order system, (5) and the Lyapunov function $V(x)$:

$$D^q V(x) \leq \left(\frac{\partial V}{\partial x}\right)^T \cdot D^q x = \left(\frac{\partial V}{\partial x}\right)^T \cdot f(x.t) \quad (10)$$

2.1.2. Adaptive Synchronization Between One Master System and Several Slave Systems

Fig. 1 shows the synchronization between one master system and several slave systems.

The master hyper-chaotic system with unknown parameters is as follows [40]:

$$D^q x_1(t) = g_1(x_1) + M_1(x_1)\theta_1(t) \quad (11)$$

where $x_1(t) = [x_{11}.x_{12}.\cdots.x_{1n}]^T$the state

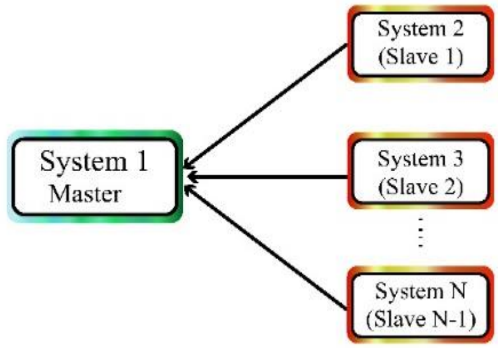

Fig. 1. Transmission multi-mode synchronization.

vectors of the system, $g_1(x_1(t)) = [g_{11}.g_{12}.\cdots.g_{1n}]^T$ a continuous function, $M_1(x_1(t)) = [M_{11}.M_{12}.\cdots.M_{1n}]^T$the matrix function, and $\theta_1 = [\theta_{11}.\theta_{12}.\cdots.\theta_{1n}]^T$ are the basic parameters of the master system that are unknown.

N-1 number of the slave hyper-chaotic systems with control function are as follows [40]:

$$D^q x_i(t) = g_i(x_i) + M_i(x_i)\theta_i(t) + u_{i-1}(t) \quad i = 2.3.\cdots.N \tag{12}$$

where $u_{i-1}(t) = [u_{i-1.1}(t).u_{i-1.2}(t).\cdots.u_{i-1.n}(t)]^T$ are control function of i^{th} slave system. Therefore, SOCS with the control function is stated as follows:

$$\begin{cases} D^q x_1(t) = g_1(x_1) + M_1(x_1 \\ D^q x_2(t) = g_2(x_2) + M_2(x_2)\theta_2 \\ \vdots \\ D^q x_N(t) = g_N(x_N) + M_N(x_N)\theta_N \end{cases} \tag{13}$$

In multi-mode synchronization form, the synchronization error is given as follows: $e_{i-1}(t)=x_i(t)-x_1(t)$ i=2.3.⋯.N.

Definition 1: For N FOCS expressed by (13) if adaptive controllers $u_{i-1}(t)$ exist such that for dynamic systems, the error is given by:

$$D^q e_{i-1}(t) = g_i(x_i) - g_1(x_1) + M_i(x_i)\theta_i - M_1(x_1)\theta_1 + u_{i-1}(t) \tag{14}$$

i=2,3,..,N-1

The required conditions are:

$$\lim_{t\to\infty}\|e_{i-1}(t)\| = \lim_{t\to\infty}\|x_i(t) - x_1(t)\| \to 0 \quad i = 2.3.\cdots N$$

If satisfied, then the adaptive transmission multi-mode synchronization between N chaotic systems with unknown parameters is realized. The controller law for $u_1(t).u_2(t).u_3(t).\cdots.u_{N-1}(t)$is designed as below:

$$u_{i-1}(t) = -g_i(x_i) + g_1(x_1) - M_i(x_i)\hat{\theta}_i + M_1(x_1)\hat{\theta}_1 + K_{i-1}e_{i-1} \quad i = 2.3.\cdots.N-1 \tag{15}$$

Therefore, dynamics of errors are given

as follows:

$$D^q e_{i-1}(t) = M_i(x_i)\tilde{\theta}_i - M_1(x_1)\tilde{\theta}_1 + K_{i-1}e_{i-1} \quad i=2.3.\cdots.N-1 \tag{16}$$

where $\hat{\theta}_I$ is the estimation of $\theta_i$and $\tilde{\theta}_i(t) = \theta_i(t) - \hat{\theta}_i(t)$ is an approximation error and:

$$K_{i-1} = -\mathrm{diag}(k_{i-1.1}, k_{i-1.2}, \cdots, k_{i-1.n}),\ k_{i-1.j} > 0 \qquad j = 1.2.\cdots.n\,.$$

Theorem 4: Transmission multi-mode synchronization of FOCS (13) with control laws (14), error dynamics (16), and update rules (18) and (19) is guaranteed.

Proof: Consider the following Lyapunov function:

$$V(e.\tilde{\theta}) = \frac{1}{2}\left(\sum_{i=2}^{N}[e_{i-1}^T K_{i-1}e_{i-1} + \tilde{\theta}_i^T\tilde{\theta}_i] + \tilde{\theta}_1^T\tilde{\theta}_1\right)$$

The fractional derivative of the mentioned Lyapunov function is yielded according to equations (8) and (15):

$$D^q V \le \sum_{i=2}^{N}[e_{i-1}^T\left(M_i(x_i)\tilde{\theta}_i - M_1(x_1)\tilde{\theta}_1 \ldots + \frac{1}{2}\left(K_{i-1} + K_{i-1}{}^T\right)e_{i-1}\right) + \tilde{\theta}_i^T D^q\tilde{\theta}_i] + \tilde{\theta}_1^T D^q\tilde{\theta}_1 \tag{17}$$

The parameters adaption rules are appointed as follows:

$$D^q\tilde{\theta}_i = -\left(M_i(x_i)^T e_{i-1} + \sigma_i\tilde{\theta}_i\right), \quad \sigma_i > 0 \qquad i = 2.3.\cdots.N \tag{18-1}$$

$$D^q\tilde{\theta}_1 = \sum_{i=2}^{N-1} M_1(x_1)^T e_{i-1} - \sigma_1\tilde{\theta}_1, \quad \sigma > 0 \tag{18-2}$$

Assuming that $\theta_i$are constant values, $D^q\theta_i = 0$ and$D^q\hat{\theta}_i = -D^q\tilde{\theta}_I$, so the rules for parameters estimation are defined as follows:

$$D^q\hat{\theta}_i = M_i(x_i)^T e_{i-1} + \sigma_i\tilde{\theta}_i \quad i = 2.3.\cdots.N \tag{19-1}$$

$$D^q\hat{\theta}_1 = -\sum_{i=2}^{N-1} M_1(x_1)^T e_{i-1} + \sigma_1\tilde{\theta}_1 \tag{19-2}$$

Substituting the estimation rules in (17), we will have:

$$D^q V \le \sum_{i=2}^{N}[e_{i-1}^T K_{i-1}e_{i-1} - \sigma_i\tilde{\theta}_i^T\tilde{\theta}_i] - \sigma_1\tilde{\theta}_1^T\tilde{\theta}_1 \le -\varphi V \tag{20}$$

where: $\boldsymbol{\varphi} = \min_{I,jk}(\sigma_I, -k_{i-1,j}) > 0$ and $K_{i-1}$are Hurwitz. Therefore, exploiting theorems (1) and (2), system (15) is asymptotically stable according to Mittag-Leffler [41] and$\|e_i\| \to 0$. Therefore, adaptive synchronization is obtained between N-1 response and drive systems.

