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Discharge Estimation by using Tsallis Entropy Concept

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ABSTRACT

Flow-rate measurement in rivers under different conditions is required for river management purposes including water resources planning, pollution prevention, and flood control. This study proposed a new discharge estimation method by using a mean velocity derived from a 2D velocity distribution formula based on Tsallis entropy concept. This procedure is done based on several factors which reflect the basic hydraulic characteristics such as river bed slope, wetted perimeter, width, and water level that are easily obtained from rivers. This method avoids putting the environment at risk and significantly reduces time and costs. Validation of the method was carried out by comparing the results with measured values in the experimental sites. Predicted results are in good agreement with the measured data in a cross section of the Tiber River, Italy. Extended usage of this method will make it possible to measure discharge and better estimate the flow rate conveyed from rivers under different hydraulic conditions.

Keywords

Mean velocity, hydraulic characteristics, rivers, probability

1. Introduction

A fast and accurate estimation of the river discharge is the most interesting issue for a large number of engineering applications such as real time flood forecasting and also water resources management. The accuracy of the discharge estimation at gauged river site depends on the velocity data accessibility for high stages and on the reliability of the model to turn recorded stages into discharge hydrographs. As far as it's related to the velocity measurement for high stages, it can be considered by sampling the maximum flow velocity which is located in the upper portion of the flow area, wherein velocity points can be easily sampled during high flow conditions (Chiu and Said, 1995). Indeed, many studies have been shown that the two-dimensional velocity distribution and, hence, the mean flow velocity can be obtained by starting the maximum flow velocity, such as proposed by Chiu (1987, 1988) who derived the probability distribution function of velocity through the entropy theory (Shannon, 1948).

Discharge estimation by using the methods based on hydraulic routing of the recorded flood stage hydrograph can be considered as a stronger and more reliable tool than the other current and common methods. However, there is the lack of

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topographical data of river cross-sections along with the issue of the Manning's roughness coefficient calibration and often inhibits the use the hydraulic models. The easiest way is to relate the Manning's roughness coefficient to the physical characteristics of river channel such as bed material, irregularity, vegetation, etc. and the suitable value is selected from tables (Chow et al., 1988), formulae (Cowan, 1956) or photographs (Barnes, 1967). Specifically, the calibration is performed for each velocity measurement by assigning the observed discharge and minimizing the error in simulating stage and/or mean flow velocity (Moramarco and Singh, 2010). Previous studies on discharge estimation are as follows. Leon et al. (2006) analyzed the relation between stage and discharge and also proposed a discharge estimation method by using the Muskingum. Sahoo et al. (2006) analyzed the correlation between stage and discharge for the Hawaii Basin by applying Artificial Neural Network (ANN) and developed a model for estimating the discharge of natural rivers. In addition, Je-seung et al. (2005) estimated the mean velocity of entire cross-section and then estimated discharge by using linear continuity equation in order to improve the conventional estimation of flood discharge based on the stage- discharge curve for most of Korean rivers. Moreover, Lee, Chan-joo et al. (2009) analyzed the results of field measurements by using an electronic float system developed with GPS and RF communication, and proposed a discharge measuring method. On the other hand, Choo (2002) implemented velocity distribution by using point velocity in Chiu's 2-D velocity distribution formula, and proposed a river discharge estimation method by applying the velocity distribution to Chiu's 2-D mean velocity formula. In addition, Kim et al. (2008) proposed a flow rate estimation method for natural rivers by using Chiu's velocity distribution formula and maximum velocity estimation. Choo et al. (2011) developed new discharge estimation method by using the Manning and Chezy equations reflecting hydraulic characteristics.

Previous studies showed limitations in reflecting the hydraulic characteristics of the rivers. Therefore, the aim of this work is to address the discharge estimation at a gauged river by using the Tsallis entropy. A formula is proposed for estimating river mean velocity by using factors which are easily obtained from rivers including the unique hydraulic characteristics of a river such as area, width, wetted perimeter and river bed slope. The formula was derived using 2-D velocity formula based on tsallis entropy concept. Then the measured data obtained from published work are used to support the validity and applicability of the model. Predicted values of cross sectional discharges are in good agreement with field measured data.

2. Concept and Theory of Entropy

Entropy, as the second law of thermo-dynamics, is a macroscopic property of a system which measures the microscopic disorder within the system. In informational theory, Shannon (1948) formulated the concept of entropy as a measurable infor-mation or uncertainty associated with the random variable or its probability distribution. About a decade later Jaynes (1957) showed that an equilibrium system under steady constraints tends to maximize its entropy.

