

# A fuzzy approach to solve a multi-objective linear fractional inventory model

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## **Abstract**

In this paper, the researchers present a multi-objective linear fractional inventory model using fuzzy programming. The proposed method in this paper is applied to a problem with two objective functions. The resulted fuzzy model is transformed into an ordinary linear programming. The implementation of the developing model presented in this paper is demonstrated through the use of some numerical examples.

**Keywords:** Multi-objective linear fractional programming; Inventory model; Fuzzy programming; Min operator

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## **1. Introduction**

Inventories are normally idle materials that are waiting to be used. For most organizations, the expense associated with financing inventories is a substantial part of the cost of doing business [2]. In order to effectively manage the inventories, the primary question is to determine optimal lot size. For many years, classical Economic Order Quantity (EOQ) problem has been solved in different forms and under various assumptions [1, 2, 15, 16, 18, 21, 22]. Classical inventory model assumes the ideal circumstances where demand is deterministic. However, many real case problems deal with the stochastic problems (e.g. [15, 16]). The concept of Multi-Objective Programming (MOP), on the other hand, has become popular among researchers during the past few years due to the fact that many single objective optimization methods are not able to help practitioners reach desirable solutions [6, 9, 10, 11]. The concept of MOP combined with fractional programming is an interesting area of research which incorporates many production planning applications [19, 20]. In this paper the researchers present a multi-objective problem formu-

lation with fractional terms in each objective function. The resulted model is then transformed into a multi-objective linear programming. The researchers provide a lower limit for each objective function by considering each one separately. Finally, the Zimmermann's min values concept [26] is implemented to transform the multi-objective function into an ordinary linear programming. There are different methods for solving Multi-Objective Linear Fractional Programming Problem (MOLFPP). Kornbluth and Steuer [13] develop an algorithm which chooses an efficient solution from different effective solutions. They also present a general method to solve goal programming when we have linear fractional objectives. MOLFPP has also been studied by other people (e.g. [6, 20]). However, their approaches tend to be complicated in terms of computational complexity. In order to resolve the complexity which appears on the solution of MOLFPP, many have tried to combine the concept of fuzzy programming with it [10, 11, 19]. Luhandjula [19] in his implementation uses linguistic variables in order to show the aspiration level of decision maker. Duta et. al. [11] criticizes his approach since in his method compensatory

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operator is implemented for the union of the membership function which does not guarantee the optimality of the model. Lee and Tcha [29] present a method to solve the same problem which ended up having a highly non-linear non-convex problem. Such a problem is often very complicated to solve specially for large number of variables. Charles and Cooper [5] also develop a method to solve MOLFP using an iterative approach which reaches the global solution after several iterations. In this paper the researchers attempt to present a new method where the resulted optimization problem is linear and reaches the global solution in only two stages by transforming MOLFP into a Linear Programming problem (LP) which can be easily solved. This paper is organized as follows. The researchers first present some of the necessary definitions. Then the problem formulation of the resulted model is presented in the context of an inventory problem. Later on, the implementation of the proposed method of this paper is demonstrated in the following section using a numerical example. Finally conclusion remarks are given at the end to summarize the contribution of this paper.

## 2. Problem formulation

In this section, the researchers explain some of the necessary definitions and the transformation which is needed to implement for their proposed case study. Also, the transformation they need to prepare a multi-objective into an ordinary linear programming based on the algorithm developed by Chakraborty and Gupta[3] is presented in this section.

**Definition 1. The linear fractional programming.** A general linear fractional programming (LFP) can be formulated as the following form,

$$\max \left\{ \frac{c'x + \alpha}{d'x + \beta} \mid Ax = b, \quad x \geq 0 \right\}, \quad (1)$$

where  $x, c, d \in R^{n \times 1}$ ,  $A \in R^{m \times n}$ ,  $b \in R^{m \times 1}$  and  $\alpha, \beta \in R$ . For some values of  $x$ ,  $d'x + \beta$  may be equal to zero. Therefore, we need to make an additional assumption.

**Assumption 1.** If  $Ax=b$  and  $x \geq 0$  then  $d'x + \beta > 0$  or  $d'x + \beta < 0$ .

**Definition 2. Multi-objective linear fractional programming problem.** A general multi-objective linear fractional programming (MOLFPP) can be defined as the following form,

$$\max \quad z(x) = \{z_1(x), \dots, z_n(x) \mid Ax \leq b \}, \quad (2)$$

where  $x \in R^{n \times 1}$ ,  $A \in R^{m \times n}$ ,  $b \in R^{m \times 1}$  and

$$z_i(x) = \frac{c'_i x + \alpha_i}{d'_i x + \beta_i}, \quad c_i, d_i \in R^{n \times 1} \text{ and } \alpha_i, \beta_i \in R.$$

## 3. Transformation of MOLFPP to linear programming (LP)

Charnes and Cooper [4, 5] develop a programming model for a fractional function. Craven gives a discussion about fractional programming [7, 8]. Kornbluth and Steuer [13, 14, 23, 24] develop a goal programming approach which consists of linear fractional criteria. In this section the researchers use the procedure of transformation of MOLFPP into a linear programming problem developed by Chakraborty and Gupta [3]. The researchers also use Zimmermann's min operator [26] to solve deterministic multi-objective linear programming problem presented in the following section.

