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Quantitative models for determining prices in a remanufacturing system with exclusive and competitive market structure

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Abstract: Remanufacturing is an industrial process that makes used products reusable. Remanufacturing is a way to establish a closed-loop supply chain. One of the important aspects in both reverse logistics and remanufacturing is pricing of returned and remanufactured products (called cores) that it has been noticed in this paper. In addition, in this paper the researchers have tried to present a mathematical model that indicates prices and inventories in a closed-loop supply chain in an exclusive market. This model has argued on acquisition price of cores and remanufactured cores. Also, in the following the researchers essay discuss about acquisition price of cores in the competitive market via fuzzy rules. Numerical results demonstrate that appropriate values of the prices are obtained by these models.

Keywords: *Reverse logistics; Remanufacturing; Pricing; Mathematical model; Fuzzy rule*

1. Introduction

Reverse Logistics has been an area of receiving attention during the last decade in real-world and in academia as its economic impact has been increasingly important and as legislation has becoming stricter. The Reverse Logistics Executive Council provides the following widely accepted definition of Reverse Logistics: The process of planning, implementing, and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin for the purpose of recapturing value or of proper disposal (Rogers and Tibben-Lembke, 1999). We can divide the RL issue into different categories regarding various methods of product recovery, such as Reuse, Remanufacture, and Recycle.

Remanufacturing is one of the recovery options for used products. It involves activities that make remanufactured products or major modules be marketable again and potentially as good as new. Product remanufacturing has developed rapidly in recent decades due to the intensified environmental legislations and economic concerns. Through re-manufacturing, products/ components that would otherwise head to land-fill or incineration will instead go through a set of values and material recapturing processes, including distribution, inspection, disassembly, repair, redistribution, and remarketing or recycling. Remanufacturing allows reusable components and recoverable materials reenter the supply chain for future reuse or new product fabrication (Zhang *et al.*, 2004).

Remanufacturing of used products is not a new term but the scale and unique processes have made remanufacturing an important topic in research. Especially, the acquisition of used products, called cores, for remanufacturing becomes an important issue (Guide and Van Wassenhove, 2001). Product acquisition is one of the few areas that management can proactively influence and, as a result, determine whether reuse activities will be economically attractive. Product acquisition is a common problem for companies offering remanufactured products in a dynamic market, where supply and demand change rapidly on a global scale. Due to the increasing pressure from the legislations, inherent value cores are being collected for parts and material recovery. Before considering any recovery options for the cores, the cores must be acquired regardless of their future recovery options. In order to attract the returns of cores, certain incentive has to be offered. Therefore, one of the important aspects in remanufacturing is pricing of returned and remanufactured products (called cores). A successful remanufacturing firm must carefully manage its product acquisition process, i.e., buy the right quantities of the right qualities for the right prices, so as to maximize profits In this paper, the researchers focus on pricing the cores and remanufactured cores.

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There is an important aspect in determination of prices that is market structure. In this paper two structures are in mind: a market with a large number of rivals (competitive market) and a market with a limited number of rivals (exclusive market). In the first market the competitors are considered as determiners of prices, but in the latter one we are the determiner ourselves. We propose a mathematical model and a fuzzy rule base for pricing in the exclusive and competitive markets respectively. This paper is organized as follows: related literature is reviewed in Section 2 and the proposed mathematical model and the fuzzy rules are given in Sections 3 and 4, respectively. Numerical results are presented in Section 5. The paper is concluded in Section 6.

2. Literature review

According to Guide (2000), product acquisition management acts as an interface between reverse logistic activities and production planning and control activities for firms. There are two most commonly used product acquisition systems: waste stream system and the market driven system (Guide, 2000; Guide and Pentico, 2003). In waste stream, the firms encouraged by the legislation passively accept all product returns from the waste stream. On the other hand, market-driven system employs financial incentives to encourage users to return their products to the firm. Several different forms of financial incentives are used by firms in market-driven system including deposit systems, cash paid for a specified level of quality, and the credit toward a new unit. The implementation of different forms of financial incentives and their impact on the performance of RL activities are the main research issues in product acquisition management literature. Klausner and Hendrickson (2000) present an implementation of buy-back programs in power-tools industry.

Guide *et al.* (2003), Aras and Aksen (2008) and Aras *et al.* (2008) determine optimal incentive values under a quality-dependent incentive policy. Aksen *et al.* (2009) extend *Aras et al.* (2008) by considering a governmentally subsidized collection system. Wojanowski *et al.* (2007) have investigated the use of a deposit refund system which requires payment of a certain deposit at the time of purchase, which is refunded upon the return of the used product. Kaya (2010) determines the optimal incentive value in case of stochastic demand and partial substitution between original and remanufactured products. Ostlin *et al.* (2008) have investigated seven different types of closed-loop relationships for the acquisition of used products. The relationships identified are ownership-based, service contract, direct-order, deposit-based, credit-based, buy-back and voluntary - based relationships.

The pricing problem is associated with the specific operation of remanufacturing, instead of a generic pricing policy. Researchers have also been focusing on the study of remanufacturing for a profit (Thierry *et al.,* 1995). The literature on the acquisition pricing of the used products and the sales pricing of the remanufactured parts/products is rather scarce. The effectiveness of incentive mechanisms used to facilitate the collection of used products is crucial to the success of a product recovery program, and both the structure and amount of incentive required to achieve the desired rate of product recovery are important. Although the need has been identified in several studies (Guide and Van Wassenhove, 2001; Guide, 2000; De Brito *et al.*, 2002), the number of analytical models to support this decision is relatively few. Klausner and Hendrickson (2000) have presented a simple mathematical model that can be used to estimate the optimal buy-back price for power tools for Bosch GmbH. Guide *et al.* (2003) studied the remanufacturing of cellular phones and argued that the prices for used phones of various quality levels should consider remanufacturing requirements as the quality of returned used phones can differ significantly. Ray *et al*. (2005) have developed the optimal pricing or trade-in strategy for remanufacturable products considering the durability and the age distribution of products in use. Decision on core prices can be based on the deterministic remanufacturing cost for specific quality cores and the price of core products.