2.2. Synchronization with Disturbance and Delay in the Systems

In this case, the master and slave system with disturbance are as follows:

$$\begin{cases} D^q x_1(t) = g_1(x_1) + M_1(x_1)\theta_1 + G_1\big(x_1(t-\tau_1)\big) + D_1(t) \\ D^q x_2(t) = g_2(x_2) + M_2(x_2)\theta_2 + G_2\big(x_2(t-\tau_2)\big) + D_2(t) + u_1(t) \\ \vdots \\ D^q x_N(t) = g_N(x_N) + M_N(x_N)\theta_N + G_N\big(x_N(t-\tau_N)\big) + D_N(t) + u_{N-1}(t) \end{cases} \tag{21}$$

It is assumed that disturbances are bounded but with an unknown boundary and $G_i(x_i)$ are Lipchitz functions.

$$|D_i(t)| \le d_i \qquad i = 1.2.\dots.N$$

where d_i are constant but unknown. The error dynamic is described as follows:

$$\begin{aligned} D^q e_{i-1}(t) = {} & g_i(x_i) - g_1(x_1) \\ & + M_i(x_i)\theta_i \\ & - M_1(x_1)\theta_1 \\ & + G_i\big(x_i(t-\tau_i)\big) \\ & - G_1(x_1(t-\tau_1)) \\ & + D_i(t) - D_1(t) \\ & + u_{i-1}(t) \quad i \\ & = 2.3.\cdots.N-1 \end{aligned} \tag{22}$$

Defining the control function as follows:

$$\begin{aligned} u_{i-1}(t) = {} & -g_i(x_i) + g_1(x_1) \\ & - M_i(x_i)\hat{\theta}_i \\ & + M_1(x_1)\hat{\theta}_1 \\ & + K_{i-1}e_{i-1} \\ & - G_i\big(x_i(t-\hat{\tau}_i)\big) \\ & + G_1(x_1(t-\hat{\tau}_1)) \\ & + \bar{u}_{i-1}(t) \quad i \\ & = 2.3.\cdots.N-1 \end{aligned} \tag{23}$$

where $\hat{\theta}_i, \hat{\tau}_i$ are estimations of θ_i, τ_i and $\bar{u}_{i-1}(t)$ is the section of the control function, which is introduced below. By placing the control function in (22), the error dynamics are given as follows:

$$\begin{aligned} D^q e_{i-1}(t) = {} & M_i(x_i)\tilde{\theta}_i - M_1(x_1)\tilde{\theta}_1 \\ & + D_i(t) - D_1(t) \\ & + K_{i-1}e_{i-1} \\ & + G_i\big(x_i(t-\tau_i)\big) \\ & - G_1\big(x_1(t-\tau_1)\big) \\ & - G_i\big(x_i(t-\hat{\tau}_i)\big) \\ & + G_1\big(x_1(t-\hat{\tau}_1)\big) \\ & + \bar{u}_{i-1}(t) \quad , \\ & i = 2.3.\cdots.N-1 \end{aligned} \tag{24}$$

Theorem 5: The error dynamics system (24) is under control law (28), the update rules (25) are stable, and the synchronization errors converge to zero despite the disturbance.

The rules updating estimation errors are as follows:

$$D^q\tilde{d}_i = -\left(\sum_{j=1}^{n}\left|e_{i-1}^j\right| + \beta_i\tilde{d}_i\right), \quad i = 2.3.\dots N \tag{25-1}$$

$$D^q\tilde{d}_1 = -\Big(\sum_{i=1}^{N}\sum_{j=1}^{n}\left|e_{i-1}^j\right| + \beta_1\tilde{d}_1\Big) \tag{25-2}$$

$$D^q\tilde{\tau}_1 = -l_1 \operatorname{sgn}(\tilde{\tau}_1)\sum_{i=2}^{N}|e_{i-1}| - \alpha_1\tilde{\tau}_1 \tag{25-3}$$

$$D^q\tilde{\tau}_i = -l_i \operatorname{sgn}(\tilde{\tau}_i)|e_{i-1}| - \alpha_i\tilde{\tau}_i, \quad i = 2.3.\dots N \tag{25-4}$$

Proof: By explaining Lyapunov function as follows:

$$V = \frac{1}{2}(V_e + V_\theta + V_d + V_\tau) \qquad (26)$$

Wherein:

$$V_e = \sum_{i=2}^{N} e_{i-1}^T K_{i-1} e_{i-1} \quad , V_d = \sum_{i=2}^{N} \tilde{d}_i{}^2 + \tilde{d}_1{}^2, \quad V_\theta = \sum_{i=2}^{N} \tilde{\theta}_i^T \tilde{\theta}_i + \tilde{\theta}_1^T \tilde{\theta}_1 \quad , V_\tau = \sum_{i=2}^{N} \tilde{\tau}_i{}^2 + \tilde{\tau}_1{}^2$$

where: $\tilde{d}_i = d_i - \hat{d}_i$, $\tilde{\tau}_i = \tau_i - \hat{\tau}_i$, $i = 1,2,3,\dots,N$

By calculating the fractional derivative of Lyapunov function:

$$D^q V \le \sum_{i=2}^{N} [e_{i-1}^T \Big(M_i(x_i)\tilde{\theta}_i - M_1(x_1)\tilde{\theta}_1 + D_i(t) - D_1(t) + \frac{1}{2}\big(K_{i-1} + K_{i-1}{}^T\big)e_{i-1} + G_i\big(x_i(t-\tau_i)\big) - G_1\big(x_1(t-\tau_1)\big) - G_i\big(x_i(t-\hat{\tau}_i)\big) + G_1\big(x_1(t-\hat{\tau}_1)\big)\Big) + \tilde{\theta}_i^T D^q \tilde{\theta}_i + \tilde{\tau}_i D^q \tilde{\tau}_i + \tilde{d}_i D^q \tilde{d}_i + \bar{u}_{i-1}(t)] + \tilde{\theta}_1^T D^q \tilde{\theta}_1 + \tilde{\tau}_1 D^q \tilde{\tau}_1 + \tilde{d}_1 D^q \tilde{d}_1 \qquad (27)$$

u_{i-1} and $\bar{u}_{i-1}{}^j$ are defined as follows:

$$u_{i-1}(t) = -f_i(x_i) + f_1(x_1) - H_i(x_i)\hat{\theta}_i(t) + H_1(x_1)\hat{\theta}_1(t) + K_{i-1}e_{i-1}(t) - F_i\big(x_i(t-\hat{\tau}_i)\big) + F_1(x_1(t-\hat{\tau}_1)) - (\hat{d}_i + \hat{d}_1) \cdot \mathrm{sgn}(e_{i-1}(t)), \quad i = 1.2.\cdots.N-1 \qquad (28)$$

$$\bar{u}_{i-1}{}^j(t) = -\big(\hat{d}_i + \hat{d}_1\big) \cdot \mathrm{sgn}\left(e_{i-1}^j(t)\right) \qquad (29)$$

By placing u_{i-1} (control function) and $\bar{u}_{i-1}$ in relation (27) we have:

$$D^q V \le -\sum_{i=2}^{N} e_{i-1}^T K_{i-1} e_{i-1} - \sum_{i=1}^{N} \beta_i \tilde{d}_i{}^2 - \sum_{i=1}^{N} \alpha_i \tilde{\tau}_i{}^2 - \sum_{i=1}^{N} \sigma_i \tilde{\theta}_i^T \tilde{\theta}_i < -\mu V \qquad (30)$$

where: $\mu = \min_{i.j}(\alpha_i, \beta_i, \sigma_i, -k_{i-1.j}) > 0.$ Therefore, according to the theorems (1) and (2) and being HurwitzK_{i-1}, the stability of the system, according to Mittag-Leffler is also confirmed. The tendency of synchronization errors to zero is also guaranteed despite disturbance.

