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Evolutionary Interval Type-2 Fuzzy Rule Learning Approaches for Uncertain Time-Series Prediction

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Abstract

This study presents Interval Type-2 Fuzzy Evolutionary models to manage uncertainty in the process of uncertain time-series prediction. This study presents two type-2 fuzzy evolutionary models for rule extraction that were proposed: 1) Evolutionary Interval Type-2 Fuzzy Rule Learning (EIT2FRL), and 1) Evolutionary Interval Type-2 Fuzzy Rule-Set Learning (EIT2FRLS). A ROC curve analysis was applied for performance evaluation, and the results were validated using a 10-fold cross-validation technique. The results reveal that the proposed methods have an AUC of 0.96 for EIT2FRLS and 0.93 for EIT2FRL proposed methods. The results are promising for knowledge extraction in uncertain circumstances, predicting uncertain patterns prediction, and making suitable strategies and optimal decisions.

Keywords: Evolutionary Algorithm, Type-2 Fuzzy Logic, Time-Series Prediction.

1. INTRODUCTION

A time series is a collection of observations made chronologically. Time series modeling is important because there are so many pattern analysis problems that contain a time component [1]. One of the most challenging issues in time series is their prediction. Another challenge is extracting automatic rules for a predictor system such as fuzzy time series. Prediction problems are often classified into short-term, medium-term, and long-term, containing different uncertainty orders in their characteristics. The uncertainty associated with the observed time-series data is implicit with a non-linear pattern. There have been different time series models, such as fuzzy time series investigation to improve the prediction accuracy [2]. A wide range of intelligent models has been used in the analysis of time

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series patterns. Recently, soft computing approaches such as fuzzy logic, neural networks, and genetic algorithms have been reported in many pieces of literature for time series prediction.

On the other hand, predicting a time series has a significant act in numerous discipline areas of applied and real-world application as engineering, air pollution, and stock price forecasting [3]. A time series is a collection of observations made chronologically. The importance of time-series dynamical characteristics, the study, and the forecast of disordered and chaotical time-series have been noticed by the scientific and engineering communities. Real-life glitches and problems necessitate misuse of architecture that permits coping with different levels and types of uncertainty. Also, time series is genuinely omnipresent, seeming in nearly every study and research where data and information are studied and analyzed. Though their routine analysis entails superior statistical techniques and concepts, without which flawed interpretations, inferences, and decisions may all too willingly be drawn. Once a time series is inspected and observed at quarterly or yearly intervals, its annual seasonal order (pattern) is an important feature. When we have a marked seasonal pattern, it has advanced over the years, even decades [3]. For example, the first quarter of a year may has the lowest rainfall or air pollution, there is usually more rainfall or pollution in the fourth quarter. In contrast, the comparative extent of the second and third quarters of the mentioned events seems to fluctuate over the observation duration, which is the seasonality feature of a time series.

*-The motivation for this study:*The nature of time-series and chaotic form comprises big data, high-dimensionality, high order uncertainty, and required to bring up-to-date uninterruptedly. Additionally, the pattern and data of time-series, which is considered by its continuous and numerical structure, are continually measured as a whole instead of the discrete numerical field. The growing use of time series has started an inordinate pact of study and development efforts in pattern recognition and data mining. The plentiful exploration and study of time-series patterns in the last couple of years could hamper the admission of interested scientists due to its complexity. So, it is a system needed in complex challenges and problem areas like pollution prediction, pattern recognition, and chaotic time-series prediction. This study's main objective is to introduce an intelligent evolutionary type-2 fuzzy time model to predict long-term temporal data alongside the optimal rule and rule-sets extraction from a raw dataset. The model is split into two parts: Type-2 fuzzy time series prediction and evolutionary rule mining [4]. The Type-2 time series prediction consists of several steps. We applied Karnik and Mendel's (KM Algorithm) operators for the defuzzification [5]. The complex min-max composition operators are applied to all predictions. Then the prediction performance is evaluated by Root Mean Square Error (RMSE), ROC curve analysis alongside a statistical evaluation under the left-tailed t-test.

The rest of this paper is organized as follows: Section 2 presents the theoretical and mathematical model of the research background; Section 3 describes the detailed structure of the proposed EIT2FRL and

EIT2FRLS models; performance evaluation of the proposed models based on ROC curve analysis, experimental and comparative results are presented in Section 4; The paper is concluded in section 5.

