



## Two Efficient Algorithms for Increasing OFDM Performance with Highly Complicated Fading Channel

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### Abstract

Equalization is a kind of the preferred methods for increasing the efficiency of modern digital communication systems. In this paper, a modified version of Sliced Multi-Modulus Algorithm (S-MMA) and genetic algorithm (GA) is applied for updating per-tone equalizations taps in the OFDM modulation using too long complicated fading channel, present of different noises and ISI impairment. For more efficiency, it is assumed that the channel impulse response is longer than the cyclic prefix length and as a result, the system will be more efficient but at the expense of the high ISI impairment. Both mathematical analysis and computer simulations show the appropriate performance of the new proposed application of the S-MMA and GA in comparison with the commonly used equalizer methods such as the LMS, RLS and multi modulus algorithm (MMA) in the channels with AWGN, burst noises and ISI impairment simultaneously. Therefore, the new modified S-MMA equalization and GA are good candidates for high speed and real-time applications such as DAB, WiFi, WiMAX, and DVB systems.

**Keywords:** Adaptive equalizer, Genetic Algorithm, ISI, LMS, OFDM, RLS, S-MMA.

### 1. INTRODUCTION

Nowadays, multicarrier transmission is very popular because of high data rate requirement for wireless systems. Orthogonal frequency-division multiplexing (OFDM) is a special

case of multicarrier transmission. It is as an effective technique for frequency-selective channels because of its spectral efficiency, its robustness in different multipath propagation environments, and its ability of combating inter-symbol interference (ISI) [1]. The

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OFDM system is a high-speed transmission technique, which occupies less bandwidth than the other multicarrier transmission systems. The broadband communication channels are frequency-selective; utilizing OFDM overcomes the frequency selective nature of broadband channels, by converting them to multiple flat fading sub-channels [2]. The OFDM systems mitigate ISI and frequency-selective fading caused by multipath propagation in modern high-data-rate wireless communication [3]. Achieving more efficiently OFDM performance, with lower bit error rate (BER), only by modifying the equalization updating method is the main contribution of this paper.

In digital communication systems, temporal dispersion caused by non-ideal characteristics of the channel frequency response and multipath phenomena may cause the ISI. Definitely, ISI is a limiting factor in many communication environments where it causes an irreducible degradation of BER, thus imposing an upper limit on the data symbol rate. The ISI is highly dynamic and changes according to the environment. As a result, the equalizer needs to employ efficient adaptive algorithms for adjusting equalizers coefficients [4, 5, 6].

Different equalization methods [7] for using in digital communication channels have gained increased attention over the last two decades. Among various equalizers, adaptive equalizer is most attractive and widely used in communication systems [6]. The conventional channel equalization methods, in order to calculate the equalizer coefficients, require the transmission of a reference signal to the receiver. On the other hand, blind equalization methods operate

without the desired signal and a training sequence [5], retrieving the information and some information about the system or the transmitted sequence [4].

In blind equalization, some statistical property of the signal is used in order to determine the instantaneous error  $e(n)$ . It obtains how much the output of the equalizer deviates from the desired property and calculates the instantaneous error. Then, the instantaneous error parameter is used to update the adaptive filter coefficient vector [8]. In the blind equalization, as shown in Fig. 1, the channel input  $a(m)$  is unavailable at the receiver input. In this system an equalizer with transfer function of  $\mathbf{W}(z)$  is applied at the receiver side. Decision directed scheme updates its coefficients according to,

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu(\hat{a}(m) - y(m))\mathbf{u}^*(m) \quad (1)$$

where  $\mathbf{w}(m)$  is the tap-weight vector at  $m$ -th iteration,  $\mu$  is the step size and  $\mathbf{u}(m)$  is the equalizer input vector. Also,  $y(m)$  is the equalizer output and  $\hat{a}(m)$  is a prediction of  $a(m)$ . The ‘\*’ symbol indicates the complex conjugate operator.

Under a high ISI, the convergence behavior of the decision directed equalizer is very poor. Adaptive equalization algorithms do not directly involve the input  $a(m)$ .

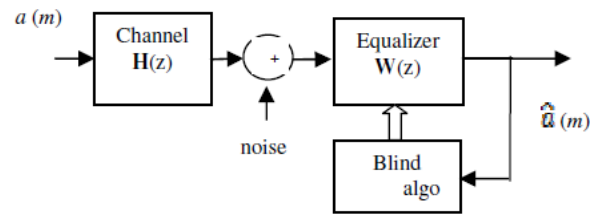
Convergence rate, residual ISI, misadjustment and BER are different criteria for performance measuring of an equalization [9]. In this paper, BER is the criterion.

In most digital communication systems, the ISI occurs due to band-limited channels or multipath propagation effect. The channel equalization is a technique for reducing the effect of the ISI. A cost-effective way for decreasing the ISI in a multicarrier system

comes at the expense of the bandwidth efficiency reduction. One method for the ISI decreasing is inserting the cyclic prefix (CP) with the length longer than the length of the channel impulse response. In recent years, the problem of alleviating the insufficient-CP length distortion has received a great deal of attention. It is apparent that OFDM modulation schemes that can perform well at short CP lengths are highly desired [10].

Chow and Cioffi [11] proposed a time domain equalizer (TEQ) for the digital subscriber line (DSL) systems. In [12], the distortion caused by insufficient-CP eliminated using a pre-coder at the transmitter. Moreover, the pre-coder essentially performs a matrix inversion and thus is prohibitively complex. The work in [12] did not fully take into account the inherent receiver noise and the transmitter power constraint. For some channels, this pre-coder results increasing the transmitter power budget. Authors [11] showed that its complexity was significantly reduced, but it was applicable only for systems with zero CP length [13].

The quadrature amplitude modulation (QAM) is an attractive technique for the multicarrier systems as well as in the next generation wireless access [14]. There are different rectangular QAMs in different multicarrier modulations. In the US, 64-QAM and 256-QAM are used for digital cable. In the UK, the 16-QAM and 64-QAM have standardized by Digital Video Broadcasting- Terrestrial (DVB-T) standard [15]. In this paper, based on DVB-T standard, a 16-points constellation is chosen.



