Comparison of the performances of neural networks specification, the Translog and the Fourier flexible forms when different production technologies are used

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Abstract

This paper investigates the performances of artificial neural networks approximation, the Translog and the Fourier flexible functional forms for the cost function, when different production technologies are used. Using simulated data bases, the author provides a comparison in terms of capability to reproduce input demands and in terms of the corresponding input elasticities of substitution estimates. The results suggest that ANN provide a better approximation than other traditional functional forms only when a single technology is used. However, when elasticities of substitution are calculated, the Translog approximate batters the true technology in both single and mixed technology.

Keywords: Artificial neural networks; Cost function; Flexible functional forms

1. Introduction

In the empirical studies on production and cost functions, the true technology relating the inputs to the output is generally little known. Several studies were carried out to compare the performances of a large number of functional forms and the one which is adjusted best with the studied case was selected. In 1928, Cobb and Douglas [12] proposed the first specification characterizing the productive combination. This function became the concern of several specialists and was the subject of many controversies giving place to a first generation of functional forms such as the Leontief and the Constant Elasticity of Substitution forms. However, this type of functional forms presents very severe restrictions on studied technology and in particular on the possibilities of substitution between the production inputs; these forms supposed fixed proportions or constant elasticities of substitution.

The developments of the duality theory and their implications were at the origin of a renewed interest concerning the development of a new generation of functional forms called the flexible type. These forms have the advantage of reproducing in a more general way a production technology without imposing prior constraints on the possibilities of substitution be-

tween inputs; they are interpreted as second order approximations of any twice differentiable unspecified technology (Fuss *et al.* [22] and Chambers [10]). Among the most popular flexible forms are Translog (TL) proposed by Christensen *et al.* [11], the Generalized Leontief form (GL) proposed by Diewert [15], the Generalized Square Root Quadratic (GSRQ) form suggested by Diewert [16], the Generalized Cobb-Douglas form (GCD) suggested by Diewert [15] and Generalized Box-Cox (GBC) proposed by Berndt and Khaled [6].

The Flexible Functional Forms (FFF) presented by definition the same theoretical properties and the economist does not dispose of any criterion allowing to decide, in a decisive way, in favor of one or the other of the suggested forms. One is then often brought to discriminate them empirically. Several empirical studies were proposed in the literature to select the functional form which best approximate reality (Berndt *et al.* [5], Wales [36], Appelbaum [2], Berndt and Khaled [6], Caves and Christensen [8], Gallant [23], Guilkey and Lovell [27], Barnett and Lee [3], Lau [30], Berndt *et al.* [5], Dévazeaux de Lavergne [14]). The technique consists in estimating different forms using the same data base, and then the choice of the best one is based on the significance of the parameters, the relevance of elasticities ob-

tained and on the maintenance of the regular conditions.

However, although FFF allows to describe an unknown technology and to estimate elasticities of substitution, it has some limitations. The most important one is that it can only provide a second order approximation of the true data generation function at a single point, and then fail to be globally flexible. Global approximation means that the FFF is capable, in the limit, of approximating the unknown underlying generating function at all points and thus of producing accurate elasticities at all data points. There are several methods to impose the conditions of global regularity. But unfortunately, imposing further restrictions on the parameters destroys flexibility (Diewert and Wales [17]). To overcome these problems, Gallant [23,24] proposes another approximation to an unknown cost or production function. His idea was to use a Fourier series which approximates the true function in the so-called Sobolev norm to achieve, asymptotically, global approximation to the complex economic relations. However, in spite of its theoretical superiority, the comparative studies between the two types of FFF do not make it possible to decide in favour of Fourier form in a categorical way.