2. THEORETICAL RESEARCH BACKGROUND

This section presents a brief review of the interval type-2 fuzzy sets (IT2FS) in 2.1. In 2.2, a short background of evolutionary algorithms has been presented. In section 2.3, the time-series structure has been reviewed.

2.1. Interval Type-2 Fuzzy System

A type-2 fuzzy system, represented as $\tilde{A}$, is characterized through a type-2 MF $\mu_{\tilde{A}(x,u)}$ where $x \in X$ and $u \in J_x \subseteq [0,1]$ as follows [5]:

$$\tilde{A} = \left\{\left((x,u), \mu_{\tilde{A}(x,u)}\right) | \forall x \in X, \forall \in J_x \subseteq [0,1]\right\} \quad (1)$$

where $0 \leq \mu_{\tilde{A}(x,u)} \leq 1$, X is the domain of fuzzy set and J_x is the domain of the secondary MF at x. $\tilde{A}$ is as follows:

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} \mu_{\tilde{A}}(x,u)}{x, uJ_x} \subseteq [0,1] \quad (2)$$

and, where $\iint$ represents union over all admissible x and u.

$$\tilde{A} = \frac{\int_{x\in X} \int_{u\in J_x} 1}{x,u} = \frac{\int_{x\in X}\left[\frac{\int_{u\in J_x} 1}{u}\right]}{x} \quad (3)$$

where x is the main variable, J_x , an interval in [0,1], is the primary MF of x, u is the second variable, and $\int_{u\in J_x}$ is the secondary MF at x [6].

2.2. The Evolutionary Algorithm

Evolutionary Algorithms (EA) are inspired by Darwinian evolution. EA consists of individuals with a fitness value and a genome

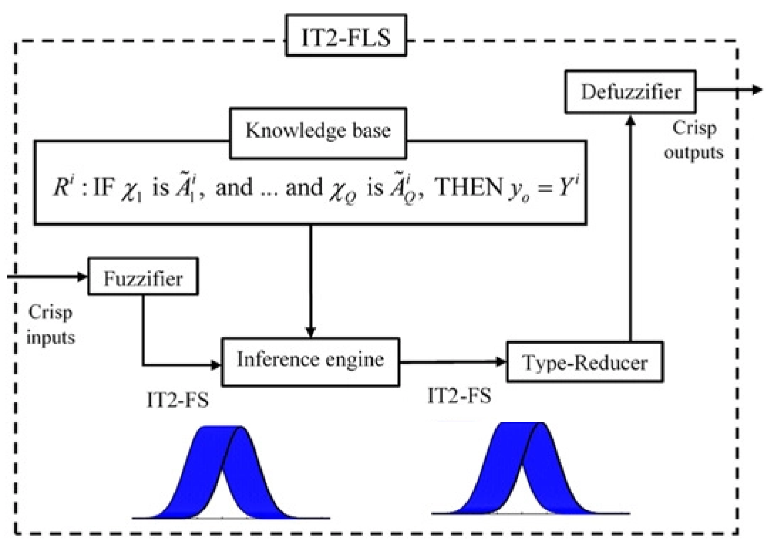

Fig. 1. The structure of the IT2FLS [7].

representing one candidate solution to the given problem [8]. In an evolutionary algorithm, the researcher chooses a representation scheme to define the set of solutions that form the search space for the algorithm. Many individual solutions are created to form an initial population. The population-based collective learning process, self-adaptation, and robustness are some of the chief features of evolutionary algorithms compared to other global optimization techniques. One of the successful methods of EAs is the genetic algorithm (GA). GA has many significant applications such as intelligent optimization, search in large problem spaces, and rule extraction. The pseudo-code of the genetic algorithm is shown in Algorithm.1.

2.3. Time-Series Modeling

Time-series observations are denoted as, $x_{(t)} = \{x_{t_1}, x_{t_2}, \dots, x_{t_n}\} \mid t \subseteq R$, where x_{t_i} is the value of the variable x in the time t_i, where $\{i = 1, 2, \dots, n\}$. One of the most challenging issues in time series is their forecasting. If the past observations of any phenomenon are denoted as a sequence, $Y_1, Y_2, \dots, Y_n$, it is possible to calculate its various computational parameters. There are three-time series types, which have been mostly proven as an appropriate approximation to represent actual associations between the elements of the observed time pattern as follows [9]:

Additive: $X_{(t)} = T_{(t)} + S_{(t)} + I_T$

Multiplicative: $X_{(t)} = T_{(t)}.S_{(t)}.I_T$

Mixed: $X_{(t)} = T_{(t)}.S_{(t)}.I_T$

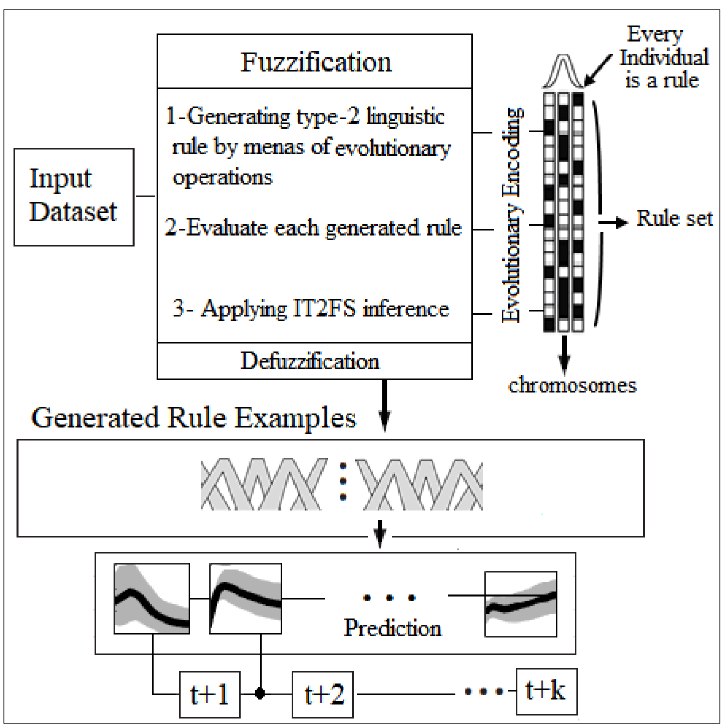

Fig.2 The architecture of the proposed evolutionary-fuzzy rule learning model.

where $X_{(t)}$ is the observed series in time t, $T_{(t)}$ represents the trend factor, $S_{(t)}$represents the seasonal factor, I_T denotes irregular or random factor, and I_T is a random or white noise component [10].

3. RESEARCH METHODOLOGY

This section presents the proposed automatic evolutionary interval type-2 fuzzy rule learning (EIT2FRL) methods. Two different studies were conducted to extract the patterns of the time-series using an evolutionary algorithm and type-2 fuzzy set as follows:

1) Evolutionary Interval Type-2 Fuzzy model for a rule (EIT2FRL)
2) Evolutionary Interval Type-2 Fuzzy model for learning a rule set (EIT2FRSL)

In the rest of this section, the EIT2FRL rule learning model has been presented in 3.2. Next, the EIT2FRSL model has been discussed in 3.3. The proposed fuzzy time series prediction models have four steps:

a. *The footprint of uncertainty (FOU) for the time series is identified in several intervals.*
b. *The type-2 fuzzy sets are defined based on the length and number of intervals.*
c. *EIT2FRL rules and EIT2FRLS rulesets are established.*

Predicting the next timestep using EIT2FRL and EIT2FRLS rules.

3.1. General Structure of Proposed Model

Fuzzification is the first step in the proposed model. According to the number of non-zero MF values and inputs, the proposed architecture's fuzzifier can be categorized into two types, singleton and non-singleton. In this study, the TSK fuzzy rule type was considered. The product and minimum t-norms have been introduced as acceptable inference methods to compute multiple antecedents' firing strength. The membership degree of the input data or measurements to interval type-2 fuzzy sets is defined. The Gaussian membership function follows the normal distribution of the Gaussian function. The Gaussian membership function assumes that the time steps are typically distributed.

To design the inference systems of the DT2FTW model, we applied a TSK type with three inputs and one output; therefore, each input-output variable is corresponded to Gaussian MFs and has 2^n if-then rules. in this study, 2^n optimal fuzzy rules have been applied for the experiments to obtain better performance and minimize the prediction error rates in different scenarios. The defuzzification process is performed to understand DIT2FTW perdition’s output better. The proposed model's output processing used the *Karnik-Mendel* (KM) algorithm. For this reason, the time steps (data points) have to be assigned to a class with the highest membership. During defuzzification, the decision rules are set to guide the process of assigning time-steps to crisp sets. A threshold value for MFs is defuzzified and assigned to a class with the highest membership in the last part.