### 3.1. Modeling Transformation

In this section the researchers describe the procedure for transforming MOLFPP to a linear programming problem.

*Step 1.* Let  $z_i(x) = \frac{c'_i x + \alpha_i}{d'_i x + \beta_i} = \frac{N_i(x)}{D_i(x)}$ . Let  $I$  be the

index set such that  $I = \{i \mid N_i(x) \geq 0, \text{ for some } x\}$  and  $I^c = \{i \mid N_i(x) < 0, \text{ for some } x\}$ .

*Step 2.* Let  $y=tx$  with  $t \geq 0$ , therefore, using the procedure explained by Chakraborty and Gupta [3] we have,

$$\max \quad \left\{ (tN_i(\frac{y}{t})) \quad i \in I, \quad (tD_i(\frac{y}{t})) \quad i \in I^c \right\}$$

Subject to:

$$tD_i(\frac{y}{t}) \leq 1, \quad i \in I$$

$$\begin{aligned}
 -tN_i\left(\frac{y}{t}\right) &\leq 1, & i \in I^c \\
 A\left(\frac{y}{t}\right) - b &\leq 0, \\
 t &\geq 0, & y \geq 0.
 \end{aligned}
 \tag{3}$$

Step 3. Using Zimmermann’s min operator [26] to solve the deterministic multi-objective linear programming problem yields,

max  $\lambda$   
 Subject to:

$$\begin{aligned}
 \mu_i(t.N_i\left(\frac{y}{t}\right)) &\geq \lambda & \text{for } i \in I \\
 \mu_i(t.D_i\left(\frac{y}{t}\right)) &\geq \lambda & \text{for } i \in I^c \\
 t.D_i\left(\frac{y}{t}\right) &\leq 1 & \text{for } i \in I \\
 -t.D_i\left(\frac{y}{t}\right) &\leq 1 & \text{for } i \in I^c \\
 A\left(\frac{y}{t}\right) - b &\leq 0 & t \geq 0, y \geq 0,
 \end{aligned}
 \tag{4}$$

where

$$\mu_i(t.N_i\left(\frac{y}{t}\right)) = \begin{cases} 0 & \text{if } (t.N_i\left(\frac{y}{t}\right)) \leq 0 \\ \frac{t.N_i\left(\frac{y}{t}\right) - 0}{\tilde{z}_i - 0} & \text{if } 0 < (t.N_i\left(\frac{y}{t}\right)) \leq \tilde{z}_i \\ 1 & \text{if } (t.N_i\left(\frac{y}{t}\right)) > \tilde{z}_i \end{cases} \tag{5}$$

and

$$\mu_i(t.D_i\left(\frac{y}{t}\right)) = \begin{cases} 0 & \text{if } (t.D_i\left(\frac{y}{t}\right)) \leq 0 \\ \frac{t.D_i\left(\frac{y}{t}\right) - 0}{\tilde{z}_i - 0} & \text{if } 0 < (t.D_i\left(\frac{y}{t}\right)) \leq \tilde{z}_i \\ 1 & \text{if } (t.D_i\left(\frac{y}{t}\right)) > \tilde{z}_i \end{cases} \tag{6}$$

The reason for using *min* operator compared with two other operators called “*and* and *product*” to identify the intersection of an attribute in a statistical population is due to research accomplished by Thole et. al. [ 24]. Zimmerman and Zysno [27] in another work show similar results on using *min* operator.

Let’s assume that  $I, I^c, i$  and  $N_i(x)$  are known for  $i=1, \dots, k$ . However, suppose we only know that the denominators are positive in the feasible region, in order to find the index sets and the maximum aspiration levels

$\tilde{z}_i$ , we need to take the following steps,

1. Maximize each objective function  $z_i(x)$  subject to the given set of constraints. Let  $z_i^*$  be the optimal value for  $z_i(x)$  for  $i=1, \dots, k$ .
2. Examine the nature of  $z_i^*$  for all values of  $i$ , (e.g. if  $z_i^* \geq 0$  then  $i \in I$  and if  $z_i^* < 0$  then  $i \in I^c$ ).
3. Similarly we use  $\tilde{z}_i = z_i^*$  if  $i \in I$  and  $\tilde{z}_i = -\frac{1}{z_i^*}$  when  $i \in I^c$ .