The Artificial Neural Networks (ANN) constitutes a new technique which is relatively recent. It consists of a mathematical model that emulates the behaviour of the human brain and has an interesting capacity to identify patterns among a group of variables without any assumption about the underlying relationships. According to Dreyfus (1997), the advantage of ANN compared to the other techniques of processing data is that they are parsimoniously universal approximators of function[1]. They were used and proved reliable in various research areas such as character and voice recognition, medical and financial diagnosis, economic and agricultural research. Moreover, several studies compare the performance of the FFF and a new form based on ANN. Guermat and Hadri [26] carried out a Monte Carlo experiment in order to analyse the effects of functional form misspecifications and the performance of neural networks versus Translog model for approximating different theoretical production functions like Cobb Douglas, CES function and Generalized Leontief model. They have found that neural networks are a serious alternative to the Translog specification. Fleissig *et al.* [20] employ

neural networks for the cost functions estimation and compare its performance with four FFF. For the first one, they find convergence problems when the properties of symmetry and homogeneity are imposed. Santin *et al.* [34] use ANN for a simulated production function and compare its performance with traditional efficiency techniques like stochastic frontier and DEA. Authors suggest that ANN is a promising alternative to traditional approaches.

The analysis of the technology based on the specification of a functional form for the cost of production function rests often on the fundamental hypothesis that the relationship is the same during the period and shared by all productive units; this supposes only one functional form with the same vector of parameters. In a previous study (Feki [19]), the researchers have tried to deal with this problem for the difference of technologies between the firms in the same industry. A new approach of specification and estimation of the cost function based on a switching regression model for panel data is proposed. The retained model allows to take into consideration the possible difference of technologies between productive units, and consequently to find the best representation of the economic reality. Finally, this technique provides an endogenous choice method of the adequate functional form. This approach has been applied to a panel of firms operating in the Tunisian textile industry. Results show that at least two technologies are used and identified by two different functional forms.

The main aim of this study is to compare the performances of ANN for the cost functions with two FFF, Translog and Fourier, when different production technologies are used. Following Guilkey *et al.* [28] who argued that for the evaluation of functional forms, *its better to begin with known technology and examine the ability of various forms to track that technology*, the author has used data provided by Nerlove 1960. His data base concerned 145 firms operating in the United States electric power industry and were largely used in the comparisons of functional forms. The author then generated data for two different but known technologies. Mixed technology is then obtained by mixing the two data sets. The rest of this paper is organised as follows:

In the next section, the author presents the three retained functional forms for the cost function, Translog, Fourier and the neural networks approximation. Section 3 describes the data construction procedure and the experimental design. The results from the empirical study are summarized in Section 4, and the concluding remarks and suggested areas for further research are provided in Section 5.

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[1] With equal precision, the neural networks require less adjustable parameters than the universal approximators usually used.

2. Functional forms

2.1. The Translog cost function

The Translog (Transcendental Logarithmic) function was first proposed by Christensen *et al.* [11] and has been used by many authors, who demonstrated that it is an excellent representation of the technology in many cases. The Translog technology is defined as follows:

$$Log\,C(P,Y) = \alpha_0 + \sum_{i=1}^{n} \alpha_i Log(P_i)$$

$$+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} Log(P_i)Log(P_j). \quad (1)$$

For a well-behaved cost function, the following restrictions need to be imposed on the Translog cost function so that it is symmetric and linear homogeneous in input prices:

$$\beta_{ij} = \beta_{ji} \qquad i,j=1,2,...,n\,, \qquad (2)$$

$$\sum_{i=1}^{n} \alpha_i = 1, \qquad (3)$$

$$\sum_{i=1}^{n} \beta_{ij} = 0 \qquad j=1,2,...,n\,. \qquad (4)$$

Differentiating with respect to each input price and applying Shephard's lemma yields a series of input cost shares stated as:

$$S_i = \frac{\partial Log\,C(P,Y)}{\partial Log(P_i)} = \frac{\partial C(P,Y)}{\partial p_i}\cdot\frac{p_i}{C(P,Y)}$$

$$= \frac{x_i p_i}{C(P,Y)} = \alpha_i + \sum_{j=1}^{n} \beta_{ij} Log(P_j). \qquad (5)$$

Following Diewert and Wales [17], we express the logarithmic second order derivatives of a cost function in term of its first and second order partial derivatives:

$$\frac{\partial^2 Log\,C(P,Y)}{\partial Log(P_i)\partial Log(P_j)} = \frac{\delta_{ij}\,p_i\,C_i(P,Y)}{C(P,Y)}$$

$$- \frac{p_i p_j C_i(P,Y)C_j(P,Y)}{C^2(P,Y)} + \frac{p_i p_j C_{ij}(P,Y)}{C(P,Y)}, \quad (6)$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise.