The estimations updating rules are obtained as follows:

$$D^q\hat{d}_i = \sum_{j=1}^{n}\left|e_{i-1}^j\right| + \beta_i\tilde{d}_i\,, \quad i = 2.3.\dots N. \tag{31-1}$$

$$D^q\hat{d}_1 = \sum_{i=1}^{N}\sum_{j=1}^{n}\left|e_{i-1}^j\right| + \beta_1\tilde{d}_1. \tag{31-2}$$

$$D^q\hat{\tau}_1 = l_1 \mathrm{sgn}(\tilde{\tau}_1)\sum_{i=1}^{N}\sum_{j=1}^{n}\left|e_{i-1}^j\right| + \alpha_1\tilde{\tau}_1) \tag{31-3}$$

$$D^q\hat{\tau}_i = l_i \mathrm{sgn}(\tilde{\tau}_i)\sum_{j=1}^{n}\left|e_{i-1}^j\right| + \alpha_i\tilde{\tau}_i, \quad i = 2.3.\dots N \tag{31-4}$$

2.3. Synchronization of Coullete Chaotic System with Proposed Method

The Coullete chaotic system with delay is given as follows [42]:

$$\begin{cases} D^q x_1(t) = y_1(t-\tau_1) \\ D^q y_1(t) = z_1(t) \\ D^q z_1(t) = a_1 x_1(t) - b_1 y_1(t) - c_1 z_1(t) - x_1^3(t) \end{cases} \tag{32}$$

The system in Eq. (32) is considered the master system. In this system, $[x(t), y(t), z(t)] \in R^3$represent the state variables. The system parameters are$a_1 = 5.5, b_1 = 3.5, c_1 = 1$. The initial conditions of the master system are $x_1(0) = 0.145, y_1(0) = 0.625\,, z_1(0) = 0.925$.The first slave system is considered as in Eq. (33).

$$\begin{cases} D^q x_2(t) = y_2(t-\tau_2) + u_{11}(t) \\ D^q y_2(t) = z_2(t) + u_{12}(t) \\ D^q z_2(t) = a_2 x_2(t) - b_2 y_2(t) - c_2 z_2(t) - x_2^3(t) + u_{13}(t) \end{cases} \tag{33}$$

The initial conditions of the system (33) are as follows $x_2(0) = 0.145, y_2(0) = 0.625\,, z_2(0) = 0.925$. The parameters of this system are$a_2 = 6, b_2 = 3.25, c_2 = 1.25$.The second slave system is considered as in Eq. (34):

$$\begin{cases} D^q x_3(t) = y_3(t-\tau_3) + u_{21}(t) \\ D^q y_3(t) = z_3(t) + u_{22}(t) \\ D^q z_3(t) = a_3 x_3(t) - b_3 y_3(t) - c_3 z_3(t) - x_3^3(t) + u_{23}(t) \end{cases} \tag{34}$$

The initial conditions of the system (34) are$x_3(0) = 0.8, y_3(0) = 0.9, z_3(0) = 0.6$.System parameters are$a_3 = 5, b_3 = 3.75, c_3 = 1.75$. The time delays $\tau_i, i = 1,2,3$ are constant and unknown.

Synchronization errors are defined as follows:

$$e_{11} = x_2 - x_1\,, e_{12} = y_2 - y_1\,, e_{13} = z_2 - z_1, e_{21} = x_3 - x_1\,, e_{22} = y_3 - y_1\,, e_{23} = z_3 - z_1 \tag{35}$$

Dynamic errors are written as follows:

$$\begin{cases} D^q e_{11} = y_2(t-\tau_2) - y_1(t-\tau_1) + u_{11}(t) \\ D^q e_{12} = -z_2(t) - z_1(t) + u_{12}(t) \\ D^q e_{13} = a_2 x_2(t) - b_2 y_2(t) - c_2 z_2(t) - x_2^3(t) + u_{13}(t) - \\ (a_1 x_1(t) - b_1 y_1(t) - c_1 z_1(t) - x_1^3(t)) \end{cases} \quad (36)$$

$$\begin{cases} D^q e_{21} = y_3(t-\tau_3) - y_1(t-\tau_1) + u_{21}(t) \\ D^q e_{22} = -z_3(t) - z_1(t) + u_{22}(t) \\ D^q e_{23} = a_3 x_3(t) - b_3 y_3(t) - c_3 z_3(t) - x_3^3(t) + u_{23}(t) - \\ (a_1 x_1(t) - b_1 y_1(t) - c_1 z_1(t) - x_1^3(t)) \end{cases} \quad (37)$$

Fig. 2 depicts the multi-mode synchronization errors during disturbance. As shown, very little fluctuation is observed.

Further, the errors are large in the beginning and then reach to zero with very little fluctuation despite the parameter disturbance (Fig. 2). There are more fluctuations in these control efforts.

The estimation errors of the system parameters and time delay errors are shown in Fig. 3.

Phase diagrams for basic and follower systems are shown in Fig.4 in the following.

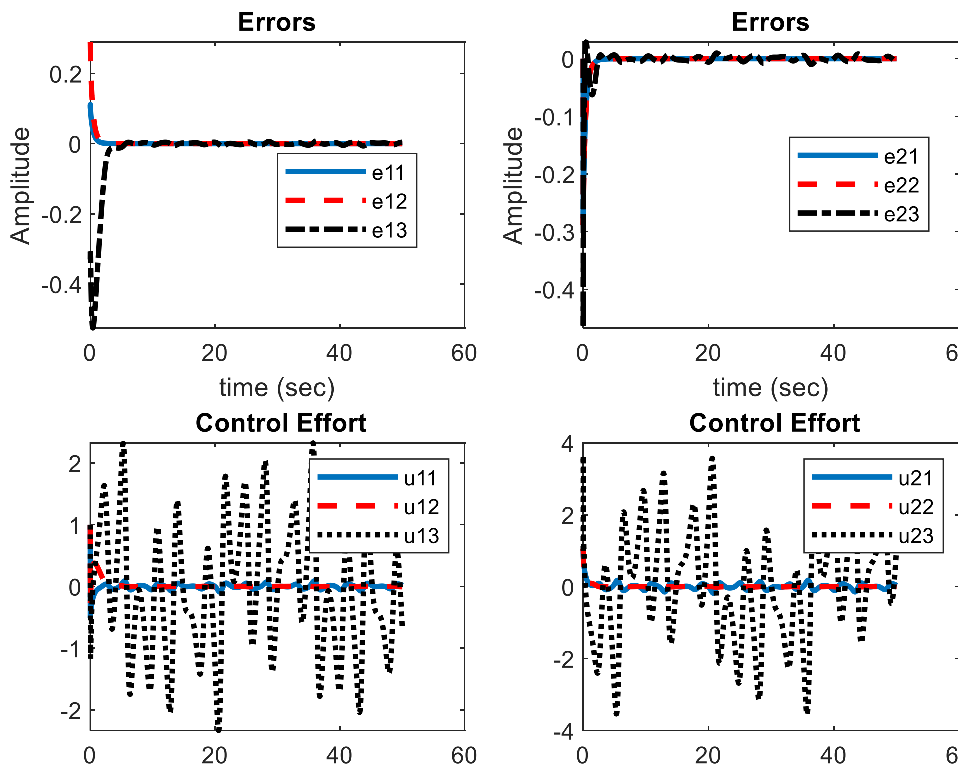

Fig. 2. Curves of synchronization errors obtained during disturbance and parameter uncertainty.

