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Adaptive Barrier Function based on Sliding Mode Control for High-Order Nonlinear Systems

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Abstract
In this paper, a new adaptive controller based on barrier function is designed for high-order nonlinear systems with a sum of the uncertainty. Accordingly, in this paper, a sliding mode controller is used, which can create asymptotic convergence and at the same time can deal with disturbances. The main drawbacks of sliding mode control can be considered as asymptotic convergence, chattering phenomenon, stimulus saturation, adaptive gain estimation and failure to deal with oscillating uncertainties. In this paper, the sliding mode control is used to deal with the phenomenon of asymptotic convergence and chattering and the barrier function is used to overcome the uncertainties of interest and fluctuation. The advantages of the proposed method include elimination of the chatting phenomenon, convergence in finite-time, compatibility with time-varying uncertainties, no use of estimation and no need for high information of disturbances. Stability analysis showed that under the proposed controller the tracking errors approach the convergence region close to zero and provide faster convergence. Finally, to prove the efficiency of the controller, based on hyperchaotic synchronization, we apply the proposed controller to the new 5D hyperchaotic system. The results showed that the proposed controller, despite the disturbances applied to the system, provides fast convergence and eliminates the chatting phenomenon.

Keywords: adaptive controller based on barrier function, BFAFTSM control, chatting phenomenon, new 5D hyperchaotic system, hyperchaotic synchronization.

1. INTRODUCTION

In many engineering structures, including mechanical structures, there are many physical constraints, such as angular rotation constraints, pressure constraints, and other constraints. In this case, the control input under the effect of these physical behaviors

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will have nonlinear constraints such as dead zone and saturation [1-6]. Stimulus saturation, which is an unwanted input, disrupts applications. This unwanted input is unavoidable in many structures and is usually applied to the system. These limitations, in particular, impair the dynamic guidance of systems and make matters worse. Hence, the need to design a controller to overcome the above problems is necessary. Much research has been done in recent years to address the above problems [7-10]. For example, in [11] the LMI method is performed to improve the performance and stability of linear control systems under the influence of stimulus saturation for the nonlinear system. A backstepping architecture is investigated to stabilize a belt system where a nonlinear auxiliary system is introduced to deal with the impacts of actuator saturation [12]. In [13], the neural network method of radial base function has been used to approximate the information of the unknown amplitude information of the control input. In [14] an output-feedback control method is used to overcome indefinite saturation with time delay. In this paper, the input saturating problem of several UAVs in a control scheme with distributed error is discussed. In [15] an adaptive saturated finite time stabilizing controller scheme is formulated for rigid spacecraft by associating with an integrated sliding mode control. In [16] an adaptive sliding mode control is designed for a multi-input and multi-output spacecraft system with a disturbance observer. The use of fuzzy logic system for a control scheme for a particular class of nonlinear systems with external disturbances and input saturation is proposed in [17]. In this research, sliding mode control has also been developed for the saturated systems. The development of sliding mode control has been done in much research. For example, in [18], a desaturation scheme is proposed for a specific class of indeterminate and nonlinear systems using second-order sliding mode control. Composite nonlinear feedback and super-twisting are combined together for the tracking control of a helicopter under the propeller saturation [19]. However, in many studies in this field, it is assumed that the uncertainties are less than the control capacity or are unknown. In [20] a robust variable structure control is introduced to stabilize the attitudes of a three-axis flexible aircraft in the presence of the dead zone without considering the bound of disturbance. In the above research, using the proposed controller, the control gain is monotonically increasing. The disadvantage of this method lies in that when the lumped uncertainties or external disturbances become small, the control gain does not decrease that leads to overestimation. Also, the control input with large amplitude causes the chatting phenomenon, which also intensifies the saturation phenomenon. In this case, an adaptive sliding mode control scheme is used to eliminate the estimate in [21]. The disadvantage of this method is that the control scheme used is as sensitive as the dead area. The purpose of this paper is to design a finite time sliding mode control based on barrier function for input saturation systems. We apply this controller to a high-order nonlinear system. The novel nonlinear system, is a 5D hyperchaotic system. Under the proposed controller, state variables can converge to an area around the origin in the presence of stimulus saturation and then remain there. One of the advantages of the method used is the lack of need to know the indefinite upper bounded, which is usually required in the control of normal sliding mode control. Also, another feature of the proposed control scheme is the adaptation of the control gain to changes in the amplitude of the perturbations so that the overestimation is eliminated. Finally, a comparison will be made to determine the performance of the controller.

Chaotic systems are used in many engineering applications. Chaotic synchronization is one of the recent challenges in the field of chaotic and hyperchaotic systems that has attracted the attention of researchers. The larger the system and the more complex the system, the better the chaos synchronization and secure communication and the more efficient the system [22]. For an anonymous receiver, chaos-based decryption is difficult without knowing the dynamics of the system and the initial conditions. One way to increase the security is to increase the dynamics of the nonlinear oscillation system. This is due to the fact that it is difficult to recover messages for unauthorized sources using recovery methods. Another way to increase the security is the dynamic complexity of the system, as this makes it difficult to decrypt. For example, in [23], a general chaos-based synchronization scheme between two nonlinear systems of correct order oscillation and fractional order has been studied. New control signals are generated using the stability theory technique and the fractional order tracking controller. In [24], chaos-based finite time synchronization of 4D memristor (MCS) oscillation systems has been studied. First, a memristor simulator circuit is created to implement MCS. Then, based on the simulator circuit provided, the MCS model is presented. In [25], chaos-based synchronization of nonlinear Lu systems with disturbance and a partial control scheme according to the dimensions of the system using an ASMC are presented. In the first step, an integrated sliding mode control (I-TSMC) is proposed for chaos-based synchronization of nonlinear Lu systems with specified positive parameters. In the second step, the new control signal is used to synchronize master-slave Lu system. In this case, the uncertain positive parameters are estimated using a new adaptive control law. Finally, the stability of the proposed control scheme is proved using the Lyapunov stability method.

