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A fuzzy reliability model for series-parallel systems

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Abstract: Fuzzy set based methods have been proved to be effective in handling many types of uncertainties in different fields, including reliability engineering. This paper presents a new approach on fuzzy reliability, based on the use of beta type distribution as membership function. Considering experts' ideas and by asking operators linguistic variables, a rule base is designed to determine the level of reliability of each component. Hence, we can determine the level of reliability of components with expending low costs. Also in this work a simple approach is presented for reducing the number of rules. The outputs of the presented model are fuzzy sets representing the reliability levels of components. In order to determine the level of reliability as linguistic variables, a new method is presented. Also the validity of the model is investigated by two methods. After determining the level of reliability of each component, the reliability of the composed system can be determined by using t-norm and s-norm functions. The system can be parallel, series, parallel-series or series-parallel. The presented model has been applied in a glass manufacturing company.

Keywords: *Beta distribution; Fuzzy reliability; Series-parallel; S-norm; T-nom*

1. Introduction

The concept of fuzzy reliability has been proposed and developed by several authors (Cai *et al.*, 1991; Cai *et al.*, 1995; Chen, 1994; Chen, 1994). The conventional reliability is considered under the probability and binary-state assumptions. Cai *et al.* (1991 and 1995) have given a different insight by introducing the possibility and the fuzzy-state assumptions to replace the probability and binary-state (Jiang and Chen, 2003).

In the conventional systems, we always give an exactly failed or functioning probability for each component. However, in practice, when the stress or the strength or both of them are fuzzy variables, it is very difficult to compute the exact value for each component. Currently, by investigating the fuzzy reliability of a system, the researchers always assume that the reliability of each component is a fuzzy variable (Mon and Cheng, 1994; Utkin and Gurov, 1996; Utkin *et al.*, 1995; Wu H-c, 1997).

The design of a fuzzy logic system (FLS) includes the design of a rule base, input scale factors, output scale factors, and membership functions. Input scale factors transform the real inputs into normalized values, and output scale factors transform the normalized outputs into real values (Simon, 2002).

Some studies have shown that FLS performance is more dependent on membership function design than rule base design (Aytekin, 2003; Cerrada *et al.*, 2005; Cordon *et al.*, 2000). Other studies have discussed rule base design (Cordon *et al.*, 2001; Procyk and Mamdani, 1979; Xian-Tu, 1990).

In this paper, a new approach is introduced for designing membership functions with Beta type distribution and designing the rule base. Finally we can determine the reliability of each component based on experimental data.

2. Limitations of conventional reliability theory

Although the probability approach has been applied successfully for many real world engineering reliability problems, there are some limitations to the probabilistic method:

1. Probabilistic methods are based on mass collection of data to achieve the requisite confidence level, but in real world applications, sometimes there is insufficient data to accurately handle the statistics of parameters. This is particularly true at the tail of the distributions, where reliability is very high and therefore failure observations are extremely rare. Also, at early stages of new product development, the available data (numbers of testing samples, recorded failures on test) is limited, so the required confidence level may not be met if probability methods are used.

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2. If using probability (random data) only, one cannot use certain types of information that are important in reliability analysis. Such information includes experts' opinions, outcome from Failure Modes and Effects Analysis (FMEA), data from robustness studies, and results from functional and performance testing.
3. The two-state assumption does not provide information of any intermediate state: the product's state of performance is always either functioning (reliability = 1) or failure (reliability = 0).

Due to these limitations, the use of the probability approach in reliability analysis has been much criticized. The most significant limitations are input and output that must be in a precise probabilistic format and the model to deal with this information must also be precise (Haofu, 2004).

3. Uncertainty

Uncertainty exists wherever and whenever human beings interact with the real world, in any situation in which a person does not quantitatively and qualitatively possess the appropriate information to describe, prescribe or predict a system, its behavior, or other phenomena deterministically and numerically.

In the history of science, there has been a gradual transition in the way uncertainty is viewed. Traditionally, the scientific way has strived for certainty in all its manifestations.

The precise laws of Newtonian mechanism are good examples of this process. The first stage of the transition from the traditional view to the modern view of uncertainty began in the late nineteenth century, when physics began to study the process at the molecular level, which contributed to the development of the statistical method. Newtonian mechanism, which involves only certainty, is replaced in statistical mechanisms by probability theory, a theory whose purpose is to capture uncertainty of a certain type. The second stage in the transition of the traditional view to modern view of uncertainty began in 1965 when Zadeh (1965) published his paper on fuzzy sets. The paper challenged the notion that probability theory was the sole agent for uncertainty.

