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Vibrational analysis of a rotating nanotube conveying fluid based on Eringen's nonlocal elasticity theory

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Abstract: In recent years, a wide range of potential applications of nano-tubes have been reported. Vibrational behavior of rotating nano-tubes can be a challenging aspect of such structures. Due to their low dimension, classical theories fail to predict accurate behavior of nano-structures. In the present paper, Eringen's non-local theory has been employed to study the vibration characteristics of a rotating nanotube conveying fluid. The effects of the scale parameter, mass, fluid velocity and angular velocity on the vibration characteristics of a rotating nanotube conveying fluid were investigated. The stability of the structure in different conditions was also discussed. The results show that dimensionless mass parameter do not affect the critical fluid velocity in a non-rotating nanotube conveying fluid. At the same time, critical velocity decreases by increasing nonlocal parameter. Also in a rotating nanotube, this point is a function of mass parameter and decreases and then increases to somewhere near the moving start point as β increases. A reciprocal relation was noticed between critical fluid velocity and critical angular velocity. Moreover, assuming constant mass, the area of the stability zone becomes wider as the nonlocal parameter increases.

Keywords: nanotube, vibrations, Eringen nonlocal theory, stability

1. Introduction

Discovered by Ijima [1] in 1991, carbon nanotubes (CNTs) are one of the carbon allotropes along with other spatial structures such as graphene sheets (GSs). Ever since this discovery, there has been intensive research on the potential applications of these unique nanostructures. Exhibiting unique electric, chemical, mechanical, optical, thermal and mechanical properties, these structures hold substantial promise as building blocks for nano-electronics, nano-sensing, nano-actuating devices and nano-composites [2-5].

Owing to their hollow cylindrical shapes and extremely high elasticity and flexibility, nanotubes have been appropriate candidates for nano-electro-mechanical systems such as nano-sensors [6], nano-robots [7], nano-motors [8], etc. In addition, nanotubes have been widely employed in nano-pipes [9], nano-containers [10], and nano-fluid and nano-gas storage devices [11]. Among different applications, the use of nanotubes to convey fluid is a special issue which requires further attention.

Toward analyzing nanostructures, their mechanical properties need to be determined. To this end, extensive research has been carried out to extract these properties accurately and reliably. Since controlled experimental investigation at nano scale is still difficult to achieve, various analytical and numerical methods of analysis have been presented recently. These methods can be generally classified as atomistic-based methods and continuum-based methods.

Even with today's computational advances, atomistic-based methods are expensive in computational requirements and most of the researchers are limited in the use of these methods. As a result, researchers have been constantly searching for convenient computational methods. Recently, scholars have increasingly turned their attentions to continuum structural mechanics models. Many continuum-based models have been presented during the last decade. These include the beam modeling by Govindjee and Sackman [12] and Yoon et al.[13], the cylindrical shell model by Ru et al.[14, 15] and the space truss model by Li and Chou[16, 17].

Experimental and atomistic simulation results have proven that there exists a significant effect in mechanical properties due to the small size of the structures. However, the classical theory of elasticity ignores these effects and as a result, fails to capture the small scale effects when dealing in nano scale [18, 19].

The continuum theories reflecting the size-dependency are therefore necessary for reliable prediction of the behavior of nanostructures. Various size-dependent continuum theories capturing this small-scale effect are reported. Some of these include couple stress elasticity theory [20], strain gradient theory [21] and modified couple stress theory [22].

In the present investigation, employing Eringen's nonlocal elasticity theory, a nonlocal beam model is employed to study the vibration characteristics of a rotating nanotube conveying fluid. This theory considers long-range interatomic interaction and efficiently avoids the classic continuum theory flaws in nano scale. Among previous works in this trend, there are only a few works using nonlocal elasticity theory. For instance, Ansari et al. [23] studied size-dependent nonlinear vibration of embedded fluid conveying single-walled boron nitride nanotubes. Also Pradhan and Murmu [24] studied flap-wise bending vibration of a rotating nano-cantilever using nonlocal theory.

