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The Effect of Parameters of Winkler-Pasternak Elastic Foundations on Stress Analysis of Rectangular Plates Subjected to a Moving Load

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Abstract: In this study, the stress analysis of rectangular plates resting on Winkler Pasternak model of elastic foundations under a moving concentrated load with constant velocity and the impact of parameters related to the elastic foundations on normal stresses are investigated. The strain components are assumed to be linear and the Poisson's ratio is kept constant. Based on first order shear deformation theory (FSDT) and by employing Hamilton's principle, the theoretical equations of motion and boundary conditions are derived. Dimensionless discrete equations and boundary conditions have been achieved by using two dimensional generalized differential quadrature method (DQM) and Newmark procedure. The convergence and accuracy of the present formulation and method of the solution, where possible, are demonstrated by comparing with the work of other investigators. With these results, the effect of Winkler foundation modulus and stiffness of Pasternak shear layer foundations on normal stresses of plates have been investigated. The analysis provides for both simply supported and clamped boundary conditions at edges. It is discovered that the Pasternak shear layer has a predominant influence over Winkler elastic modulus on the plates.

Keywords: Rectangular plates, Elastic foundations, Moving load, First order shear deformation theory, Differential quadrature method

1. Introduction

The analysis of plates resting on elastic foundation subjected to moving loads is interesting and important, as some of the results may be applicable in understanding the dynamic behaviour of roadways and runways. In the analysis of roadways and runways of airports, the structure is usually modelled as a plate resting on an elastic foundation. In general, loads on these types of structures are moving loads or moving masses such as the wheel loads from moving vehicles and planes. Shahbaztabar A, Ranji AR. [1] extended the applications of differential quadrature element method to study the vibrational response of rectangular plates resting partially on elastic foundations. DinhDuc N, Quang VD, Nguyen PD, Chien TM. [2] presented an analytical approach to evaluate natural frequencies and nonlinear dynamic responses of a simply supported plate resting on the elastic foundation subjected to thermal and mechanical loading using FSDT by using Von Karman geometrical nonlinearity formulations. Hien TD, Lam NN. [3] presented an analytical solutions for the dynamic responses of FG rectangular plates supported with a viscoelastic foundation subjected to concentrated moving loads. They assumed that material properties vary through the thickness according to the power-law function and by using Hamilton’s principle derived constitutive relations based on higher-order shear deformation and solved them by using state-space methods. They also discussed various structural parameters on displacement and stresses of the plate. Idowu AS, Are EB, Gbadeyan JA. [4] studied the vibration of the dynamic behaviour of a damped orthotropic rectangular plate resting on a Winkler foundation subjected to dynamic loads. Their studies showed that damping plays a very significant role in the vibration of solid structures, and also graphically presented viscous damping foundation effects on stability of deflection profile. The dynamic response of moderately thick antisymmetric cross-ply rectangular laminated plates resting on Pasternak elastic foundation is investigated based on the higher order shear deformation theory by Vosoughi AR, Malekzadeh P, Razi H. [5]. Huang MH, Thambiratnam DP. [6] developed a procedure using the finite strip method together with a spring system to investigate the dynamic response of rectangular plates resting on elastic Winkler foundation. They discussed plates' response to moving accelerated concentrated loads by considering the effects of initial moving velocity, acceleration and initial load position.

Although various analytical methods have been used to study the dynamic response of rectangular plates on elastic foundations under a moving load, few studies have been focused on the stress analysis of plates in these circumstances. In this study, the forced vibration response and stress analysis of rectangular thin plates resting on Winkler-Pasternak elastic foundation and subjected to moving load is investigated. The linear equations of motion are derived based on the FSDT by utilizing Hamilton's principle. The governing equations are solved by using Newmark and differential quadrature methods, and the effects of stiffness of elastic foundations in various boundary conditions on stress components of rectangular plates are studied.

2. Fundamental Equations of Rectangular Plates on Elastic Foundation

2.1Rectangular Plates on Elastic Foundation

As shown in Fig. 1, a rectangular plate resting on Winkler-Pasternak foundations is considered. The geometry of the plate and coordinate system are illustrated in Fig. 1.