Harrison (1985), Ito (2004), Sigman (2005), and Beichelt (2006) have presented on GBM, its characteristics and applications in different market settings. In the investment market, the stock price is a Geometric Brownian Motion (GBM). We believe that the major mathematical studies on pricing at remanufacturing have been done by: Mitra (2007), Vorasayan and Ryan (2006), Liang *et al.* (2007), and Guide, Teunter and Wassenhove (Guide *et al.*, 2003). In Table 1 we have compared four models that are introduced by the mentioned researchers.

3. Exclusive market: Pricing via mathematical model

In this section, the researchers propose a non-linear mixed integer mathematical model to optimize acquisition price of cores, remanufactured cores, and inventory in the exclusive markets. In this market, we assume that there are a limited number of

Table 1: Major studies in pricing at remanufacturing.

Author	Objective function	Deterministic/Probabilistic
Mitra	Income maximization	Probabilistic
Assumptions: 1) The demand is linear function of price, 2) The inventories are divided into: manufactured product, remanufactured product, 3) The quality of remanufactured product are classified into two levels, 4) The level of inventories is determined, 5) Product will be sold entirely, 6) The amount of sale is a decreasing function of price, 7) The unsold remanufacture product can be sold in lower price.		

Author	Objective function	Deterministic/Probabilistic
Vorasayan and Ryan	Profit maximization	Probabilistic
Assumptions: 1) The rate of return has Poisson distribution, 2) Rebuilt time has exponential distribution, 3) Inspection time has exponential distribution, 4) Production time has exponential distribution, 5) Modeling with open queue network.		

Author	Objective function	Deterministic/Probabilistic
Liang, Pokharel and lim	Profit maximization	Deterministic
Assumptions: 1) There is a time interval [0, T] between core entrance and core product exit, 2) The sale price has GBM pattern, 3) The entire purchased core can be used for remanufacturing, 4) There are disassembly, inspection, quality assurance, logistic, purchased core and remanufacturing cost.		

Author	Objective function	Deterministic/Probabilistic
Guide, Teunter and Wassenhove	Profit maximization	Deterministic
Assumptions: 1) There is a balance between demands and returns, 2) The rate of return is a function of purchase price, 3) Demand is a function of sale price, 4) The number of quality classes is confined, 5) Complete testing, 6) There is no fixed cost, 7) The rate of return depends on the rate of sale, 8) Model is considered for single period, 9) There are no constraints for demand and supply.		

rivals; therefore, we can determine the prices individually. Our model takes advantage of combining pricing of cores and remanufactured cores with an inventory control. While suppliers compete with suppliers, factories with factories, distributors with distributors and retailers with retailers conventionally, the researchers have changed this approach in their model. In other words, competition in the market is not between companies but between supply chains. For instance, they consider whole supply chain instead of only each individual element.

The model consists of a supplier, factories $\{1,2,...,M\}$, a collection and disassembly center, a distributor (central warehouse) and customers. It is remarkable that returned products (cores) are shipped from these three channels: manufacturer, distributor, and customer. It means that a product may be returned after either used by the customer or defected in production or distribution processes. There is one centralized facility for collection and disassembly of cores to different factories. In this model, cores are sent to collection and disassembly center and sorted according to quality. Cores can be completely disassembled to primal parts.

The central warehouse receives the demand for remanufactured cores that the factories have manufactured. Each factory can manufacture either all or a part of the demand. For each product the bill of materials is known. Factories acquire the materials and the parts from supplier, collection and disassembly center. They should determine the quantity of materials that are purchased at each period. Having been produced in the factories, remanufactured cores are sent to the central warehouse.

Upon reception, remanufactured cores are sent to customers. The central warehouse contains only the final products, but it is allowed to be stock out in the case when products are not sufficient to meet the demand. The researchers assume there is stock out costs. The model here considers that cores are returned to the collection and disassembly center, where they will be classified based on their quality. In this model, the researchers examine the case of remanufacturer which acquires cores in k quality classes. The first class has high quality and the last class has low quality. With respect to their quality, they will be disassembled and the resulted parts are prepared to be shipped to the factories, where they will be used by remanufactures.

The production time is affected by uncertainty in quantity and quality of cores. Therefore, one product can be produced by different processes. Cores are collected from customers via paying acquisition price f_{itk} per used product i in quality group k at period t to final user. The researchers present the supply for cores with quality group k from customer channel as a linear function of acquisition price paid to customer, and thus we have $CF_{itk} = \alpha_{itk} + \beta_{itk} f_{itk}$, where $\alpha_{itk} \geq 0, \beta_{itk} > 0$. Also, the researchers model the demand for remanufactured cores as a linear function of the corresponding price, and thus they have $d_{it} = a - b \times p_{it}$ where $a, b > 0$. The model discuses the following decisions:

a) Pricing of the cores and the remanufactured cores.
b) Determining the amount of inventories.
c) Determining the amount of purchased materials from suppliers.
d) Determine the process for producing.