2.2. The Fourier cost function

The Fourier approximation of the cost function proposed by Gallant [23,24] consists of two components, the first one corresponds to the Translog cost function, while the second is a nonparametric Fourier expansion. This function is based on a logarithmic transformation of prices:

$$x_i = Log(p_i) + Log(a_i) \qquad i=1,2,...,n. \qquad (7)$$

The constants a_i are location parameters chosen to ensure that the minimum value of the scaled log-input price x_i will be slightly greater than zero. The choice of a_i is arbitrary and does not affect the result. Following Chalfant and Gallant [9], we may consider:

$$Log(a_i) = -\min\{Log(p_i)\} + 10^{-5}. \qquad (8)$$

The logarithmic version of the Fourier approximation of the cost function is expressed as:

$$g_K(x/\theta) = u_0 + b'x + \frac{1}{2}x'\psi x \qquad (9)$$

$$+ \sum_{\alpha=1}^{A}[u_{0\alpha} + 2\sum_{j=1}^{J}(u_{j\alpha}\cos(jz_\alpha) - v_{j\alpha}\sin(jz_\alpha))],$$

where $g_K(x/\theta)$ represents the logarithm of the true cost function and $z_\alpha = \lambda k'_\alpha x$. K is the order (degree) of approximation that can be chosen freely (the only limiting factor is the sample size).

Let $\theta'_{(\alpha)} = (u_{0\alpha},u_{1\alpha},v_{1\alpha},...,u_{J\alpha},v_{J\alpha})$, then $\theta = (u_0,b',\theta'_{(1)},\theta'_{(A)})'$ represents the vector of $1+n+A(1+2J)$ parameters to be estimated. The matrix ψ is defined by $\psi = -\lambda^2 \sum_{\alpha=1}^{A}[u_{0\alpha}k_\alpha k'_\alpha]$. The sequence $\{k_\alpha\}$ is that of so-called elementary multi-indexes (vectors with integer components of dimension n). The number A depends on the order of ap-

proximation, and its length is $\left| k_\alpha \right|^* = \sum_{i=1}^{n} \left| k_{i\alpha} \right|$. The multi-indexes are constructed by increasing their length and they reduce the complexity of the notation of high-order partial differentiation and multivariate Fourier series expansions. λ is a scaling factor chosen a priori so that all x_i are in the interval $[0, 2\pi]$, and it is computed as $\lambda = (2\pi - \varepsilon)/\max\{x_i\}$. Gallant [18] suggested that a reasonable choice is $(2\pi - \varepsilon) = 6$.

Finally, constant A (number of terms) and J (degree of the approximation) determines the degree of Fourier polynomials. The input cost shares and the Hessian matrix are stated as follows:

$$S = \frac{\partial g_K(x/\theta)}{\partial x} = b - \lambda \sum_{\alpha=1}^{A} [u_{0\alpha} z_\alpha$$

$$+ 2\sum_{j=1}^{J} (u_{j\alpha} \sin(jz_\alpha) + v_{j\alpha} \cos(jz_\alpha))] k_\alpha, \quad (10)$$

$$H = \frac{\partial^2 g_K(x/\theta)}{\partial x \partial x'} = -\lambda^2 \sum_{\alpha=1}^{A} [u_{0\alpha}$$

$$+ 2\sum_{j=1}^{J} j^2 (u_{j\alpha} \cos(jz_\alpha) - v_{j\alpha} \sin(jz_\alpha))] k_\alpha k'_\alpha. \quad (11)$$

Parameters A and J must be selected for estimation. Chalfant and Gallant [9] and Eastwood and Gallant [18] suggested that, for reliable asymptotic, the number of parameters to be estimated in Fourier functional function be equal to the effective sample size raised to the two thirds in power.