3.2. The Evolutionary Interval Type-2 Fuzzy rule learning (EIT2FRL)

In the EIT2FRL model, each fuzzy rule is considered a classifier. This methodology progressively goes forward and constructs a single rule set through the whole population of fuzzy rules. A fitness function is designed to assess the proficiency of each rule in the whole population. The GA works on the stage of individual rules and selects the fittest rules to reproduce. Simultaneously, rules inside the population need to collaborate to resolve the problem. The knowledge base, composed of the MF parameters, is renewed using the newly extracted fuzzy rules. Consider the structure of a type-2 fuzzy rule as:

Rule R_i: if x_1 is Z_{i1} and… x_k is Z_{ik} then C_i, where R_i is the label of ith rule, Z_{i1} and Z_{ik} are the linguistic fuzzy terms and C_i is the output. Degree of compatibility for each training sample as, $X = x_1, x_2, \dots, x_k$, is computed through a fuzzy if-then rule R_i through the minimum t-norm operator as:

$$\mu_i(X) = \min\{\mu_1(x_1), \mu_2(x_2), \dots, \mu_k(x_k)\} \tag{7}$$

where $\mu_i(X)$ is the membership function of fuzzy linguistic term Z_{ik}, for $i = 1$ to m where n_i is the number of samples with ith class and is computed for each rule R_i as follows:

$$R_i = \frac{\sum_{class(X)=i} \mu_{i_X}}{n_i} \tag{8}$$

3.3. The Evolutionary Interval Type-2 Fuzzy rule-set learning (EIT2FRLS)

The proposed evolutionary interval type-2 fuzzy ruleset learning method preserves an entire population as a rule set. In this model, each chromosome encodes the entire ruleset. The EIT2FRSL algorithm evaluates each individual as a rule set and chooses parent rule sets for recombination. In the EIT2FRS model, each individual indicates a whole set of rules. Interconnecting individual rule codes typically achieve the chromosome structure of the ruleset. Therefore, the fuzzy sets in the database of L will be defined as follows:

$$L = L_a + L_c, L_a = \sum_{i=1}^{n} N_i\,,\ L_c = \sum_{j=1}^{m} M_j \tag{9}$$

The structure of the proposed model comprises as many elements $n + m$ as there are fuzzy variables in the scheme. Hence, each element models a label j that leads to a specific fuzzy set $C_{y_L^U}$ in the fuzzy partition *{Ci1, Ci2…, Cn}* of variable X_i. The population size after one iteration has been defined by:

$$PopSize_{(t+1)} = PopSize_{(t)} + M_{(t)} - E_{(t)} \tag{10}$$

where $E_{(t)}$ is chromosome numbers of current time which expire off throughout generation t and $M_{(t)}$ is the number of offspring throughout the generation t. The number of offspring $M_{(t)}$ is relative to the population size with assumed generation t, while the individual numbers "to expire" $E_{(t)}$ relies on the stage of each individual in chromosome structure.

3.4. Measuring Uncertainty

Uncertainty in a model affects the confidence of a prediction model and its accuracy with a distribution. This distribution depends on the data points as $D = \{X, Y\}$, where D is the distribution and X, Y are the datapoints sample on a 2-D area of predicted measures in whole distribution D, where

$$X = \{x_1, x_2, \dots, x_n\} \text{ and } Y = \{y_1, y_2, \dots, y_n\} \quad (11)$$

Therefore, the weight of the distribution after predicting the time series can be written as $p(\omega|X, Y)$. In order to approximate this distribution, a Monte-Carlo based approach collects weights using the Bernoulli rate, which requires calculation as follows:

$$p(\omega|x, y) \approx q(\omega; \alpha) = Bern(\omega; \alpha) \quad (12)$$

where α is the Bernoulli rates on the weights. Hence, the model uncertainty is the variance of T Monte-Carlo samples (Data points) as follows:

$$\mathrm{Var}_{p(\mathbf{y}|\mathbf{x})}^{\mathrm{model}}(\mathbf{y}) = \frac{1}{T}\sum_{t=1}^{T} (\mathrm{y}_t - \overline{y})^2 \quad (13)$$

where $\{\mathrm{y}_t\}_{t=1}^{T}$ is a set of Tsampled outputs of DIT2FLSTM model for weights instances given by $\overline{y} = \frac{1}{T\sum_t \mathrm{y}_t}$.