In the modern view, uncertainty is considered essential to science. It is not only an unavoidable issue, but also many potential applications, including those in reliability engineering (Maglaras, and Nikolaidis, 1997).

4. Uncertainty in reliability engineering

In reliability engineering, there are several kinds of uncertainties, such as imprecise data caused by failure time, incomplete data resulted from censored tests, and vague descriptions about failure. The most important aspects of uncertainty are:

1. Type of uncertainty,
2. Causes of uncertainty,
3. Theory used to model uncertainty.

Reliability engineering theory was developed in the 1960s when probability theory and application made great progress in many engineering areas. Probability theory was used as a primary tool to deal with all kinds of uncertainties encountered in reliability engineering. The different types of uncertainties in reliability engineering are enumerated in Table 1.

5. Theories for modeling uncertainty

At present there are various theories for modeling uncertainty, such as probability theory, possibility theory, fuzzy set theory, evidence theory, rough set theory, and convex modeling among others. In modeling uncertainty, each of these theories focuses on either a specific type of uncertainty or a cause of uncertainty or even the specific type of information needed to process the data. An appropriate theory for modeling a specific uncertainty situation should be determined by the property of the situation as specified by cause and type of uncertainty and by the requirement of the observers.

Due to the fact that each theory has its own assumptions about available information and contains a certain calculus by which the information or data is measured and processed, it is obvious that each of the mentioned theories can only be appropriate for modeling a limited number of the causes or types of uncertainty.

There is no single theory that can model all types of uncertainty and include all kinds of causes of uncertainty.

Probability theory has been widely used as a traditional approach to model the real world problems in reliability engineering.

Table 1: Uncertainties in Reliability Engineering.

Types of Uncertainty	Engineering Example
Imprecision	Failure time Load simulation in the lab Measurement accuracy Modeling in simplification due to the proposed distribution Maintenance Operational profile External environment
Incomplete data	Censored testing data Lack of data Suspended test
Vagueness	Material property Soft failure criteria, Software failure, Human error, Operator (customer) description for the malfunction phenomenon, Linguistic description of characteristics of performance such as "good", "unacceptable", etc. Maintenance
Randomness	Component geometry variation Material property variation Loading and variation Input signal and variation External environment Operating environment Frequency of usage Measurement error Component failure
Subjectivity	Lack of knowledge Expert judgment Engineering experience
Complexity	Relationships between system and components Interaction between subsystems Heuristic algorithms

Since probability theory associated with statistics, is only adequate for processing random types of uncertainty, there might exist certain degrees of risk by only using probability theory in dealing with all kinds of uncertainties in reliability analysis. They are less effective in modeling other types of uncertainties such as vagueness and imprecision.

Probability theory has been widely used as a traditional approach to model the real world problems in reliability engineering. Since probability theory associated with statistics, is only adequate for processing random types of uncertainty, there might exist certain degrees of risk by only using probability theory in dealing with all kinds of uncertainties in reliability analysis. They are less effective in modeling other types of uncertainties such as vagueness and imprecision.

The fuzzy set concept (Maglaras and Nikolaidis, 1997) was introduced to model linguistic-like variables. It has been found a wide range of applications in dealing with uncertainties involving vagueness, subjectivity, incompleteness, and imprecision in nature. Fuzzy set based methods have been proved to be effective in handling multiple types of uncertainties in different areas, including reliability engineering (Kanagawa and Ohta, 1990; Wang *et al.*, 1998; Wang *et al.*, 1999; Zadeh, 1965).

6. Developing the membership function

An important aspect about fuzzy modeling is defining the membership functions. There are many articles, where the different types of membership functions such as triangular and bell shaped are studied, but all of them are considered as static models. In this paper, a new approach is introduced to assign more flexible membership functions under different situations with optimistic and pessimistic conditions. A modified Beta type distribution function is used for this purpose.

The beta distribution shows the optimistic and pessimistic states that can be presented by:

$$\beta(\mu_a(x)) = \frac{\Gamma(a).\Gamma(b)}{\Gamma(a+b)} x^{wl}(1-x)^{wh} \tag{1}$$

where *Wl* and *Wh* are the pessimistic and optimistic values.