This is commonly known as the Principle of Maximum Entropy (POME). Entropy can be considered as a useful characteristic of any probability distribution and is widely used in environmental engineering and water including geomorphology, hydrology, and hydraulics.

Shannon (1948) defined a quantitative measure of uncertainty associated with a probability distribution of a random variable in terms of entropy, H, called Shannon entropy or informational entropy as follows (Chiu, 1978):

$$H = -\int_0^{u_{max}} f(u) \log f(u) du \tag{1}$$

Where *f(u)* is the continuous probability density function of random variable *u, and* u_{max} is the maximum velocity. Shannon entropy along with the principle of maximum entropy (POME), can be applied to determine the probability distribution of a given random variable.

Many studies have shown that the two-dimensional velocity distribution and, hence, the mean flow velocity can be obtained starting from the maximum flow velocity, such as proposed by Chiu (1987, 1988) who derived the probability distribution function of velocity through the entropy theory (Shannon,1948). Chiu and his associates derived a relationship between the mean velocity and the maximum velocity at the cross section according to the probability and Shannon entropy theories as follows:

$$\frac{u_{mean}}{u_{max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \Phi(M) \tag{2}$$

Where $\Phi(M)$ is the entropy function.

Tsallis entropy is a generalization of the standard Boltzmann–Gibbs entropy, which was proposed by Tsallis (1988) as a general form of the Shannon entropy. The entropy, H(u), for a continuous variable, u, is expressed quantitatively in terms of the probability as (Luo and Singh, 2011):

$$H = \frac{1}{m-1}\left[1 - \int_0^{u_{max}} \left(f(u)\right)^m du\right] \tag{3}$$

Where m is a real number. For m=1, Eq. (3) reduces to the Shannon entropy. Thus the Tsallis entropy is defined as a generalization of the Shannon entropy. Similar to Shannon entropy, Tsallis entropy can be coupled with the principle of maximum entropy (POME) to achieve the probability distribution of a given random variable.

3. Velocity Distribution based on Tsallis Entropy

The probability density function *f(u)*, must satisfy the properties of probability space (Chiu, 1987). There are two constraints, probability and continuity on *f(u)*. Since integration of the probability density function of velocity must always be unity, one can write:

$$C_1 = \int_0^{u_{max}} f(u) du = 1 \tag{4}$$

The second constraint C_2 can be obtained from the mass conservation as:

$$C_1 = \int_0^{u_{max}} u f(u) du = \bar{u} \tag{5}$$

Where $\bar{u}$ is the cross-sectional mean velocity or Q/A, where Q is the discharge passing through a cross-sectional area of A.

In order to obtain the least biased probability distribution of u, f(u), the method of Lagrange multipliers and constraints given by Eq. (4) and (5) for m>0 Tsallis entropy (Eq. (2)) becomes as:

$$H=\int_0^{u_{max}}\frac{f(u)}{m-1}\{1-[f(u)]^{m-1}\}du$$
$$+\lambda\left(\int_0^{u_{max}}f(u)du-1\right)+\lambda_1\left(\int_0^{u_{max}}uf(u)du-\bar{u}\right) \quad (6)$$

Where λ_0 and λ_1 are Lagrange parameters. By differentiating Eq. (6) with respect to *f(u)* and equating the derivative of the function to zero, probability density function is achieved:

$$f(u)=\left\{\frac{m-1}{m}\left[\frac{1}{m-1}+(\lambda_0+\lambda_1 u\right]\right\}^{m-1} \quad (7)$$

In which:

$$\lambda_V=\lambda_0+\frac{1}{m-1} \quad \Rightarrow$$
$$f(u)=\left\{\frac{m-1}{m}[\lambda_V+\lambda_1 u]^{\frac{1}{m-1}}\right\} \quad (8)$$

Eq. (8) defines the least biased probability distribution of the velocity that satisfies Eq. (4) and (5) and is based on Tsallis entropy.