The summary of this paper is as follows: In section 2 the problem formulation and the new barrier function adaptive finite-time sliding mode (BFAFTSM) control scheme will be introduced. In Section 3, a new 5D nonlinear system will be introduced and nonlinear analysis will be performed. Section 4 will be devoted to the application of the proposed method and numerical simulation. Finally, in Section 5, conclusions will be drawn from the previous sections.

2. PROBLEM FORMULATION AND CONTROLLER DESIGN

2.1. Problem Formulation

Consider the nonlinear system of n^{th} order as follows:

$$\begin{cases} \dot{x}_1 = f(x_2) \\ \dot{x} = f(x_3) \\ . \\ . \\ \dot{x}_{n-1} = f(x_n) \\ \dot{x}_n = F(x,t) + D(x,t) + B(x,t)u \end{cases} \quad (1)$$

where $x = [x_1, ..., x_n]^T$, $F(x,t), B(x,t)$, are the state variables, nonlinear functions are non-zero, respectively. d (x, t) is the sum of the uncertainties and disturbances of the system (1).In this case:

(1): the sliding mode control scheme based on barrier function is proposed. In the proposed control, a high level of uncertainty is required. The size of the area can also be predefined, and the new controller is independent of the high level of uncertainty. Finally, unwanted oscillations and the chatting phenomenon will be eliminated.

(2): Consider the BFAFSM controller as follows:

$$U = \frac{1}{B(x,t)}(u_1 + u_2) \quad (2)$$

where u_1, u_2 were discontinuous control inputs, which will be designed later.

To design the sliding surface controller, we consider the following [20]:

$$\delta^{\delta} = \left(\frac{d}{dt} + \lambda\right)^{n-1} x_1 \quad (3)$$

where y is a positive constant and is used to determine the performance in the sliding phase. We will have [20]:

$$s = \begin{bmatrix} A^T & 1 \end{bmatrix} x \quad (4)$$

where

$$A = \left[\lambda^{(n-1)}, (n-1)\lambda^{(n-2)}, ..., (n-1)\lambda\right]^T$$

Derived from the sliding surface and using equations (2) and (4) we have:

$$\begin{aligned} \dot{\delta} &= \begin{bmatrix} 0 & A^T \end{bmatrix} x + \dot{x}_n = \begin{bmatrix} 0 & A^T \end{bmatrix} x \\ &+ F(x,t) + D(x,t) + B(x,t) = \\ &\begin{bmatrix} 0 & A^T \end{bmatrix} x + F(x,t) + D(x,t) \\ &+ x(u_1 + u_2) \end{aligned} \quad (5)$$

By placing equations (2) and (6) in equation (5), we will have:

Where $0 \le x(u(t)) \le 1$ is the index coefficient of indeterminacy in the control input. assuming $D(x,t) = 0,\ x = 1,\ u_1 = 0$ control input $u_{1'}$ is equal to:

$$u_1 = -F(x,t) - \begin{bmatrix} 0 & A^T \end{bmatrix} x \quad (6)$$

By placing equations (2) and (6) in equation (5), we will have:

$$\begin{aligned} \dot{\delta} &= \begin{bmatrix} 0 & A^T \end{bmatrix} x + F(x,t) \\ &+ D(x,t) + X(u_1 + u_2) \\ &= \begin{bmatrix} 0 & A^T \end{bmatrix} x + F(x,t) \\ &+ X\left(-F(x,t) - \begin{bmatrix} 0 & A^T \end{bmatrix} x\right) \\ &+ Xu_1 + D(x,t) = (1 - X) \\ &\left(F(x,t) - \begin{bmatrix} 0 & A^T \end{bmatrix} x\right) + Xu_1 \\ &+ D(x,t) = Xu_1 + \gamma \end{aligned} \quad (7)$$

where γ is equal to:

$$\begin{aligned} \gamma &= (1 - X)\left(F(x,t) + \begin{bmatrix} 0 & A^T \end{bmatrix} x\right) \\ &+ D(x,t) \end{aligned} \quad (8)$$

It is assumed that:

$$|\gamma| < k \tag{9}$$

where k is a positive constant for which there is an upper bound but it is unknown. To overcome the uncertainties of γ, we select a positive constant number from the error curve so that the sliding surface variables converge to that around origin. Given the convergence around origin, we consider two states for γ given the size of the indeterminates $(\gamma \leftrightarrow \tau)(\delta \leftrightarrow \gamma)$.

Theorem (2): For $|\delta \geq \varepsilon|$ we design the control input as follows:

$$u_2 = -\alpha \hat{\tau} \hat{k} \tanh(\delta) \tag{10}$$

where $\hat{\tau}\, \hat{k}$ are the adaptive parameters of the system and are defined as follows:

$$\begin{aligned} &\dot{\hat{k}} = \eta^{-1}|\delta| \\ &\dot{\hat{\tau}} = \alpha \hat{\tau}^3 k\, |\delta| \\ &\alpha > 1, \eta > 0, \hat{\tau}(0) > 0 \end{aligned} \tag{11}$$

Theorem (3): For $|\delta| \leq \varepsilon$ we design the control input as follows:

$$u_2 = -F_b(\delta) \tanh(\delta) \tag{12}$$

where $F_{b(\delta)}$ is defined as follows:

$$F_b(\delta) = \frac{|\delta|}{\varepsilon - |\varepsilon|} \tag{13}$$

where $\delta \in (-\varepsilon, \varepsilon)$.