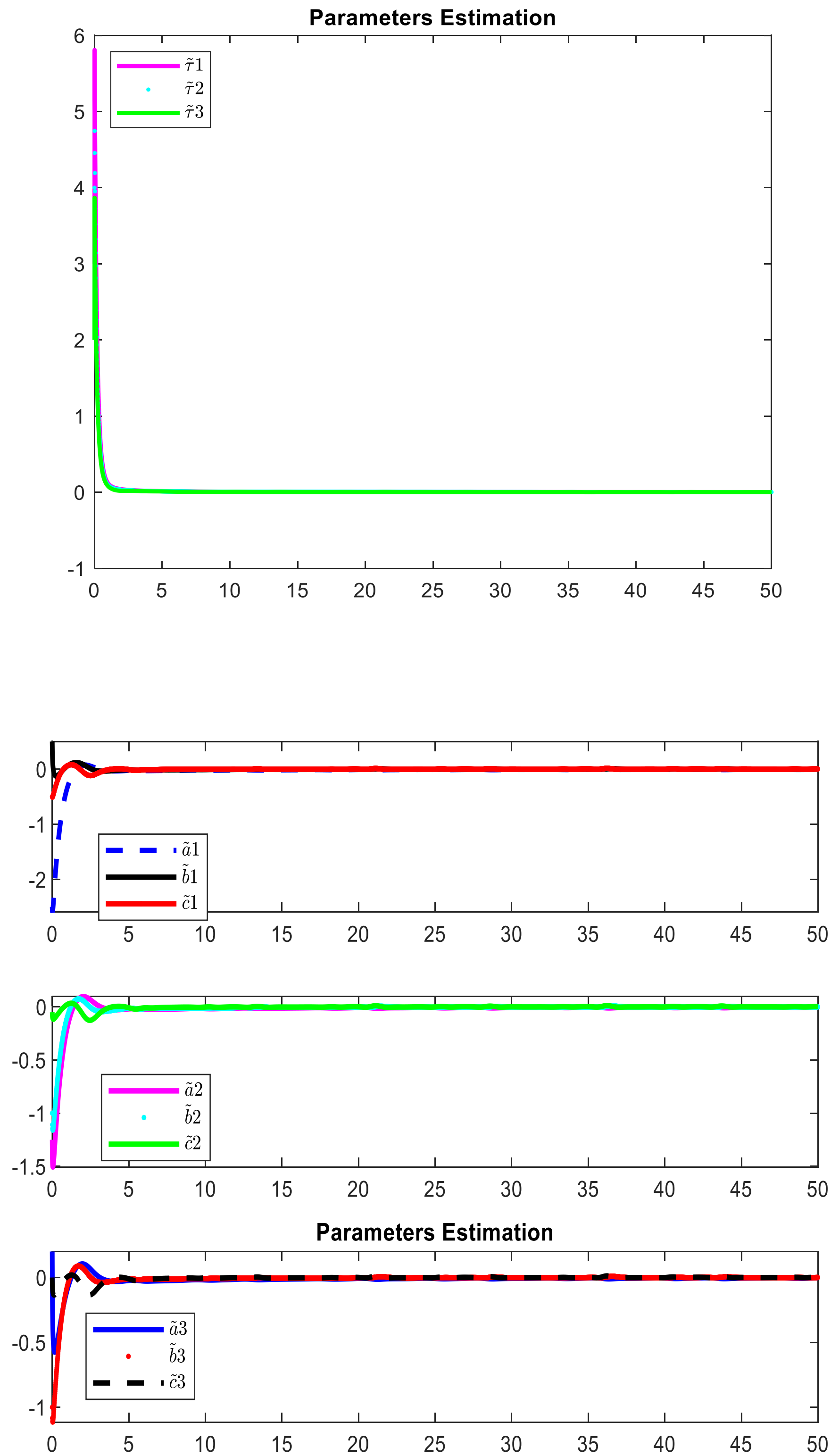

Fig.3. Estimation of parameters and time delay errors in multi-mode synchronization.

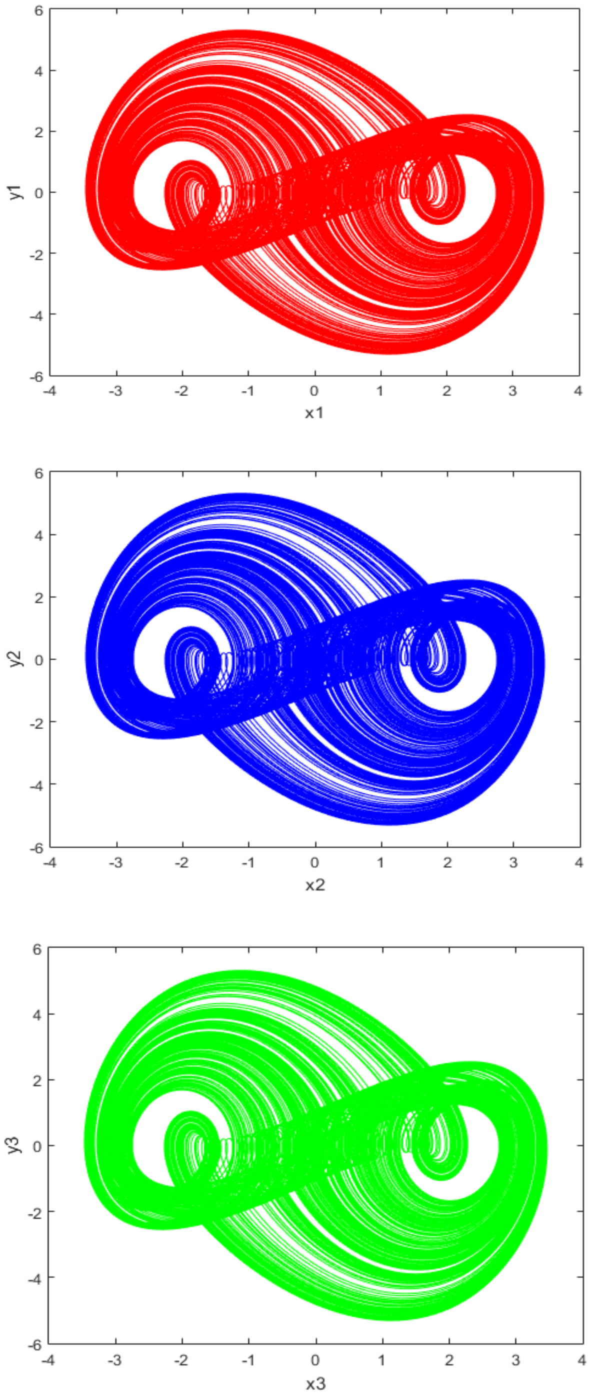

Fig. 4. Phase diagrams for master and slave systems.

3. NEW ENCRYPTION METHOD WITH CHAOTIC MASKING

Chaotic signals have complex behavior which makes them difficult to predict. Using chaotic signals as message carriers in secure communications and cryptography is a suitable and secure solution [43]. In this approach, the message signal is added to a linear combination of state vector components of the master system. In other words, the message signal information is concealed within the chaotic behavior of the state components, which can enhance the communication channel security. Regarding the receiver side, the message signal can be recovered by synchronizing the slave system with the master system. Ensuring the synchronization error convergence to zero and the presence of disturbance signals and unknown parameters in the master and slave systems can increase security in the communication channel.