The following indexes are used in the proposed model:

a: the number of manufacturing processes; $\{1,2,..., A\}$

i: the number of product types; $\{1,2,..., N\}$

j: the number of factories; $\{1,2,..., M\}$

t: the number of programming periods; $\{1,2,...,T\}$

k: the number of quality groups; $\{1,2,..., K\}$

p: the number of materials or parts; $\{1,2,..., P\}$

l: the number of collection centers; $\{1,2,..., L\}$

The researchers consider the following parameters:

C_{aijt} : Cost of a unit production i in factory j by process a at period t

MR_{aip} : Number of the material (part) p for producing a unit of product i by process a

H_{ij} : Cost of holding a unit of remanufactured core i in factory j

W_{it} : Cost of holding a unit of remanufactured core i in central warehouse at period t

L_{jpt} : Cost of holding a unit of material (part) p in factory j at period t

S_{it} : Cost of shortage a unit of remanufactured core i at period t

R_{jpt} : Cost of purchasing a unit of material (part) p in factory j at period t

TR_{ijt} : Cost of transporting a unit of remanufactured core i shipped from factory j to central warehouse at period t

SS_{ij} : Safety stock of remanufactured core i in factory j

V_{jp} : Safety stock of material (part) p in factory j

U_{jt} : Available time for remanufacturing in factory j at period t

PT_{ai} : Required time for producing a unit of remanufactured core i by process a

B_{ij} : Maximum inventory of remanufactured core i in factory j

CF_{ip} : Number of material (part) p that obtained from core i

MI_{jp} : Maximum inventory of material (part) p in factory j

G_{pjt} : Cost of transporting a unit of material (part) p from collection and disassembly canter to the factory j at period t

HC_{pt} : Cost of holding a unit of material (part) p at period t in collection and disassembly center

d_{it} : Demand of remanufactured core i at period t ($d_{it} = a - b \times p_{it}$)

Ca_p : Holding capacity of material (part) p in collection and disassembly canter

π_{ipk} : Probability of obtaining a unit of material (part) p through disassembling a unit of core i with quality k

DC_{iptk} : Cost of disassembling material (part) p from core i at period t with quality k

f'_{itkl} : Acquisition price of core i with quality k at period t from collection center l

F_{tl} : Cost of changing the sale place of core from current center to center l at period t

CF_{itk} : Number of returned core i by customer to collection and disassembly center with quality k at period t ($CF_{itk} = \alpha_{itk} + \beta_{itk} f_{itk}$)

DF_{itk} : Number of returned core i by central warehouse to collection and disassembly center with quality k at period t

MF_{ijtk} : Number of returned core i by manufacturer to collection and disassembly center with quality k at period t

RAV_{itk} : Remaining added value of returned core i with quality k at period t

The researchers consider the following variables:

X_{aijt} : Number of remanufactured core i produced in factory j by process a at period t

XP_{jpt} : Amount of purchased material (part) p for using in factory j at period t

XQ_{ijt} : Number of remanufactured core i shipped from factory j to central warehouse at period t

II_{ijt} : Amount of remanufactured core i inventory in factory j at the end of period t

M_{jpt} : Amount of material (part) p inventory in factory j at the end of period t

YB_{it} : Amount of remanufactured core i shortage at period t in central warehouse

YI_{it} : Amount of remanufactured core i surplus at period t in central warehouse

O_{jpt} : Amount of material (part) p shipped from collection and disassembly center to the factory j at period t

IIN_{pt} : Amount of material (part) p inventory at period t in collection and disassembly center

f_{itk} : Acquisition price of core i with quality k at period t

p_{it} : Price of remanufactured core i at period t

In this model we have three objective functions: maximization of both revenue and remaining added value of returned cores by customer, and minimization of costs. Therefore, the objective function is proposed as fallowed:

Minimize Z = -W1 (remaining added value of returned cores) - W2 (revenue) + W3 (costs)

Which W1 is the weight of the first objective function (remaining added value of returned cores), W2 is the weight of the second objective function (revenue), and W3 is the weight of the third objective function (costs).

$$
\begin{aligned}
Min\ Z = & -W_1[\sum_{i=1}^{n}\sum_{t=1}^{T}\sum_{k=1}^{K} CF_{itk} RAV_{itk} \\
& -W_2[\sum_{i=1}^{n}\sum_{t=1}^{T}(d_{it} - YI_{it})p_{it}] \\
& +W_3[\sum_{a=1}^{A}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{t=1}^{T} C_{aijt} X_{aijt} \\
& +\sum_{t=1}^{T}\sum_{i=1}^{n}\sum_{p=1}^{P}\sum_{k=1}^{K}\pi_{ipk}(CF_{itk} + DF_{itk} + \sum_{j=1}^{m} MF_{ijtk}) \\
& DC_{iptk} + \sum_{i=1}^{n}\sum_{t=1}^{T}\sum_{k=1}^{K} CF_{itk} f_{itk} \\
& +\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{t=1}^{T} H_{ij} II_{ijt} + \sum_{i=1}^{n}\sum_{t=1}^{T} S_{it} YB_{it} \\
& +\sum_{i=1}^{n}\sum_{t=1}^{T} W_{it} YI_{it} + \sum_{j=1}^{m}\sum_{p=1}^{P}\sum_{t=1}^{T} R_{jpt} XP_{jpt} \\
& +\sum_{j=1}^{m}\sum_{p=1}^{P}\sum_{t=1}^{T} L_{jpt} M_{jpt} + \sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{t=1}^{T} TR_{ijt} XQ_{ijt} \\
& +\sum_{j=1}^{m}\sum_{p=1}^{P}\sum_{t=1}^{T} G_{jpt} O_{jpt} + \sum_{p=1}^{P}\sum_{t=1}^{T} HC_{pt} IIN_{pt}]
\end{aligned} \tag{1}
$$

The researchers consider following constraints:

(1) Inventory equation for materials (part) in collection and disassembly center

$$
\begin{aligned}
IIN_{pt} = & \\
& IIN_{p(t-1)} + \sum_{i=1}^{n}\sum_{k=1}^{K}\pi_{ipk}(\alpha_{itk} + \beta_{itk} f_{itk}) \\
& CF_{ip} + \sum_{i=1}^{n}\sum_{k=1}^{K}\pi_{ipk}(DF_{itk} + \sum_{j=1}^{m} MF_{ijtk})CF_{ip} \\
& -\sum_{j=1}^{m} O_{jpt} \quad ;\ p = 1,\ldots,P\ ,\ t = 1,\ldots,T
\end{aligned} \tag{2}
$$

(2) Control of shipment

$$
\sum_{j=1}^{m} O_{jpt} \le IIN_{p(t-1)}\ ; \quad p = 1,\ldots,P\ ,\ t = 1,\ldots,T \tag{3}
$$

(3) Inventory capacity for materials (parts) in collection and disassembly center

$$
IIN_{pt} \le Ca_p \quad ; p = 1,\ldots,P\ ,\ t = 1,\ldots,T \tag{4}
$$

(4) Inventory equation for materials (parts) in factory

$$
\begin{aligned}
M_{jpt} = & M_{jp(t-1)} + XP_{jpt} \\
& -\left[\sum_{i=1}^{n}\sum_{a=1}^{A} MR_{aip} X_{aijt}\right] + O_{jpt} \\
& j = 1,\ldots,m\ ,\ t = 1,\ldots,T\ ,\ p = 1,\ldots,n
\end{aligned} \tag{5}
$$

(5) Inventory capacity for materials (parts) in factories

$$
V_{jp} \le M_{jpt} \le MI_{jp}\ ; \quad j = 1,\ldots,m\ ,\ t = 1,\ldots,T\ ,\ p = 1,\ldots,P \tag{6}
$$

(6) Control of materials (parts)

$$\sum_{a=1}^{A}\sum_{i=1}^{n} MR_{aip} X_{aijt} \le M_{jp(t-1)} + XP_{jpt} \quad (7)$$

$j = 1,...,m$, $t = 1,...,T$, $p = 1,...,P$)

(7) Inventory equation remanufactured cores

$$II_{ijt} = II_{ij(t-1)} + \sum_{a=1}^{A} X_{aijt} - XQ_{ijt} - \sum_{k=1}^{K} MF_{ijtk} \quad (8)$$

$i = 1,...,n$, $j = 1,...,m$, $t = 1,...,T$

(8) Shipment control

$$\sum_{a=1}^{A} X_{aijt} + II_{ij(t-1)} \ge XQ_{ijt} + \sum_{k=1}^{K} MF_{ijtk} \quad (9)$$

$i = 1,...,n$, $j = 1,...,m$, $t = 1,...,T$

(9) Inventory capacity for remanufactured cores in factory

$$SS_{ij} \le II_{ijt} \le B_{ij} \quad (10)$$

$i = 1,...,n$, $j = 1,...,m$, $t = 1,...,T$

(10) Stock out or inventory units at central warehouse

$$YI_{i(t-1)} + YB_{i(t-1)} + \sum_{j=1}^{m} XQ_{ijt} - d_{it} - \sum_{k=1}^{K} DF_{itk} = \quad (11)$$

$YI_{it} + YB_{it}$; $i = 1,...,n$, $t = 1,...,T$

(11) Production capacity

$$\sum_{a=1}^{A}\sum_{i=1}^{n} PT_{ai} X_{aijt} \le U_{jt} \quad (12)$$

$j = 1,...,m$, $t = 1,...,T$

(12) Maximum of acquisition price

$$f_{itk} \le f'_{itkl} + F_{tl}$$

$i = 1,...,n$, $t = 1,...,T$ (13)

$, k = 1,...,K$, $l = 1,...,L$

(13) Minimum of price of remanufactured core

$$p_{it} \ge \sum_{p=1}^{P} R_{jpt} + max\{C_{aijt}\} + TR_{ijt} \quad (14)$$

$i = 1,...,n$, $t = 1,...,T$, $j = 1,...,m$

In pricing of cores with various qualities, we have to pay attention to the fact that the vendor of cores can sell these products to collection centeres other than ours, which their purchasing price is higher than ours. However, this selection has some cost for vendor like transportation costs to more distant collection centers, etc. So, our price of cores should be less than prices of other vendors along with the cost of changing the sale place from our center to their centers (F_{tl}). This subject is presented in constraint 12. Preventing garrulity, the researchers ignore explaining other constraints

4. Competitive market: Pricing via fuzzy rules

In this section, the researchers are going to propose a method for determining acquisition price of core in a competitive market. In competitive market, prices are determined by rivals. In the other words, prices are deducted from collected data of market. So, in this kind of market, the researchers only reveal the prices not set them. Therefore, knowledge of price determination exists in the market that the researchers are going to discover it. This knowledge could be presented as set of if-then rules. The uncertainty of collected data will approach us to fuzzy if-then rules for determining of prices. To continue, fuzzy rule generation is discussed.

4.1. Fuzzy rules

In this paper the researchers use the following type of fuzzy rule for a multi-input and single-output system:

$$R^i : if \ x_1 \ is \ A_1^i \ and \ x_2 \ is \ A_2^i ... \ and \ x_n \ is \ A_n^i \quad then \ y \ is \ B_i$$

Where R^i is the *i*th rule ($1 \le i \le m$), x_j ($1 \le j \le n$) are input variables, and y is the output. Also, A_j^i and B^i are fuzzy numbers. We can rewrite the above rule as:

$$R^i : if \ x \ is \ A^i \ then \ y \ is \ B_i$$

Where

$x = (x_1, x_2, \ldots, x_n)$ and

$A = (A_1, A_2, \ldots, A_n)$.