2.2. Stability Analysis

Assumption (1): There is a positive constant τ that satisfies the following conditions:

$$0 < \tau \leq X\, (u(t)) \leq 1 \tag{14}$$

Assumption (2): We define errors as follows:

$$\tilde{\tau} = \tau - \hat{\tau}^{-1} \tag{15}$$

$$\tilde{k} = k - \hat{k} \tag{16}$$

Proof of theorem (2): Choose a Lyapunov function v as:

$$v = 0.5\left(\delta^2 + \eta \tilde{k}^2 + \tilde{\tau}^2\right) \tag{17}$$

Using equation (5) and using the derivative of the Lyapunov function (17) and placing it in the control input (2) we will have:

$$\begin{aligned} \dot{v} &= \delta\dot{\delta} + \eta\tilde{k}\dot{\tilde{k}} + \tilde{\tau}\dot{\tilde{\tau}} = \delta\left(XU_2 + \gamma\right) \\ &-\eta\tilde{k}\dot{\hat{k}} + \tilde{\tau}\dot{\tilde{\tau}} = \delta\left(-X\alpha\hat{\tau}\hat{k}\tanh(\delta) + \gamma\right) \\ &-\eta\tilde{k}\dot{\hat{k}} + \tilde{\tau}\hat{\tau}^{-2}\dot{\hat{\tau}} \leq -X\alpha\hat{\tau}\hat{k}|\delta| + |\gamma||\delta| \\ &-\tilde{k}|\delta| - \hat{k}|\delta| = -(X - \tau)\alpha\hat{\tau}\hat{k}|\delta| \\ &+|\gamma||\delta| - \tilde{k}|\delta| - \alpha\hat{k}|\delta| \leq |\gamma||\delta| \\ &-\left(k - \hat{k}\right)|\delta| - \alpha\hat{k}|\delta| \\ &= -\left((\alpha - 1)\hat{k} + \left(k - |\gamma|\right)\right)|\delta| \end{aligned} \tag{18}$$

With $\kappa \geq |\gamma|$and $\alpha \geq 1$ as a result $\dot{v} \leq 0$ and in this case the system modes will reach the sliding surface δ. In this case, with the adaptive law $|\delta| \geq \varepsilon$ the modes of the system will reach the area$(-\varepsilon, \varepsilon)$.

Proof of Theorem (3): For $|\delta| \leq \varepsilon$and under the controller (13) we define a variable as follows:

$$\theta = \varepsilon \frac{|\gamma|}{|\gamma| + X} \tag{19}$$

It should be noted that the variable θ is modulated by perturbation with time γ.

As a result, we seek that with $\theta \le |\delta| \le \varepsilon$ the controller (12) estimates that δ is $[-\theta, \theta]$. Select the candidate Lyapunov function w as follows:

$$w = 0.5\left(\delta^2 + \left(F_b(\delta) - F_b(0)\right)^2\right) \tag{20}$$

Using equation (5) and using the derivative of Lyapunov function w and placing it in the control input (12) we will have:

$$\begin{aligned} w &= \delta\dot{\delta} + F_b(\delta)\dot{F}_b(\delta) = \delta\left(XU_2 + \gamma\right) \\ &+ F_b(\delta)\frac{\varepsilon}{\left(\varepsilon - |\delta|\right)^2}\tanh(\delta)\dot{\delta} \\ &= \delta\left(-XF_b(\delta)\tanh(\delta) + \gamma\right) \\ &+ F_b(\delta)\frac{\varepsilon}{\varepsilon - |\delta|^2}\tanh(\delta) \\ &\left(-XF_b(\delta)\tanh(\delta) + \gamma\right) \le \\ &- XF_b(\delta)|\delta| + |\gamma||\delta| + F_b(\delta) \\ &\frac{\varepsilon}{\left(\varepsilon - |\delta|\right)^2}\left(-XF_b(\delta) + |\gamma|\right) \\ &= -\left(XF_b(\delta) - |\gamma|\right)|\delta| \\ &- F_b(\delta)\frac{\varepsilon}{\left(\varepsilon - |\varepsilon|\right)^2}\left(XF_b(\delta) - |\gamma|\right) \end{aligned} \tag{21}$$

We explain:

$$\gamma_1 = XF_b(\delta) - |\gamma| \tag{22}$$

$$\gamma_2 = \left(XF_b(\delta) - |\gamma|\right)\frac{\varepsilon}{\left(\varepsilon - |\delta|\right)^2} \tag{23}$$

Using the equations (13) and (19), it can be concluded that $F_b(\delta) \ge F_b(\theta) = X^{-1}|\gamma|$. for all $\theta \le |\delta| \le \varepsilon$ and $\gamma_1, \gamma_2 \ge 0$. So equation (18) can be rewritten as follows:

$$\begin{aligned} \dot{W} &\le -\gamma_1 \frac{\sqrt{2}}{\sqrt{2}}|\delta| - \gamma_2 \frac{\sqrt{2}}{\sqrt{2}} F_b(\delta) \le \\ &- \min\left\{\gamma_1\sqrt{2}, \gamma_2\sqrt{2}\right\} \\ &\left\{\frac{1}{\sqrt{2}}|\delta| + \frac{1}{\sqrt{2}} F_b(\delta)\right\} \le -\phi W^{0.5} \end{aligned} \tag{24}$$

where:

$$\phi = \min\left\{\gamma_1\sqrt{2}, \gamma_2\sqrt{2}\right\} \tag{25}$$

With $\phi \ge 0$ there is a positive constant $\bar{\phi}$ such that:

$$\bar{\phi} \le \phi, \forall \theta < |\delta| < \varepsilon \tag{26}$$

So, we will have: $\dot{w} \le -\bar{\phi} w^{0.5}$

Using the results for $\theta \le |\delta \le \varepsilon|$ the sliding surface converges with the initial conditions $\delta(0)$ to $[-\theta, \theta]$ in a finite time.