In the following, we will discuss our proposed method in secure communication for chaotic masking. Suppose m(t) is a message. We encrypt this message with a proper map:

$$m_0(t) = \Lambda(m(t), f(t), a) \tag{38}$$

where $\Lambda(m(t), f(t), a)$ is a definite and continuous function as the map, and $f(t)$ is a definite and continuous signal as a coder. For instance, we can define $\Lambda(m(t), f(t), a)$ as follows:

$$\Lambda(m(t), f(t), a) = \tanh\big(a \cdot m(t) + f(t)\big), \quad f(t) = 0.2\sin(10t) + 0.1\sin(20\pi t) + 0.05\cos(2\pi t), a \in R \tag{39}$$

where a is a coefficient so that $|a \cdot m(t) + f(t)| \leq 4$ will stand.

Signals $m_0(t)$ and $f(t)$ will be masked as follows and transmitted in two components different from the chaotic system:

$$\tilde{m}(t) = m_0(t) + \sum_{i=1}^{n} \lambda_i x_i \tag{40}$$

$$\tilde{f}(t) = f(t) + \sum_{i=1}^{n} \mu_i x_i$$

The receiver initially obtains the estimation of signals $m_0(t)$ and $f(t)$. Then, it will calculate $\hat{m}_0(t)$, and ultimately, we will calculate $\hat{m}(t)$ as follows:

$$\hat{m}_0(t) = \tilde{m}(t) - \sum_{i=1}^{n} \lambda_i y_i = m_0(t) + \sum_{i=1}^{n} \lambda_i x_i - \sum_{i=1}^{n} \lambda_i y_i = m_0(t) + \sum_{i=1}^{n} \lambda_i e_i \rightarrow m_0(t) \tag{41}$$

$$\hat{f}(t) = \tilde{f}(t) - \sum_{i=1}^{n} \mu_i y_i = f(t) + \sum_{i=1}^{n} \mu_i x_i - \sum_{i=1}^{n} \mu_i y_i = f(t) + \sum_{i=1}^{n} \mu_i e_i \rightarrow f(t) \tag{42}$$

To recover the signal of message $m(t)$, we can do as follows:

$$m_0(t) = \Lambda(m(t), f(t), a) \rightarrow \hat{m}_0(t) = \Lambda\big(\hat{m}(t), \hat{f}(t), a\big) = \tanh\big(a \cdot \hat{m}(t) + \hat{f}(t)\big) \tag{43}$$

$$\Rightarrow \hat{m}(t) = \frac{1}{a}(\tanh^{-1}(\hat{m}_0(t) - \hat{f}(t))$$

Fig. 5 shows the chaotic masking in multi-mode synchronization.

Various statistical methods demonstrate the efficiency of synchronization systems of chaotic systems using image encryption. The histogram, correlation, NPCR, UACI, PSNR, and information entropy parameters are used in this paper to demonstrate the efficacy of the proposed synchronization method for synchronization of chaotic order systems. The description of the image evaluation parameters is given in reference [44].

4. EXPERIMENTAL RESULTS

In this section, the proposed method's results are presented. Standard benchmark images and then color medical images make up the test data. Various statistical parameters are presented in the following sections to demonstrate the efficacy of the proposed technique. These parameters are used to indicate the effectiveness of the proposed plan for the values of the variable q in the fractional-order chaos system using chromatographic images. simulations have been performed on two image models and two fractional-order formats with different time delays.

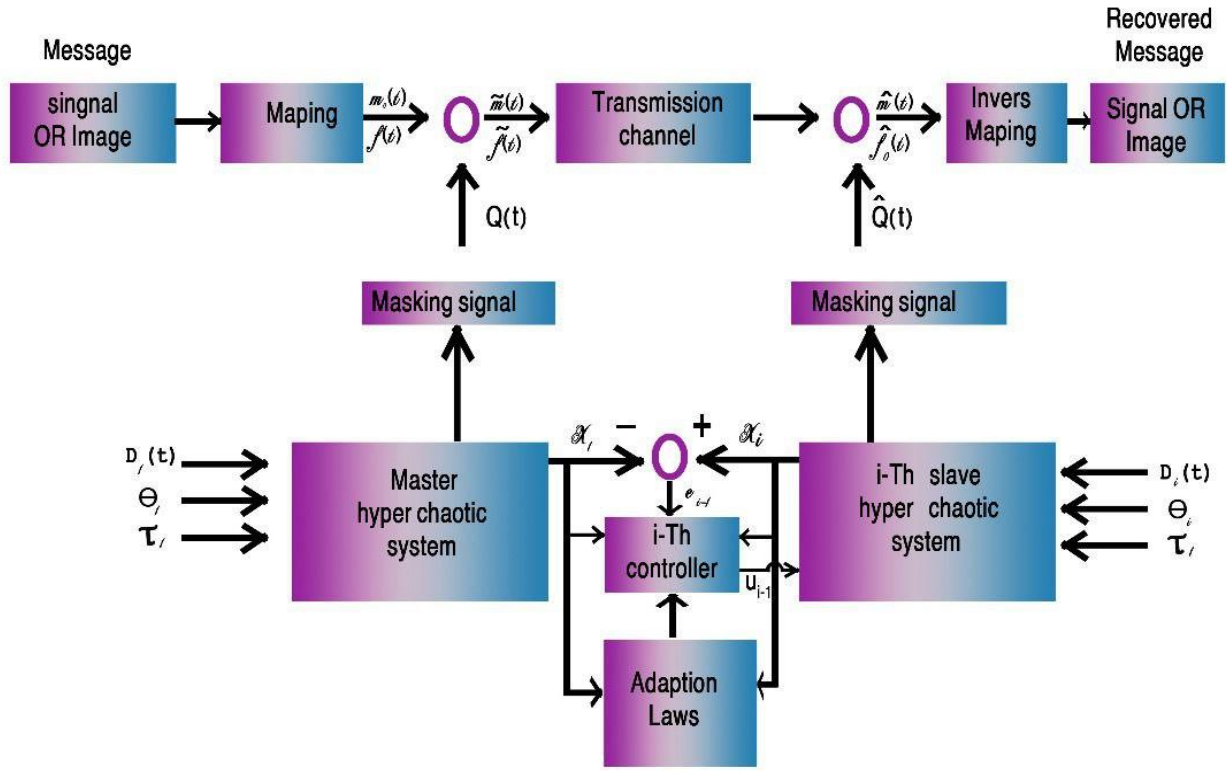

Fig. 5. Block diagram of proposed method.