The membership function of A_j^i is denoted $A_j^i(\bullet)$. For reasoning, we may use the following steps:

1) Given the inputs $x_1^0, x_2^0, \ldots,$ and x_n^0, calculate the degree of mach, w^i, in the premises for the ith rule, $1 \le i \le m$ as:

$$w^i = A_1^i(x_1^0) \times A_2^i(x_2^0) \times \ldots \times A_n^i(x_n^0) \qquad (15)$$

2) Then defuzzify B^i in the consequents by taking the center of gravity:

$$b^i = \int B^i(y)ydy \Big/ \int B^i(y)dy \qquad (16)$$

3) Calculate the inferred value, $\hat{y}$, by taking the weighted average of b^i with respect to w^i:

$$\hat{y} = \sum_{i=1}^{m} w^i b^i \Big/ \sum_{i=1}^{m} w^i \qquad (17)$$

Where m is the number of rules.

As we find in the process of reasoning, the rule R^i translate to the form:

R^i: *if x is* A^i *then y is* b^i.

Usually in the generation of fuzzy rules we first pay attention to rule premises and find an optimal partition based on a certain criterion. Sugeno and Yasukawa (1993) proposed a different method; that is, they first paid attention to the consequents of the rules and then find a partition concerning the premises.

In this paper the researchers use the Sugeno-Yasukawa method for rule generation. In their method, they do not take an ordinary fuzzy partition of the input space for if they take this kind of partition, the number of rules increases exponentially with the number of inputs. For this reason, they proposed fuzzy C-mean method, abbreviated FCM, for rule generation. Using FCM, we make fuzzy clustering of the output data. As a result, every output *y* is associated with the grade of membership belonging to a fuzzy cluster *B*. Therefore; we have the following data associated with the grade of the membership of y^i *in* $B^j (1 \le j \le c)$:

$$(x^i, y^i), B^1(y^i), B^2(y^i), \ldots, B^c(y^i)$$

We can induce a fuzzy cluster *A* in the input space (See figure 1). By making the projection of cluster *A* onto the axes of the coordinates x_1 and x_2, we obtain the fuzzy sets A_1 and A_2 as shown in figure 2. We have the following relation: $A_1(x_1^i) = A_2(x_2^i) = B(y^i)$ where *B* is the output cluster. Now this cluster gives a fuzzy rule:

$$\text{if } x_1 \text{ is } A_1 \text{ and } x_2 \text{ is } A_2, \text{ then } y \text{ is } B.$$

4.2. Clustering validation criteria

Clustering is an unsupervised method that despite classification method has no predefined class. It is necessary to clustering validation before rules generation. In former researches, clustering validation was achieved by individual criterion, such as: PC, PE, FSm, and FH. In this paper, the researchers have merged four mentioned criteria in literature: PC, PE, FSm, and FH. In the following paragraph are introduced these indexes.

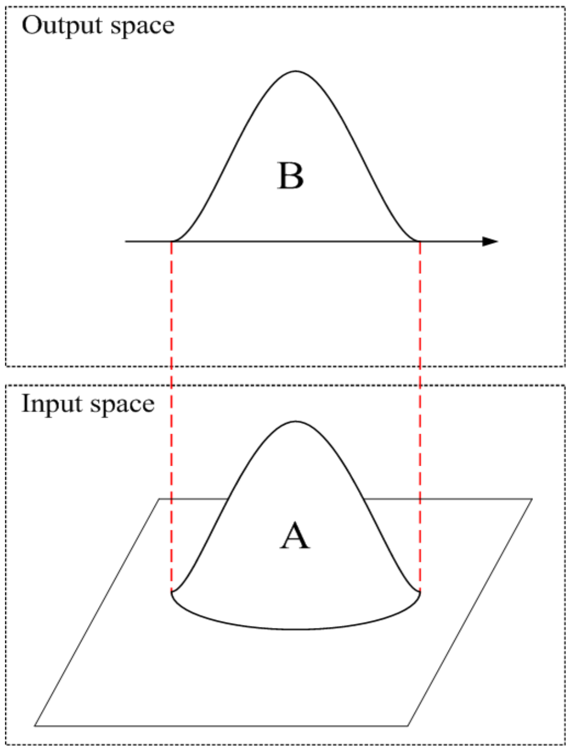

Figure 1: Fuzzy cluster in the input space.

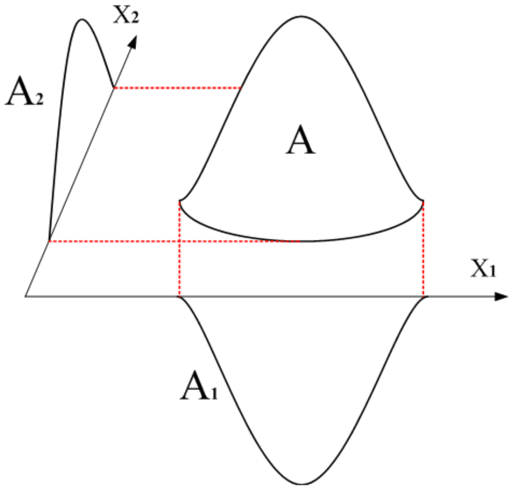

Figure 2: Projection of a fuzzy cluster.

4.2.1. Partition coefficient criterion (PC)

This criterion was suggested by Bezdek and defines as:

$$PC = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{n_c} u_{ij}^2 \tag{18}$$

PC is changed in $[1/n_c, 1]$ interval. n_c is cluster number and μ_{ij} is fuzzy membership degree of datum i to cluster j. If PC tends to 1, it means that we have a good clustering, but if the value of PC approaches to $1/n_c$, it means that the clustering is doing badly.

4.2.2. Partition entropy coefficient criterion (PE)

This criterion is defined as follows:

$$PE = \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{n_c} u_{ij}.log_a(u_{ij}) \tag{19}$$

PE is changed in $[0, \log_a^{nc}]$ interval. n_c is cluster number and μ_{ij} is fuzzy membership degree of datum i to cluster j. If PE tends to 0, it means that we have a good clustering.

