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ORIGINAL RESEARCH

Parallel computation framework for optimizing trailer routes in bulk transportation

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Abstract

We consider a rich tanker trailer routing problem with stochastic transit times for chemicals and liquid bulk orders. A typical route of the tanker trailer comprises of sourcing a cleaned and prepped trailer from a pre-wash location, pickup and delivery of chemical orders, cleaning the tanker trailer at a post-wash location after order delivery and prepping for the next order. Unlike traditional vehicle routing problems, the chemical interaction properties of these orders must be accounted for to prevent risk of contamination which could impose complex product-sequencing constraints. For each chemical order, we maintain a list of restricted and approved prior orders, and a route is considered to be feasible if it complies with the prior order compatibility relationships. We present a parallel computation framework that envelops column generation technique for large-scale route evaluations to determine the optimal trailer-order-wash combinations while meeting the chemical compatibility constraints. We perform several experiments to demonstrate the efficacy of our proposed method. Experimental results show that the proposed parallel computation yields a significant improvement in the run time performance. Additionally, we perform sensitivity analysis to show the impact of wash capacity on order coverage.

Keywords Vehicle routing problem · Stochastic transit times · Compatibility constraints · Column generation · Parallel computation

Introduction

According to American Chemical Council (ACC), the chemical industry accounts for a $797B enterprise that is projected to increase its capacity by 18% in 2020, resulting in complexity in transportation (Baldwin 2017). For chemical and liquid bulk transportation companies such as Schneider National, a fleet could be comprised of thousands of trailers, across which hundreds of new orders per day are dispatched. Over the course of a year, tens of thousands of distinct orders may be transported. Unlike classical transportation problems (Dantzig and Ramser 1959), chemical transportation involves two additional constraints: hazardous interaction properties among chemicals, and washing decisions (e.g., location, wash type) for trailers after delivery. These constraints need to be addressed in addition to those involved with standard vehicle routing problems, making the problem complex. Typically, the customer orders represent requests to freight chemicals that are characterized by a set of attributes consisting of an origin and destination locations, pickup and delivery time windows, an order specification, restrictions based on prior orders. The execution of each task requires several inter-dependent sourcing decisions such as: (1) determine suitably cleaned, and configured tanker trailers that are compatible with chemical order requirements, (2) check whether the previous contents of the trailer meet compatibility rules for the new chemical order, (3) select another tank-wash facility (post-wash location) where the trailer will be washed and prepped for the subsequent order.

There are two main contributions of this research. Firstly, to address large-scale nature of this problem, we show how parallel computation framework can be implemented for large-scale route evaluations with order compatibility checks to significantly reduce the total run time and number of iterations required for traditionally used column generation approach to such problems and determine the optimal trailer-order-wash combinations. Secondly, using numerical experiments, we show the performance of our approach

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under different scenarios with varying number of trailers, orders and tank-wash locations. We also show how order incompatibilities impact the choice to trailer and wash selection. Additionally, we analyze the impact of wash constraints (capacity, location, etc.) on the order coverage.

The rest of the paper is organized as follows. "Problem description" section describes the system model and assumptions. "Mathematical model" section presents the mathematical model formulation of the proposed method. Using these specialized set of mathematical models, we develop an exact solution methodology to solve complex tanker trailer routing problems with stochastic transit times. "Solution approach" section presents an approach which combines column generation technique with parallel computation to determine the optimal solution. "Numerical experiments" section summarizes numerical experiments conducted for the proposed system. It also includes the details of sensitivity analysis performed to analyze the impact of several factors on the performance of the model. Finally, "Conclusion" section summarizes model insights and conclusions.

Literature review

We briefly review the literature related to rich vehicle routing and stochastic travel times (Table 1).

One of the classes of rich vehicle routing problems deals with systems of heterogeneous fleet of vehicles (Sherali et al. 2013; Yousefikhoshbakht et al. 2013; Goel and Vidal 2014; Cacchiani and Salazar-González 2017). Heterogeneous vehicle routing problem was first introduced by Golden et al. (1984) that operates under an unlimited fleet of vehicles that differ in terms of vehicle type, capacity, and costs. The authors presented several heuristic approaches as well as techniques to determine a lower bound and underestimate of the optimal solution. Later, Taillard (1999) introduced heterogeneous fixed fleet vehicle routing problem (HFVRP) operating under pre-defined vehicles; more relevant to our research. Ceselli et al. (2009) propose a column generation based algorithm to solve a rich vehicle routing problem (VRP) in which they compute a daily plan for a heterogeneous fleet of vehicles that depart from various depots and must visit a set of customers to deliver certain goods. For a detailed review of the application of column generation in vehicle routing problems, we suggest the reader to refer Feillet (2010). Choi and Tcha (2007) develop an integrated column generation and dynamic programming based schema approach to generate tight bounds on the optimal solution for heterogeneous vehicle routing problem. Unlike these works, our research investigates a large-scale rich vehicle routing problem for chemical transportation while considering prior order compatibility relationships and provides optimal trailer-order-wash combinations.