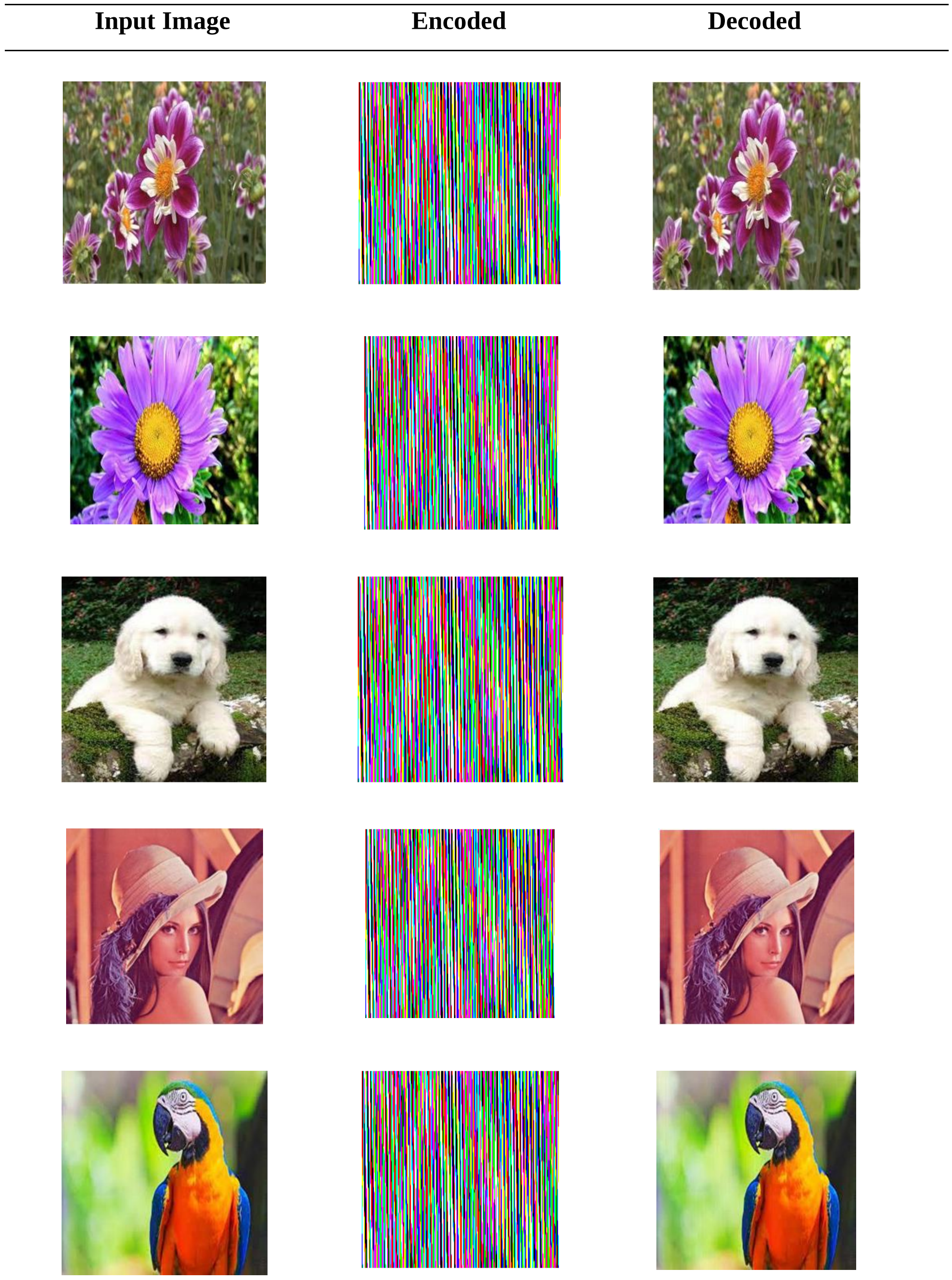

Fig. 6. Standard benchmark images encryption using synchronization of the fractional-order Coullete systems.

4.1. Results For Standard Color Images Benchmark

The results of FOCS synchronization method for standard benchmark images are shown in this section. The proposed fractional-order chaos system synchronization method's coded and reconstructed images are shown in Fig. 6. The proposed method successfully encrypts and reconstructs benchmark color images in Fig.6. which also shows that the proposed method works well on a variety of color images [51].

Histograms for input and reconstructed images are shown in Fig. 7. The method of synchronizing the proposed fractional-order system encrypts benchmark images successfully, as shown in Fig. 7. The histograms of the input and reconstructed images are very similar, as can be seen.

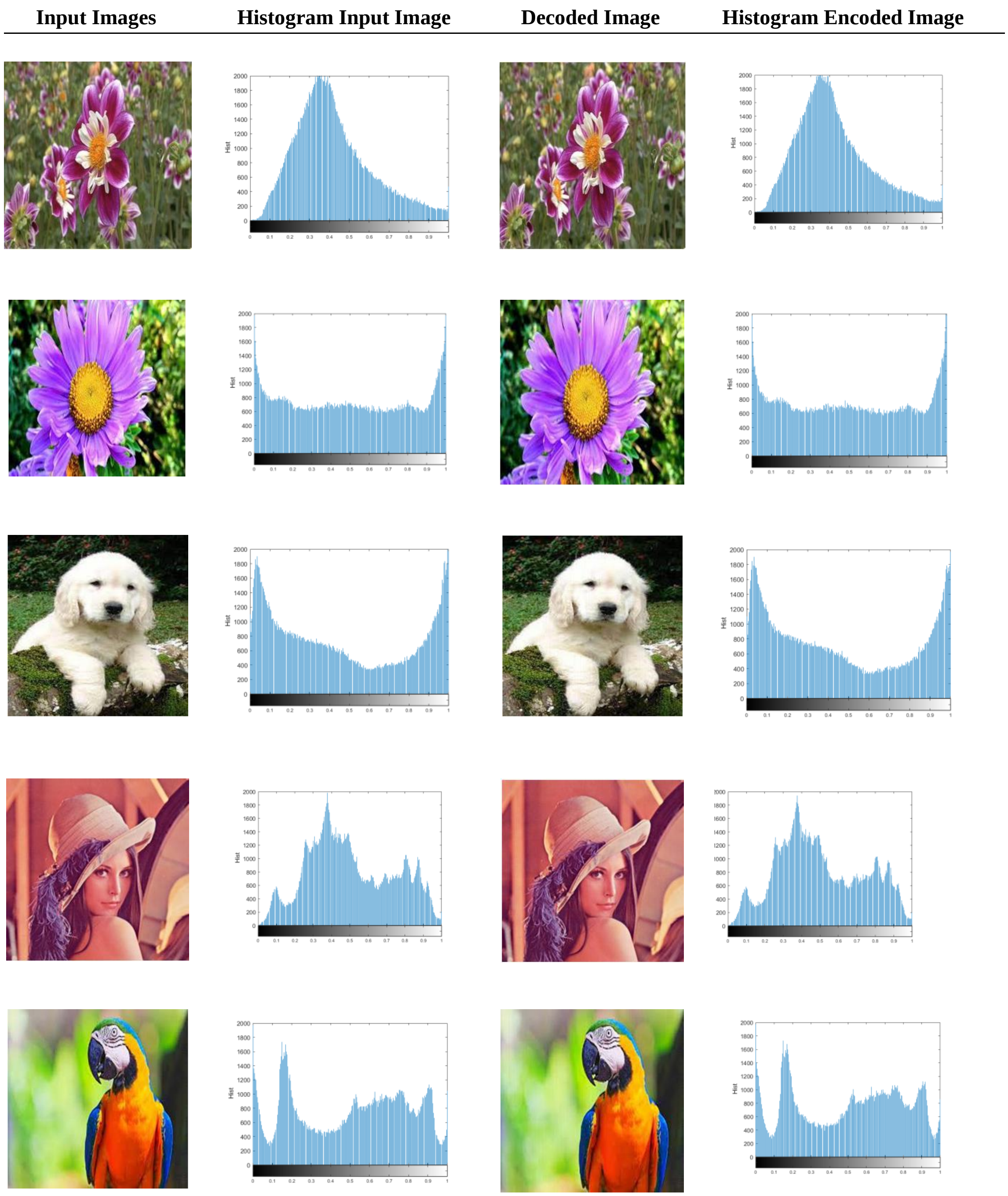

Fig. 7. Histograms input and output image (q=0.98,$\tau_1 = 2, \tau_2 = 3, \tau_3 = 4$).