4.2.3. Fukuyama-Sugeno criterion (FSm)

This criterion is defined as follows:

$$FS_m = \sum_{i=1}^{N}\sum_{j=1}^{n_c} u_{ij}^m (\|x_i - v_j\|_A^2 - \|v_j - v\|_A^2) \tag{20}$$

That v is average vector of input vector X. It is obvious that the less value of FSm the better the clustering.

4.2.4. Fuzzy hyper volume (FH)

This criterion is defined as follows:

$$\sum_j = \frac{\sum_{i=1}^{N} u_{ij}^m (x_i - v_j)(x_i - v_j)^T}{\sum_{i=1}^{N} u_{ij}^m}$$

$$V_j = \left|\sum_j\right|^{1/2}$$

$$FH = \sum_{i=1}^{n_c} V_j \tag{21}$$

I) V_j : Fuzzy hyper volume of cluster j

II) $\left|\sum_j\right|$: Determinant value of $\sum_j$

It is clear that the less value of FH, the better the clustering (See Zhang et al., 2008).

For clustering validation we use the following steps:

Step 1: Set the number of clusters equal to 2.

Step 2: Calculate the values of PC, PE, FSm, and FH.

Step 3: Add one unit to clusters.

Step 4: Repeat steps 2 and 3 until the number of clusters is equal or less than a predetermined value (n_{c_0}).

Step 5: Generate decision matrix D that the rows and columns present the number of clusters and the criteria respectively. Each entry of matrix shows the criterion value instead of determined clusters (See table 2).

Step 6: Determine the weight of criteria:

$$W = (W_{PC}, W_{PE}, W_{FSm}, W_{FH}).$$

Table 2: Decision matrix.

	Criteria			
The number of clusters	PC (+)	PE (-)	FSm (-)	FH (-)
2	r_{11}	r_{12}	r_{13}	r_{14}
3	r_{21}	r_{22}	r_{23}	r_{24}
.	.	.	.	.
N	r_{n1}	r_{n2}	r_{n3}	r_{n4}

Step 7: Make matrix D' (dimensionless matrix of D) as follows:

Step 7.1: Positive criterion:

$$r'_{ij} = r_{ij} \big/ r_j^* \,,\; r_j^* = \max_i \{ r_{ij} \} \tag{22}$$

Step 7.2: Negative criterion:

$$r'_{ij} = r_j^* \big/ r_{ij} \,,\; r_j^* = \min_i \{ r_{ij} \} \tag{23}$$

Definitions:

a) Positive criterion: is a criterion that its higher value is desired.

b) Negative criterion: is a criterion that its lower value is desired.

Step 8: Calculate scores of alternatives (the number of clusters) as follows:

$$S_{n\times 1} = D'_{n\times 4} \times W^T_{4\times 1} \tag{24}$$

Step 9: The highest entry value in matrix S shows the number of clusters.

After running the above steps, we derive the rules by mentioned method in section 4.1.

5. Numerical results

5.1. Example 1: Exclusive market

In Table 3, the size of the proposed mathematical models in section 3 for some of system configurations are shown. For example, if manufacturing processes=2, product types=5, factories=3, programming periods=4, Quality groups=1, Materials or parts=1, and Collection centers=1 then the number of constraints=402, total number of variables=360, and nonlinear variables=60.

To illustrate the mentioned model in section 3, let's give an example. The researchers have solved the mathematical model for this configuration: manufacturing processes=2, product types=3, factories =1, programming periods=3, Quality groups =1, Materials or parts=1, Collection centers=1, and $W_1 = W_2 = W_3 = 1$. We changed the value of W_1 and depicted changes in two of decision variables: f_{111}=Acquisition price of core 1 with quality 1 at period 1 and f_{121}=Acquisition price of core 1 with quality 1 at period 2.

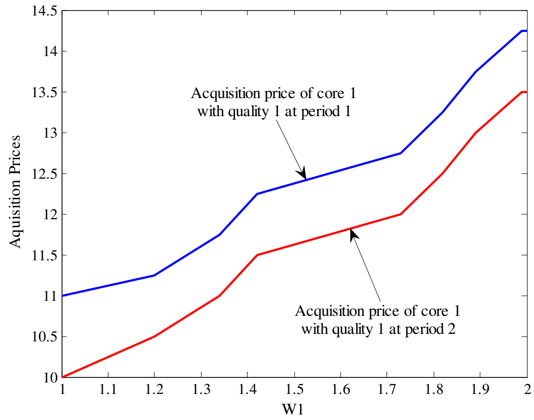

Figure 3: Variations of f_{111} and f_{121} vs. W_1 .

According to figure 3, acquisition price of core1 with quality 1 at period 1 is always higher than acquisition price of core 1 with quality 1 at period 2 for different W_1's. Therefore we pay the less amount of money for acquisition over the time.

The researchers changed the value of W_2 and depicted changes in following decision variables:

p_{11} =Price of remanufactured core 1 at period 1,

p_{12} =Price of remanufactured core 1 at period 2,

p_{13} =Price of remanufactured core 1 at period 3,

p_{31} =Price of remanufactured core 3 at period 1,

p_{32} =Price of remanufactured core 3 at period 2, and

p_{33} =Price of remanufactured core 3 at period 3.

According to figures 4 and 5, product 1 is in growth stage of its life cycle while product 3 is in decline stage.

The researchers changed the value of W_3 and depicted changes in following decision variables:

p_{11} =Price of remanufactured core 1 at period 1,

p_{12} =Price of remanufactured core 1 at period 2,

p_{13} =Price of remanufactured core 1 at period 3.