Table 1 Tabular representation of literature review

Authors	Homogeneous or heterogeneous VRP	Deterministic or stochastic transit times	Research methodology and findings
Golden et al. (1984)	Heterogeneous	Deterministic	Presented several heuristic approaches as well as techniques to determine a lower bound and underestimate of the optimal solution
Ceselli et al. (2009)	Heterogeneous	Deterministic	Proposed a column generation based algorithm in which they compute a daily plan for vehicles that depart from various depots and must visit a set of customers to deliver certain goods
Choi and Tcha (2007)	Heterogeneous	Deterministic	Developed an integrated column generation and dynamic programming based schema approach to generate tight bounds on the optimal solution
Afshar-Bakeshloo et al. (2016)	Heterogeneous	Deterministic	Developed a mixed integer linear programming (MILP) model which efficiently uses piecewise linear functions (PLFs) to linearize a nonlinear fuzzy interval in order to incorporate customer satisfaction into other linear objectives
Woensel et al. (2003)	Homogeneous	Stochastic	Developed a heuristic approach that combines the ant colony optimization algorithm with congestion component that was modeled using a queuing approach to traffic flows
Jula et al. (2006)	Homogeneous	Stochastic	Proposed a solution approach that uses a dynamic programming based approximate solution method to find the best route with minimum expected cost
Tas et al. (2013)	Homogeneous	Stochastic	Solved a problem that considers both transportation costs and service costs by a Tabu search algorithm. Further improvements have been made by using a post-optimization method
Errico et al. (2016)	Homogeneous	Stochastic	Solved the problem using a two-stage recourse model with priori optimization

Unlike dispatch decisions with pickups and deliveries under deterministic travel times (DellAmico et al. 2006; Bianchessi and Righini 2007; Qu and Bard 2014; Kır et al. 2017; Santillan et al. 2018), stochastic transit times (Li et al. 2010; Tavakkoli-Moghaddam et al. 2012; Lei et al. 2012; Yan et al. 2013; Errico et al. 2013) create additional complexity. Woensel et al. (2003) develop a heuristic approach to solve a vehicle routing problem with stochastic travel times due to potential traffic congestion. The approach combines the ant colony optimization algorithm with congestion component that was modeled using a queuing approach to traffic flows. Jula et al. (2006) analyze a stochastic traveling salesman problem with time windows (STSPTW) under stochastic travel and service times. The solution approach uses a dynamic programming based approximate solution method to find the best route with minimum expected cost. Tas et al. (2013) propose a method to solve a vehicle routing problem with stochastic travel times and soft time windows. The problem that considers both transportation costs and service costs has been solved by a Tabu search algorithm and further improvements have been made by using a post-optimization method. Errico et al. (2016) analyze a vehicle routing problem under hard time windows using a two-stage recourse model with priori optimization. Our research analyzes intricacies in chemical transportation under soft time windows for pickup and deliveries with stochastic transit times.

The main gap in the literature related to rich vehicle routing decisions is that, in chemical transportation, an integral component of selecting a feasible tanker trailer is to ensure that its previous contents are chemically compatible with that of a prospective order. Even with consideration of intervening washes, this compatibility may need to be checked with up to three previous orders, and requirements for seemingly identical chemical compounds may vary according to customers, making it much harder to solve.

Problem description

In the chemical and liquid bulk transportation, the fleet dispatch problem which addresses the matching of tanker trailers to customer orders is a complex problem. Figure 1 highlights the issues with selecting trailer, wash locations for chemical order transportation.

The solid lines of Fig. 1 illustrate a single episode (coverage of a single order) in the life cycle of tanker trailer. In this case, a customer order requires order pickup at customer location P_1 and delivery at consignee location D_1. A suitable tanker trailer is identified at wash location W_1 and moved clean to location P_1 for loading. Next, the loaded tanker trailer is transported to consignee location D_1. Once the unloading is complete at location D_1, the dirty tanker trailer is repositioned to wash location W_4, where it is cleaned and prepped for its next order. The dashed lines of Fig. 1 depict an alternative (non-selected) assignment choice for the selected trailer to a different order (W_1–P_2–D_2–W_4), or use of a different trailer for this order (W_2–P_1–D_1–W_4). The task of the trailer route optimization is to determine optimal solutions from the large amount of feasible combinations. Pre-wash location alternatives (e.g., W_2–P_1) are considered in the context of both costs (e.g., distance) and constraints (e.g., inventory balancing). Post-wash alternatives (e.g., D_1 –W_3) attempt to account for future value considerations (next orders) based on tanker types/attributes.

Customer-specific requirements also drive the prior-order compatibility rules that together with trailer physical attributes (lining material, heaters, pumps, etc.) determine the subset of tanker trailers that are feasible for a particular order. Note that the selection of post-wash location could strategically position the tanker trailer to serve the subsequent order. However, the chemical interaction properties of these orders must be accounted for which limits the feasibility and selection of next order in the route. Each order

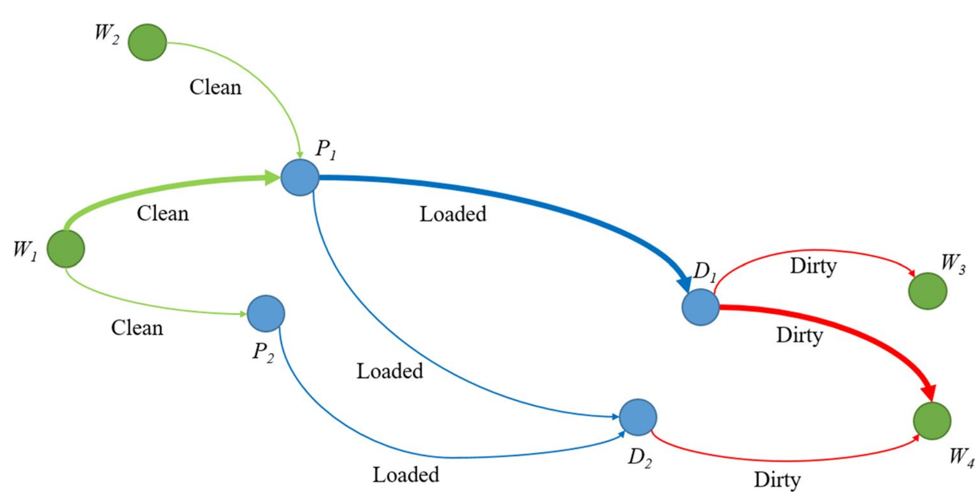


Fig. 1 An interpretation of a trailer route

has a list of restricted and approved prior orders (as shown in Fig. 2a). Typically, two to three prior order compatibility checks are required before serving any order. One such example addressing the issue with prior order relationships is shown in Fig. 2b.