Accordingly, velocity for a specific point in the 2-D cross-section coordinate system is defined as:

$$\int_0^u f(u)du=$$
$$\int_0^u\left[\frac{(m-1)}{m}(\lambda_V+\lambda_1 u)\right]^{\frac{1}{m-1}}=\frac{r-r_0}{r_{max}-r_0} \quad (9)$$

Where *u*=velocity at r; r =independent variable with which *u* develops such that each value of r corresponds to a value of *u*; r_{max} =maximum value of r where the maximum velocity u_{max} occurs; and r_0 =minimum value of *r* which occurs at the channel bed where *u* is zero. Equation (9) means that if r is randomly sampled a large number of times within the range of $\{r_0, r_{max}\}$ and the corresponding velocity samples are obtained, the probability of velocity falling between *u* and *u*+*du* is *f* (*u*) *du*.

Using Eq. (8), the probability density function is defined as Eq. (10):

$$f(u)=\frac{dF(u)}{du}=$$
$$\frac{df(u)}{dr}\frac{dr}{du}=[(r_{max}-r_0)\frac{du}{dr}]^{-1} \quad (10)$$

If Eq. (7) is solved and rearranged using Eq. (9), a 2-D velocity distribution formula is obtained as in Eq. (10).

$$u=\left(\frac{m}{m-1}\right)\frac{1}{\lambda_1}$$
$$\left\{\left(\lambda_1\frac{r-r_0}{r_{max}-r_0}\right)+\left[\left(\frac{m}{m-1}\right)\lambda_V\right]^{\frac{m}{m-1}}\right\}^{\frac{m-1}{m}} \quad (11)$$

If Eq. (8) is substituted into Eq. (4) and rearranged, Eq. (12) is obtained:

$$\int\left[\frac{(m-1)}{m}(\lambda_V+\lambda_1 u)\right]^{\frac{1}{m-1}}du=1 \quad (12)$$

By integration:

$$\left[\frac{m-1}{m}(\lambda_V+\lambda_1 u_{max})\right]^{\frac{m}{m-1}}=$$
$$\lambda_1+\left[(\frac{m-1}{m})\lambda_V\right]^{\frac{m}{m-1}} \quad (13)$$

If Eq. (8) is substituted into Eq. (5) and rearranged, Eq. (14) is obtained:

$$\int u\left[(\frac{m-1}{m})(\lambda_V+\lambda_1 u)\right]^{\frac{1}{m-1}}du=\bar{u} \quad (14)$$

By integration:

$$\frac{u_{max}}{\lambda_1}\left[(\frac{m-1}{m})(\lambda_V+\lambda_1 u_{max})\right]^{\frac{m}{m-1}}-\frac{1}{\lambda_1^2} \quad (15)$$

$$(\frac{m}{2m-1})\left[(\frac{m-1}{m})(\lambda_V+\lambda_1 u_{max})\right]^{\frac{2m-1}{m-1}}+$$
$$\frac{1}{\lambda_1^2}(\frac{m}{2m-1})\left[(\frac{m-1}{m})(\lambda_V)\right]^{\frac{2m-1}{m}}=\bar{u}$$

Based on Luo and Sing (2001) proposition and using both field and experimental data, the feasible range of *m* is from 0 to 2. For fixing *m*= 2, the two parameters λ_1 and λ_V have simple analytical expressions obtained by solving Eq. (13) and (15) as:

$$\lambda_1=-\frac{12}{u_{max}^3}(u_{max}-2\bar{u}) \quad (16)$$

$$\lambda_V=\frac{4-\lambda_1 u_{max}^2}{2u_{max}} \quad (17)$$

With $u_{\max}$ and $\bar{u}$ known, the two parameters can be easily obtained by substituting these two terms into Eq. (16) and (17). To make the velocity distribution equations simpler, a new dimensionless parameter *M* is introduced, which is mathematically defined as:

$$M=\lambda_1 u_{max}^2 \quad (18)$$

Based on mathematical definition of the parameter *M*, its mathematical range lies between (-12, 12). *M* is directly linked to the ratio between mean and maximum velocity, serving as a new key hydraulic parameter it can play an important role in understanding open channel flow.