Table 1. Statistical metrics for color benchmark images (q=0.98,$\tau_1 = 2, \tau_2 = 3, \tau_3 = 4$)

Images	Histogram			Correlation	Differential Attack		P.S.N.R	Information Entropy
	Main	Encrypted	Decoded		NPCR (%)	UACI (%)		
Image 1	48131.35	8035425.30	48045.82	0.9999	99.61	33.46	50.34	7.4436
Image 2	16469.89	7544608.2	16153.14	0.9999	99.23	33.34	48.39	7.8596
Image 3	29938.64	8017574.60	29900.25	0.9999	98.75	33.34	48.40	7.7604
Image 4	59637.08	72032698.	59001.98	0.9997	98.55	32.46	46.93	7.2801
Image 5	314062.50	8024938.11	309549.18	0.9993	98.42	32.21	49.68	7.0423

The types of statistical parameters used to demonstrate the efficacy of the fractional-order chaos system synchronization method for color benchmark images are shown in Table (1). For each benchmark color image, statistical parameters such as histogram, cryptography, N.P.C.R., U.A.C.I., P.S.N.R., and information entropy are calculated in the Table (1).

4.2. Results for Medical Color Images

At this stage, medical images are tested and simulated for a fractional-order of q=0.95 and time delays of$\tau_1 = 4, \tau_2 = 5, \tau_3 = 6$, and the image processing and histogram results are presented in the figures below.

Fig. 8 shows the original image and the image recovered from the cryptographic channel, which indicates the correlation between the two histogram images. Now Fig. 9, shows the encrypted image for a fractional-order of q=0.95 with different time delays of the chaotic system.

In the following, the Table of evaluation of parametric criteria of medical images for the fraction-order of q=0.95 and different time delays for master and slave chaotic systems is given below, which indicates the security of the proposed cryptographic channel and the quality of recovered images [50].

By comparing the criteria in the parameter tables, we find that the Coullete chaotic system, with a fractional-order of 0.98, has performed better in terms of cryptographic security and no data leakage.

5. DISCUSSION

In this paper, to encrypt medical images, Coullete chaotic fractional-order multi-state synchronization with unknown time delay has been used. In other articles, unknown time delay for this system has not been considered. Also, the combination of this synchronization based on the transmission mechanism Secure communications has received less attention, which allows us to operate freely in the encoder function for better encryption. To improve the entropy, in addition to criteria such as the degree of fractional order, we can test different coding functions in the mapping section. In this

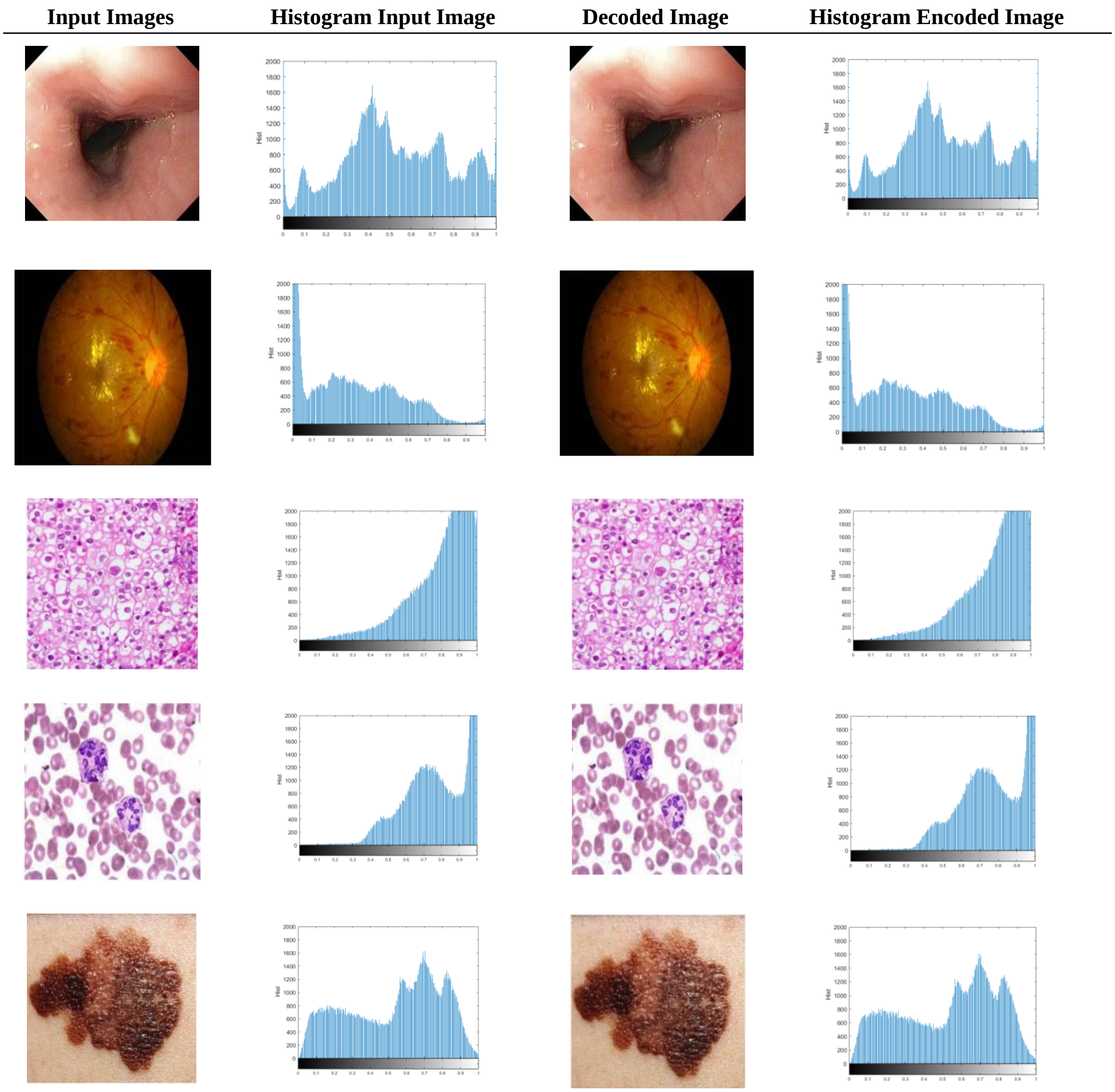

Fig.8. Histograms input and output image (q=0.95 ,$\tau_1 = 4, \tau_2 = 5, \tau_3 = 6$).

study, we used a simple coder. Our major innovation in this area is the existence of unknown delays in the Coullete system, which complicates the transmission of medical images

The use of chaos techniques in encrypting various data, including medical data, has increased significantly in recent years [45]. The use of chaos theory-based cryptography techniques is widely recognized as one of the most important ways to improve information security in telecommunication systems [46]. Recently, synchronization methods have been used in conjunction with chaotic systems to improve information security, such as data, signals, and images. According to research, synchronization systems in data encryption are very efficient [47].