#### 4. Case study

Many realistic inventory problems deal with more than one objective function which may be in conflict with each other. In this section, the researchers study an inventory problem model where they are simultaneously interested in maximizing the profit ratio to holding cost and minimizing the back orders ratio to total ordered quantities. For the sake of simplicity, the researchers assume there is only one period in the cycle time. Finally they consider two constraints in our modeling formulation: warehouse space and budget. Therefore we have,

$$\begin{aligned}
 \max Z_1 &= \frac{\sum_{i=1}^n (S_i - P_i)Q_i}{\sum_{i=1}^n \frac{h_i Q_i}{2}}, \\
 \min Z_2 &= \frac{\sum_{i=1}^n (D_i - Q_i)}{\sum_{i=1}^n Q_i},
 \end{aligned}
 \tag{7}$$

Subject to:

$$\begin{aligned}
 \sum_{i=1}^n P_i Q_i &\leq B, \\
 \sum_{i=1}^n f_i Q_i &\leq F,
 \end{aligned}$$

where:

$Q_i$  : Ordering quantity of product  $i$ .

$P_i$  : Purchasing price of product  $i$ .

$S_i$  : Selling price of product  $i$ .

$h_i$  : Inventory holding cost of product  $i$ .

$D_i$  : Demand for product  $i$  per unit time.

$B$  : Maximum available budget.

$f_i$  : Space required for product  $i$ .

$F$  : Maximum available space.

We now demonstrate the implementation of the proposed application with some numerical example. Consider the following information,

**Table 1.** The input information for the numerical example.

Item	$h_i$	$p_i$	$S_i$	$D_i$	$f_i$	B	F
i=1	20	600	650	8000	1	400000	240
i=2	24	720	750	4000	2	400000	240

Based on the information of table 1, The MOLF inventory problem with two objective functions is summarized as follows,

$$\begin{aligned} \max \quad & z_1 = \frac{50Q_1 + 30Q_2}{10Q_1 + 12Q_2} \\ \min \quad & z_2 = \frac{8000 - Q_1 + 4000 - Q_2}{Q_1 + Q_2} \end{aligned}$$

Subject to: (8)

$$\begin{aligned} 600Q_1 + 720Q_2 &\leq 400000, \\ Q_1 + 2Q_2 &\leq 240, \\ Q_1, Q_2 &\geq 0. \end{aligned}$$

We may observe that  $z_1(x) \geq 0$  for all  $x$  in feasible region and  $z_2(x) < 0$  for some  $x$  in feasible region using the transformation,  $y=tQ$ . Problem (8) can be also formulated to the following MOL inventory problem.

$$\begin{aligned} \max \quad & f_1(y,t) = 50y_1 + 30y_2 \\ \max \quad & f_2(y,t) = y_1 + y_2 \end{aligned}$$

Subject to:

$$10y_1 + 12y_2 \leq 1$$

$$\begin{aligned} -y_1 - y_2 - 12000t &\leq 1 \\ 600y_1 + 720y_2 - 400000t &\leq 0 \\ y_1 + 2y_2 - 240t &\leq 0 \\ y_1, y_2, t &\geq 0. \end{aligned}$$

Let,  $f_1(y,t) \geq 5$  and  $f_2(y,t) \geq 0.1$ . The resulted fuzzy model (9) can be reduced to the following crisp model using the membership functions defined in [5, 6].

$$\max \lambda$$

Subject to:

$$\begin{aligned} 50y_1 + 30y_2 - 5\lambda &\geq 0 \\ y_1 + y_2 - 0.1\lambda &\geq 0 \\ 10y_1 + 12y_2 &\leq 1 \\ -y_1 - y_2 + 12000t &\leq 1 \\ 600y_1 + 720y_2 - 400000t &\leq 0 \\ y_1 + 2y_2 - 240t &\leq 0. \end{aligned}$$

The Optimal solution of (10) is as follows,  
 $\lambda^* = 0.2041, \quad t^* = 0.0001, \quad y_1^* = 0.0204, \quad y_2^* = 0.$

Therefore we have,

$$Q_1^* = 204, \quad Q_2^* = 0, \quad z_1^* = 5, \quad z_2^* = 57.82.$$

### 5. Conclusions

The researchers have presented a method to solve multi-objective linear fractional inventory problems based on set theoretic approach and fuzzy programming. Using a transformation, a multi-objective linear fractional inventory model has been changed into a linear programming problem. The implementation of the researchers' methodology has been shown using a numerical example. The inventory model explained in this paper can be extended to some more complicated inventory problems such as models with deteriorations, discount, dynamic demand, replenishment, etc.

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