**Fig.1. Blind equalizer block diagram.**

There are different algorithms for updating the equalizer taps weight values. The most common used adaptive algorithm for blind channel equalization is constant modulus algorithm (CMA). Where, the input to the channel is modulated signal that has constant amplitude at every instant in time [8]. CMA is a favorite method because of its LMS-like complexity and desirable robustness properties [16]. But it independently convergence of carrier recovery has a phase error [16]. CMA needs the separate phase recovery system. It is used for QAM signals where the amplitude of the modulated signal is not the same at every instant. In CMA, the BER range is moderate and convergence rate is low [17].

In [17], the modified CMA proposed no need the separate system for phase recovery. The authors in [8] proposed a variable step size modified CMA where it speeds up the convergence rate, and decreases the steady state MSE and corrects the phase error and the frequency offset simultaneously. The authors in [18] proposed an adaptive multi-modulus as equalizer. The authors in [19] proposed an algorithm that does not converge to any wrong solutions under time varying and noisy conditions.

In order to improve its performance, Abrar and Axford proposed the multi-modulus algorithm (MMA) with reliable initial convergence [7]. However, there are

many MMA for minimizing the dispersion of received QAM signals [20]. In [16] two new MMA i.e. the dual-mode MMA and the stop-and-go dual-mode MMA proposed for blind decision-feedback equalization of high order QAM signals. Its cost function was calculated according to an updated CMA [18]. In [21] an adaptive equalizer, named the multi-constant modulus algorithm (MCMA), for blind equalization of QAM signals was introduced. They considered a new cost function, where the fast convergence performance of the CMA, as well as the ability of joint blind equalization and carrier-phase recovery of the MMA were achieved.

To overcome the high mis-adjustment exhibited by CMA and MMA, many algorithms have been proposed so far [22]. One of these algorithms, called Sliced MMA (S-MMA), incorporates the sliced symbols (outputs of the decision device) in the multimodulus-based coefficient adaptation process. Although the steady-state mis-adjustment of S-MMA is lower than that of MMA for high-order QAM signals, it is still relatively large in comparison with the achieved value of the equalization of constant modulus signals with MMA [22]. In [23], a regional multi-modulus algorithm was introduced for blind equalization of QAM signals, independent of the QAM order. In [24], for channel estimation of a MIMO system with OFDM, soft outputs of soft data detection fed back to S-MMA. The authors in [10] proposed the S-MMA algorithm to a system with only a three-taps channel with AWGN noise and ISI impairments. However, they tested the algorithm with channels with AWGN and burst noises simultaneously in the presence of the ISI due to an insufficient

CP length in high fading channels. The applied channel, in this work, is spread and too long complicated fading channels with 42 of taps consist of uncorrelated Rayleigh fading taps with an exponential delay profile are considered. It consist of 6 uncorrelated Rayleigh fading taps with a uniform delay profile. Also, in this paper, for more qualifying the proposed modified system, it is compared with three commonly used algorithms.

Combining neural network with evolutionary algorithms leads to evolutionary artificial neural network. Evolutionary algorithms like Genetic algorithms (GA) train the neural networks structures or design related aspects like the function of their neurons [20].

David Goldberg, in 1989 offered the definition of GA as: "Genetic algorithms are search algorithms based on the mechanics of natural selection and natural genetics". This method combines Darwinian style survival of the fittest among binary string "artificial creatures" with a structured. So, the GA consists of binary bit strings. Each combination of ones and zeros is a possibility in the complex space and the relation between them is found in an evaluation function that will return a "fitness" or ranking for that particular bit string [20]. In general, Genetic algorithms have three main operations [20]:

1. Reproduction (or selection is a process in which individual strings are coping according to their fitness.
2. Crossover is a process with two steps. First, pairs of bit strings is to be mated randomly to become the parents of two new bit strings. The second part consists

of choosing a place in the bit string and exchanges all characters of the parents after that point.

3. Mutation is the probability that a certain bit can't be changed by the previous operations due to its absence from the generation, either by a random chance or because it has been discarded. It only implies the change of a 0 for a 1 and vice versa.

During the process of construction cost estimation, back propagation (BP) neural network has great applications. For GA optimization, the BP proposed to aim at handling locality minimum and to use GA to improve the ability of BP. Such cost estimations allow owners and planners to evaluate project feasibility and control costs effectively [25].

In [26], a hybrid GA for solving nonlinear channel blind equalization problems was presented. They estimated the output states of a nonlinear channel, based on the Bayesian likelihood fitness function, instead of the channel parameters. Authors in [27] derived a hybrid simplex GA for nonlinear channel blind equalization using RBF networks. They proposed an effective method for nonlinear channel blind equalization based on RBF networks which was constructed from channel output states instead of the channel parameters. Authors in [28] formulated the baud-spaced CMA blind equalization in the presence of Gaussian noise. Their approach [28] was robust to the adopted initialization strategy. In [29] a simplex GA for blind equalization, using RBF networks, was proposed. In order to reduce the computation cost, the algorithm searches the center's elements instead of centers because of the

inter-relation between the centers. In [30] a novel method for blind FIR channel estimation, based on higher order statistics, was proposed. In [31], a novel direct blind equalization based on GA with fitness function-formatted was proposed. Authors in [32] proposed a blind equalization based on GA for digital communication systems. They [32] used a linear programming based on modelling, so the approach allows a simplification of the cost function by improving the performance of the GA.

In this work, a new modified of the sliced multi-modulus algorithm (S-MMA) and GA, for updating the per-tone equalization taps in the OFDM multicarrier modulation with a complicated channel, is proposed.

The paper organized as follows. An analysis of the per-tone equalization in the OFDM modulation is explained in Section 2. In Section 3, channel shortening, and using neural networks, are considered. The S-MMA equalization performance analysis is developed in Section 4. Sections 5 gives the simulation results and finally in Section 6, we present the paper conclusions.