4. Definition of r Coordinate System

By defining r in terms of physical plane coordinates, Eq. (10) can describe two-dimensional velocity distribution. Eq. (10) indicates that $r-r_0/r_{\max}-r_0$ is equal to the cumulative distribution function, or the probability of velocity being less than or equal to u. Therefore, the probability concept is needed to identify an expression for r. If a large number of r values are randomly generated within the range of $(r_0, r_{\max})$ and substituted into Eq. (11) to obtain a set of velocity samples, the probability of velocity being between u and u+ du is p(u) du. By such a concept, $r-r_0/r_{\max}-r_0$ is equal to the area to the total cross-sectional area ratio, that in the former the velocity is less than or equal to u(r). For example, for a wide $r-r_0/r_{max}-r_0=(BY)/(BD)=y/D$, where B is channel width, D is flow depth, and y is vertical distance from the channel bed. For an ax symmetric flow in a circular pipe in which curves are concentric circles, $r-r_0/r_{max}-r_0=1-r^2/R^2$ where *r* is radial distance from the pipe center; and *R* is pipe radius (Chiu et al. 1993). In such a way, $r-r_0/r_{\max}-r_0$ may be defined to suit flows in various channels and conduits. In both cases $r_0=0$; $r_{\max}=1$ and hence, $r-r_0/r_{max}-r_0=r$. Equations of $r-r_0/r_{\max}-r_0$ for two-dimensional velocity distributions in open channels are shown in Fig. 1.

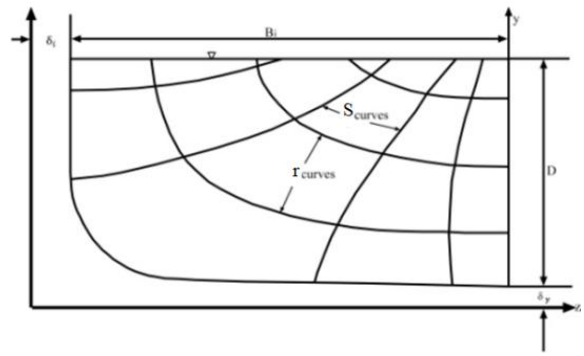
The following equations proposed by Chiu and Chiou (1986) are found to be suitable for the orthogonal curvilinear r-s coordinates.

$$r=Y(1-Z)^{\beta}\ \exp\ (\beta_i Z-Y+1) \quad (19)$$

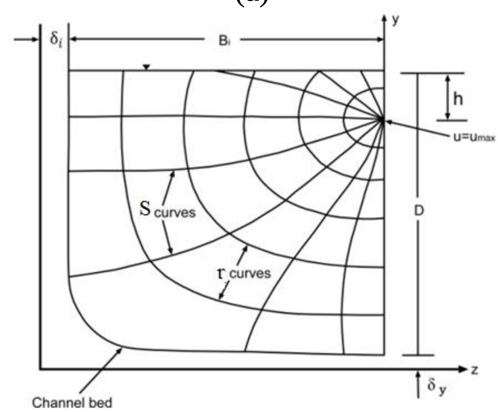
In which

$$Z=\frac{|Z|}{\beta_i+\delta_i} \qquad Y=\frac{y+\delta_y}{D+\delta_y+h} \quad (20)$$

Eq. (19) represents a set of curves. Each curve has a value of r. The channel bed is a curve itself in which $r=r_0$. In Eq. (19) and (20), as shown in Fig. 1, y is the vertical coordinate measured from the channel bed along the y – axis which is defined as the special vertical coordinate that passes through the point where the maximum velocity in the channel cross section occurs; D



(a)



(b)

Fig. 1. Velocity distribution and curvilinear coordinate system (a) h>0, and (b) h<0

is the water depth at the y-axis; z is the coordinate in the transverse direction, β_i for i equal to either 1 or 2 is the transverse distance on water surface; and δ_y, δ_i, β_i and h are parameters characterizing geometry.

Among these parameters, δ_y and δ_i approach zero if the channel cross section tends towards the rectangular shape. These values increase as the cross-section shape deviates from the rectangular shape. Parameter h controls the shape and slope of the curve especially near the water surface and in the vicinity of the point of maximum velocity. The value of h may vary from –D to +∞. If h<0, $u_{\max}$ occurs below the water surface and $|h|$ is the depth of $u_{\max}$ below the water surface; and along the y-axis, velocity increases with y only up to y=D-h, and decreases with y in the region (D+h) < y ≤ D. If h ≥ 0, $u_{\max}$ occurs at the water surface. If h=0, curves are perpendicular to the water surface. If h>0, h is a parameter that can be used to finely tune the slope of the curve. If the magnitude of h is very large, curves are parallel horizontal lines such that the velocity varies only with y and r approaches y/D. Such a situation tends to occur in very wide channels. S curves shown in Fig. 1 are orthogonal trajectories of r curves that can be derived from Eq. (19) as:

$$S = \pm\frac{1}{Z}(\left|1-Z\right|)^{\beta_i\{(D+\delta_i-h)/(\beta_i+\delta_i)\}^2} \exp\left[Z+\beta_i(\frac{D+\delta_i-h}{\beta_i+\delta_i})^2 Y\right] \tag{21}$$

In which S is negative only when y>$D-h$ and h > 0. In other cases S is positive.