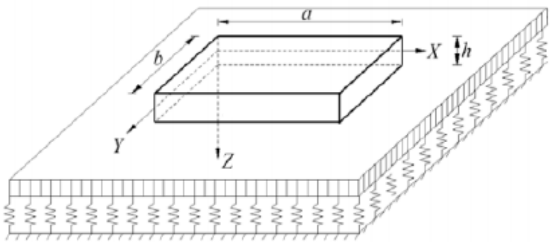

Fig. 1. Geometry and coordinate system of the Rectangular Plates resting on Winkler-Pasternak

The Poisson ratio is assumed to be constant $(\nu(z) = \nu)$and the reaction-deflection relation of Pasternak foundations is given by

$$q_{\text{elastic}} = \mathrm{k_w} w - \mathrm{k_p} \nabla^2 w \tag{1}$$

In which$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, w is the deflection of the plate, $k_w$is Winkler foundation modulus and $k_p$is shear layer stiffness of Pasternak foundation.

2.2Governing Equations of Motion

The fist-order shear deformation theory is used to establish governing equations based on displacements components of the plate as follows [7]

$$u(x,y,z,t) = u_0(x,y,t) + z\theta\,(x,y,t)$$
$$v(x,y,z,t) = v_0(x,y,t) + z\psi(x,y,t)$$
$$w(x,y,z,t) = w_0(x,y,t) \tag{2}$$

Where u_0, v_0 and w_0 are the displacement components of the middle surface in the direction of x,$y$and z, respectively. Moreover θ and ψ present the rotations of a transverse normal about x and y directions.

The linear strain components are computed from displacement fields (2) and can be used in constructing the strain and kinetic energies expressions. By using the Hamilton's principle, the equations of motion are driven as follow

$$A_{11}\left[\left(\frac{\partial^2 u_0}{\partial x^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 u_0}{\partial y^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 v_0}{\partial x \partial y}\right)\right] + B_{11}\left[\left(\frac{\partial^2 \theta}{\partial x^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 \theta}{\partial y^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)\right] - I_0\frac{\partial^2 u_0}{\partial t^2} - I_1\frac{\partial^2 \theta}{\partial t^2} = 0$$

$$A_{11}\left[\left(\frac{\partial^2 v_0}{\partial y^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 v_0}{\partial x}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 u_0}{\partial x \partial y}\right)\right] + B_{11}\left[\left(\frac{\partial^2 \psi}{\partial y^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 \psi}{\partial x^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 \theta}{\partial x \partial y}\right)\right] - I_0\frac{\partial^2 v_0}{\partial t^2} - I_1\frac{\partial^2 \psi}{\partial t^2} = 0$$

$$A_{11}\left[\kappa\frac{1-\nu}{2}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2} + \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial y}\right)\right] - k_w w_0 + k_p\nabla^2 w_0 - I_0\frac{\partial^2 w_0}{\partial t^2} = -\mathrm{P}$$

$$A_{11}\left[-\kappa\frac{1-\nu}{2}\left(\frac{\partial w_0}{\partial x} + \theta\right)\right] + B_{11}\left[\left(\frac{\partial^2 u_0}{\partial x^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 u_0}{\partial y^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 v_0}{\partial x \partial y}\right)\right] + D_{11}\left[\left(\frac{\partial^2 \theta}{\partial x^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 \theta}{\partial y^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 \psi}{\partial x \partial y}\right)\right] + -I_1\frac{\partial^2 u_0}{\partial t^2} - I_2\frac{\partial^2 \theta}{\partial t^2} = 0$$

$$A_{11}\left[-\kappa\frac{1-\nu}{2}\left(\frac{\partial w_0}{\partial y} + \psi\right)\right] + B_{11}\left[\left(\frac{\partial^2 v_0}{\partial y^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 v_0}{\partial x^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 u_0}{\partial x \partial y}\right)\right] + D_{11}\left[\left(\frac{\partial^2 \psi}{\partial y^2}\right) + \frac{1-\nu}{2}\left(\frac{\partial^2 \psi}{\partial x^2}\right) + \frac{1+\nu}{2}\left(\frac{\partial^2 \theta}{\partial x \partial y}\right)\right] - I_1\frac{\partial^2 v_0}{\partial t^2} - I_2\frac{\partial^2 \psi}{\partial t^2} = 0$$