## 2. ANALYSIS OF PER-TONE EQUALIZATION IN THE OFDM MODULATION

The OFDM modulation is robust to multipath interference and frequency selective fading effects. Also, it has a relatively simple receiver structure compared to the single carrier transmission in frequency selective fading channels [33]. However, the OFDM utilizes the spectrum efficiently as it spaces the channels closely [2]. OFDM has good performances of anti-ISI, anti-decline, interference resisting of narrow-band, fitting

for asymmetrical transmission and robustness to multipath fading [11]. Because the mentioned advantages, OFDM has adopted in both wireless and wired systems as well.

In the block diagram of an OFDM transceiver, as shown in Fig. 2, a sampled analog signal bit-stream divided into a number of parallel blocks with a serial to parallel (S/P) converter. These blocks are the input of the constellation mapper. After IFFT transformation and CP insertion, the resulting sub-channels are orthogonal to each other as long as the CP is longer than the channel impulse response. Otherwise, the system will suffer from insufficient-CP length distortion [15]- [34].

In conventional OFDM, to assign parallel data to orthogonal sub-channels the IFFT algorithm is used [35]. This method has some advantages listed as following.

- 1-Parallel transmission occurs over different frequencies in order to support digital data transmission in multipath fading channels since this spreads the fading effect on many bits and degrades its effects.
- 2-Due to orthogonality between sub-channels, their spectrums are allowed to be overlapped with each other resulting in bandwidth efficiency increasing [36], [37].
- 3-Parallel transmission also increases symbol length leading to more robustness of system against inter symbol interference (ISI), multipath fading channels and wideband channels with impulse noise characteristics [36]- [38].

4-Because of using the FFT, traditional OFDM is a comparable low complexity MCM technique [39], [40].

The main drawback of IFFT is its high levels side-lobes which causing high level interferences. In order to mitigate this kind of interferences, the OFDM modulation adds a guard interval (in special cases CP) at the beginning of the data frames. But, the CP uses extra bandwidth and power without carrying any additional information, and this leads to a bandwidth/power inefficiency.

In this paper, the applied channel is modeled so that it adds two forms of interference i.e. ISI and inter carrier interference (ICI) impairments. It is illustrated that not only ISI but also ICI is caused by the collapse of orthogonality in the received signal. As a result, both the channel identification and equalization become difficult, and the communication performance cannot be guaranteed [41]. The ISI and ICI impairments are removed by turning the linear convolution into a cyclic convolution via insertion a CP at the beginning of each input data stream blocks [4].

In the receiver side, the received signal is again broken up into parallel blocks. The CPs are removed and then the FFT of each block is calculated. The equalizer attempts to reduce the ISI in the received signal and maximizes the SNR at the input of the decision circuit. A constellation de-mapper converts the complex values to a bit stream. For recovering the received bit-stream, a nearest-neighbor approximation method at each point is calculated. The blocks of bits concatenates back into a single bit-stream. With the aid of IFFT and appending a CP

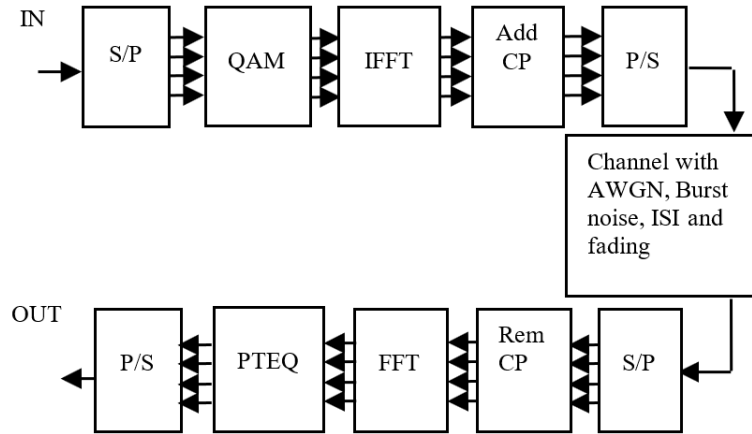


Fig. 2. The OFDM transceiver block diagram.

between the individual blocks at the transmitter and using the FFT at the receiver, a broadband frequency-selective channel converts into a set of parallel flat fading sub-channels [42]. Practical MCM systems use a relatively short CP and employ equalization to compensate for the channel effects.

A way for channel shortening is using of a time domain equalizer (TEQ) with  $T$ - real tap at the receiver. In this case, the TEQ outputs are fed into an FFT block that is followed by a complex 1-tap frequency domain equalizer (FEQ) for compensating the channel amplitude and phase effects. Authors in [34] proposed a per-tone equalizer in a DMT multicarrier modulation in 2001. In that work, the structure of a  $T$ -tap TEQ in combination with a complex 1-tap FEQ per-tone was modified into a structure with a complex  $T$ -tap FEQ per-tone. As a result, each tone is equalized separately and this leads to a higher bit rate [34]. In this process, the channel impulse response length may be shorter or longer than the CP length. In the former, the ISI is removed and only the FEQ is required, whereas for the latter both the

FEQ and TEQ are needed. For considering the effect of the CP length compared to channel impulse response length, three successive OFDM symbols  $X_{L:N}^{(c)}$  for time  $C = k-1, k, k+1$  are considered. Thus, the received signal [34] is written as,

$$\begin{aligned}
 \begin{bmatrix} \mathbf{Y} \\ y_{k,s+v-T+2} \\ \vdots \\ y_{(k+1),s} \end{bmatrix} &= \begin{bmatrix} \mathbf{h} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{h} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{L:N}^{(k-1)} \\ \mathbf{X}_{L:N}^{(k)} \\ \mathbf{X}_{L:N}^{(k+1)} \end{bmatrix} \\
 &\times \begin{bmatrix} \mathbf{P} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} \end{bmatrix} \begin{bmatrix} I_{NFFT} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{NFFT} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{NFFT} \end{bmatrix} \\
 &+ \begin{bmatrix} n_{S,k+v-T+2} \\ \vdots \\ n_{(k+1),s} \end{bmatrix} \\
 &= \mathbf{H}\hat{\mathbf{x}} + \mathbf{n} \\
 &\text{with } \mathbf{P} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_\epsilon \\ \mathbf{I}_N & \mathbf{0} \end{bmatrix}
 \end{aligned} \quad (2)$$

where  $I_{NFFT}$  is an  $N \times N$  IFFT matrix that modulates the input symbols and  $\mathbf{h} = [h_0, \dots, h_L]$  is the channel impulse response. Also,  $\mathbf{I}$  and  $\mathbf{0}$  matrices are 'identity' and 'zero' matrices respectively and their indexes show the size of the matrices [12]. In addition,  $\mathbf{O}_{(1)}$