To be able to use the beta distribution function as the membership function of fuzzy reliability, Equation (1) is normalized such that the probabilistic distribution function (1) is changed to possibility function as (Khanmohammadi *et al*., 2000):

$$\beta(\mu_a(x)) = \frac{x^a(1-x)^b}{(\frac{a}{a+b})^a(1-\frac{a}{a+b})^b} \tag{2}$$

where $a=(1-Wl)/2$ and $b=(1-Wh)/2$. The parameters *a* and *b* may be multiplied by a suitable factor α to have appropriate shapes depending on *Wl* and *Wh*. Figure 1 shows some typical beta shaped membership functions for different situations.

7. Determining the fuzzy reliability of a mechanical system as a case study

Figure 2 shows a conceptual modeling of reliability for each component of a mechanical system. The reliability of each component depends on different factors in various conditions. Some of them are mutually dependent and some others are independent. The reliability of component i can be presented as a function of different factors as follow:

$$R_i = f(M, E_i, E, L) \tag{3}$$

where R_i is reliability of ith component, M is the material of each component, E_i is the expert's ideas, on the levels of failure.

It can be defined by linguistic variables such as completely failure, failure, semi-failure, healthy and completely healthy - that can be determine by membership function with beta type distribution as presented by Equation (2) - and *L* is the lifetime of each component at the time of determining the reliability. It is a quantitative variable, but in order to coordinate with other variables, we are considering it as a linguistic variable. It can be very old, old, medium, new and very new. *E* is failure signal such as sound, frequency of vibrations and smell.

Sounds of components are used to determine the reliability of components. They can be quantitative variables in terms of decibel or linguistic variables such as very much, much, medium, a little and feeble, calculated by bell shaped membership function:

$$\mu_A(u,t) = bell(u,c,d) = \frac{1}{1+d(u-c)^2} \tag{4}$$

where $\mu_A(u,t)$ is membership function of each element u of universe in fuzzy value *A* at time *t*, *d* is a parameter that determines shape of function. It is selected 0.0625 by trial and error in this work; *U* is the array of universe; *c* is the median of fuzzy value *A* at time *t*. Vibration, smell and lifetime of components can be defined by linguistic variables calculated by bell shaped membership function Equation (4). The algorithm for determining the reliability of component is denoted by the following stages at each time *t*.

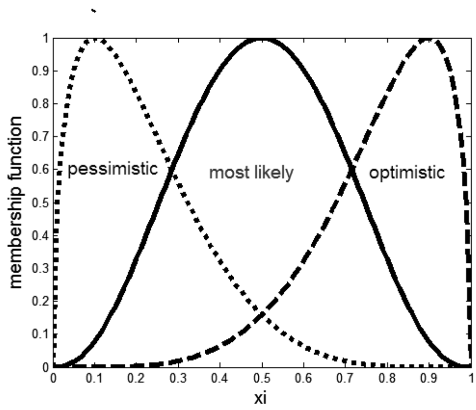

Figure 1: Typical beta type membership functions.

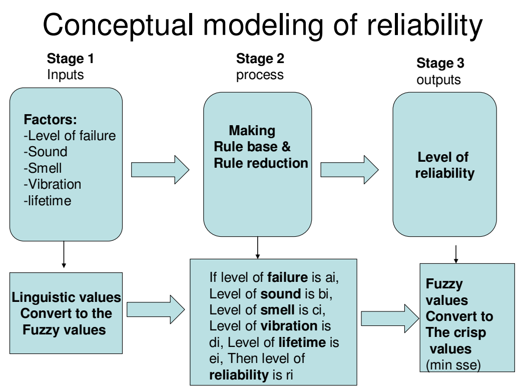

Figure 2: Conceptual modeling of reliability.

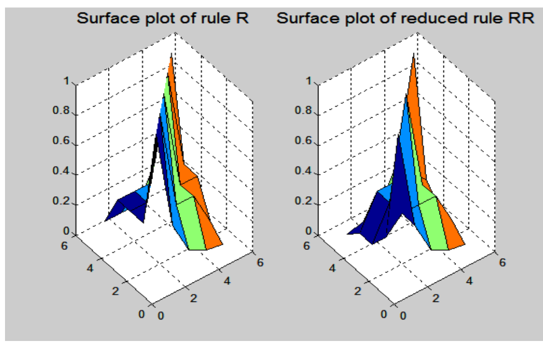

Figure 3: Surface plots of total rule R and reduced rule RR.

7.1. Stage 1

Step 0. Determine the factors affecting reliability. In this work we have factors, such as failure, sound, smell, vibration and life time.

Step 1. Determine the universes of discourses for factors.