4. Mean Velocity Estimation

By substituting the two parameters of λ_1, λ_v into Eq. (7), the probability density function is obtained as:

$$f(u)=\frac{1}{2}(\frac{4-M}{2u_{max}}+\frac{M}{u^2_{max}}u) \tag{22}$$

If Eq. (21) is substituted into Eq. (4) and solved, a 2-D mean velocity equation is obtained as follows:

$$\frac{u}{u_{max}}=\frac{12+M}{24}=\phi(M) \tag{23}$$

This equation can be simplified to Eq. (24) as:

$$\bar{u}=\phi(M)u_{max} \tag{24}$$

Where $\phi(M)$ is an indicator showing the linear relation between the mean velocity and the maximum velocity as in Eq. (23) and is called equilibrium state $\phi(M)$. If Eq. (5) is substituted into Eq. (17) and rearranged, Eq. (25) is obtained.

$$\lambda_V\bar{u}=\frac{4-M}{2}\times\frac{24}{12+M} \tag{25}$$

5. Proposed Mean Velocity Equation

If the bottom shear stress of channel is expressed as Eq. (26) and $\frac{du}{dr}$ is estimated from Eq. (9) and rearranged, results are as follows:

$$\tau_0=\mu\left[\frac{du}{dy}\right]_{y=y_0}=\mu\left[\frac{1}{h_r}\right]\left[\frac{du}{dr}\right]_{r=r_0} \tag{26}$$

Where τ_0 is bottom shear stress, μ is viscosity coefficient of the fluid, and h_r is unit conversion factor indicating length unit dy multiplying by h_r and by the relation of:

$$h_r=(D+h)\left[(1-\frac{y}{D+h})\exp(1-\frac{y}{D+h})\right]^{-1}$$

In which y is the vertical distance from the channel bed, D is the water depth at y-axis and h is the depth of $u_{\max}$ below water surface. In addition, the mean shear stress can be expressed as Eq. (27).

$$\bar{\tau}_0=\mu\left[\frac{1}{h_r}\right]\left[\frac{du}{dr}\right]_{r=r_0}=\rho gRl_f \tag{27}$$

Where $\overline{\tau_0}$ is mean shear stress of the bottom boundary layer, $\overline{h_r}$ is mean value of h_r according to the channel boundary layer, ρ is water density, g is gravitational acceleration, R is hydraulic radius, and l_f is energy gradient.

$\frac{du}{dr}$ in Eq. (26) can be expressed as Eq. (28).

$$\frac{du}{dr}=\left[\frac{1}{(r_{max}-r_0)f(u)}\right]=$$

$$\left[\frac{1}{(r_{max}-r_0)(\frac{1}{2}(\lambda_V+\lambda_1 u))}\right] \quad (28)$$

Because u = 0 in the bottom boundary layer of channel, $r_0=0$ and $r_{\max}=1$ and as a result $r_{\max}-r_0=1$. Accordingly, Eq. (18) is rearranged to Eq. (20).

$$\left[\frac{du}{dr}\right]_{r=r_0}=\left[\frac{2}{\lambda_V}\right] \quad (29)$$

If Eq. (29) is substituted into Eq. (27) and rearranged, Eq. (30) is obtained:

$$\frac{2\mu}{\bar{h}_\zeta}=\rho gRl_f\times\lambda_V \quad (30)$$

If Eq. (30) is substituted into Eq. (25) and rearranged, new mean velocity is derived as in Eq. (31).

$$\bar{u}=\frac{h_r gRl_f}{v}\times F(M) \quad (31)$$

Where:

$$F(M)=\frac{6(4-M)}{(12+M)} \quad (32)$$

Accordingly, Eq. (32) means that if there are measured values of *F* (*M*), $\overline{h_r}$ ،g ،R ،l_f ،v indicating the hydraulic characteristics of the river, the mean velocity can be calculated and the flow rate can be estimated by multiplying the velocity by the cross-sectional area.