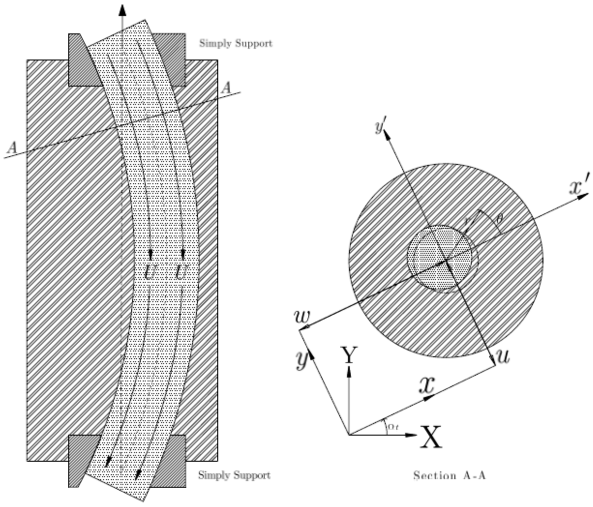
A real-world application case for such an analysis is drug injection nanotubes. Rotational speed helps the injection needle when dipped in the tissue. Drug injection rate has a close relation with the speed of conveying fluid. In such a problem, some significant parameters like fluid velocity, angular velocity and induced mass are critical.

In the current investigation, the effects of the small-scale or nonlocal parameter, dimensionless mass parameter, fluid velocity and angular velocity on the vibration characteristics of a rotating nanotube conveying fluid are examined and discussed.

2. Governing equations

Nonlocal elasticity theory is based on the idea that the stress field at any reference point of a structure is dependent not only on strain at that point but also on strains at all other points in the domain. Taking into account scale effects, internal size is considered as a material parameter in nonlocal elasticity theory. Murmu and Pradhan [25] presented governing equations of the vibration of nano-plates by using nonlocal continuum model.

Figure. 1 depicts front and cross-sectional views of the structure under study.



Figurer 1) Front view (left) and cross-section (right) of the rotating nano-tube

Considering Euler-Bernoulli beam theory, nonlocal constitutive relation for a beam can be written as:

$$\mathcal{L}\sigma = E\varepsilon \tag{1}$$

Where E, σ, ε, and $\mathcal{L}$ are elasticity modulus, nonlocal stress, nonlocal strain and nonlocal differential operator respectively. $\mathcal{L}$ is defined as:

$$\mathcal{L} = 1 - (e_0 a)^2 \frac{d^2}{dx^2} \tag{2}$$

Where e_0a (denoted by μ hereafter) is the nonlocal parameter. This parameter depends on the material type. The partial differential equations of motion in the explicit form are:

$$\begin{aligned} m\ddot{\omega} - 2m\Omega\dot{u} - m\Omega^2\omega + EI\omega^{(4)} = F_\omega \\ m\ddot{\omega} - 2m\Omega\dot{u} - m\Omega^2\omega + EI\omega^{(4)} = F_\omega \end{aligned} \tag{3}$$

Where u and w are transverse displacements in two directions normal to the nanotube axis and Ω is the angular velocity. $\hat{F}_{\widehat{w}}$ and $\hat{F}_{\hat{u}}$ are fluid induced forces along w and u directions, respectively. Considering $\Delta = w + iu$ and using Laplace transformation, Eqs. (3) can be written as:

$$EI\Delta^{(4)} + m(s^2 + 2\Omega is - \Omega^2)\Delta = \hat{F}_{\widehat{w}} + i\hat{F}_{\hat{u}} \tag{4}$$

Forces inserted by the fluid can be written as [26]:

$$\begin{aligned} \hat{F}_{\widehat{w}} &= -M(s^2\widehat{w} + 2Us\widehat{\omega}' + U^2\widehat{\omega}'') \\ \hat{F}_{\hat{u}} &= -M(s^2\hat{u} + 2Us\hat{u}' + U^2\hat{u}'') \end{aligned} \tag{5}$$