As shown in figure 6, the higher value of W_3, the higher price of remanufactured core.

5.1. Example 2: Competitive market

To illustrate the mentioned method in section 4, let's give an example. The example here is about acquisition price of K750 Sony Ericsson cell phone of Tehran cell phone market. The number of rule premises must be determined generating rules. In fact, in the case of cell phone, these premises are equal to principal parts of cell

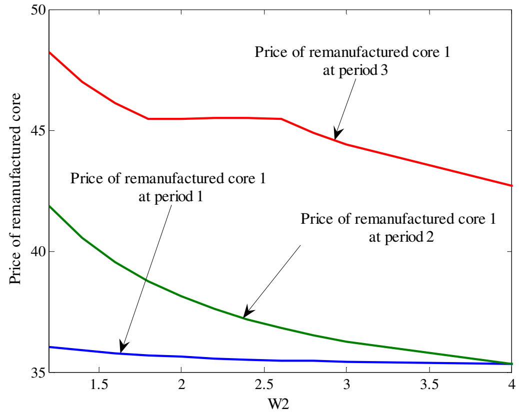

Figure 4: Variations of p_{11}, p_{12}, and p_{13} vs. W_2.

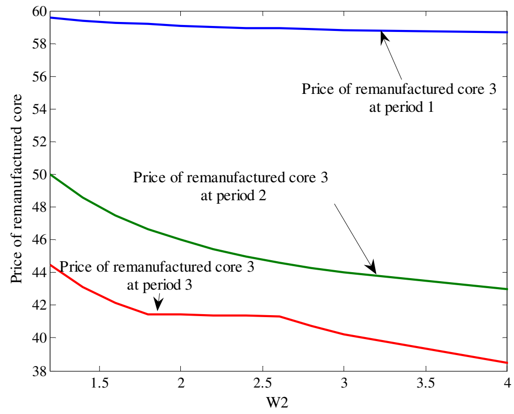

Figure 5: Variations of p_{31}, p_{32}, and p_{33} vs. W_2.

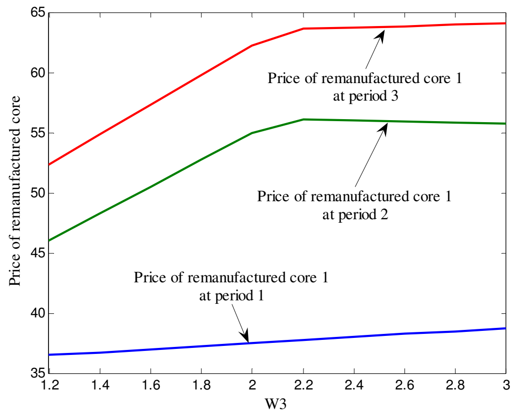

Figure 6: Variations of p_{11}, p_{12}, and p_{13} vs. W_3.

phone that affect its acquisition price. Therefore, the tallying reached is about effective principal parts on acquisition price by collected data from experts, researches, and markets. Principal parts of cell phone: Housing, Printed wiring board (PWB), Antenna, Display, Keypad, Microphone, Speaker, Battery, Camera, and Bluetooth. After cell phone disassembly, PWB is usually useless, so, the researchers consider 9 remaining parts as principal parts.

Therefore generated rules are as follows:

R_i : If worth of housing is A_1^i, worth of antenna is A_2^i, worth of display is A_3^i, worth of keypad is A_4^i, worth of microphone is A_5^i, worth of speaker is A_6^i, worth of battery is A_7^i, worth of camera is A_8^i, and worth of bluetooth is A_9^i then acquisition price of cell phone is B^i.

Where A_j^i and B^i are fuzzy numbers. Also, the number of rules is equal to the number of acquisition price clusters. According to the collected data about K750 Sony Ericsson cell phone, the results of clustering are shown in Table 4.

Based on mentioned steps in Section 4.2, the maximum score is gained in 4th alternative (See Tables 5 and 6). Therefore, the number of clusters equal is to five.

Membership functions of clusters are shown in Figure 7. For all premises, there will be five clusters. For example, membership functions for clusters of battery are shown in Figure 8. These member functions are presented as trapezoidal fuzzy numbers by regression.

We have five rules:

R_i : If worth of housing is cluster i, worth of antenna is cluster i, worth of display is cluster i, worth of keypad is cluster i, worth of microphone is cluster i, worth of speaker is cluster i, worth of battery is cluster i, worth of camera is cluster i, and worth of bluetooth is cluster i then acquisition price of cell phone is cluster i (i=1, 2, 3, 4, and 5). These rules are shown in Figure 9.

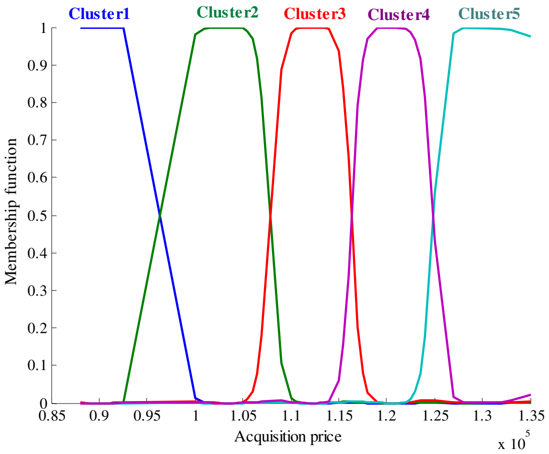

Figure 7: Membership functions of acquisition price clusters.