6. Discharge Estimation based on the Mean Velocity Formula

Based on the 2-D velocity formula using the Tsallis entropy concept, a mean velocity formula was derived as Eq. (31) from the relation between the sum of kinematic coefficient of viscosity and velocity gradient perpendicular to the channel boundary and the mean shear stress formula. The hydraulic characteristic factors in this equation are easily measured from rivers. *F*(*M*) is estimated from Eq. (31) by substituting the measured values of mean velocity, river energy slope, hydraulic radius, kinematic coefficient of viscosity, etc., and then entropy parameter *M* is calculated. Using the calculated *M*, $\phi(M)$ is calculated from Eq. (23) and $u_{\max}$ is also calculated by using Eq. (24). With all the data, $\phi(M)$ at the whole river equilibrium was calculated. Through this process, the mean velocity was estimated using the relation between maximum velocity $u_{\max}$ and overall equilibrium state $\phi(M)$. Therefore, the discharge estimation process using the proposed method is as follows:

1. Estimate M by substituting F (M), $\overline{h_{r_0}}$, g ،R, l_f , v for each cross-section of the channel, and then estimate $\phi(M)$.
2. Estimate the maximum velocity of each cross-section from Eq. (24).
3. From Eq. (22) estimate the overall equilibrium state $\phi(M)$ which means the gradient of the linear relation. Accurately estimate the mean velocity using $\phi(M)$.
4. Test the accuracy of the flow rate based on estimated and measured flows.

7. Filed Data

To evaluate the accuracy field data was used in this study. Velocity data from four gauged sections in the upper Tiber River basin in Central Italy (shown in Fig. 2) were used for evaluating 2D velocity distribution which were collected from seven floods occurred from 1984 to 1997. Mean velocity data collected during a period of 20 years at the four gauges sections, three of which are located along the Tiber river at 68 km (Santa Lucia), 109.2 km (Ponte Felcino) and 137.4 km (Ponte Nuovo) and one section along the Chiascio River, a tributary of the Tiber River, at 85 km (Rosciano) were used to evaluate the mean velocity computation using entropy. Discharge values for different events varied between $1.5 m^3/s$ and $537 m^3/s$ with the mean velocity ranging between 0.12 m/s and 2.42 m/s and the maximum water depth between 0.8 m and 6.7 m. Number of velocity measurements and flow characteristics of each station are summarized in table 1. Selected sections were equipped with a remote ultrasonic water level gauge, while the velocity measurements were made by current meter from cableways. In particular, depending on the cross-sectional flow area, the number of verticals carried out changed from 4 up to 10, and for each vertical at least 5 velocity points were sampled. The main charac-teristics of the selected flood events are shown in tables 2 (a) and (b).

8. Analysis of the Results

Applicability of the liner relationship between the mean and the maximum flow velocities estimated through Tsallis entropy model was investigated by using data collected

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Fig .2. Upper Tiber River basin with location of river gauging stations (Moramarco et al., 2004)

during a period of 20 years in four gauged river sections in central Italy. The relations between the cross sectional mean velocity and the maximum velocity estimated in four gauged sites of Ponte Nuovo, Ponte Felcino, Rosciano, and Santa Lucia along Tiber river are shown in Fig. 3(a)-3(d), respectively.

The value of the Tsallis entropy parameter M for the four gauged sections was found to be constant and equal to 3.51. The same value can be estimated for other river sections located within the investigated river reach.

It was found essential to compare the proposed model with Chiu model for the accurate estimation of the discharge. Mean flow velocity to maximum velocity ratio at selected gauged river sections was found to be constant and showed that the parameter $\Phi(M) = 0.667$ and then, Shannon entropy value based on Eq. (2) was obtained as M=2.15.

Table 1. Flow characteristics: Discharge (Q) and maximum water depth (D) of the available velocity measurements (N) for four gauges

Location	N	$Q(m^3/s)$	D(m)
S.Lucia	42	1.5-215	0.9-5.2
P.Felcino	34	2.3-412	0.8-6.2
P.Nuovo	51	5.4-537	1.1-6.7
Rosciano	38	3-160	1.3-3.3