2.3. Simulation Results

In what follows, to further assess the performance of the BFAFTSM control approach, we compare its performance to the BFASM control approach proposed in [26]. Fig. ***1*** displays the output of the system using the BFAFTSM control (2) compared to the outputs of the reference model and method in [26]. The time response of the tracking errors is shown in Fig. ***2***. From Fig. ***2***, it is demonstrated that the tracking error signal appropriately converges to the origin and provides faster tracking performance than method in [26].

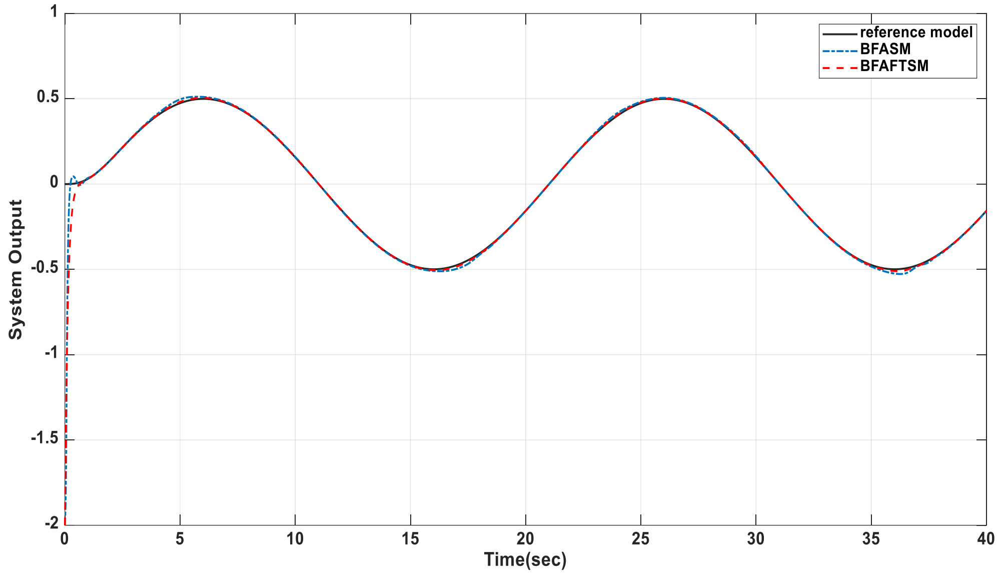

Fig. 1. System outputs.

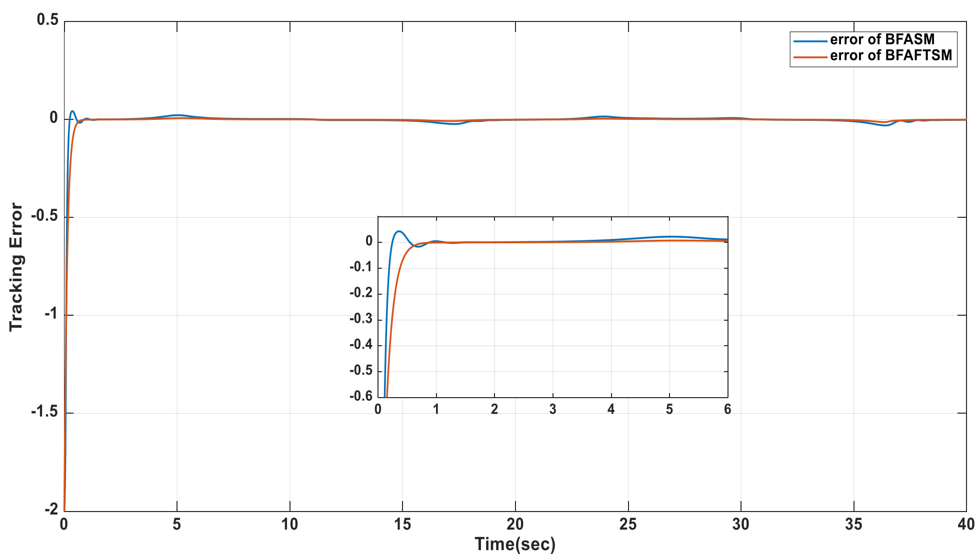

Fig. 2. Tracking errors.

3. NEW HYPERCHAOTIC DESIGN

Consider a new 5D nonlinear system designed as follows:

$$
\begin{aligned}
\dot{x}_1 &= x_2 + a_1 x_4 \\
\dot{x}_2 &= x_3 - a_2 x_2 \\
\dot{x}_3 &= a_3 x_4 - a_4 x_1 x_3 \\
\dot{x}_4 &= -x_3 - a_5 x_4 + x_1 x_2 - a_6 x_2 \\
\dot{x}_5 &= -x_4 - a_7 x_5 + x_1 x_3 - a_8 x_3
\end{aligned} \tag{27}
$$

where:

$$a_1 = 0.06, a_2 = 0.025, a_3 = 15.8,$$
$$a_4 = 0.24, a_5 = 0.48, a_6 = 0.95 \quad (28)$$
$$a_7 = .085, a_8 = 6.2$$

With initial conditions:

$$x_1(0) = -0.025, x_2(0) = -0.053,$$
$$x_3(0) = -0.0075, x_4(0) = -0.019, \quad (29)$$
$$x_5(0) = -0.007,$$

System (27) is hyperchaotic. phase portraits of the new hyperchaotic system (27) is shown in Fig. ***3***.