4. PERFORMANCE EVALUATION AND EXPERIMENTAL RESULTS

In this section, the EIT2FRL and EIT2FRLS models were applied to a real and official dataset which includes real data collected from the standards Mackey-glass time-series model. First, a ROC curve analysis of the results of applying the EIT2FRL and EIT2FRLS model to the official dataset has been presented. Then, a statistical evaluation of the EIT2FRL and EIT2FRLS has been conducted to represent the model's capability in time-series prediction.

4.1. Mackey-Glass Time-Series Modeling

This section presents the experimental results of applying the proposed prediction method to the Mackey-Glass time series. The one-step-ahead prediction capabilities of the proposed model is evaluated on a high-dimensional chaotic time-series system generated by the Mackey-Glass delay differential equations:

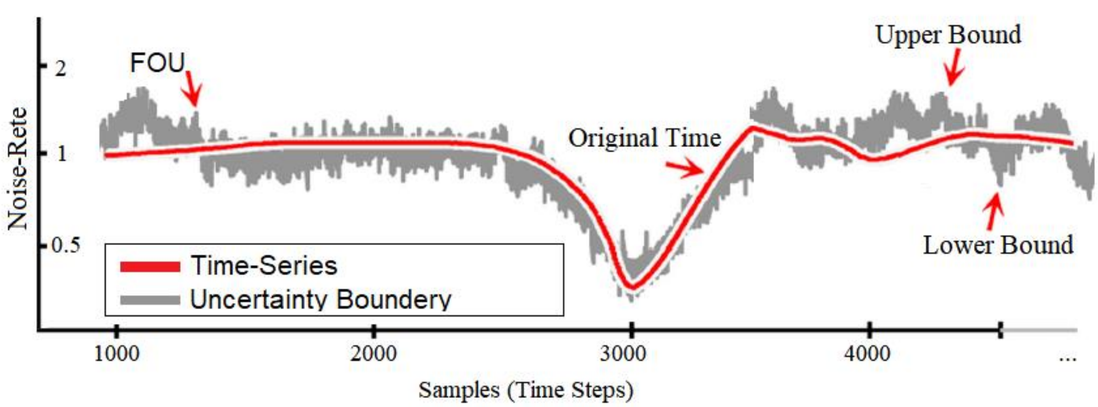

Fig.3. Modeling uncertainty in Mackey-glass time-series.

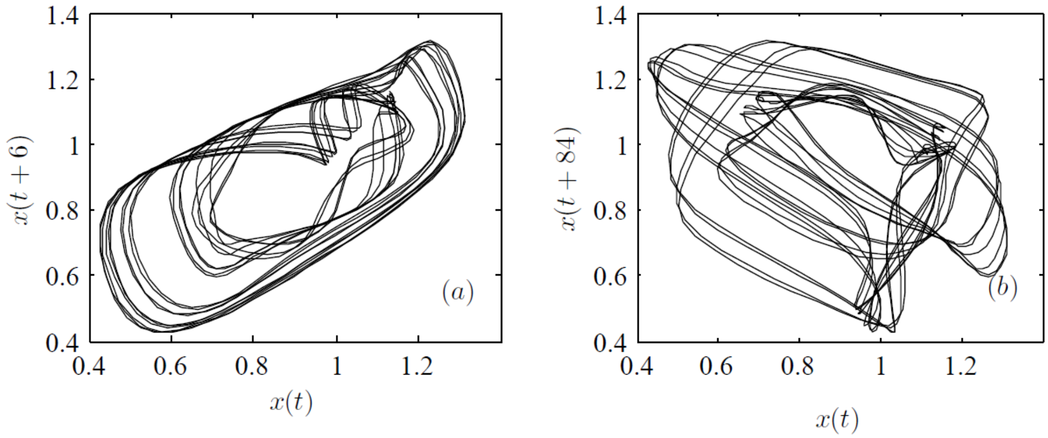

Fig.4. (a) short-range time-series representation; (b) long-range time-series representation.

$$\frac{dx(t)}{dt} = -0.1x(t) + \frac{0.2x(t-t_\Delta)}{1+x(t-t_\Delta)^{10}} \qquad (14)$$

where t_Δ is the delay of the time series of the model. Furthermore, a statistical test was conducted on applying the DIT2FLSTM to the Mackey-Glass time series through t-test. First, we calculate the mean of observation as follows:

$$t = \frac{\bar{X}-t_\Delta}{S} \times \sqrt{n} \qquad (15)$$

where, $\bar{x}$ is the sample mean, t_Δ is the time delay mean, s is the sample standard deviation, and S is the test sample size as follows:

$$S = \sqrt{\frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{n-1}} \qquad (16)$$