Step 2. Find the states of factors by asking the machines operators and obtain the linguistic variables at different times. In this work the states for linguistic variables in Table 2 are considered.

Step 3. Determine the medians and shape factors for different linguistic values that were determined in Step 1.

Step 4. Calculate the membership functions of linguistic values at any time t, using Equation (2) for failure, and Equation (4) for sound, smell, vibration, lifetime and reliability.

7.2. Stage 2

Step 1. Make the rule bases as:

If the level of failure is a_i and level of sound is b_i and level of smell is c_i and level of vibration is d_i and level of lifetime is e_i , then the level of reliability is r_i . Or simply:

If S_i then $r_i \Rightarrow R_i$

where S_i is the composition of the fuzzy sets of different factors at any particular time with special conditions, r_i is level of the reliability of component at the particular time with ith condition, and R_i is the ith rule at the particular time with ith condition.

In this work, we have 5 states and 5 factors. Hence there will be $5^5 = 3125$ conditions that must be considered but the outputs of many of them are similar or have overlaps with each other. We investigated majority of conditions by using of experts' ideas. Finally, 50 conditions were chosen and the total rule base was made.

Step 2. Rule reduction:

When the number of factors and conditions are too many, then there will be overlaps between the rules, making related calculations complex. In order to overcome this drawback the number of rules must be reduced. In this work we have compared the rule matrixes. If sum of squared errors between elements of two rules R_i and R_j is less than a predefined parameter such as α, one of them will be rejected. Hence, the new rule base is obtained. Figure 3 shows the surface plots of total rule and reduced rule, calculated in Step 1 and Step 2.

7.3. Stage 3: Determining final reliability

In this stage, with having the rule base and fuzzy values of factors we can determine the levels of reliabilities of components as fuzzy sets at various times. In order to determine the nearest standard linguistic variable of reliabilities with the obtained fuzzy sets, we can use several methods that one of them is Minimum Sum of Squared Distance that is used in this paper.

Sum of squared distance between the obtained fuzzy set and the linguistic variables for reliability, is calculated by the following formula:

$$d_{i=}[(\mu_{i1}-\mu_{r1})^2+(\mu_{i2}-\mu_{r2})^2+(\mu_{i3}-\mu_{r3})^2 + (\mu_{i4}-\mu_{r4})^2+(\mu_{i5}-\mu_{r5})^2]^{1/2} \qquad (5)$$

where μ_i is the calculated membership and μ_r is the predefined linguistic variable for reliability.

Then we choose the linguistic variable with the minimum sum of squared distance for the current calculated reliability. For example suppose the obtained reliability is:

r = 0.2373 0.2373 0.2373 0.3333 0.3333

Then by considering different levels of reliabilities as shown in Table 3, the level of reliability is "reliable".
Where:

Cu = Completely unreliable

Un = Unreliable

Fr = Fairly reliable

R = Reliable

Cr = Completely reliable

Table 2: Linguistic variables and their states.

levels of failures:	complete failure, failure, semi-failure, healthy, complete healthy.
sound:	very much, much, medium, little, feeble.
smell:	very much, much, medium, little, feeble.
vibration:	always, usually, some times, seldom, never.
lifetime:	very old, old, medium, new, very new.
reliability:	completely reliable, reliable, fairly reliable, unreliable, completely unreliable.

8. The model validity

We can consider the validity of the model with two methods:

- Studying input-output of the model,
- Sensitivity analysis method.

8.1. Studying input-output of the model

When the model behaves logically, if all inputs are inappropriate (which means they are in the worst situation), the output of the model will be expected to be completely unreliable.

And if all inputs are appropriate (which means they are in the best situation), the output of the model will be expected to be completely reliable.

And if all inputs are moderately appropriate (which means they are in the medium situation), the output of the model will be expected to be moderately reliable.

After these situations have been designated as inputs of the model, the outputs were obtained as it was expected.

8.2. Sensitivity analysis method

In this method, the level of effect on reliability of each factor is being considered. Two different states are considered.

In state one, it is supposed that the situation of the reliability is "completely reliable" ($r = 0.9$). That means all factors have appropriate situation ($x_i = 0.1$). If all factors are fixed and one of them changes, then the levels of reliabilities will be changed as shown in Figure 4.

In this Figure, plot 1 shows the variations of the levels of failures, that means all factors are fixed and only the level of failure changes, plot 2 represents variations of life time, plot 3 shows variations of sound, plot 4 represents variations of the smell and plot 5 shows the variations of the vibration. As it is clear in the Figure, with increasing of each factor, the level of reliability decreases, although the levels of effects are different. Plot 1 has the most effect and plot 5 has the least effect.