Where U, M and s are fluid velocity, fluid mass and Laplace variable. Substituting Eqs. (5) in Eq. (4) and applying the nonlocal differential operator, the nonlocal equation of motion of a fluid conveying nanotube can be written as:

$$EI\hat{\Delta}^{(4)} + m(s^2 + 2\Omega is - \Omega^2)\hat{\Delta} - \mu^2\big(m(s^2 + 2\Omega is - \Omega^2)\hat{\Delta}''\big) + M\big(s^2\hat{\Delta} + 2Us\hat{\Delta}' + U^2\hat{\Delta}''\big) - \mu^2\left(M\big(s^2\hat{\Delta}'' + 2Us\hat{\Delta}''' - U^2\hat{\Delta}^{(4)}\big)\right) = 0, \mu = \frac{e_0 a}{l} \quad (6)$$

Equation (6) can be rewritten in dimensionless form as:

$$(1 - \mu^2u^2)\delta^{(4)} - 2\mu^2u\beta^{\frac{1}{2}}\tilde{s}\delta''' + (-\mu^2\tilde{s}^2 - 2\mu^2(1-\beta)i\Theta\tilde{s} + \mu^2(1-\beta)\Theta^2 + u^2)\delta'' + \left(2\beta^{\frac{1}{2}}u\tilde{s}\right)\delta' + (\tilde{s}^2 + 2(1-\beta)\Theta i\tilde{s} + (\beta - 1)\Theta^2)\delta = 0 \quad (7)$$

Dimensionless parameters in Eq. (7) are defined as:

$$\tilde{s} = \frac{1}{L^2}\left(\frac{M+m}{EI}\right)^{\frac{1}{2}} s, \beta = \frac{M}{M+m}, \quad \Theta = (\frac{M+m}{EI})^{\frac{1}{2}}L^2\Omega, u = \left(\frac{M}{EI}\right)^{\frac{1}{2}} LU, \delta = \frac{\tilde{\Delta}}{L}, \xi = \frac{z}{L}, \mu = \frac{e_0 a}{L} \quad (8)$$

3. Results and discussion

In order to attain the solution of the current problem, a pure harmonic vibration is assumed as:

$$w(x,t) = \overline{w}(x)e^{i\omega_n t} \quad (9)$$

Using the Galerkin's technique, solution of the governing equations can be expressed by the following modal expansion series

$$\overline{w}(x) = \sum_{i=1}^{N} c_i\phi_i(x) \quad (10)$$

Where c_i denote the modal coordinates and $\phi_i(x)$ shows the trial functions satisfying boundary conditions. The dimensionless trial functions are assumed to be the mode shapes of simply supported beam as follows:

$$\phi_i(\xi) = \sin(i\pi\xi) \quad (11)$$

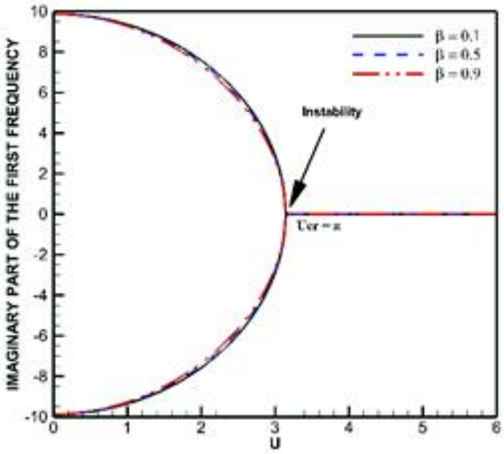
Substituting Eq. (10) in Eq. (7) and integrating over the domain, the solution leads to:

$$\left(\bar{k} + \bar{c}s + \bar{m}s^2\right)[\bar{c}_i] = 0 \quad (12)$$

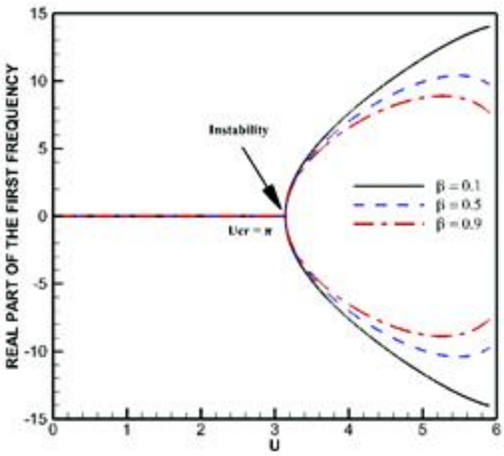
The frequencies can be obtained by calculation of the determinant of the following matrix:

$$\begin{bmatrix} 0 & \bar{I} \\ -\bar{m}^{-1}(\bar{k}) & -\bar{m}^{-1}(\bar{c}) \end{bmatrix} \tag{13}$$

Present problem has been solved by Paidoussis and Michael [40] using local elasticity theory. In Paidoussis's work, the vibration of a non-rotating slender structure conveying fluid is studied. Neglecting terms containing μ and θ in Eq. (7), the effect of increasing fluid velocity on real and imaginary parts of the frequency is exhibited in Figure. 2 a, b.



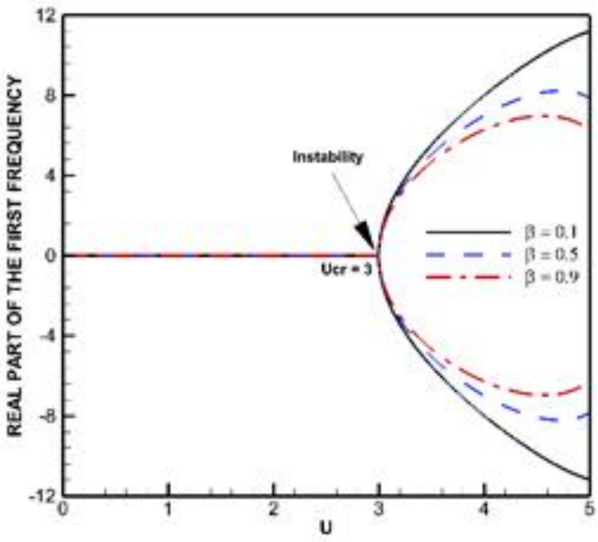
(a)



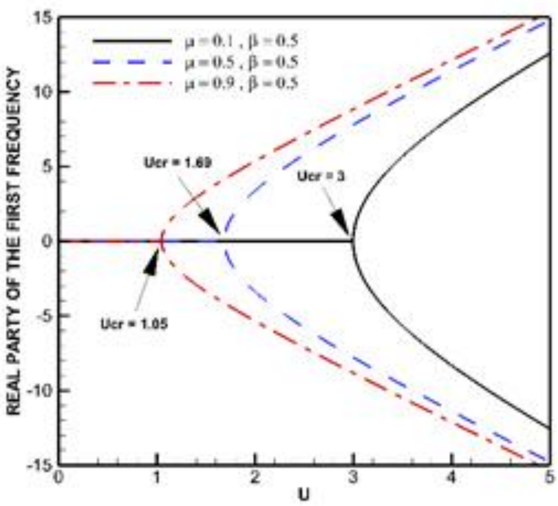
(b)

Figurer 2) Effect of increasing fluid velocity on a) imaginary and b) real parts of the first frequency for non-rotating tube conveying fluid using local theory

It can be easily noticed that frequency does not depend on β and the critical fluid velocity is π for any value of β. This result is compatible with the results presented by Paidoussis and Michael [40]. Considering μ=0.1, same diagram as Figure. 2b is plotted in Figure. 3a for a nanotube using nonlocal elasticity theory. Although instability start point has changed due to applying nonlocal effect, this point is not a function of β as in local elasticity theory. The effect of nonlocal parameter on instability start point is studied in Figure. 3b. In this diagram 0.5 is assigned to the dimensionless mass parameter while μ is changing.