(3)

Where P is the moving load and $\left(N_{ij}, M_{ij}, Q_{ij}\right)$ are the stress resultants and the inertias (I_i) are defined by [7]:

$$(N_{xx}, N_{yy}, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_{xx}, \sigma_{yy}, \sigma_{xy})\, dz$$

$$(M_{xx}, M_{yy}, M_{xy}) = \int_{-h/2}^{h/2} z(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})\, dz$$

$$(Q_{xz}, Q_{yz}) = \kappa \int_{-h/2}^{h/2} (\sigma_{xz}, \sigma_{yz})\, dz$$

$$I_i = \int_{-h/2}^{h/2} \rho z^i\, dz, \qquad i = 0,1,2$$

$$A_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{E}{1-\nu^2} dz;\ B_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{zE}{1-\nu^2} dz;\ D_{11} = \int_{\frac{-h}{2}}^{\frac{h}{2}} \frac{z^2 E}{1-\nu^2} dz$$

(4)

In which κ is called the shear correction factor and its most commonly used value is $\pi^2/12$.

The plate is subjected to a moving concentrated load, $P(x,y,t) = P_0 \delta(x - x_{mov}(t)) \delta(y - y_{mov}(t))$, where P_0 is the magnitude of the concentrated moving load and $(x_{mov}(t), y_{mov}(t))$ are the coordinates of the load. Symbol δ represents the Dirac delta function, integral of which is equal to unity in any neighborhood of $(x_{mov}(t), y_{mov}(t))$ and zero elsewhere [8].

Boundary Conditions

Two cases of boundary condition will be considered

- Case 1. Four edges of plate are simply supported (SSSS):
-

$$u_0 = v_0 = w_0 = \psi = M_{xx} = 0; \text{ at } x = 0, x = a$$

$$u_0 = v_0 = w_0 = \theta = M_{yy} = 0; \text{ at } y = 0, y = b \tag{5}$$

- Case 2. Four edges of plate are clamped (CCCC):

$$u_0 = v_0 = w_0 = \theta = \psi = 0; \text{ at } x = 0, x = a$$

$$u_0 = v_0 = w_0 = \theta = \psi = 0; \text{ at } y = 0, y = b \tag{6}$$

3. Solution Methodology of the Equations

In this section, differential quadrature method (DQM) along with Newmark method which can solve the differential equations is proposed. In the following, the Newmark method and DQM are explained.

3.1 Differential Quadrature Method

Differential quadrature is a numerical solution method that rapidly converges to fairly accurate numerical solution of differential equations in engineering problems.

Let $\zeta(x,y)$be a solution of a differential equation and $x_1 = 0, x_2, x_3, \dots, x_N = a$ be a set of sample points in the direction of x −axis. According to DQM, the r[th] –order derivative of the function $\zeta(x,y)$ at point $x = x_i$ along any line $y = y_j$ parallel to the x −axis can be approximated by [9]

$$\zeta^{(r)}\Big|_{x = x_i} = \sum_{n=1}^{N} C_{in}^{(r)} \zeta_{nj} \tag{7}$$

where N is the number of sample points and $C_{in}^{(r)}$ is the weighting coefficients of r[th] –order derivative. , Quan and Chang obtained the following algebraic formulations to compute the fist-order weighting coefficients [10]

$$C_{in}^{(1)} = \frac{M^{(1)}(x_i)}{(x_i - x_n)M^{(1)}(x_n)} \; ; \; i,n = 1,2,\dots,N \; and \;\; i \neq n$$

$$C_{ii}^{(1)} = -\sum_{\substack{m=1 \\ i\neq m}}^{N} C_{im}^{(1)} \; ; \qquad i = 1,2,\dots,N \tag{8}$$

where $M^{(1)}(x_i)$ is defined as

$$M^{(1)}(x_i) = \prod_{\substack{k=1 \\ k\neq i}}^{N} (x_i - x_k) \tag{9}$$

3.2 The Newmark Method

In this technique, the velocity and displacement matrices at time t_{k+1} (k is the number of time steps) are approximated in terms of their values at time t_k [11]