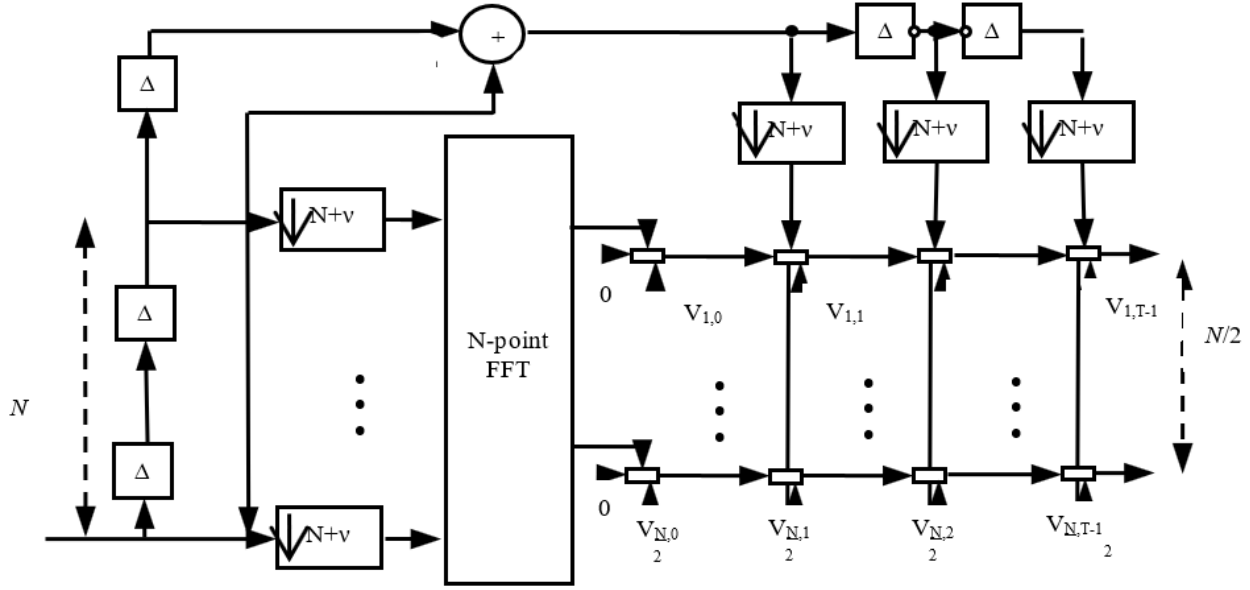


Fig. 3. Per-tone equalizer for the OFDM receiver.

and  $O_{(2)}$  are zero matrices of size  $(N+T-1) \times (N+v-T+1-L+v)$  and  $(N+T-1) \times (N+v-K)$  respectively. Finally,  $\bar{\mathbf{h}} = [h_L, \dots, h_0, \dots, h_k]$  is the channel impulse response in reverse order,  $y_i$  and  $n_i$  for  $i = 1, 2, \dots, N$ , are the  $i$ th component of the received and the noise vectors.

The equalizer attempts to reduce the ISI in the received signal and maximizes the SNR at the input of the decision circuit. Practical systems use a relatively short CP and employ equalization for compensating the channel effects. In the per-tone equalizer [34], each tone is equalized separately which leads to a higher bit rate and a reduced sensitivity to the synchronization delay [41].

In this work, the per-tone equalizer, as shown in Fig. 3, in the OFDM receiver part, is used and instead of the well-known algorithms such as LMS, we have used the MMA and S-MMA algorithms for updating the per-tone equalizer taps. So, every CP bit

and the corresponding bit are compared for determining the channel effects on the data bit stream. In this case,  $\Delta = N+v$  is the OFDM symbol length, where  $N$  is the symbol size and  $v$  is the CP length. Also  $V_{i,l}$  is the coefficient of the per-tone equalizer and  $\downarrow_{N+v}$  denotes the down-sampling with a period of  $N+v$  samples.

The modified per-tone equalizer is defined as  $\mathbf{v}_i = [v_{i,0}, \dots, v_{i,T-1}]^T$ , and therefore  $W_i$  coefficients [34] is computed as,

$$\begin{bmatrix} v_{i,0} \\ v_{i,1} \\ \vdots \\ v_{i,T-1} \end{bmatrix} = \begin{bmatrix} 1 & -\alpha^{i-1} & 0 & \dots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \cdot & \ddots & \ddots & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & -\alpha^{i-1} \\ 0 & \cdot & \cdot & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} w_{i,0} \\ w_{i,1} \\ \cdot \\ \cdot \\ w_{i,T-1} \end{bmatrix} \quad (3)$$

In following, Equation (3) is written in recursive form as,

$$v_{i,t+1} \cdot \alpha^{i-1} + w_{i,t} = v_{i,t} \quad (4)$$



For tone  $i= 1, \dots, N$ , with  $t = 0, \dots, T-2$

$$\bar{\mathbf{w}}_i^T = [w_{i,T-1} \dots w_{i,0}] \quad (5)$$

In this paper, for more efficiency, we assumed that the channel impulse response,  $\mathbf{h}=[h_0, \dots, h_L]$ , is longer than the CP length. So, the transmitted symbol at time  $k-1$  contribute to the received symbol at time  $k$  [34],