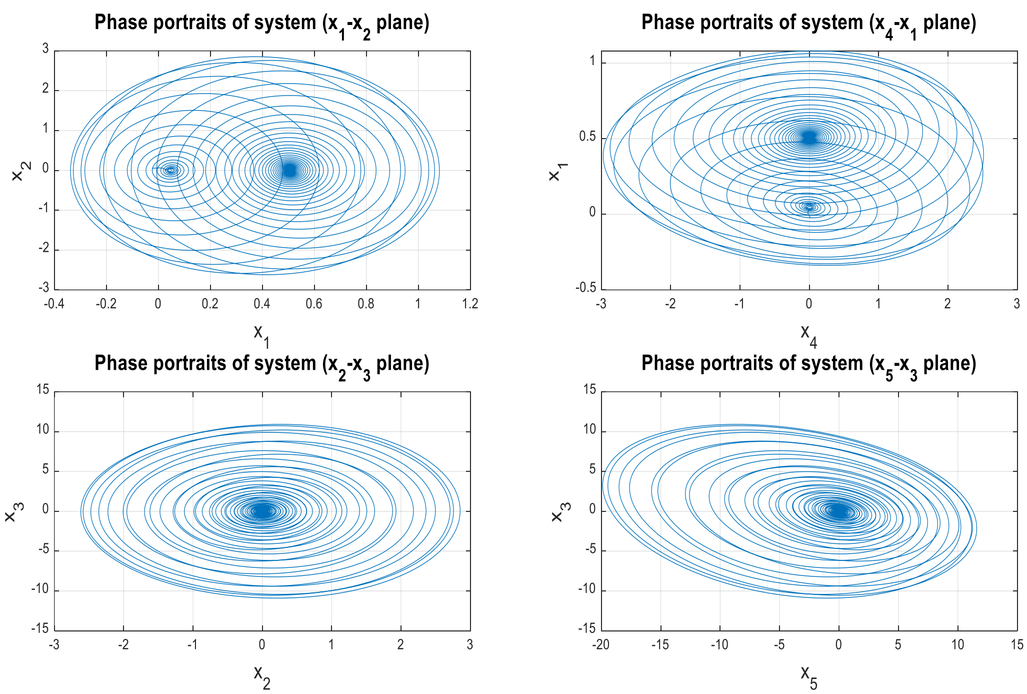

Fig. 3. 2D phase portraits.

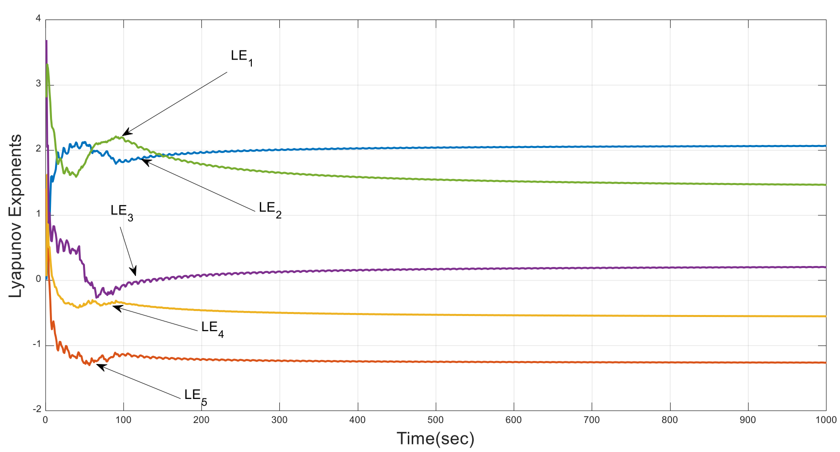

Fig. 4. Lyapunov exponents of new hyperchaotic system.

One of the main characteristics of hyperchaotic systems is having at least two positive LEs. Fig. **4** shows the Lyapunov exponents (LEs) of the proposed hyperchaotic system.

As it turns out, the system has the following LEs:

$$\begin{aligned} &LE_1 = 2.192,\ LE_2 = 1.969, LE_3 = 0, \\ &LE_4 = -0.3605,\ LE_5 = -1.189 \end{aligned} \tag{30}$$

One of the characteristics for introducing hyperchaotic systems is having at least two positive LEs [27]. As it is known, the system has two positive LEs, one zero LE and two negative LEs.

4. HYPERCHAOTIC SYNCHRONIZATION

In this section, finite-time synchronization and its theorems are presented between two new and overly hyperchaotic systems with homogeneous parametric uncertainties and unknown disturbances. At this point, we use the system (27) by modifying the initial conditions and its parameters, both as the master and the slave systems for hyperchaotic synchronization. Consider the hyperchaotic master system as follows:

$$\begin{aligned} &\dot{x}_{1m} = x_{2m} + a_{1m}x_{4m} \\ &\dot{x}_{2m} = x_{3m} - a_{2m}x_{2m} \\ &\dot{x}_{3m} = a_{3m}x_{4m} - a_{4m}x_{1m}x_{3m} \\ &\dot{x}_{4m} = -x_{3m} - a_{5m}x_{4m} + x_{1m}x_{2m} - a_{6m}x_{2m} \\ &\dot{x}_{5m} = -x_{4m} - a_{7m}x_{5m} + x_{1m}x_{3m} - a_{8m}x_{3m} \end{aligned} \tag{31}$$

where:

$$\begin{aligned} &a_{1m} = 0.06, a_{2m} = 0.025, \\ &a_{3m} = 15.8, a_{4m} = 0.24, \\ &a_{5m} = 0.48, a_{6m} = 0.95, \\ &a_{7m} = 0.085, a_{8m} = 6.2 \end{aligned} \tag{32}$$