Medical data entails essential information from patients, and therefore, their transferring and receiving are of particular significance. Furthermore, medical information involves images, signals, and other data. The synchronization method of the proposed FOCS has been implemented on the gray images, and its performance has been investigated. Therefore, in future work, this scheme can be utilized for medical gray image encryption such as MRI [48], X-Ray, CT, mammography, and so on. Additionally, this approach can encrypt various medical signals such as EEG signals [49].

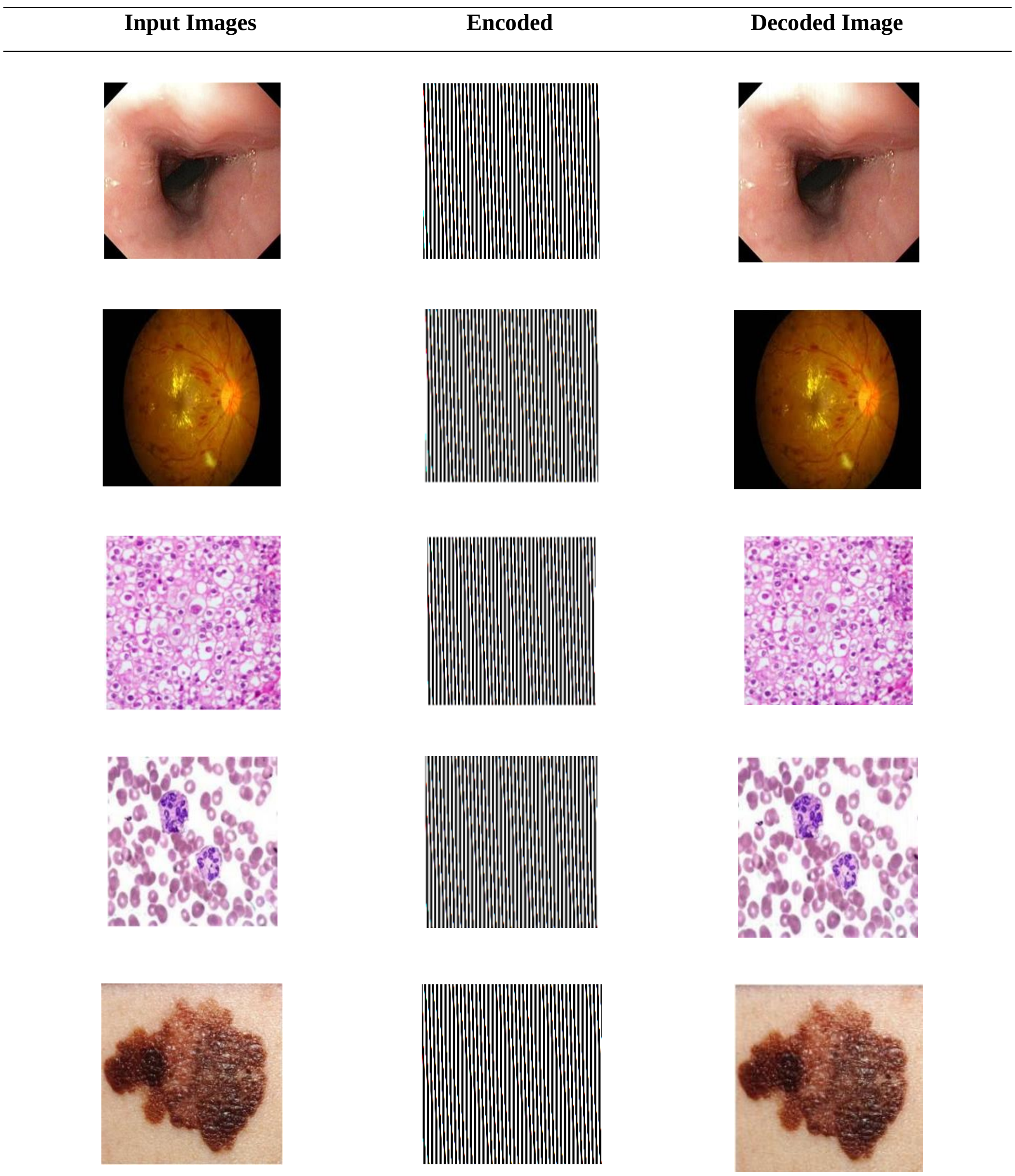

Fig. 9. Displayed medical images encryption using synchronization of the fractional-order Coullete systems(q=0.95,$\tau_1 = 4, \tau_2 = 5, \tau_3 = 6$).

Table 2. ***Statistical metrics medical images without noise.(q=0.95,$\tau_1 = 4, \tau_2 = 5, \tau_3 = 6$).***

Images	Histogram			Correlation	Differential Attack		PSNR	Information Entropy
	Main	Encrypted	Decoded		NPCR (%)	UACI (%)		
Image 1	57023.41	7916925.39	56145.76	0.9998	99.61	33.34	47.8344	7.456
Image 2	1213744.68	71264683.7	1274817.53	0.9998	99.18	33.98	52.7832	7.3099
Image 3	184451.61	7912454.97	183376.82	0.9996	99.67	33.46	45.8202	7.3864
Image 4	460595.93	7915940.89	430325.88	0.9994	99.08	33.18	45.7726	7.1473
Image 5	63980.75	71232348.49	63396.71	0.9998	99.61	33.31	48.0901	7.3972

6. CONCLUSION

In this study, a new cryptographic method based on Coullete fractional-order turbulence synchronization is presented. In this work, the synchronization method is based on multi-mode adaptive control and the Coullete fractional-order chaos system to encode color images. In this work, experiments on 5 benchmark color images and 5 medical color images with size 300 * 300 have been used. Next, the cryptographic method based on synchronization of Coullete fractional-order chaos system is applied to benchmark color images and medical color images. The fractional-order chaos system has different delay and q values in this section. Moreover, the presence of an unknown delay factor in the systems complicates the synchronization problem.

In two separate cases (q=0.98,$\tau_1 = 2, \tau_2 = 3, \tau_3 = 4$) and (q=0.95,$\tau_1 = 4, \tau_2 = 5, \tau_3 = 6$) simulation on standard images and various medical images has been done. The method of synchronizing the Coullete fractional-order chaos system for different values of q has been tested for image encryption. In the next step, the masking technique of chaotic systems is also used to encode color images. Then, to demonstrate the efficiency of the proposed synchronization method, the parameters of histogram, correlation, N.P.C.R., U.A.C.I., P.S.N.R., and information entropy for benchmark and medical color images were obtained. In addition, histogram diagrams for medical input and encrypted images for the proposed synchronization method for different q values are displayed. The experimental results show that the Coullete fractional-order synchronization method for q = 0.95 has succeeded. Derivative change changes the behavior of the system as a whole. This is an essential point in cryptography that reduces the possibility of decryption. On the other hand, the proper performance of the method for different values of q indicates its ability.

In this paper, a controlling mechanism has been introduced based on synchronization of FOCS. This mechanism could be useful in the case of transmission of medical data or IOMT. Considering that the medical data are combined with chaotic signals, the amount of security in encryption is sufficiently high. In this mechanism, parameters of the system and time delays are unknown. Moreover, the presence of nonlinear uncertainty makes its detection harder.

Results obtained from the rigorous security analysis prove the robustness of the proposed method.

In encryption problems, computational complexity can help to improve the method's security. On the other hand, due to the computational complexity of fractional derivatives, the correct use in synchronization problems improves security indicators. In addition, the possibility of sequential change of the derivative is available to the user as a degree of freedom, which certainly complicates the discovery of the password

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