In addition, the following indices, the root mean squared error (RMSE), have been applied to evaluate the performance of the DIT2FLSTM model in the Mackey-Glass time-series. The detailed calculation of RMSE is given by [11]:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} (\hat{y}^{(k)} - y^{(k)})^2} \qquad (17)$$

where $\hat{y}^{(k)}$ represents the predicted value, and $y^{(k)}$ is the real value. The smaller the values of RMSE represent the better prediction accuracy performance. Figure 4 shows that the system operates in a high-dimensional administration with the mentioned parameters in the proposed model.

4.2. ROC Curve Analysis of The EIT2FRL and EIT2FRLS Models

In order to have a reliable estimate of the performance of the EIT2FRL and EIT2FRLS models, a ROC curve analysis was conducted and the results were statistically verified. The following equations were used for assessing the performance through a ROC curve analysis of the proposed method where μ_i and μ_j are the means of the accuracy of the ROC curve for the 10-fold cross-validation**.** Also, the standard metrics, such as precision, recall, and measurement function, were applied to evaluate the proposed model as follows:

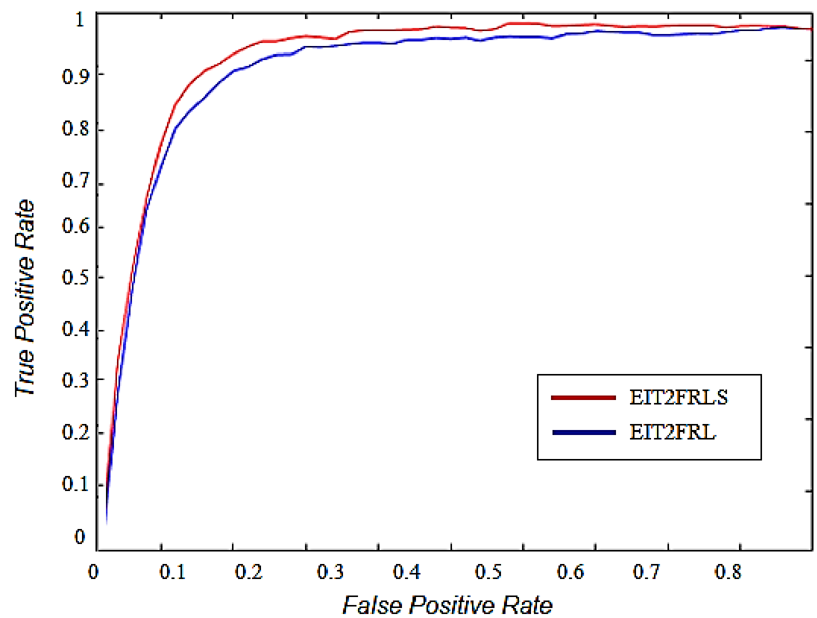

Fig.5. Comparison of ROC curves of the proposed models.

$$\text{Precision} = \frac{TP}{(TP+FP)} \times 100\% \quad (18)$$

$$\text{Recall} = \frac{TP}{(TP+TN)} \times 100\% \quad (19)$$

These results reveal the proficiency of evolutionary type-2 fuzzy rule learning models for automatic rule extraction from the imprecise and time-series training dataset. The area under the curve (AUC) in Fig.5 illustrates the performance of evolutionary EIT2FRL and EIT2FRLS on the Mackey-glass time series. According to the obtained results, the rule sets extracted from the EIT2FRLS method give a better performance than the EIT2FRL rules in terms of ROC curve analysis, which is revealed in Fig.5.

4.3. Null Hypothesis and Left Tailed T-Test Evaluation

$$\text{Accuracy} = \frac{(TP+TN)}{(TP+FP+FN+TN)} \quad (20)$$

$$\mu_i = \frac{1}{10}\sum_{k=1}^{10} AUCj \quad (21)$$

To represent the proficiency of the proposed method and its robustness, a two-sample t-test (left tailed) was applied. The null hypothesis is defined as: $H_0 = \mu_i > \mu_j$ and, H_1: $\mu_i < \mu_j$, where μ_i and μ_j are the means of the area under the ROC curve (AUC) of EIT2FRL, EIT2FRLS and interval type-2 fuzzy logic system (IT2FLS) for 10 different runs of the cross-validation technique, respectively [12]. The t-test results in Table 1 reveal the superiority of the proposed models for Mackey-glass time-series prediction. The t-test (according to the defined hypothesis testing) failed to reject the defined null hypothesis. The EIT2FRL and