$$\begin{aligned} \begin{bmatrix} Y_1^{(k)} \\ \vdots \\ Y_N^{(k)} \end{bmatrix} &= F_N \begin{bmatrix} \mathbf{0} & \begin{bmatrix} h_L & \dots & h_{v+1} \\ 0 & \ddots & \vdots \\ \vdots & 0 & h_L \\ 0 & \dots & 0 \end{bmatrix} \end{bmatrix} \cdot \mathbf{P} \cdot \begin{bmatrix} X_1^{(k-1)} \\ \vdots \\ X_N^{(k-1)} \end{bmatrix} \\ + F_N &\begin{bmatrix} h_v & \dots & \dots & h_0 & 0 & \dots & \dots & \dots & 0 \\ h_{v+1} & h_v & \dots & \dots & h_0 & 0 & \dots & \dots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ h_L & \dots & \ddots & \ddots & \ddots & \ddots & h_0 & 0 & \vdots \\ 0 & h_L & \dots & \ddots & \ddots & \ddots & \ddots & h_0 & 0 \\ 0 & 0 & h_L & \dots & \dots & \dots & \dots & \dots & h_0 \end{bmatrix} \cdot \mathbf{P} \cdot \begin{bmatrix} X_1^{(k)} \\ \vdots \\ X_N^{(k)} \end{bmatrix} \quad (6) \end{aligned}$$

where  $X_i^{(k)}$  and  $Y_i^{(k)}$ , for  $i= 1, 2, \dots, N$  are the FFTs of the  $i$ th element of the transmitted and received symbols at time  $k$ , also  $\mathbf{I}$  and  $\mathbf{0}$  are the ‘identity’ and ‘zero’ matrices respectively. Therefore,  $\mathbf{I}_v$  and  $\mathbf{I}_N$  are identity matrices with sizes  $v \times v$  and  $N \times N$ . Also,  $F_N$  is the  $N \times N$  DFT matrix. Suppose that  $\mathbf{I}_N$  is an  $N \times N$  IDFT matrix, then the demodulated received symbol in Eq. (6) becomes

$$\mathbf{Y} = F_N \mathbf{T}^{(k-1)} \mathbf{P} \mathbf{I}_N \mathbf{X} + F_N \mathbf{T}^{(k)} \mathbf{P} \mathbf{I}_N \mathbf{X} \quad (7)$$

The two terms,  $\mathbf{T}^{(k)} \mathbf{P}$  and  $\mathbf{T}^{(k-1)} \mathbf{P}$  in the right hand side of Eq. (7) are not circulant. Therefore, for the case  $h_L > v$ , the ISI is not removed and the undesired contributions in  $Y_i^{(k)}$  from sub-symbols which differ from  $X_i^{(k)}$  are the interferences [34].

### 3. S-MMA EQUALIZATION PERFORMANCE ANALYSIS

The relation between the transmitted symbol  $x(m)$  and the received signal  $u(m)$  in a blind equalizer is,

$$u(m) = \sum_{i=0}^{L-1} h_i x(m-i) + n(m) \quad (8)$$

where  $h_i$  is the  $i$ -th tap of the channel impulse response with length of  $L$  and  $n$  denotes the AWGN noise.

Least mean square (LMS) algorithm is a conventional algorithm for updating the adaptive filter weights. LMS algorithm introduced by ‘Widrow’ and ‘Hoff’ for adaptive signal processing [1]. In general, LMS algorithm performs poorly when the condition number is relatively high due to the step size [44]. The LMS cost function is,

$$J = J_{\min} + 2 \operatorname{Re} \left\{ E \left( e_o^*(m) \boldsymbol{\varepsilon}_o^H(m) \mathbf{u}(m) \right) \right\} + E \left( \boldsymbol{\varepsilon}_o^H(m) \mathbf{u}(m) \mathbf{u}^H(m) \boldsymbol{\varepsilon}_o(m) \right) \quad (9)$$

where  $J_{\min}$  and  $e_o(m)$  are the minimum mean-square error and the error of Wiener filter, and  $\boldsymbol{\varepsilon}_o(m)$  is the zero-order weight-error vector [4]. Also,  $\mathbf{u}(m)$  denotes the LMS input vector and  $e_o(m)$  denotes the difference between the desired data and the filter output. Furthermore, ‘\*’ and  $\mathbf{H}$  refer to the complex conjugate and hermitian operator.

The LMS tap updating algorithm with the step size of  $\mu$  is defined as,

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu \mathbf{u}(m) e^*(m) \quad (10)$$

where  $\mathbf{u}(m)$  is the LMS input vector,  $\mathbf{w}(m)$  is the tap-weight vector at  $m$ -th iteration [41], and  $e(m)$  denotes the error of adaptive algorithm.

The RLS tap updating algorithm with inverse correlation matrix  $\mathbf{P}(m)$  is [42],

$$\begin{aligned} \mathbf{w}(m+1) &= \mathbf{w}(m) + \mathbf{P}(m)\mathbf{u}(m)\xi^*(m) \\ &\hat{=} \mathbf{W}(m) + \mathbf{k}(m+1)\xi^*(m+1) \end{aligned} \quad (11)$$

where  $\mathbf{P}(m)$  is the inverse of the correlation matrix and has a recursive form as bellow

$$\mathbf{P}(m) = \lambda^{-1}\mathbf{P}(m-1) - \lambda^{-1}\mathbf{k}(m)\mathbf{u}^H(m)\mathbf{P}(m-1) \quad (12)$$

where  $\xi(m)$  is defined as a priori estimation error and it is calculated as follows,

$$\xi(m) = d(m) + \hat{\mathbf{w}}^H(m-1)\mathbf{u}(m) \quad (13)$$

and  $\mathbf{k}(m)$  is the gain vector which can be updated via the Riccati equation,

$$\mathbf{k}(m) = \frac{\lambda^{-1}\mathbf{P}(m-1)\mathbf{u}(m)}{\lambda^{-1}\mathbf{u}^H(m)\mathbf{P}(m-1)\mathbf{u}(m)} \quad (14)$$

Typical simple initialization is also defined as,

$$\begin{aligned} \mathbf{P}(0) &= \delta \mathbf{I} \\ \hat{\mathbf{w}}(0) &= \mathbf{0} \end{aligned} \quad (15)$$

where  $\mathbf{I}$  and  $\mathbf{0}$  matrices are ‘identity’ and ‘zero’ matrices respectively.