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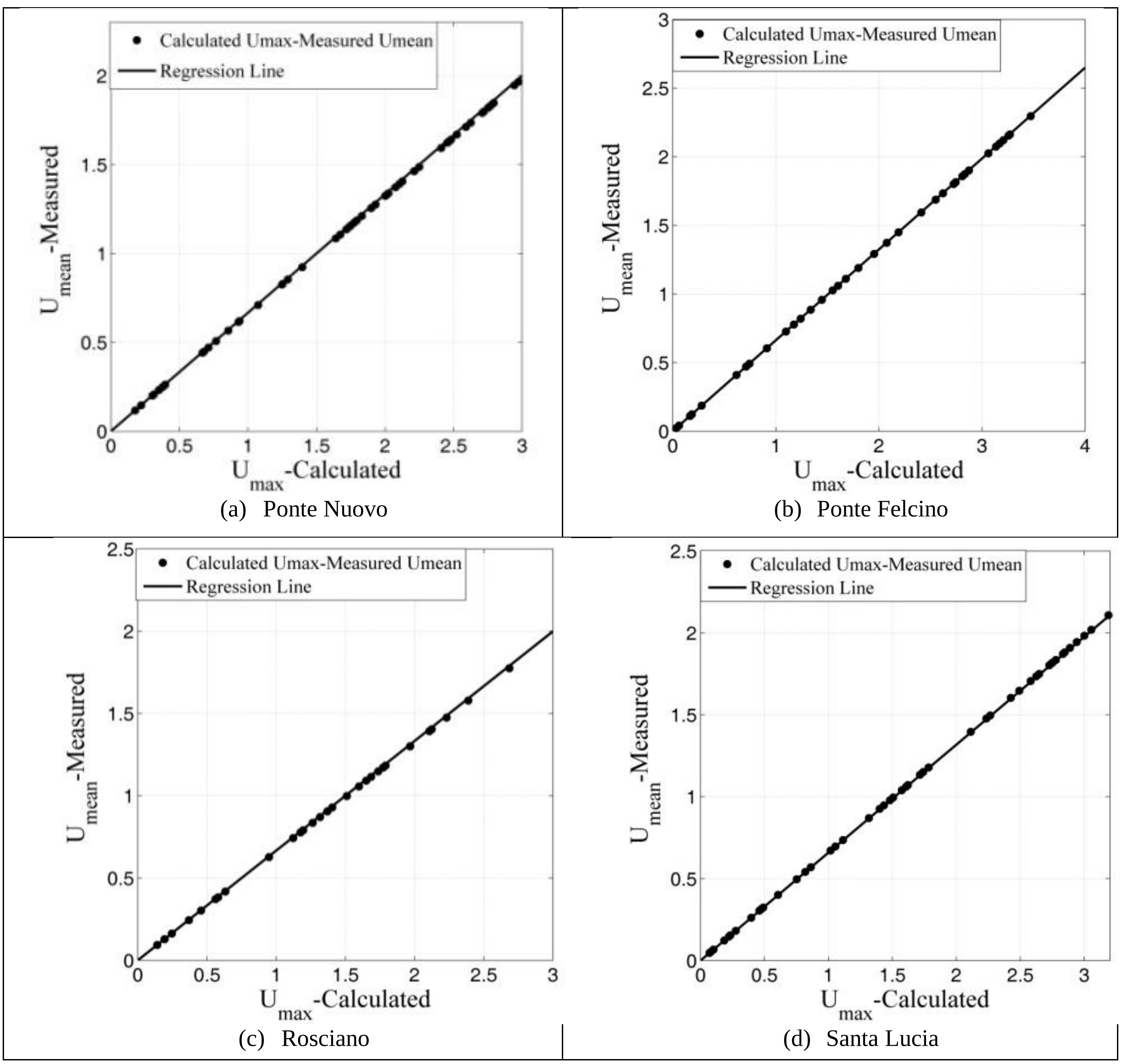
Table. 2(a): Main characteristics of selected events: Maximum velocity $u_{\max}$, mean velocity u_m

Location	Data	$u_{\max}(m/s)$	$u_{\mathrm{m}}(m/s)$
P.Nuovo	Nov. 15, 1982	2.023	1.085
P.Nuovo	Nov. 18, 1996	2.597	1.736
P.Nuovo	Jun. 03, 1997	2.719	1.820
Rosciano	May 28, 1984	2.583	1.784
Rosciano	Nov. 20, 1996	2.447	1.525
P.Felcino	Apr. 21, 1997	3.365	2.120
S.Lucia	May 28, 1984	2.437	1.873



(a) Ponte Nuovo

(b) Ponte Felcino

(c) Rosciano

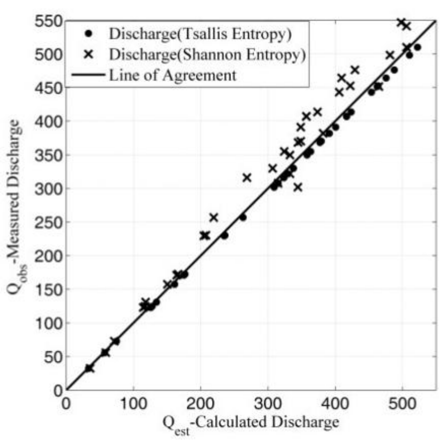
(d) Santa Lucia

Fig. 3. Relationship between measured mean velocity and calculated maximum velocity in Tiber River sections.