EIT2FRLS rule learning models were applied to extract the patterns for uncertain time-series prediction. Table 2 represents the results of the proposed models, with an accuracy of 93% with a 95% confidence interval of [91-95] % for EIT2FRL and 95% with a 95% confidence interval [93-95] % for EIT2FRLS for uncertain time-series prediction. Table 2 shows the superiority of the proposed model with higher accuracy and lower complexity compared to its counterpart methods.

Table 1. ***T-test analysis of the EIT2FRL, EIT2FRL and IT2FLS.***

Fold#	EIT2FRLS	EIT2FRL	IT2FLS
1	0.9071	0.9123	0.7434
2	0.9212	0.9052	0.7291
3	0.9252	0.9129	0.6359
4	0.8905	0.885	0.6301
5	0.9421	0.9387	0.7218
6	0.9312	0.9329	0.7925
7	0.9651	0.9649	0.7651
8	0.9426	0.9407	0.7271
9	0.9667	0.9561	0.8112
10	0.9821	0.9729	0.8214
Mean	**0.9673**	**0.93216**	**0.8277**

Table 2. ***Comparison of evolutionary type-1 and type-2 fuzzy rule learnings***.

Parameter and Metric	ET1FRL	ET1FRLS	EIT2FRL	EIT2FRLS
Initial Population size	500	500	500	500
Rule/Ruleset	15	8	9	4
STD	0.0157	0.0129	0.0109	0.0107
Precision	74%	76%	91%	92%
Recall	79%	80%	92%	94%
AUC%	72	74	0.93	0.96
Confidence Interval%	[68-73] %	[70-75] %	[90-95] %	[91-96] %

Table 3. ***Comparison of related works and the proposed models.***

Methodology	Parameters	Rule Extraction
Markov chain and Chen-logical [16]	6	Subjective
Hybrid fuzzy C-means and NN [17]	6	Subjective
Group decision making under generalized fuzzy soft sets [18]	7	Semi-Auto
Type-2 Mutual Subsethood Fuzzy Neural (IT2MSFuN) [19]	4	Automatic
Fuzzy-Genetic [20]	5	Automatic
Intuitionistic fuzzy-genetic set [21]	6	Subjective
Loop Fuzzy Pattern Tree Evolution [22]	4	Subjective
EIT2FRL (This work)	3	Automatic
EIT2FRLS (This work)	3	Automatic

5. CONCLUSION

This study presents an automatic evolutionary model based on type-2 fuzzy rule learning to predict complex time-series problems. For this purpose, two main models with two different ways of learning rules of a type-2 fuzzy set through evolutionary techniques were proposed; EIT2FRL and EIT2FRLS. The proposed approaches can automatically extract patterns in terms of rule sets for the problems associated with uncertainty and lack of expert knowledge. The key benefits of the proposed rule learning methods are the high interpretability of the patterns in terms of rules for uncertain time-series prediction. The results show the superiority, robustness, and reliability of models based on 100 runs of 10-fold cross-validation techniques used in [13-15] for rule learning models. The comparison of the obtained results of the proposed models shows the superiority and robustness of the ET1FRLS and ET1FRL approaches compared to the related methods [16]-[20]. Further studies are suggested to extend to extract patterns of recovery and its duration for each patient.

REFERENCES

[1] Kuo I H, Horng S J, Chen Y H, et al, Forecasting TAIFEX based on fuzzy time series and particle swarm optimization, Expert Systems with Applications, Vol.37, No.2, 1494-1502, (2020).

[2] Chen S M, Chung N Y, forecasting enrollments using high order fuzzy time series and genetic algorithms, International Journal of Intelligent Systems, Vol.21, No.5, 485-501, (2016).

[3] LuW, Chen X, PedryczW, Liu X, Yang J. Using interval information granules to improve forecasting in fuzzy time series, International Journal of Approximate Reasoning, Vol.57, 1-18,

(2019).

[4] Arindam Dey, Anita Pal, Tandra Pal. "Interval Type 2 Fuzzy Set in Fuzzy Shortest Path Problem", Mathematics, (2016).