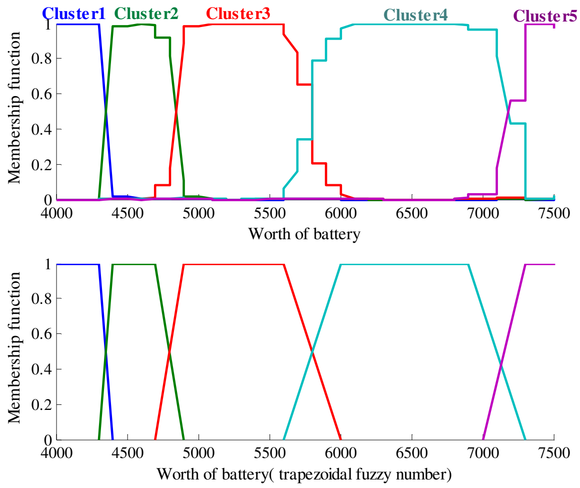

Figure 8: Membership functions of battery worth clusters.

Table 3: Model size.

Manufacturing Processes	Product types	Factories	Programming periods	*	Nonlinear variables	Total variables	Constraints
2	3	1	3	1	27	85	90
2	4	1	3	1	36	109	112
2	3	1	4	1	36	113	122
2	5	1	3	1	45	133	134
2	3	2	3	1	27	131	144
2	3	1	5	1	45	141	154
2	3	3	3	1	27	177	198
2	5	2	3	1	45	203	214
2	5	3	3	1	45	273	294
2	5	3	4	1	60	360	402 🡐
2	5	3	5	1	75	450	510

* Quality groups, Materials or parts, and Collection centers

Table 4: Clustering results.

The number of clusters	Cluster center1	Cluster center2	Cluster center3	Cluster center4	Cluster center5	Cluster center6	Cluster center7	PC	PE	FSm	FH
2	100180	118380	-	-	-	-	-	0.9233	0.0298	-2196100000	12967
3	95250	111600	122460	-	-	-	-	0.9149	0.0327	-6092600000	13093
4	90160	103580	113330	123170	-	-	-	0.915	0.0316	-7407500000	11028
5	90150	103350	112320	120330	129370	-	-	0.9295	0.0254	-8342000000	11558
6	90150	103280	111960	118420	123300	130570	-	0.9282	0.0262	-8679100000	11413
7	90140	102380	109870	113820	118890	123510	130700	0.9136	0.0344	-8810100000	11554

Table 5: Decision matrix (D).

The number of clusters	Criteria			
	PC (+)	PE (-)	FSm (-)	FH (-)
2	0.9233	0.0298	-2196100000	12967
3	0.9149	0.0327	-6092600000	13093
4	0.915	0.0316	-7407500000	11028
5	0.9295	0.0254	-8342000000	11558
6	0.9282	0.0262	-8679100000	11413
7	0.9136	0.0344	-8810100000	11554

Table 6: Dimensionless decision matrix (D').

The number of clusters	Criteria				Score
	PC (+)	PE (-)	FSm (-)	FH (-)	
2	0.99333	0.852349	0.249270723	0.850467	0.848057
3	0.984293	0.776758	0.691547202	0.842282	0.864356
4	0.9844	0.803797	0.840796359	1	0.918979
5	1	1	0.946867799	0.954144	0.985516 🡐
6	0.998601	0.969466	0.985130702	0.966267	0.982047
7	0.982894	0.738372	1	0.954475	0.905564
Criterion weight	0.4	0.3	0.1	0.2	

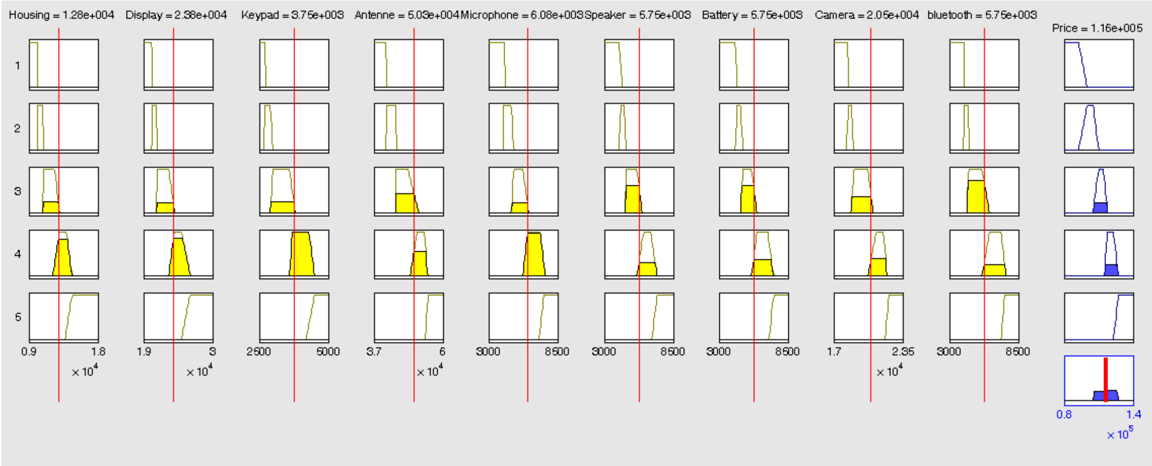

Figure 9: Fuzzy rules for acquisition price determination.

6. Conclusion

Sustainability has become a major issue in most economies, causing many leading companies to focus on product recovery and reverse logistics. Remanufacturing is an industrial process that makes used products reusable. One of the important aspects in both reverse logistics and remanufacturing is pricing of returned and remanufactured products (called cores). In this paper, the researchers focus on pricing the cores. In pricing, focusing on market structure is obligatory.

They consider two kinds of markets: exclusive and competitive. In exclusive markets, because the researchers determine prices, so we use mathematical modeling. In competitive market they obtain the prices from market data (rivals). For this, the researchers use fuzzy rule that is a proper method for uncertainty modeling.

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