In state two, it is supposed that the situation of the reliability is "completely unreliable" ($r = 0.1$) that means all factors have inappropriate situation ($x_i = 0.9$). If all factors are fixed and one of them changes, then the levels of reliabilities will be changed as shown in Figure 5.

In this Figure, plot 1 shows the variations of the levels of failures, plot 2 represents variations of life time, plot 3 shows variations of sound, plot 4 represents variations of the smell and plot 5 shows the variations of the vibration. As it is shown in the Figure, the level of reliability increases, by decreasing each factor, although the levels of effects are different. Plot 1 has the most effect and plot 5 has the least effect.

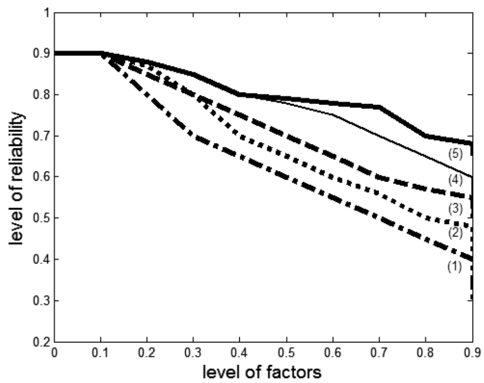

Figure 4: Effects variations of factors in state 1.

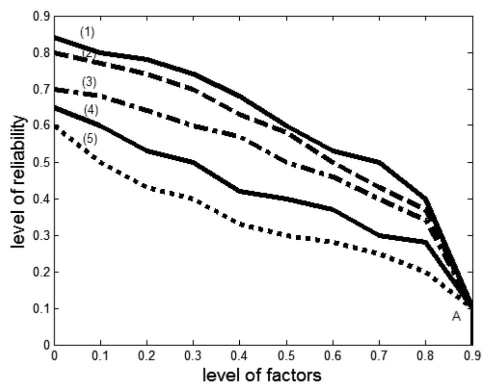

Figure 5: Effects variations of factors in state 2.

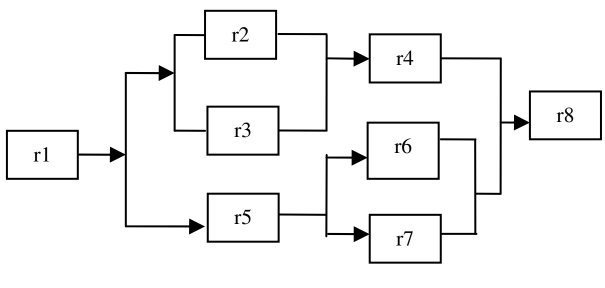

Figure 6: Block diagram of components.

Table 3: Levels of reliability.

Current reliability	0.2373 0.2373 0.2373 0.3333 0.3333	$di\sum((r-ri)^2)^{1/2}$
Cu	1.0000 0.3333 0.1111 0.0526 0.0303	0.7775
Un	0.3333 1.0000 0.3333 0.1111 0.0526	0.7283
Fr	0.1111 0.3333 1.0000 0.3333 0.1111	0.6562
R	0.0526 0.1111 0.3333 1.0000 0.3333	0.5037 *
Cr	0.0303 0.0526 0.1111 0.3333 1.0000	0.5373

Table 4: The fuzzy inputs.

components	memberships				
s1	0.0000	1.0000	0.3333	0.0642	0.0000
s2	0.0303	0.0526	0.0303	0.0010	0.0000
s3	0.0000	0.3333	1.0000	0.3333	0.0000
s4	0.0000	0.1111	0.3333	0.0526	0.0000
s5	0.0000	0.0526	0.1111	0.0526	0.0000
s6	0.0000	0.0010	0.0313	0.2373	1.0000
s7	0.0000	0.0010	0.0313	0.0526	0.0313
s8	0.0000	0.0010	0.0313	0.2373	0.3333

Table 5: The fuzzy outputs.

Componens reliabilities	memberships				
r1	0.1111	0.3333	0.3333	0.3333	0.2373
r2	0.0526	0.0526	0.0526	0.0526	0.0526
r3	0.2373	0.3333	1.0000	0.3333	0.2373
r4	0.2373	0.3333	0.3333	0.3333	0.1111
r5	0.1111	0.1111	0.1111	0.1111	0.1111
r6	1.0000	0.3333	0.2373	0.2373	0.1111
r7	0.0526	0.0526	0.0526	0.0526	0.0526
r8	0.3333	0.3333	0.2373	0.2373	0.1111

Table 6: Determining the reliability of the total system.