Similarly, for the slave system we will have:

$$\begin{aligned} &\dot{x}_{1s} = x_{2s} + a_{1s}x_{4s} + u_1 + d_1 \\ &\dot{x}_{2s} = x_{3s} - a_{2s}x_{2s} + u_2 + d_2 \\ &\dot{x}_{3s} = a_{3s}x_{4s} - a_{4s}x_{1s}x_{3s} + u_3 + d_3 \\ &\dot{x}_{4s} = -x_{3s} - a_{5s}x_{4s} + x_{1s}x_{2s} \\ &\qquad - a_{6s}x_{2s} + u_4 + d_4 \\ &\dot{x}_{5s} = -x_{4s} - a_{7s}x_{5s} + x_{1s}x_{3s} \\ &\qquad - a_{8s}x_{3s} + u_5 + d_5 \end{aligned} \tag{33}$$

where:

$$\begin{aligned} &a_{1s} = 0.042, a_{2s} = 0.065, \\ &a_{3s} = 14.9, a_{4s} = 0.3, a_{5s} = 0.5, \\ &a_{6s} = 0.88, a_{7s} = 0.05, a_{8s} = 6 \end{aligned} \tag{34}$$

In slave system (33), $d = d_1, \dots, d_5$ and $u = u_1, \dots, u_5$ are disturbances and input controls, respectively.

Assumption 1: Let dene the hyperchaotic synchronization and finite-time synchronization errors of the system (32) and system (33) as: $e_i = x_{is} - x_{im}$ (i=1,...,5).

Assumption 2: In general, consider the constraints on the disturbance and uncertainty as $|f(x(\tau))| \le \alpha_1$, $|d(\tau)| \le \alpha_2$.

Definition 1 [28]: Master system (32) and slave system (33) are synchronized, if there has a controller $u_p(\tau)$ and a constant $T > 0$ such that $\lim\limits_{x\to\infty} [z_p{}^*(\tau) - z_p{}^{**}(\tau)] = 0$, where $z_p{}^*(\tau) - z_p{}^{**}(\tau)$ for $\tau > T$, $z^*(\tau)$ and

$z^{**}(\tau)$ are the solutions of master and slave systems.

For hyperchaotic synchronization, we used the controller (2). Then, based on Assumption 1, we trust that the hyperchaotic synchronization between master system (32) and slave system (33), will be achieved in finite–time. Based on Assumption 2, we consider the disturbances as follows:

$$d = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} = \begin{bmatrix} 0.1\sin(t)+9 \\ 2\cos(3t)+1.5 \\ -0.5\sin(1.2t)-3 \\ -12\cos(1.4t) \\ 6.3\cos(2t)-5.3 \end{bmatrix} \quad (35)$$

Fig. ***5*** and Fig. ***9*** shows the hyperchaotic synchronization of the master system (32) and slave system (33).

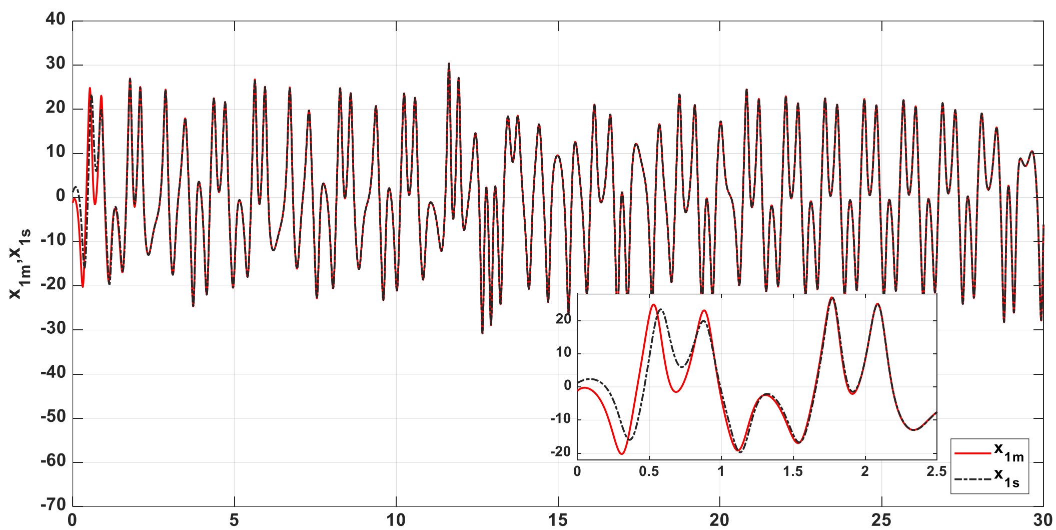

Fig. 5. Hyperchaotic synchronization of the master system (32) and slave system (33) in x_{1m} and x_{1s}.