The trailer had previously transported order O_1 which serves as one of the prior orders for the new set of orders. Also, we assume that O_1, O_4 are incompatible, O_2, O_5 are incompatible, and O_3, O_6 are incompatible which is very common in practice. Using this compatibility relationship, for any route serving O_3 as the first order, the trailer can only serve O_6 as the next order. Similarly, for any route serving O_2 as the first order, the trailer can only serve O_5 as the next order. Note that order O_4 can never be served as it conflicts with the trailer's prior order O_1. Given the number of distinct options that are often available for each decision, the overall number of trailer route combinations can be counted in billions.

Mathematical model

Let $\mathcal{T}$ be the set of tanker trailers. We define $\mathcal{N}$ as the set of network nodes comprising of wash locations, $\mathcal{W} \subseteq \mathcal{N}$, pickup locations, $\mathcal{P} \subseteq \mathcal{N}$, and final delivery locations, $\mathcal{F} \subseteq \mathcal{N}$ for customer orders. At the beginning of each time period, trailer $t \in \mathcal{T}$ is sourced from the wash location $w \in \mathcal{W}$, i.e., trailer is left at a tank wash at the end of each time period. For each trailer t, we maintain a *prior product* matrix $\mathbb{P}_t$ that keeps track of up to three prior orders that were covered using trailer t, and a *prior product requirements* matrix $\mathbb{Q}_t$ that depends on $\mathbb{P}_t$ and provide a list of orders that should not be included in the route for trailer t. Next, for a given pickup and delivery location pair $(p_{o_i}, f_{o_i}), p_{o_i} \in \mathcal{P}, f_{o_i} \in \mathcal{F}$ for customer order $o_i \in \mathcal{O}$, we define a route of a trailer t as a walk represented by $r_t = (w_1, p_{o_1}, f_{o_1}, w_2, p_{o_2}, f_{o_2}, w_3), w_1, w_2, w_3 \in \mathcal{W}, r_t \in \mathcal{R}$, where w_i is the pre-wash and w_{i+1} is the post-wash for order $o_i, i = 1, 2$. A typical route of a trailer t is shown in Fig. 3. For a route to be feasible, it must comply with the chemical compatibility relationship, i.e., each trailer t should not process any restricted set of orders Q_o corresponding to any individual order o in the prior order vector $\mathcal{P}_t$.

Let $(l_{o,p}, u_{o,p})$ be the time windows at pickup for order o and $(l_{o,f}, u_{o,f})$ be the time windows at delivery for order o. Each trailer route r_t incurs a cost C_{r_t} comprised of transit costs, empty mile costs, bonuses and penalties. Empty mile cost is the cost incurred for every mile traveled by a trailer t without carrying any orders while traveling to and fro from a wash location. A bonus may be gained in the form of discounts in some situations like a post-wash location offering some discounts for any trailer t getting washed there during specific set of times. A penalty cost will be incurred when a trailer t either delivers an order o later than the delivery time windows $(l_{o,f}, u_{o,f})$ or does not cover that order at all. Note that at the end of each delivery, trailer t is required to undergo a specialized tank-wash operation. Finally, at the end of time period, the tanker is left at the post-wash facility.

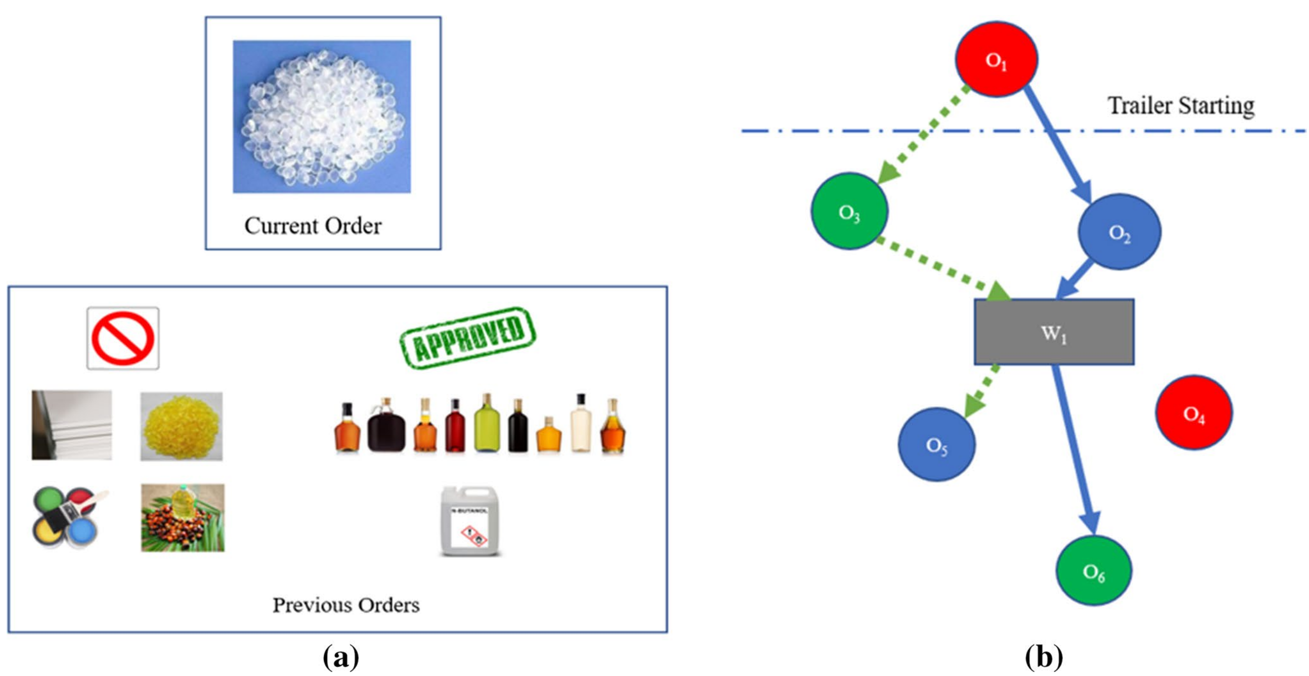


Fig. 2 **a** Restricted and approved list of orders. **b** An illustration of prior order relationships