$$\dot{x}_{k+1} = \dot{x}_k + (1-\alpha)h\ddot{x}_k + h\alpha\ddot{x}_{k+1}$$
$$x_{k+1} = x_k + \dot{x}_k h + (0.5-\beta)h^2\ddot{x}_k + h^2\beta\ddot{x}_{k+1} \tag{10}$$

where the coefficients α and β are parameters which determine the accuracy and stability of the numerical technique and h shows the time interval. By using (10), the first and the second derivatives of the displacement matrix at time t_{k+1} , in terms of $x_{k+1}, x_k, \dot{x}_k$ and $\ddot{x}_k$can be obtained.

3.3 The Numerical Implementation

To solve the governing equation, these equations must be in dimensionless form, then the derivatives with respect to x and y are discrete by using DQM and the derivatives with respect to t are discrete by employing Newmark method.

4. Solving Process

After numerical implementation, governing equations (3), boundary conditions (5) and (6), together can be rewritten in the matrix form as

$$[\mathcal{A}]\{\mathcal{X}_{k+1}\} = [\mathcal{B}] \tag{11}$$

where

$$[\mathcal{A}] = \left[\frac{1}{h^2\beta}M + K\right]$$

$$[\mathcal{B}] = \left[P + M\left(\frac{1}{h^2\beta}\mathcal{X}_k + \frac{1}{h\beta}\dot{\mathcal{X}}_k + \left(\frac{1}{2\beta} - 1\right)\ddot{\mathcal{X}}_k\right)\right] \tag{12}$$

In which $[M]$ is mass matrix, $[K]$ is stiffness matrix and $\{P\}$ presents the load vector based on the general form of equations of motion($[M]\{\ddot{x}\} + [K]\{x\} = \{P\}$). Also vector $\{\mathcal{X}\}$ includes all displacement components $(U_0, V_0, W_0, \theta, \psi)$ and their derivatives with respect to x and y of entire grid points.

By using (11), the values of $\{\mathcal{X}_{k+1}\}$ can be obtained as

$$\{\mathcal{X}_{k+1}\} = [\mathcal{A}]^{-1}[\mathcal{B}] \tag{13}$$

It should be noted the initial values for Newmark method are assumed as $\left(\{\mathcal{X}_0\} = \{\dot{\mathcal{X}}_0\} = \{\ddot{\mathcal{X}}_0\} = 0\right)$

By obtaining $\{\mathcal{X}\}$ which is including all displacement components $(U_0, V_0, W_0, \theta, \psi)$ and their derivatives with respect to x and y of entire grid points, linear strain components can be achieved and finally by using Hook's law, for the plane stress case, stresses can be driven as follow

$$\sigma_{xx} = \frac{E(z)}{1-\nu^2}\left(\varepsilon_{xx} + \nu\varepsilon_{yy}\right)\ ,\quad \sigma_{yy} = \frac{E(z)}{1-\nu^2}\left(\varepsilon_{yy} + \nu\varepsilon_{xx}\right)\ \sigma_{xy} = \frac{E(z)}{2(1+\nu)}\gamma_{xy},\ \sigma_{xz} = \frac{E(z)}{2(1+\nu)}\gamma_{xz},\ \sigma_{yz} = \frac{E(z)}{2(1+\nu)}\gamma_{yz} \tag{14}$$

In order to do this process, a computer code is written in MATLAB environment and results are presented in following sections.

5. Validation

In this section, the validity of the present study for vibrations of rectangular plates under moving loads is examined. To do this, the results obtained are compared with those obtained by Eftekhari and Jafari [12]. The parameters used here are as follows

$$\frac{\rho h}{D_{11}} = 1,\quad \frac{P_0}{D_{11}} = 1,\quad a = b = 1$$

$$k_w = k_p = 0,\ \ x_{mov}(t) = vt,\ \ y_{mov}(t) = 0.5 \tag{15}$$

In which v is the velocity of the moving load (which is constant). It should be noted the t −axis is normalized by dividing t on T_d ($T_d = {}^{a}/_{v}$). From Fig. 2 the good agreement is obtained between these two studies which means vector $\{\mathcal{X}\}$ includes all displacement components $(U_0, V_0, W_0, \theta, \psi)$ and their derivatives with respect to x and y is achieved correctly which leads to correct linear strain components and stresses.