The CMA is a special case of the Godard’s family of blind equalization algorithms [44]. Its cost function is only amplitude-dependent, and the knowledge about the signal constellation is discarded. For signal constellations for which all signal points have the same magnitude, the performance of the CMA is reasonable [9].

Numerous variants of the MMA have been presented in past in order to overcome the misadjustment caused by the CMA. Some of these MMA schemes, specifically for the QAM constellations, fix the phase offset error without needing any rotator at the end of the equalizer stage.

In general, MMA algorithm minimizes the dispersion of the real and the imaginary parts,  $y_R$  and  $y_I$ , of the equalizer output of  $y(m)$  separately [45]. Unlike CMA, MMA ignores the cross term  $y_R y_I$  between the in-phase and quadrature components. As a result, the MMA cost function is not a two-dimensional function, but a pseudo two-dimensional case, because it contains only  $y_R(m)$  and  $y_I(m)$  [46]. The MMA cost function is,

$$J = E\{(y_R^2(m) - R_R)^2 + (y_I^2(m) - R_I)^2\} \quad (16)$$

$$R_R = \frac{E[x_R^4]}{E[x_R^2]}, \quad R_I = \frac{E[x_I^4]}{E[x_I^2]} \quad (17)$$

where  $y_R(m)$  and  $y_I(m)$  are the real and the imaginary parts of  $y(m)$ , and  $x_R$  and  $x_I$  are the real and the imaginary parts of the channel input of  $x(m)$ . Also,  $R_R$  and  $R_I$  are defined as the dispersion constants for the real and the imaginary parts of the transmitted signal. The corresponding MMA tap updating algorithm is,

$$\mathbf{w}(m+1) = \mathbf{w}(m) + \mu \left\{ \begin{aligned} &(y_R(m)(R_R - y_R^2(m))) \\ &+ j y_I(m)(R_I - y_I^2(m)) \end{aligned} \right\} \mathbf{u}^*(m) \quad (18)$$

S-MMA cost function satisfies a number of desirable properties, including multiple-modulus, symmetry, and (almost) uniformity. The S-MMA cost function exhibits a much lower misadjustment compared to CMA and MMA [46].

In this paper, for the first time, we propose using the S-MMA algorithm for updating the taps of per-tone equalizer of the OFDM applications. The proposed S-MMA algorithm is devised by embedding the sliced symbols in the dispersion constants. The S-MMA updating mechanism is aware of the

dispersion of  $y(n)$  away from the closest symbol  $\hat{x}(m)$  in some statistical sense. The S-MMA cost function is [9],

$$J = E \left\{ \begin{aligned} & \left( y_R^2(m) - |\hat{x}_R(m)|^c R_R \right)^2 \\ & + \left( y_I^2(m) - |\hat{x}_I(m)|^c R_I \right)^2 \end{aligned} \right\} \quad (19)$$

where  $\hat{x}(m)$  is the predicted symbol and  $c$  is a positive constant. The S-MMA tap updating is,

$$\begin{aligned} \mathbf{w}(m+1) = \mathbf{w}(m) \\ + \mu \left\{ \begin{aligned} & (y_R(m) (|\hat{x}_R(m)|^c R_R - y_R^2(m))) \\ & + j \cdot y_I(m) (|\hat{x}_I(m)|^c R_I - y_I^2(m)) \end{aligned} \right\} \cdot \mathbf{u}^*(m) \end{aligned} \quad (20)$$

#### 4. CHANNEL SHORTENING USING NEURAL NETWORK

In this paper, for channel shortening and as a result of achieving better performance, another method based on neural network and GA is proposed. For channel shortening, a supervised version of neural network [48], [49] is applied to the simulated system. In the supervised learning, which is illustrated in Fig. 4, the learning rule is provided with a set of learning data (training set) which are

$\{(p^i, t^i), t=1, \dots, L\}$ , where  $p^i$  is the input of the

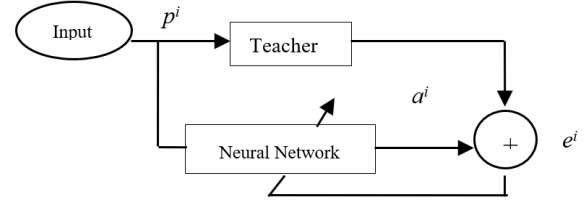


Fig. 4. Supervised learning

network and  $t^i$  is the corresponding correct output. The learning rule is used for adjusting the parameters of the network in order to move the network outputs as close as to the targets [42]. For this propose, the recursive equation is expressed [48],

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \eta(\mathbf{t} - \mathbf{a}(k))\mathbf{p}(k) \quad (21)$$

where  $\mathbf{w}$  is the weight matrix,  $\mathbf{p}$  is the network input and  $\mathbf{t}$  is the desired output of the network for input  $\mathbf{p}$ . Also,  $\eta$  is the learning rate which is a positive constant smaller than one and  $\mathbf{a}(k)$  is the network output. The notations  $\mathbf{h} = [h(1), \dots, h(m)]^T$ ,  $\mathbf{c} = [c(1), \dots, c(m+n-1)]^T$  and  $\mathbf{w} = [w(1), \dots, w(n)]^T$ , are used for the channel impulse response, the effective channel impulse response and the TEQ weights respectively. Therefore, the effective channel impulse response is defined as:

$$\begin{bmatrix} \mathbf{H} \\ \hline H(1,1) & \cdots & H(1,n) \\ \vdots & \ddots & \vdots \\ H(m+n-1,1) & \cdots & H(m+n-1,n) \end{bmatrix}_{(m+n-1) \times n} \cdot \begin{bmatrix} w(1) \\ \vdots \\ w(n) \end{bmatrix}_{n \times 1} = \begin{bmatrix} c(1) \\ \vdots \\ c(m+n-1) \end{bmatrix}_{(m+n-1) \times 1} \quad (22)$$

where  $\mathbf{H}$  is the channel toeplitz matrix. Generally, the TEQ design method attempts to minimize the rate of the energy of the effective channel samples beyond the CP

length. For this propose, a single-layer neural network is used. Each sample of the effective channel ( $\mathbf{c}(i)$ ) can be described as:

$$\begin{bmatrix} w(1) & \cdots & w(n) \end{bmatrix}_{1 \times n} \cdot \begin{bmatrix} H(i,1) \\ \vdots \\ H(i,n) \end{bmatrix}_{n \times 1} = c(i), \quad 1 < i < m+n-1 \quad (23)$$