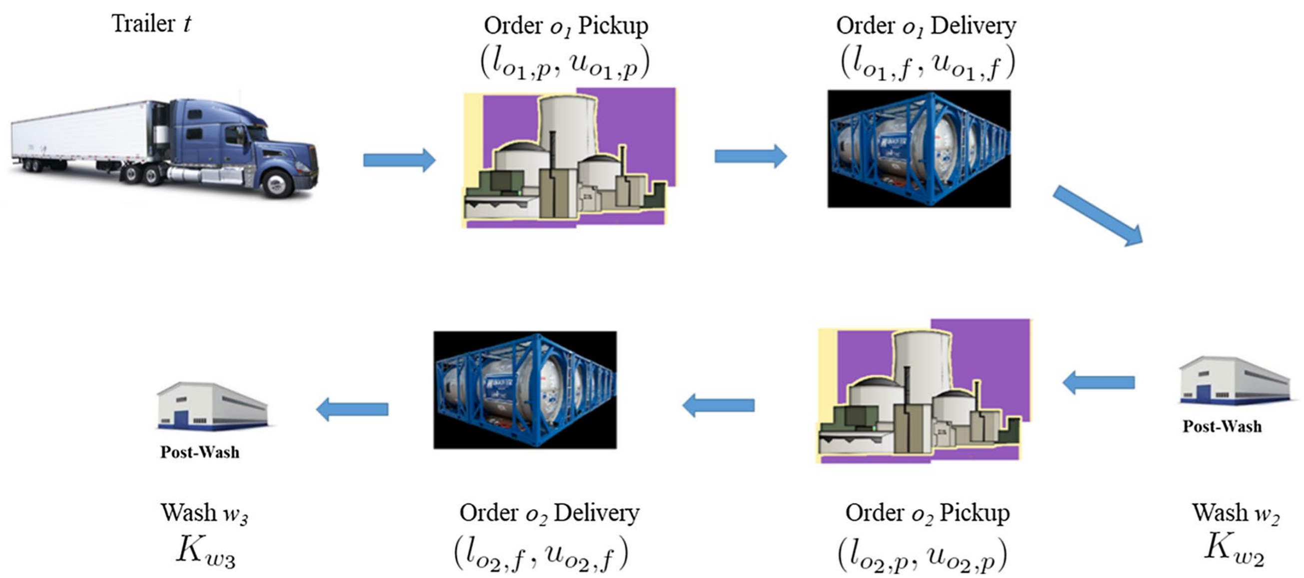


Fig. 3 A typical route of a trailer t

Expected earliness and expected delay

We assume stochastic transit times to capture the impact of traffic/weather delays. We also assume negligible waiting time for the trailers at the wash stations to get cleaned. However, this can be generalized by shifting the arrival times or changing the distribution parameters of the transit times. Some of the most commonly used distributions for the transit times are normal, log-normal and gamma distributions (Fan et al. 2005; Lecluyse et al. 2009; Tas et al. 2013).

We assume that travel time τ_{g,h,r_t} follows Gamma distribution $\Gamma(\alpha(g,h,r_t), \beta(\cdot))$ where $\alpha(g,h,r_t)$ is the shape parameter, $\beta(\cdot)$ is the inverse scale parameter, and g, h refer to the two nodes of the arc (g, h) traveled in route r_t by the trailer t. Then, the cumulative distribution function of the transit times for the arc (g, h) can be given by: $\Gamma_{\alpha(g,h,r_t),\beta(\cdot)}(\theta) = \int_0^{\theta} \frac{(\mathrm{e}^{-q/\beta(\cdot)})q^{\alpha(g,h,r_t)-1}}{\Gamma(\alpha(g,h,r_t))(\beta(\cdot))^{\alpha(g,h,r_t)}}\mathrm{d}q$. Let $d_{k_{o_i}}$ be the total distance traveled by the trailer t from its starting location to cover order o_i at location type $k, k = p,f$ (pickup or delivery location). Using the additive property of Gamma distribution and our assumption of negligible waiting times, the shape and scale parameters of the arrival time of trailer t for order o_i at location type $k, k = p,f$ can be given as: $\alpha_{ik} = \alpha d_{k_{o_i}}$, and $\beta_{ik} = \beta$.

Let $\mathbb{E}_{ik}$ be the expected earliness for order $o_i, i = 1,2$ at location type $k, k = p,f$. Then $\mathbb{E}_{ik}$ can be defined in Eq. (1) as:

$$\begin{aligned}
\mathbb{E}_{ik} &= \int_0^{l_{o_i,k}} (l_{o_i,k} - q)\frac{(\mathrm{e}^{-q/\beta_{ik}})q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}}\,\mathrm{d}q \\
&= l_{o_i,k}\int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}}\,\mathrm{d}q - \int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{\alpha_{ik}}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}}\,\mathrm{d}q \\
&= l_{o_i,k}\int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}}\,\mathrm{d}q - \int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{(\alpha_{ik}+1)-1}}{\frac{\Gamma(\alpha_{ik}+1)}{\alpha_{ik}}\frac{(\beta_{ik})^{(\alpha_{ik}+1)}}{\beta_{ik}}}\,\mathrm{d}q \\
&= l_{o_i,k}\int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}}\,\mathrm{d}q - \alpha_{ik}\beta_{ik}\int_0^{l_{o_i,k}} \frac{(\mathrm{e}^{-q/\beta_{ik}})q^{(\alpha_{ik}+1)-1}}{\Gamma(\alpha_{ik}+1)(\beta_{ik})^{\alpha_{ik}+1}}\,\mathrm{d}q \\
&= \left(l_{o_i,k}\Gamma_{\alpha d_{k_{o_i}},\beta}(l_{o_i,k}) - \alpha\beta\Gamma_{\alpha d_{k_{o_i}}+1,\beta}(l_{o_i,k})\right)
\end{aligned} \tag{1}$$