Therefore, with three equations (the cost function and two cost shares) and 145 observations, the cost function should include about 58 parameters.

With A=17 and J=1, we have 55 parameters to estimate. Finally, for the linear homogeneity of the Fourier cost function, we need to impose restriction $\sum_{i=1}^{n} b_i = 1$ on the parameters and set equal to zero all $u_{0\alpha}, u_{j\alpha}, v_{j\alpha}$ for witch $\sum_{i=1}^{n} k_{i\alpha} \neq 0$.

Finally, the Hessian matrix of the Fourier cost function is:

$$H = \frac{\partial^2 g_K(x/\theta)}{\partial x \partial x'} = -\lambda^2 \sum_{\alpha=1}^{A} [u_{0\alpha}$$

$$+ 2\sum_{j=1}^{J} j^2 (u_{j\alpha} \cos(z_\alpha) - v_{j\alpha} \sin(z_\alpha))] k_\alpha k'_\alpha. \quad (12)$$

2.3. The neural networks approximation

An Artificial Neural Networks is composed of a large number of highly interconnected elements (neurons) working in parallel. The most commonly used ANN is the Multilayer Perceptron (MLP). It consists of an input layer, an output layer and one or more intermediary layers called hidden layers. The hidden layers can capture the nonlinear relationship between variables. Each layer is composed of a number of neurons. The information progress from the input layer to the output layer without feedbacks, for this reason this kind of ANN is called a feedforward neural network. For a system of three demand equations, the MLP with one hidden layer can be expressed as:

$$x = f\{\beta_0 + \sum_{j=1}^{h} g[\alpha_j + \sum_{i=1}^{3} \gamma_{ij} p_i] \beta_j\}, \quad (13)$$

where, x is the vector of input demands and p_i, $i = 1, 2, 3$ are the associated input prices. h is the number of neurons on the hidden layer determined empirically. g and f are respectively the hidden and the output activation functions, usually chosen to be monotonous without decreasing. In this paper the function g is the sigmoid and f is the linear function. β_0, β_j, α_j and γ_{ij} for $i = 1,...,n$ and $i = 1,...,h$, are the weights (or parameters) to be usually adjusted (estimated) adjusted iteratively by a supervised learning algorithm, the back propagation algorithm, proposed by Rumelhart *et al.* [31]. The learning is guided by specifying the desired response to the network for each training input pattern through the comparison with the actual output computed by the network in order to adjust the weights.

2.4. Elasticity of substitution

There are different measures of input substitutability proposed in the literature when the production process has more than two inputs. Empirical research in production usually utilizes Allen-Uzawa partial

elasticity (AES). For a twice-differentiable cost function, the AES between inputs i and j is defined as [2]:

$$AES_{ij} = \frac{C(P,Y)\,C_{ij}(P,Y)}{C_i(P,Y)\,C_j(P,Y)}. \qquad (14)$$

When the cost function is expressed in logarithmic form, AES can be computed as follow:

$$AES = [AES_{ij}]_{i,j=1,...,n} = ss' + S^{-1}HS^{-1} - S^{-1}, \quad (15)$$

where s in an n vector of 1, S is a diagonal matrix of inputs cost shares: $S_{(n,n)} = diag[S_1, S_2, ..., S_n]$, and H is the Hessian.