Current reliability	0.0012 0.0113 0.0274 0.0167 0.0060	$di\sum((r-ri)^2)^{1/2}$
Cu	1.0000 0.3333 0.1111 0.0526 0.0303	1.0529
Un	0.3333 1.0000 0.3333 0.1111 0.0526	1.0919
Fr	0.1111 0.3333 1.0000 0.3333 0.1111	1.0325
R	0.0526 0.1111 0.3333 1.0000 0.3333	1.0718
Cr	0.0303 0.0526 0.1111 0.3333 1.0000	1.0213*

9. Determining the series-parallel systems

In order to determine the reliability of a series-parallel system, first we can divide the network to several subsystems and determine the reliability of each one and then with connecting the subsystems, the reliability of the system is determined.

In this paper, the levels of reliabilities are calculated as fuzzy sets. Besides, there are t-norm and s-norm functions in the fuzzy field (Xu *et al.*, 2003) that are similar to classic series-parallel systems and they have some specifications that match those of classic systems. Hence, t-norm and s-norm functions are respectively used for series and parallel systems. Finally the reliability of the system is determined as a fuzzy set that is converted to the linguistic variables by using Minimum Sum of Squared Distance method as explained before.

10. Case study

As a case study, the presented model has been applied to a part of the production line of “Jamgin Sardar Glass” company, a glass manufacturer company in Iran.

The mentioned production line consists of components that its block diagram is shown in Figure 6. According to the presented model composition of fuzzy input factors for components are as Table 4. The designed rule base is:

R =

$$\begin{matrix} 1.0000 & 0.3333 & 0.1111 & 0.0526 & 0.0303 \\ 0.2373 & 0.3333 & 0.3333 & 0.3333 & 0.1111 \\ 0.1111 & 0.3333 & 1.0000 & 0.3333 & 0.1111 \\ 0.1111 & 0.3333 & 0.3333 & 0.3333 & 0.2373 \\ 0.0303 & 0.0526 & 0.1111 & 0.3333 & 1.0000 \end{matrix}$$

Hence the outputs for components are as shown in Table 5.

After the reliabilities of components are determined, the system will be divided to the sub systems where the whole reliability is calculated by using t-norm and s-norm functions. These functions are of different types such as: Drastic product, Bounded difference, Einstein product, Product Algebraic, Hamacher product and Minimum.

Considering that each of these types have their own special specifications, after several tests it is found that the Einstein product and Hamacher product types are more appropriate for t-norm (series) and s-norm (parallel) in reliability systems.

Hence by considering the two functions s-norm (*a, b, ‘h’*), t-norm (*a, b, ‘e’*) and according to the relations between components, the reliability of the total system is calculated as:

L1 = s-norm (r2,r3,’h’)

= 0.2683 0.3571 1.0000 0.3571 0.2683

L2= t-norm (L1,r4,’e’)

= 0.0298 0.1190 0.3333 0.1190 0.0637

L3= s-norm (r6,r7,’h’)

= 0.1529 0.2683 0.2683 0.3571 1.0000

L4 = t-norm (L3,r5,'e')

= 0.0170 0.0298 0.0298 0.0397 0.1111

L5 = s-norm (L2,L4,'h')

= 0.0458 0.1423 0.3467 0.1500 0.1618

L6= t-norm (r1,L5,'e')

= 0.0109 0.0474 0.1156 0.0500 0.0180

L7= t-norm (L6,r8,'e')

= 0.0012 0.0113 0.0274 0.0167 0.0060

Finally by using Minimum Sum of Squared Distance Method the reliability of the total system is determined "completely reliable" as shown in Table 6.

11. Conclusion

In this paper, a new fuzzy reliability model was proposed that does not have the limitations of conventional reliability. Although the concept of fuzzy reliability had been previously presented and developed by several authors, the proposed fuzzy model in this paper is based on membership functions and has been designed based on beta type distribution that it is more flexible than the other models and the new model could determine the reliability of each component with expending minimum time and cost. It also models the uncertainties caused by imprecise, incomplete and vague data.

The proposed model is not only usable in the case study, but can be also applied in all companies that are faced with imprecise, incomplete and vague data.

At last, after the reliabilities of components were determined, the reliability of a series-parallel system was obtained by using t-norm and s-norm functions.

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