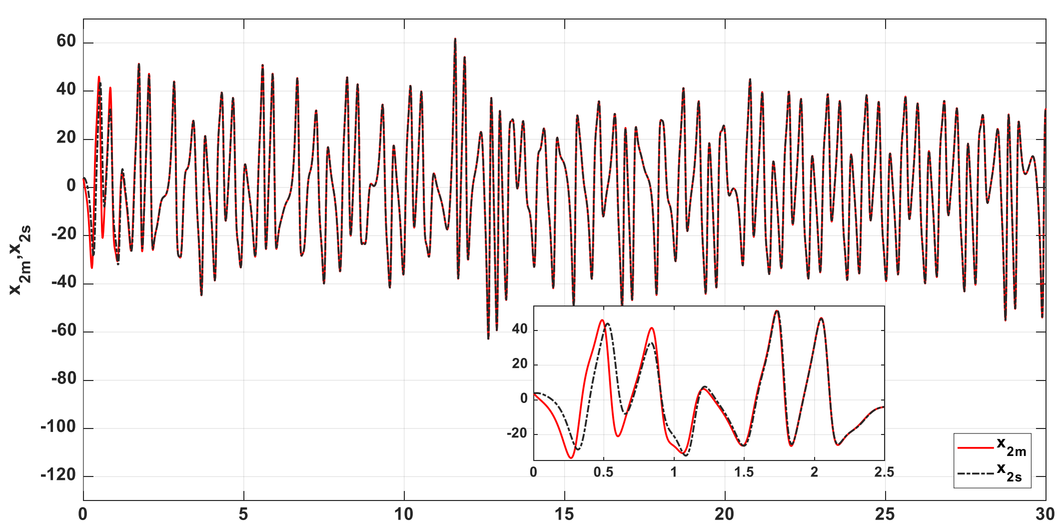

Fig. 6. Hyperchaotic synchronization of the master system (32) and slave system (33) in x_{2m} and x_{2s}.

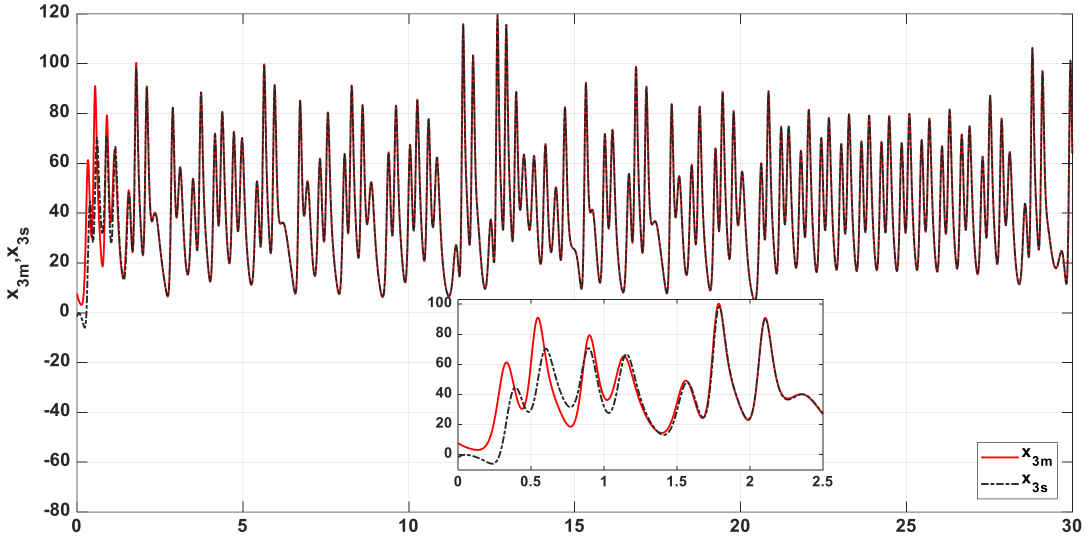

Fig. 7. Hyperchaotic synchronization of the master system (32) and slave system (33) in x_{3m} and x_{3s}.

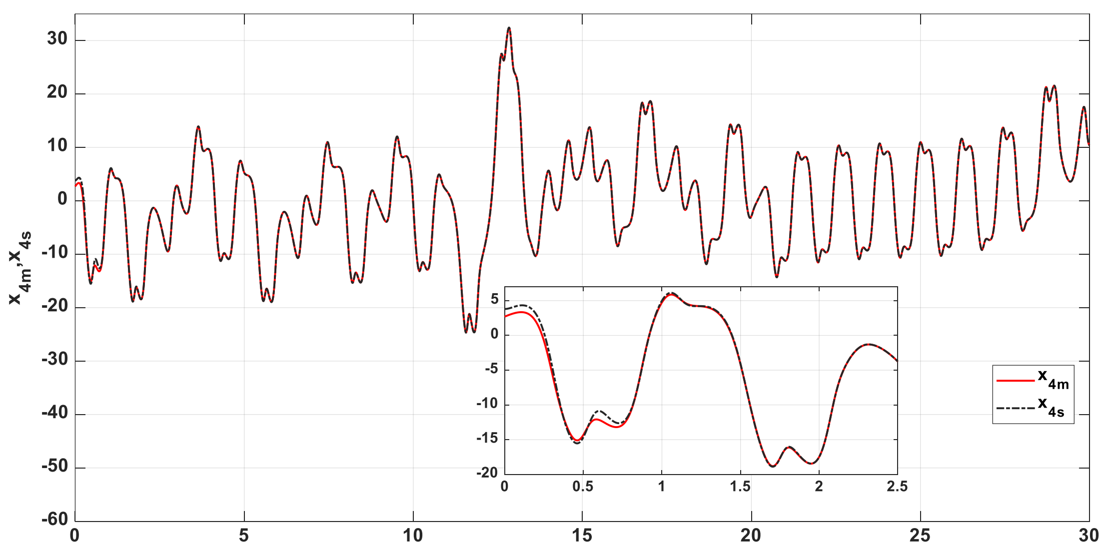

Fig. 8. Hyperchaotic synchronization of the master system (32) and slave system (33) in x_{4m} and x_{4s}.

The errors of hyperchaotic synchronization are shown in Fig. ***10***.