AES provide information on input substitutability by measuring changes in the demand of input i with respect to a change in the price of another input j. It has been employed to measure substitution behaviour and structural instability in different contexts. However, when there are more than two inputs the AES may be uninformative. In this case The Morishima elasticity of substitution (MES) is viewed as more appropriate measure of substitutability [7]. It is defined as follows:

$$MES_{ij} = \frac{p_i\,C_{ij}(P,Y)}{C_j(P,Y)} - \frac{p_i\,C_{ii}(P,Y)}{C_i(P,Y)}. \qquad (16)$$

MES measures the percentage change in the ratio of input i to input j, given a one-percent change in the price of input j. Note that AES is symmetric while the MES is not because changes in the input ratio induced by the price of input j can be different from those induced by the price of input i. Estimation of the "true" elasticity substitution has been the main purpose of different studies [7,21,35]. It seems that both the AES and the MES can be useful measures of input substitutability depending on the purpose of the analysis. Finally, MES can be expressed in term of the AES [7].

3. Experimental design

In order to investigate the performance of the neural network specification, when different production technologies are used, against the Translog and the Fourier Functional Forms, we use cross-section data set on individual firms in the United States electric power industry. This data set corresponds to 145 firms observed in the year 1955 and used by Nerlove (1960). We retain the price indices for capital (P_K), labour (P_L) and energy (P_E). Those data are widely used for the comparison of production and cost functional forms.

Taking each input price indices, the author constructed two series by adding normally distributed errors to obtain two data sets. Then, the author generated total cost and input demands were generated from CES form. The CES cost function used in this study and its derived input demands are:

$$C(Y,P) = Y[\sum_{i=1}^{n} p_i^{\rho/(\rho-1)}]^{(\rho-1)/\rho}, \qquad (17)$$

and

$$x_i = C(Y,P)^{\rho/(\rho-1)} p_i^{1/(\rho-1)} \quad i = 1,...,n. \qquad (18)$$

Two values for ρ are considered yielding to different technologies. For the first data set we consider $\rho = -4$ to define a first technology characterised by small elasticity (0,2). A second technology with large elasticity (4) was defined from the second data set by setting $\rho = 0.75$. A multiple technology is then obtained by mixing data for the two technologies.

4. Results

In order to improve the efficiency of the estimates, total cost function is estimated along with share equations[3] for both the Translog and the Fourier using maximum likelihood estimation. Neural networks estimation is carried out with Matlab Numeric Computation Software and we report, for each technology results corresponding to the best MLP. Homogeneity and symmetry are imposed only for the two first functions.

For The neural network, Fleissig *et al.* [20] used a penalty functions to impose those restrictions and concluded that no improvements were detected over the unconstrained model. Comparison is focused on the average root square error (RMSE) for the tree functional forms and for the tree technologies.

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[2] The subscripts denote partial derivatives with respect to input prices.

[3] The energy share equation is arbitrarily dropped from the system estimation to overcome the problem of singularity.

Table 1. RMSE for total cost and input demands for the tree functional forms.