Equation (23) is considered as a multi-input neuron, in which  $[H(i,1), \dots, H(i,n)]_{1 \times n}^T$ ,  $[w(1) \dots w(n)]_{1 \times n}$  and  $c(i)$  are the input vector, the weight matrix, and the output of neural network. Rayleigh fading model [49] is used for the channel description. For computing

$$\begin{aligned} & (a(1)r(i,1) - b(1)s(i,1)) + j(a(1)s(i,1) + b(1)r(i,1)) + \\ & \quad \vdots \\ & + (a(n)r(i,n) - b(n)s(i,n)) + j(a(n)s(i,n) + b(n)r(i,n)) \\ & = R(i) + jI(i) \end{aligned} \quad (25)$$

Finally, by separating these two parts, the  $R(i)$  i.e. the summation of all of the real parts and  $I(i)$  as the summation of all the imaginary parts, we obtain:

$$\begin{aligned} & a(1)r(i,1) + \dots + a(n)r(i,n) \\ & - [b(1)s(i,1) + \dots + b(n)s(i,n)] \\ & = R(i) \quad 1 < i < m+n-1 \end{aligned} \quad (26)$$

$$\begin{aligned} & a(1)s(i,1) + \dots + a(n)s(i,n) \\ & + b(1)r(i,1) + \dots + b(n)r(i,n) \\ & = I(i) \quad 1 < i < m+n-1 \end{aligned} \quad (27)$$

where  $[r(i,1), \dots, r(i,n), -s(i,1), \dots, -s(i,n)]_{1 \times 2n}^T$ ,  $[a(1), \dots, a(n), b(1), \dots, b(n)]_{1 \times 2n}$  and  $R(i)$  are the input vector, real part of the output and the weight matrix. Also,  $[s(i,1), \dots, s(i,n), r(i,1), \dots, r(i,n)]_{1 \times 2n}^T$ ,  $I(i)$  and  $[a(1), \dots, a(n), b(1), \dots, b(n)]_{1 \times 2n}$  are the input vector, the output and the weight matrix for the imaginary part. In order to limit the

the convergence of the complex inputs, the calculations are separated into real and imaginary parts,

$$\begin{aligned} & [a(1) + jb(1) \dots a(n) + jb(n)]_{1 \times n} \cdot \begin{bmatrix} r(i,1) + js(i,1) \\ \vdots \\ r(i,n) + js(i,n) \end{bmatrix}_{n \times 1} \\ & = R(i) + jI(i), \quad 1 < i < m+n-1 \end{aligned} \quad (24)$$

where  $w(1) = a(1) + jb(1)$ ,  $H(i,1) = r(i,1) + js(i,1)$ , and  $c(i) = R(i) + jI(i)$ . After simplifying Eq. (24) and separating the real and imaginary parts, we obtain:

number of the algorithm iterations, following constraint is imposed:

$$|c(CP+1)|^2 + \dots + |c(m+n-1)|^2 < \sigma \quad (28)$$

where  $c(\cdot)$  represents a sample of the effective channel (i.e.  $(c(i) = R(i) + jI(i))$ ). Also,  $c(CP+1)$  and  $c(m+n-1)$  are the first sample after the CP and the last sample in the effective channel impulse response respectively. In practice,  $\sigma$  is a small positive constant and in this research it was chosen as 0.7. In fact, Eq. (28) expresses that the neural network is modifying the weight matrix until the sum of the all-samples energies beyond the CP become smaller than  $\sigma$ . Thus, at the end of each run of the algorithm, the constraint in Eq. (28) is to be checked.

## 5. SIMULATION RESULTS AND DISCUSSIONS

In this paper, different adaptive algorithms are applied for the OFDM transceiver (see Fig. 2) with 64 subcarriers. In this system, a simple OFDM without any error correction codes is simulated. Four algorithms (i.e. LMS, RLS, MMA and S-MMA) for updating the taps of the per-tone equalizer are used for the OFDM multicarrier modulation.

In many time variant channels, because of amplitude and phase distortion, the channel impulse response leads to occurring ISI and as a result, an equalizer must be used. Basically, equalization is a process for compensating the distortion caused by the channel [48]. The channel model consists of  $h_{local,1}$  scatters near the transmitter,  $h_{mid}$  remote scatters, and  $h_{local,2}$  scatters near the receiver which is implemented as [49],

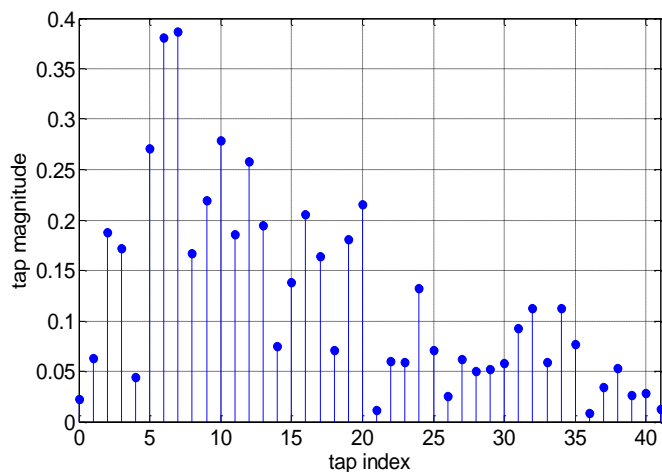
$$h = h_{local,1} * h_{mid} * h_{local,2} \quad (29)$$

where  $h_{mid}$  consists of 32 uncorrelated Rayleigh fading taps with an exponential delay profile, and  $h_{local,1}$  and  $h_{local,2}$  each consisting of 6 uncorrelated Rayleigh fading taps with a uniform delay profile. Fig. 5 shows an example of such a channel with 42 taps. For transmission efficiency, the CP

length was set to be smaller than the channel length, i.e. 36 taps, and therefore the system has the ISI impairment.