Then, the expected earliness $\mathbb{E}_{r_t}$ for route r_t for trailer t can be given by Eq. (2):

$$\mathbb{E}_{r_t} = \sum_{i=1,2}\sum_{k=p,f}\left(l_{o_i,k}\Gamma_{\alpha d_{k_{o_i}},\beta}(l_{o_i,k}) - \alpha\beta\Gamma_{\alpha d_{k_{o_i}}+1,\beta}(l_{o_i,k})\right) \tag{2}$$

Similarly, let $\mathbb{L}_{ik}$ be the expected delay for order $o_i, i = 1,2$ at location type $k, k = p,f$. Then $\mathbb{L}_{ik}$ can be defined in Eq. (3) as:

$$\begin{aligned}\mathbb{L}_{ik} &= \int_{u_{o_i,k}}^{\infty} (q - u_{o_i,k}) \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}} \quad \mathrm{d}q \\ &= \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{\alpha_{ik}}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}} \quad \mathrm{d}q - u_{o_i,k} \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}} \quad \mathrm{d}q \\ &= \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}} q^{(\alpha_{ik}+1)-1}}{\frac{\Gamma(\alpha_{ik}+1)}{\alpha_{ik}} \frac{(\beta_{ik})^{(\alpha_{ik}+1)}}{\beta_{ik}}} \quad \mathrm{d}q - u_{o_i,k} \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}} \quad \mathrm{d}q \\ &= \alpha_{ik}\beta_{ik} \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{(\alpha_{ik}+1)-1}}{\Gamma(\alpha_{ik}+1)(\beta_{ik})^{\alpha_{ik}+1}} \quad \mathrm{d}q - u_{o_i,k} \int_{u_{o_i,k}}^{\infty} \frac{(\mathrm{e}^{-q/\beta_{ik}}) q^{\alpha_{ik}-1}}{\Gamma(\alpha_{ik})(\beta_{ik})^{\alpha_{ik}}} \quad \mathrm{d}q \\ &= \left(\alpha\beta d_{k_{o_i}} (1 - \Gamma_{\alpha d_{k_{o_i}}+1,\beta}(u_{o_i,k})) - u_{o_i,k}(1 - \Gamma_{\alpha d_{k_{o_i}},\beta}(u_{o_i,k})) \right)\end{aligned} \tag{3}$$

Then, the expected lateness $\mathbb{L}_{r_t}$ for route r_t for trailer t can be given by Eq. (4):

$$\mathbb{L}_{r_t} = \sum_{i=1,2} \sum_{k=p,f} \left(\alpha\beta d_{k_{o_i}} (1 - \Gamma_{\alpha d_{k_{o_i}}+1,\beta}(u_{o_i,k})) - u_{o_i,k}(1 - \Gamma_{\alpha d_{k_{o_i}},\beta}(u_{o_i,k})) \right) \tag{4}$$

Bulk dispatch model

Next, we present the optimization model corresponding to the proposed bulk dispatch problem. Let $x_{r_t} \in x$ be a binary variable that takes the value 1 if trailer route r_t is selected, and 0 otherwise. We define additional sets of routes as shown in Table 2.

The proposed dispatch optimization problem can be best described with a set partitioning formulation with side constraints as model $P1$ as follows:

$$P1 \, : \, z_1 = \min \sum_{r_t \in \mathcal{R}} C_{r_t} x_{r_t} + \sum_{r_t \in \mathcal{R}} (\mathbb{E}_{r_t} + \mathbb{L}_{r_t}) + \sum_{o \in \mathcal{O}} B_o \xi_o \tag{5}$$

$$\sum_{r_t \in \mathcal{R}^t} x_{r_t} + \xi_t = 1, \quad \forall t \in \mathcal{T} \tag{6}$$

$$\sum_{r_o \in \mathcal{R}^o} x_{r_o} + \xi_o = 1, \quad \forall o \in \mathcal{O} \tag{7}$$

$$\sum_{r_w \in \mathcal{R}^w} x_{r_w} \leq K_w, \quad \forall w \in \mathcal{W} \tag{8}$$

In the above formulation, the objective defined by Eq. (5) minimizes the total cost that includes the following costs terms: total cost of the route, $\sum_{r_t \in \mathcal{R}} C_{r_t} x_{r_t}$, total penalty cost for not covering an order by any trailer route, $\sum_{r_t \in \mathcal{R}} (\mathbb{E}_{r_t} + \mathbb{L}_{r_t})$, and $\sum_{o \in \mathcal{O}} B_o \xi_o$, where B_o and ξ_o are the corresponding penalty cost and slack variable associated with not covering order o with any trailer route. Main constraints include: Eq. (6) which ensures that for each piece of trailer t, we can only have one route scheduled in the optimal solution, Eq. (7) which ensures that each order o can be assigned to a route or it can be unscheduled, Eq. (8) which prevents over-capacity at a tank wash w where K_w is the capacity at tank wash w.