(a)



(b)

Figurer 3) a) Effect of increasing fluid velocity on real part of the first frequency and b) Effect of nonlocal parameter on instability start point for non-rotating nonlocal nanotube conveying fluid

Stability of rotating nanotube conveying fluid is studied in the following. To this end, considering $\mu = 0.1$ and $\theta = 10$, stability of the nanotube while increasing fluid velocity is studied in Figure. 4

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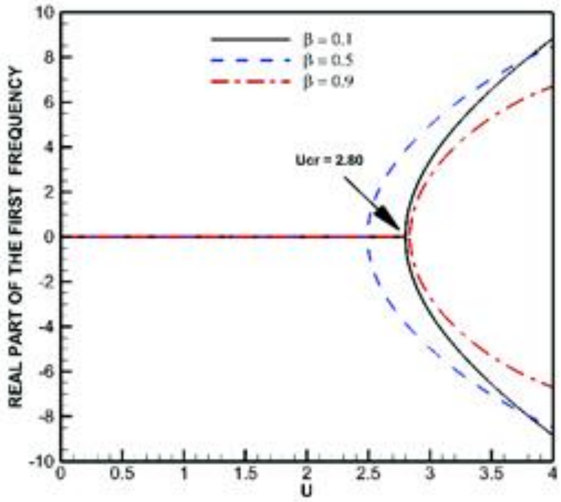


Figure 4) Effect of increasing fluid velocity real part of the first frequency for rotating nonlocal nanotube conveying fluid

As shown in Figure. 4, assigning nonzero values to θ, critical instability fluid velocity is a function of β. Also as β is increasing from 0.1 to 0.9, critical velocity reduces and then gets back to its previous start point. As a result, instability fluid velocity for $\beta = 0.1$and $\beta = 0.9$ are very close.

Considering μ=0.1 and u=2.8, Figure. 5 exhibits the effect of increasing rotational dimensionless parameter on real part of the first frequency for $\beta = 0.1$.

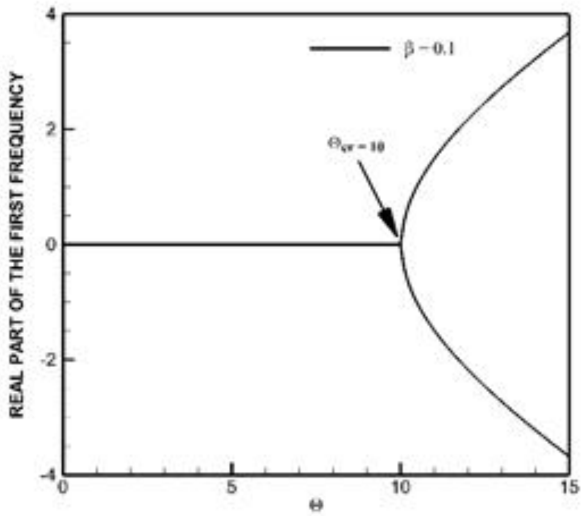
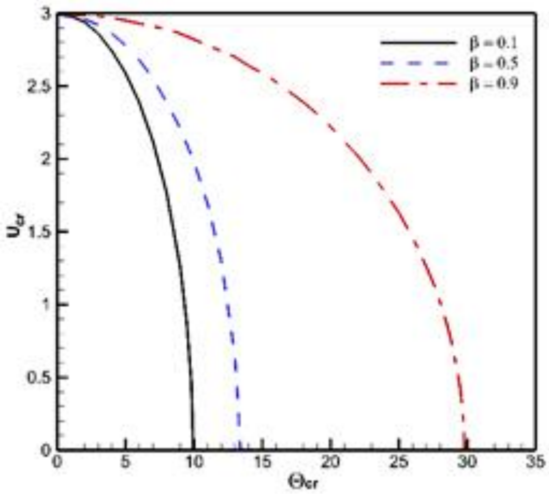