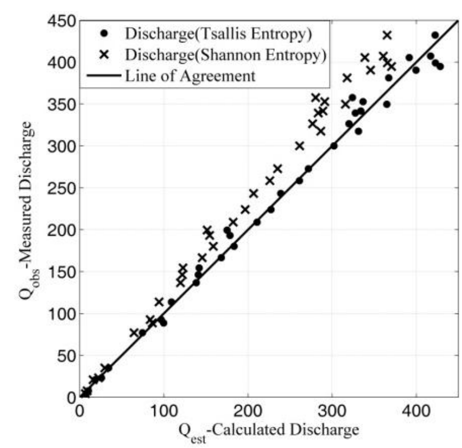
Fig. 4(a)-(d) show clearly the discharge estimation, which is the gradient in the linear relation between the mean velocity and the maximum velocity estimated through the process proposed using the data measured in gauged sections of Natural River. The mean velocity of the river was estimated by using Eq. (30) and the discharge was estimated by multiplying the estimated mean velocity by the cross-section area of each river.

Table. 2(b): Main characteristics of selected events: discharge (Q), flow area (A) and water depth along the vertical where $u_{\max}$ occurs (D)

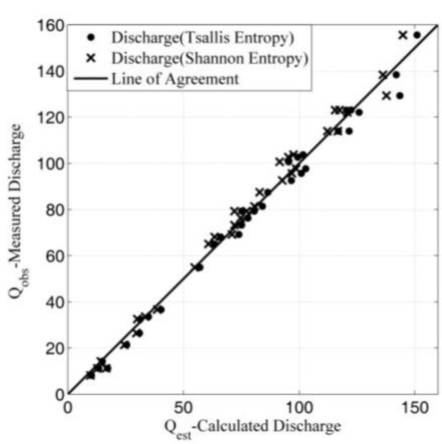
Location	$Q(m^3/s)$	$A(m^2)$	D(m)
P.Nuovo	159.19	146.74	2.9
P.Nuovo	541.58	311.91	6.64
P.Nuovo	506.39	278.16	6.07
Rosciano	156.24	87.60	3.2
Rosciano	131.20	86.03	3.11
P.Felcino	399.16	188.26	6.15
S.Lucia	69.53	51.53	2.93



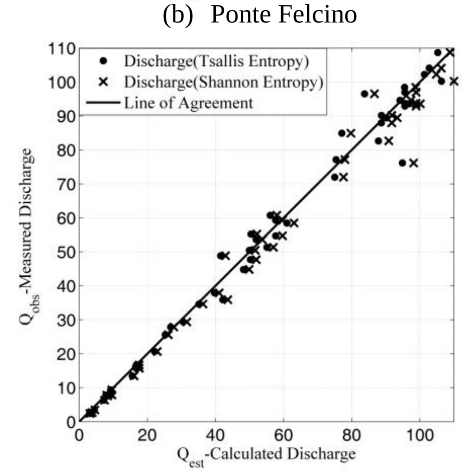
(a) Ponte Nuovo



(b) Ponte Felcino



(c) Rosciano



(d) Santa Lucia

Fig. 4. Analysis results by using the proposed mean velocity equation in Tiber River sections.

9. Conclusions

This study developed a mean velocity formula derived from 2-D velocity formula using Tsallis entropy concept and river bed shear stress of the channel. In particular, the developed new velocity formula reflects accurately the hydraulic characteristics such as water level, width, hydraulic radius and river slope that are easily obtained from rivers, and can estimate accurately the maximum velocity that is difficult to measure in natural rivers.

For this study reliable data measured from gauged sections of Natural River were used. According to the results, standard deviations for gauged sections were 0.009, 0.003, 0.033 and 0.0241, respectively and showed that estimated data are quite close to the measured data.

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