Table 2 Additional sets of routes

Notation	Description
$\mathcal{R}^t, \mathcal{R}^t \in \mathcal{R}$	The set of routes for trailer t
$\mathcal{R}^o, \mathcal{R}^o \in \mathcal{R}$	The set of routes that cover order o
$\mathcal{R}^w, \mathcal{R}^w \in \mathcal{R}$	The set of routes using tank wash w

The bulk dispatch model presented in formulation $P1$ is a combinatorial problem with TO^2W^2 feasible routes in the worst case. For instance, a problem with 500 trailers, 500 orders, and 20 wash locations could result in about 50 billion routes in the worst case. In such cases, the conventional branch-and-bound approach is not tractable. However, this class of problems have been proved to be solved to optimality using column generation approach which transforms the standard branch-and-bound approach into branch-and-price (Ceselli et al. 2009; Feillet 2010). In the column generation method, particularly we relax integrality conditions and take only a subset of the decision variables ($\mathcal{R}_1 \in \mathcal{R}$) into consideration thus transforming the Master Problem [the linear relaxation of (5–8)] into a Restricted Master Problem.

At each column generation iteration, we solve the Restricted Master Problem. Then, we search for new columns (or potential routes) with minimum negative reduced cost. The negative reduced cost δ^{-r_t} of each column (or route) r_t is given by:

$$\delta^{-r_t} = (C_{r_t} + \mathbb{E}_{r_t} + \mathbb{L}_{r_t}) - \lambda_t - \sum_{o \in \mathcal{O}} a_{r_t,o} \sigma_o - \sum_{w \in \mathcal{W}} a_{r_t,w} \mu_w \tag{9}$$

where λ_t is the non-negative dual variable associated with the t-th constraint of the set (6), σ_o is the non-negative dual variable associated with the o-th constraint of the set (7), $a_{r_t,o}$ takes the value of 1 if route r_t covers order o or 0 otherwise, μ_w is the non-negative dual variable associated with the w-th constraint of the set (8) and $a_{r_t,w}$ takes the value of 1 if route r_t contains tank wash w or 0 otherwise.

Solution approach

Note that the model under consideration entails large volume of orders and trailers which results in two competing objectives: (1) determining optimal trailer-order-wash combinations, (2) improving the run time performance for our large-scale problem. Typically in the literature, some variant of shortest path algorithm is used to determine the routes with the least reduced cost as a candidate to enter the feasible set for next iteration of column generation approach. However, this method has a limitation to only add one route at a time and adding multiple routes is combinatorial in nature, resulting in more iterations to solve the problem. To overcome this deficit, we develop efficient parallel computation framework (see "Candidate route selection" section) to significantly

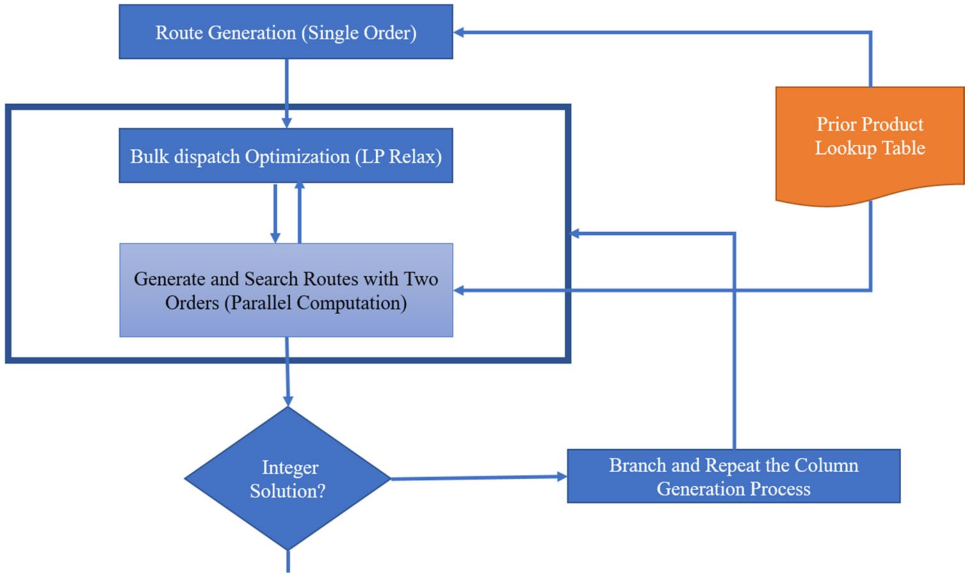


Fig. 4 Proposed solution approach

expedite the evaluation of multiple route choices with order compatibility checks while reducing the number of column generation iterations. Figure 4 provides the framework of our proposed integrated approach.

The first step in our solution approach is to generate all feasible routes with single orders and wash locations. Then, the next step is to solve the LP relaxation of Restricted Master Problem (which considers only a subset of the decision variables). Next, the single order routes and the dual values from the LP relaxation are used to generate second order routes. We employ efficient parallel computation technique in this step. While generating routes with two orders, we make sure that the orders are chemically compatible by looking at the prior orders table in which we have the details of up to three prior orders. Next, the set of single order and two order routes is fed to the optimization model. Finally, the process terminates if the solution is integral. Otherwise, we repeat the process with single order routes until we get an optimal integer solution.

Candidate route selection

This step generates and evaluates trailer routes while respecting the model constraints presented in "Mathematical model" section . Considering the fact that the route evaluation process requires several chemical compatibility checks for orders, candidate route selection process can be inherently parallelized by extending single order routes. This can be done using the parallel computation technique which is an efficient way to reduce the run time of the model by running several tasks in parallel with the help of multiple processors. The detailed procedure of the parallel computation technique is presented in Algorithm 1.