| | RMSE for Total cost and inputs demands | | | | | | | | |
| | Technology 1 | | | Technology 2 | | | Mixed technology | | |
	Translog	Fourier	ANN	Translog	Fourier	ANN	Translog	Fourier	ANN
Architecture			3-20-3			3-30-3			3-30-3
Performance			8.6612E-12			2.30713E-12			197.73614
Total cost	0.1195557	0.1386477	0.0004451	0.0892424	1.2040983	0.0002830	394.30792	394.34116	360.52892
Labor	0.0483505	0.0387100	0.0000032	0.1425040	0.1795817	0.0000016	29.11321	29.11280	23.87875
Capital	0.0055287	0.0011930	0.0000022	0.0033594	0.0024128	0.0000016	1.07585	1.07614	1.018682
Energy	0.0550754	0.0105887	0.0000033	0.0373578	0.0170522	0.0000014	5.50940	5.50530	4.64811
Mean for inputs	0.0363182	0.0168306	0.0000029	0.0610737	0.0663489	0.0000015	11.89949	11.89808	9.90456
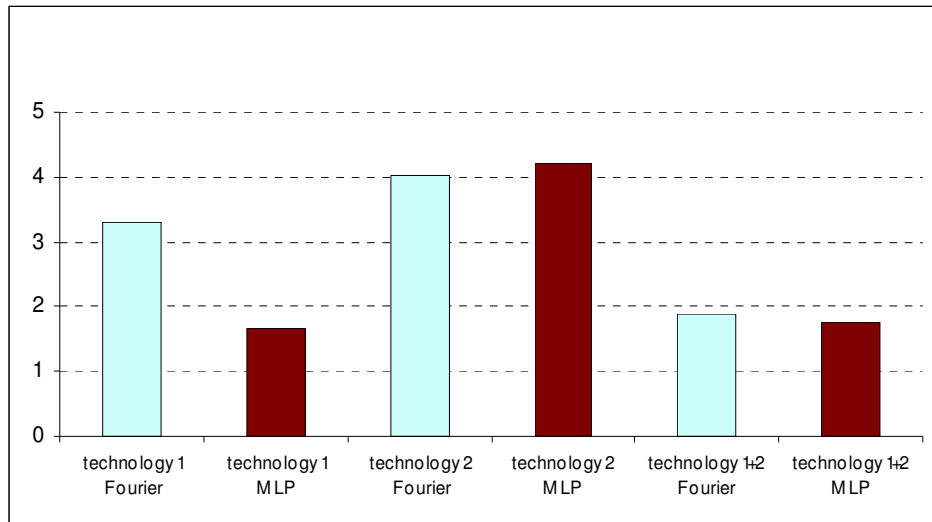
Figure 1. Ratio of RMSE for elasticities.

According to the results displayed in Table 1, MPL results best in all cases indicating that ANN better approximates the underlying technologies than both the Translog and Fourier functional forms corresponding to a local and global approximation. But this superiority to traditional approaches is highly marked when data are generated from only one technology. For the case of multiple technologies, the differences between Translog and Fourier results are negligible. For the best MLP (one hidden layer with 30 neurons), the performance is very important (197.73614) compared to the cases of one technology (8.6612E-12 and 2.30713E-12), but RMSE for the total cost function and all input demands remain slightly lower than those obtained for other two functional forms. This last result is a little surprising since one expects a clear predominance of the neural networks, especially for the mixed technology, since they are known to be better suited to modeling complex relationships.

ANN is a non-parametric approach. To compute input elasticity of substitution, we use the procedure based on the usual definition of elasticity itself developed by Gruca and Klemz [25]. To measure the effect for one input, say j, we first set all others prices to their sample mean levels. Then, estimate input demands using the trained neural networks and estimate elasticities at every observed level of p_j using the standard elasticity formula:

$$\varepsilon_{ij} = \frac{\Delta x_i}{\Delta p_j} \times \frac{p_j}{x_i} \qquad i, j = 1,2,3. \qquad (19)$$

In Figure 1, we represent average RMSE for all elasticities of substitution calculated for the Fourier functional form and the Neural networks approximation, divided by the average RMSE for the Translog form. We notice that the Translog approximate batters the true technology in both single and mixed technology. It is possible that the disappointing results obtained in the case of the neural networks approximation come from the procedure used to calculate elasticities.

5. Conclusion

This paper evaluates and compares the performances of ANN approximation, Translog and Fourier functional forms, for the cost function, the derivative input demands and the corresponding input elasticities of substitution, when data are generated from one or mixed technologies. Results show that neural networks better approximate the underlying technologies than the other two functional forms in all cases.

However, when elasticities of substitution are calculated, the Translog approximate batters the true technology in both single and mixed technology. This result is a little surprising since one expects that ANNs perform better that traditional functional forms. The author thinks that neural networks can be a useful but complementary tool for this type of analysis. Further research should, first, develop another method for estimating elasticity of substitution when using ANNs, and second, provide a comparison in term of violations of the regularity conditions for the cost function since those conditions guarantees the maintained hypothesis and validate duality theory that produces the estimated models.

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