Finally, the control input used for the hyperchaotic synchronization is shown in Fig. ***11***. According to the simulation results, it is easy to see that the master system (32) and slave system (33) are synchronized in finite–time.

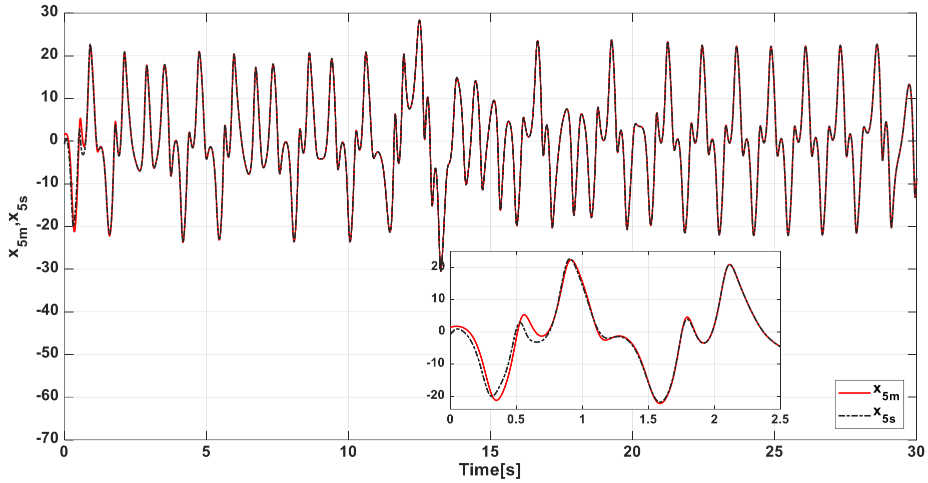

Fig. 9. Hyperchaotic synchronization of the master system (32) and slave system (33) in x_{5m} and x_{5s}.

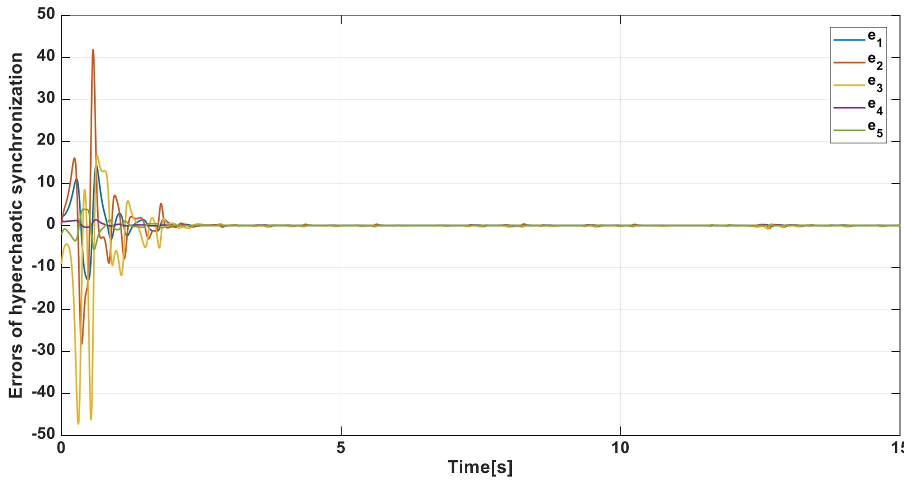

Fig. 10. Errors of hyperchaotic synchronization of the master system (32) and slave system (33).

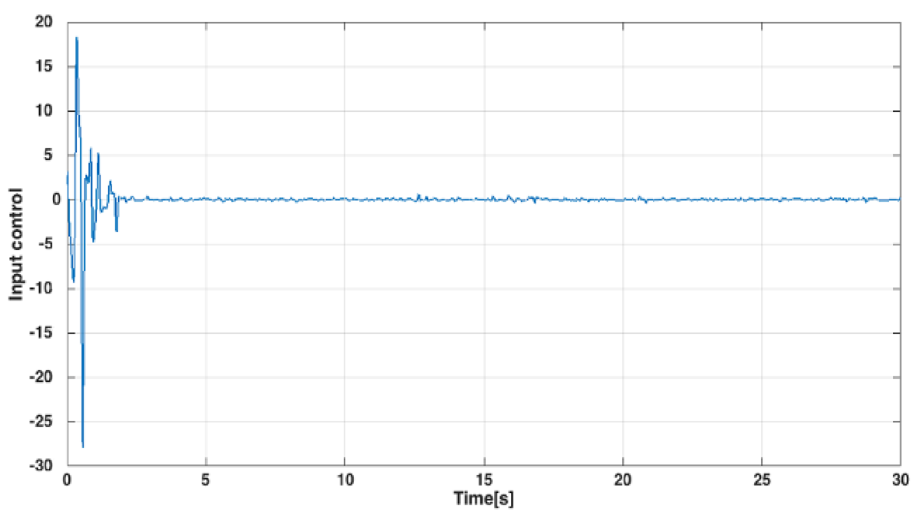

Fig. 11. The control input used for the hyperchaotic synchronization.

5. CONCLUSIONS

In this paper, a new adaptive controller based on barrier function is constructed to overcome disturbance in high-order nonlinear systems. The designed controller is of switching type and the sliding surface was determined based on the error function. One of the strengths of the proposed controller is the adaptation of the controller to the disturbance. Also, the proposed controller scheme was able to overcome the chatting phenomenon and no estimation was used in its design. Next, to demonstrate the performance of the new controller, we implemented a hyperchaotic synchronization system on the new 5D nonlinear system with the proposed controller. The results showed that the tracking errors converge around the origin in a finite time.

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