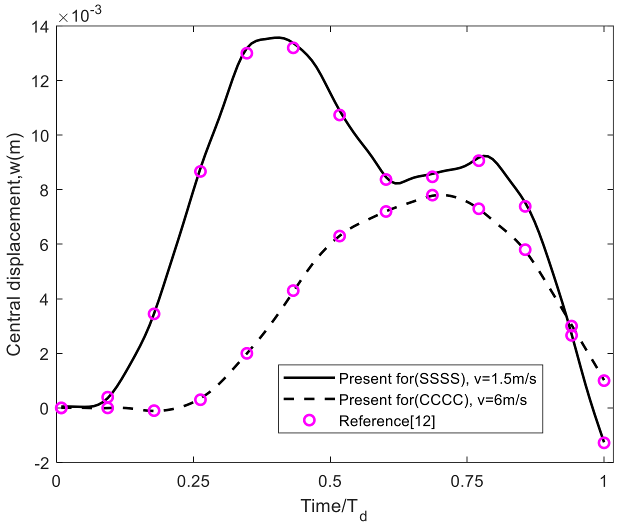

Fig.2. Comparison of numerical results for central deflection of fully simply supported and fully clamped square plates subjected to a moving load

6. Results and Discussion

To illustrate the present approach for stress analysis of rectangular plates under a moving load resting on elastic foundations, consider a square plate $(a = b = 1, h = 0.05)$ consisting of aluminum (Al) with following properties $E = 70\ GPa$, $\rho = 2702\ {}^{kg}/_{m^3}$, $\nu = 0.3$, $v = 1\ {}^{m}/_{s}$ and $x_{mov}(t) = vt$, $y_{mov}(t) = 0.5b$ also $P_0 = 0.5\ MPa$.

For observing the impact of Winkler-Pasternak elastic foundations, Three phases are considered which are phase-1($k_w = 0\ {}^{GPa}/_{m}$ and $k_p = 0\ GPa.m$), Phase-2($k_w = 0.2\ {}^{GPa}/_{m}$ and $k_p = 0\ GPa.m$) and phase-3($k_w = 0.2\ {}^{GPa}/_{m}$ and $k_p = 0.02\ GPa.m$) and as it is shown in Figs. (3) – (6), normal stresses will decrease if elastic foundations enhance from phase-1 to phase-2 and phase-3. Moreover, Pasternak's elastic foundation (k_p) has greater influences on normal stresses of rectangular plates than Winkler's foundation (k_w).

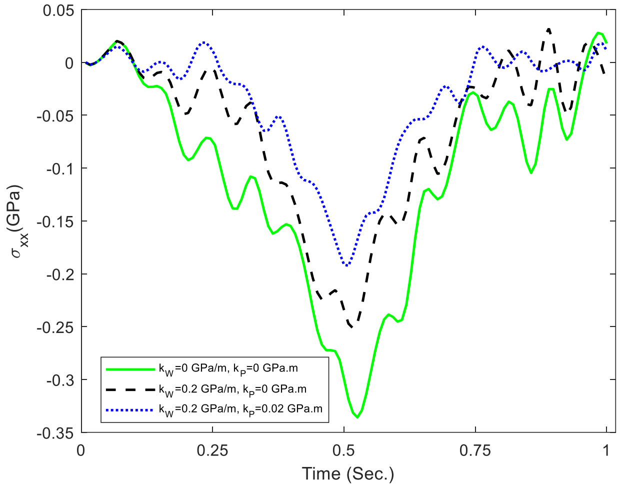

Fig .3. Effect of the Winkler-Pasternak elastic foundations on normal stress (σ_{xx}) at top surface; fully simply supported