Algorithm 1 Candiate Route Selection using Parallel Computation

```
procedure SecondOrder(route r, orders, washes)
    for order o in orders do
        r1 ← add order o to route r
        compatibilityflag ← Is order o compatible with r1
        if (compatibilityflag == True) then
            for wash w in washes do
                r2 ← add wash w to route r1
                if route r2 has minimum negative reduced cost then
                    RouteSelect ← r2
    return RouteSelect

procedure InitiateParallelTasks
    ParallelTaskList ← []
    for SingleOrderRoutes r do
        ParallelTask ← SecondOrder(r, orders, washes)
        ParallelTaskList ← ParallelTask
        execute ParallelTaskList
    for ParallelTask pTask in ParallelTaskList do
        SelectedSecondOrderRoutes ← get value from pTask
```

The first step in our technique is to generate n feasible routes with single order. Then, the next step involves route generation with two orders using each single order route. This is accomplished using the procedure SecondOrder. In order to do this task, first we look at the prior orders table to check if the orders are chemically compatible. If they are compatible, then we generate routes with two orders. Next, for every single order route, we select the best two order routes with minimum negative reduced cost. Then, the final step in our framework combines the list of all the selected two order routes by joining all the parallel threads from n single order routes. The parallelization of the entire process is done with the help of procedure InitiateParallelTasks. Note that we also have the capability to aggregate single order routes to limit the number of active threads.

Table 3 Different scenarios for experiments

Scenario	Trailers	Orders	Tank washes
Scenario 1	50	75	20
Scenario 2	75	100	20
Scenario 3	100	125	20
Scenario 4	125	150	20
Scenario 5	150	200	20
Scenario 6	175	225	20
Scenario 7	200	250	20
Scenario 8	225	275	20
Scenario 9	250	300	20
Scenario 10	300	400	20

Numerical experiments

In this section, we discuss numerical experiments to compare the performance of our solution approach under different scenarios. We conduct two sets of experiments. For each set of experiments, we consider ten different scenarios with varying number of trailers and orders (50–300 trailers and 75–400 orders) as shown in Table 3.

For each scenario, the trailers are equally spread at the wash location at the beginning, i.e., same number of trailers start from the wash locations. Each order has restrictions on two to three orders for their prior orders. The wash capacity at each tank wash is set to 10. The algorithms are coded in JAVA, and all the experiments are conducted on two hardware: (1) Workstation—Intel Xenon processor 6C, 3.6 GHz with 32 GB RAM, (2) Server—Intel Xenon 24C, 3.2 GHz with 256 GB RAM. The first set of experiments analyze the impact of thread pool size on run rime performance (see "Run time performance under different scenarios" section). The second set of experiments analyze the impact of wash constraints on order coverage (see "Impact of wash capacity" section).

Run time performance under different scenarios

In order to parallelize route generation and evaluation on the data sets, first we need to figure out the optimal number of processor threads. For instance, for Scenario 10, we perform five experiments with different thread pool sizes to analyze the impact of thread pool size on run time performance. The Scenario 10 has 300 trailers, 400 orders and 20 tank-wash

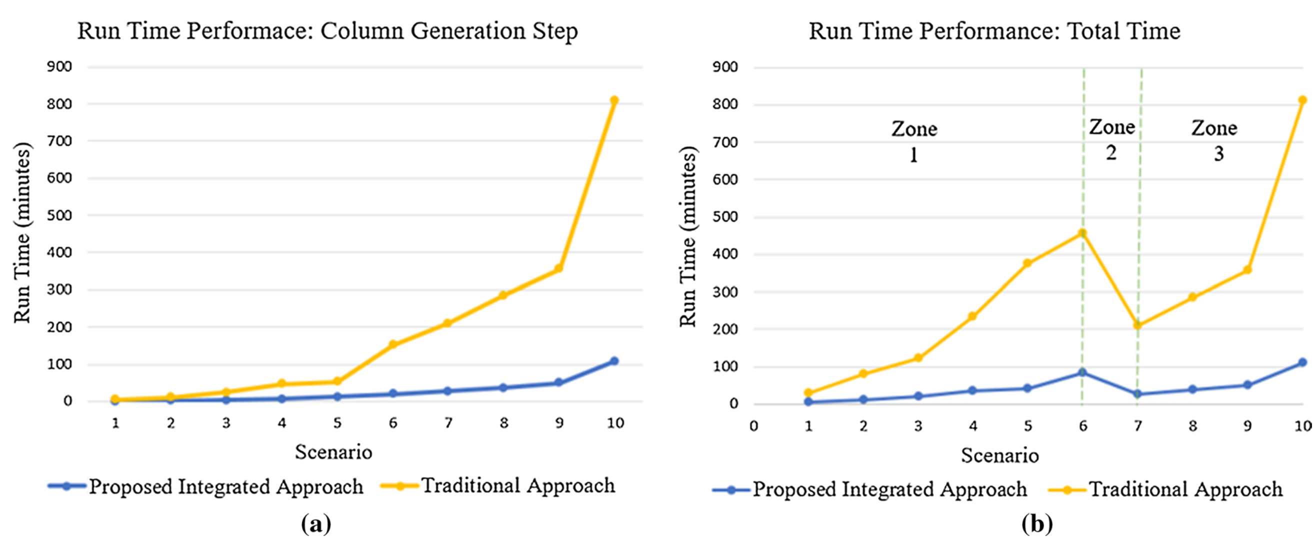


Fig. 5 **a** Run time performance: column generation step. **b** Run time performance: total time

locations. Experimental results suggest that 6 core workstation provides maximum performance with 15 threads and 24 core server provides maximum performance with 100 threads. We analyze ten scenarios to examine the performance of our model using the proposed parallel computation framework. For each scenario, we compare the run time performance of our proposed approach with the traditional approach (no parallel computation) under two cases: (1) time taken to complete single column generation iteration (see Fig. 5a) (2) total time taken to complete the algorithm (see Fig. 5b).