For verifying the advantages of our proposed method, two experiments are performed. In the first experiment, a 42 taps channel with an AWGN noise (with zero mean) employed and in the second experiment, the burst noise used. The burst noise produced with stochastically standard Gaussian distribution changing the duration, power and position of occurrence of the noise in the applied channels. For the two experiments, the BER is obtained from the ensemble average of 1000 independent Monte Carlo simulations runs. Where, for every run, the channels and its specifications such as the length of taps and noise are chosen according to the Gaussian probability distribution.

The results are shown in Fig. 6 and Fig. 7. As seen, the GA has a much lower BER than the S-MMA algorithm, in both AWGN and burst noisy channels, especially for high SNRs. Also, S-MMA has a much lower BER than LMS, RLS and MMA algorithms, in both AWGN and burst noisy channels, especially for high SNRs.



**Fig. 5. An example of a random channel.**

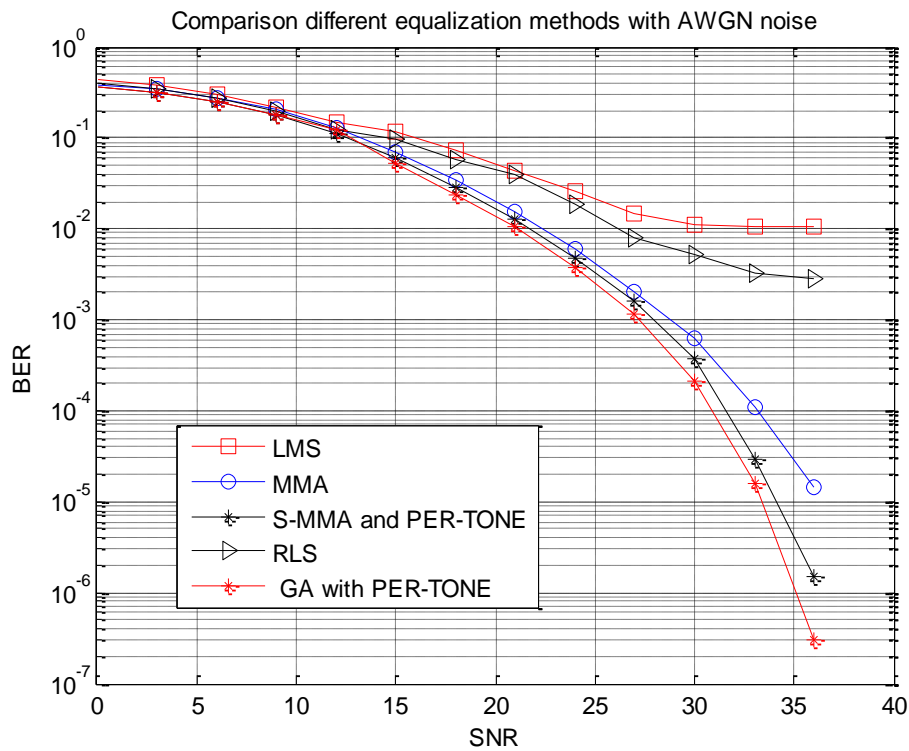


Fig. 6. The effects of the different algorithms on channel with AWGN noise.

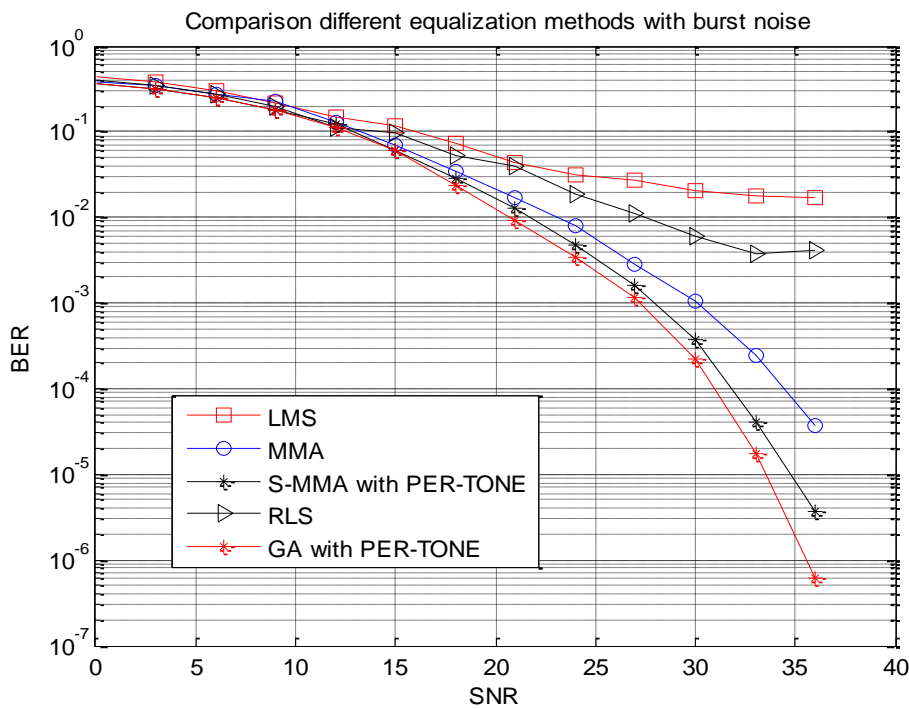


Fig. 7. The effects of different algorithms on channel with burst noise.

## 6. CONCLUSION

GA and S-MMA adaptive equalizations algorithms for a special application of OFDM modulation were proposed in this paper. Performance of the GA and S-MMA contrasted against the well-known MMA, LMS and RLS equalizations in per-tone equalizer for highly fading channels with the AWGN and burst noises. For transmission efficiency, the length of the CP was set to be smaller than the channel length, and hence the system had simultaneous ISI impairment. Both mathematical analysis and simulations results show the gains and clearly verify that the GA and S-MMA equalizations, with an insufficient length of CP, has a lower BER than the most well-known MMA, LMS and RLS equalizers across all channel SNR's. Thus, we recommend S-MMA to replace for the LMS equalization in the OFDM modulation based on per-tone equalizer.

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