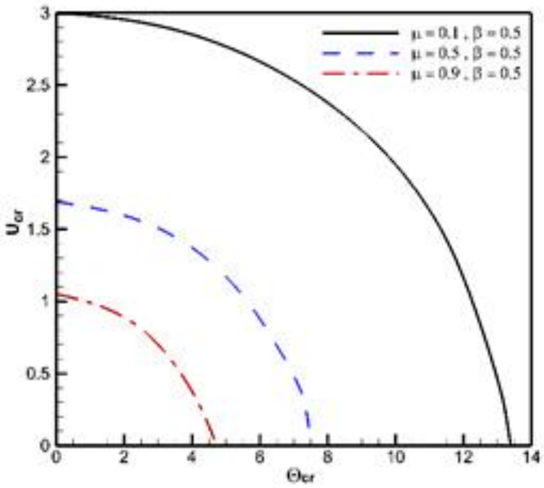
Figure 5) Effect of increasing angular velocity on real part of the first frequency for rotating nonlocal nanotube conveying fluid

Comparing Figure. 4 and Figure. 5, a reciprocal relation can be noticed between critical fluid velocity and critical angular velocity . While $\mu = 0.1$, $\theta = 10$ corresponds to the critical fluid velocity of 2.8 and considering $u = 2.8$ leads to critical angular velocity of 10. This type of instability is called flutter.

Critical fluid velocity is plotted against critical angular velocity in Figure. 6 a. It can be noticed that increasing β leads to higher critical angular velocities. Since the area under the curves is where the system is stable and the exterior area is instability region, increasing β leads to larger stability region. The effect of increasing nonlocal parameter is shown in Figure. 6 b. As μ is increased, both critical fluid velocity and angular velocity decrease drastically.



(a)



(b)

Figure 6) a) Critical fluid velocity versus critical angular velocity and b) the effect of nonlocal parameter on critical fluid velocity and critical angular velocity

4. Conclusion

The free vibration of a simply supported rotating nanotube conveying fluid was investigated. A mathematical model based on Eringen's nonlocal elasticity theory was utilized to formulate the transverse vibrations of the nanotube. The results show that dimensionless mass parameter do not affect the critical fluid velocity in a non-rotating nanotube conveying fluid. On the other hand, critical velocity decreases by increasing nonlocal parameter. In the case where the axial speed is zero, an increase in angular velocity, elevates the stiffness, resulting in higher critical speeds. Similar situation occurs when mass parameter is increased.

In a rotating nanotube, instability start point is a function of mass parameter. While increasing mass parameter, critical speed decreases firstly and then increases to somewhere near the moving start point. A reciprocal relation was noticed between critical fluid velocity and critical angular velocity. Moreover assuming a constant value for β, stability area increases as nonlocal parameter increases.

The physical highlights of the present study can be summarized as follows:

- In the special case of non-rotating nanotube, the first instability mode is static while the second mode is a dynamic instability. This pattern maintained for all values of non-local coefficient and dimensionless mass.

- Increased size effect makes the tube more flexible which results in an expansion in stability region. Hence, applying non-classical methods can improve the predicted stability
- There exists a direct relationship with rotating speed of the nanotube and it's stability
- Increased speed of flow in the tube, restricts the stability region of the system

Nomenclature

E	elasticity modulus
σ	nonlocal stress
ε	nonlocal strain
μ	the nonlocal parameter
u, w	displacements in two perpendicular directions normal to the nanotube axis
Ω	the angular velocity
$\hat{F}_{\hat{w}}, \hat{F}_{\hat{u}}$	the forces inserted by the moving fluid in u and w directions
U	fluid axial velocity
M	fluid mass
s	Laplace variable

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