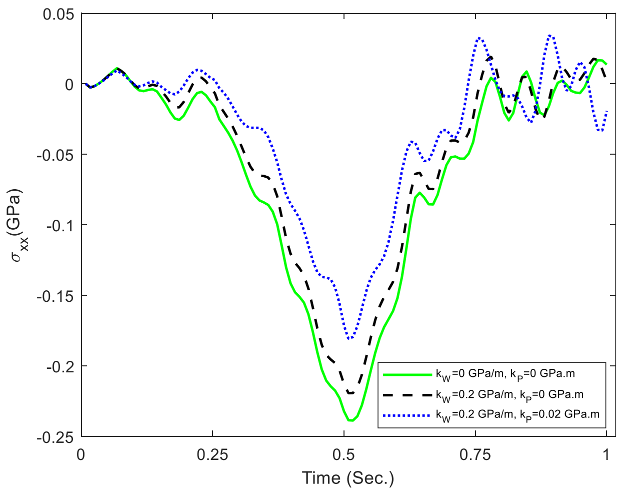

Fig .4. Effect of the Winkler-Pasternak elastic foundations on normal stress (σ_{xx}) at top surface; fully clamped

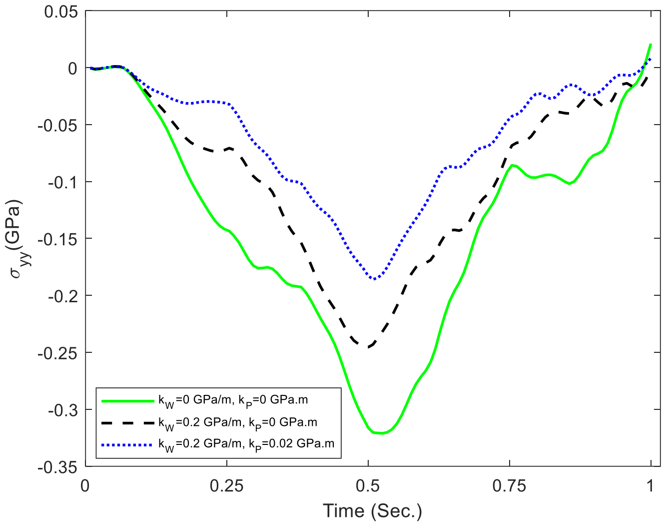

Fig .5. Effect of the Winkler-Pasternak elastic foundations on normal stress (σ_{yy}) at top surface; fully simply supported

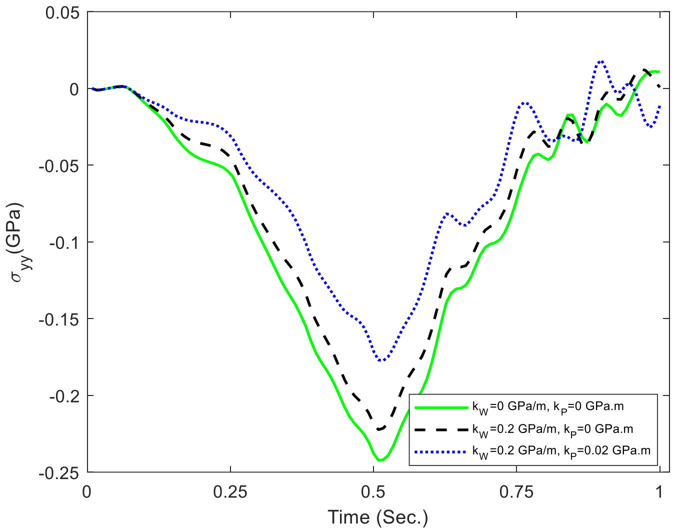

Fig .6. Effect of the Winkler-Pasternak elastic foundations on normal stress (σ_{yy}) at top surface; fully clamped

7. Conclusions

This paper employed first order shear deformation theory and used DQ and Newmark method to study the stress analysis of rectangular plates on elastic foundations under concentrated moving loads for different boundary conditions. The convergence and accuracy of the present formulation and method of the solution are demonstrated, then the influences of the stiffness of Winkler-Pasternak foundation on normal stresses are investigated. Main finding and inferences are summarized as follow

- At the same circumstances, the value of normal stresses for (SSSS) is greater than the one for (CCCC).
- Pasternak shear layer has a predominant influence over Winkler elastic modulus on the plates.

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