[5] J. M. Mendel, "on km algorithms for solving type-2 fuzzy set problems", IEEE Transactions on Fuzzy Systems, vol 21(3), pp 426–446, (2013).

[6] Safari, A., Mazinani, M., Hosseini, R. "A Novel Type-2 Adaptive Neuro-Fuzzy Inference System Classifier for Modelling Uncertainty in Prediction of Air Pollution Disaster (RESEARCH NOTE)", International Journal of Engineering, 30(11), (2017) pp1746-1751.

[7] A Safari, M Mazinani, R Hosseini, A novel type-2 adaptive Neuro fuzzy inference system classifier for modelling uncertainty in prediction of air pollution disaster (research note), International Journal of Engineering, Vol. 30 (11), pp.1746-1751, 2017.

[8] A. Safari, R. Hosseini and M. Mazinani, "A Type-2 Fuzzy Time Series Model for Pattern Similarity Analysis: A Case Study on Air Quality Forecasting", in IEEE Intelligent Systems, 2021.

[9] A. Safari, R. Hosseini, M. Mazinani, "A novel deep interval type-2 fuzzy LSTM (DIT2FLSTM) model applied to COVID-19 pandemic time-series prediction", Journal of Biomedical Informatics, Vol. 123, 2021.

[10] A. Safari, R. Hosseini and M. Mazinani, " Dynamic Type-2 Fuzzy Time Warping (DT2FTW): A Hybrid Model for Uncertain Time-Series Prediction", International Journal of Fuzzy Logic and Intelligent Systems Vol. 21, pp.338-348, 2021.

[11] A. Safari, R. Hosseini and M. Mazinani, "A Novel Type-2 Adaptive Neuro Fuzzy Inference System Classifier for Modelling Uncertainty in Prediction of Air Pollution Disaster", IJE Transactions B: Applications, Vol. 30, No. 11, pp. 1568-1577, 2017.

[12] Safari, A., Hosseini, R., Mazinani, M. (2022). An Adaptive Intelligent Type-2 Fuzzy Logic Model to Manage Uncertainty of Short and Long Time-Series in Covid-19 Patterns Prediction: A Case Study on Iran. Computational Intelligence in Electrical Engineering.

[13] Safari A, Hosseini R, Mazinani M. Dynamic Type-2 Fuzzy Time Warping (DT2FTW): A Hybrid Model for Uncertain Time-Series Prediction. IJFIS 2021; 21:338 348.

[14] J. Zhu, "Comparison of ARIMA model and exponential smoothing model on 2014 air quality index in yanqing county, Beijing, China," Applied and Computational Mathematics, vol. 4, no. 6, p. 456, (2015).

[15] Safari, A., & Hosseini, R. (2022). An Interval Type-2 Fuzzy LSTM Algorithm for Modeling Environmental Time-Series Prediction. Anthropogenic Pollution, 6(2), 62-72. doi: 10.22034/ap.2022.1963124.1133

[16] K. Ramadani and D. Devianto, The forecasting model of Bitcoin price with fuzzy time series Markov chain and chen logical method, AIP Conference Proceedings 2296, 020095 (2020).

[17] E.Egrioglu, CH.Aladag, U.Yolcu,

"Fuzzy time series forecasting with a novel hybrid approach combining fuzzy c-means and neural networks", Expert Systems with Applications, Vol. 40, Issue 3,pp. 854-857, (2018)

[18] W. Chen, Y. Zou, Group decision making under generalized fuzzy soft sets and limited cognition of decision makers, Engineering Applications of Artificial Intelligence, 87(2), (2020).

[19] S. Sumati and C. Patvardhan, Interval Type-2 Mutual Subsethood Fuzzy Neural Inference System (IT2MSFuNIS), IEEE Transactions on Fuzzy Systems, 26(1) (2018).

[20] A. Fernández, V. López, M. José del, F. Herrera, Revisiting Evolutionary Fuzzy Systems: Taxonomy, applications, new trends and challenges, Knowledge-Based Systems, Vol 80, (2019), Pages 109-121.

[21] Dejian Yu, Libo Sheng, Zeshui Xu, Analysis of evolutionary process in intuitionistic fuzzy set theory: A dynamic perspective, Information Sciences, Vol 601, (2022), Pages 175-188.

[22] Murphy, A., Murphy, G., Dias, D.M. et al. Human in the Loop Fuzzy Pattern Tree Evolution. SN COMPUT. SCI. 3, 163 (2022), Pages 137-152