For a single column generation iteration, the run time increases exponentially with the increase in number of trailers and orders due to increase in the number of route evaluations. We also observe a significant reduction in the run time in our proposed approach. However, for the total time case, we observe three main trends/zones in the graph: (1) Zone 1: Scenario 1–Scenario 6 (2) Zone 2: Scenario 6–Scenario 7 and (3) Zone 3: Scenario 7–Scenario 10. Note that the total time depends on the run time for each column generation step as well as the number of iterations needed to converge. In Zone 1, we observe an increasing trend as the problem size increases due to increase in the number of feasible routes. We also observe fairly constant number of column generation iterations to reach convergence. However, in Zone 2, the number of column generation steps start to decrease due to fairly large amount of candidate routes generated for the optimization model. This increases the chance for the column generation process to find the optimal routes in only one iteration. Finally, in Zone 3, the number of column generation steps can no longer be further reduced (only one iteration needed) and the total run time starts to increase with the problem size. Note that our proposed approach tries to balance column generation iterations and their run time to

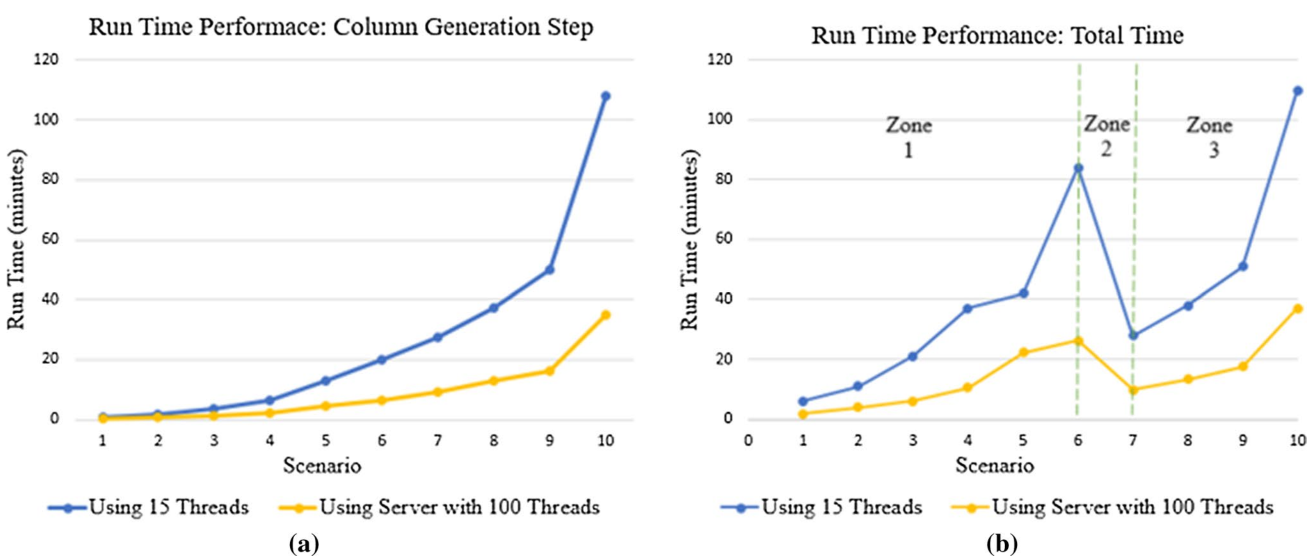


Fig. 6 Run time performance comparison on workstation and server

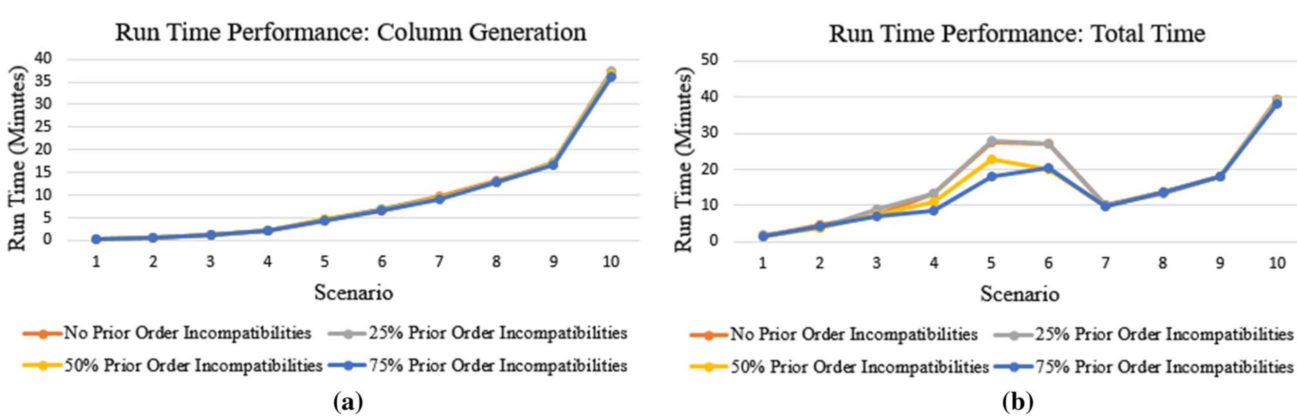


Fig. 7 Impact of prior order compatibility relationship on performance

achieve a significant improvement in the performance. The final results from Fig. 5a, b indicate a 85% improvement in run time using the proposed parallel computation approach. Given the fact that every single order route is independent, route selection process can be done parallelly by extending the single order routes contributing to a significant improvement in run time performance which is evident from the results.

Next, we compare the performance of our proposed approach under two hardware, one using 15 threads on a 6 core workstation and the other using a server (24 core server) with 100 threads (see Fig. 6a, b). For single column generation iteration and total time, we observe trends similar to the comparison made earlier. We also observe an additional 65% reduction in run time using the server. Finally, we examine the performance of our proposed approach under four different prior order compatibility relationships (see Fig. 7a, b): (1) No prior order compatibility relationships, (2) 25% of orders have compatibility relationships, (3) 50% prior order compatibility relationships and (4) 75% prior order compatibility relationships.

For single column generation iteration, changes in percentage of prior order relationships do not have any significant impact on the run time performance. However, for the total time case, increase in the percentage of prior order compatibility relationships results in the reduction of run time due to decrease in the number of route evaluations. We observe increase in optimal routes with two orders as prior order compatibility relationships are relaxed.

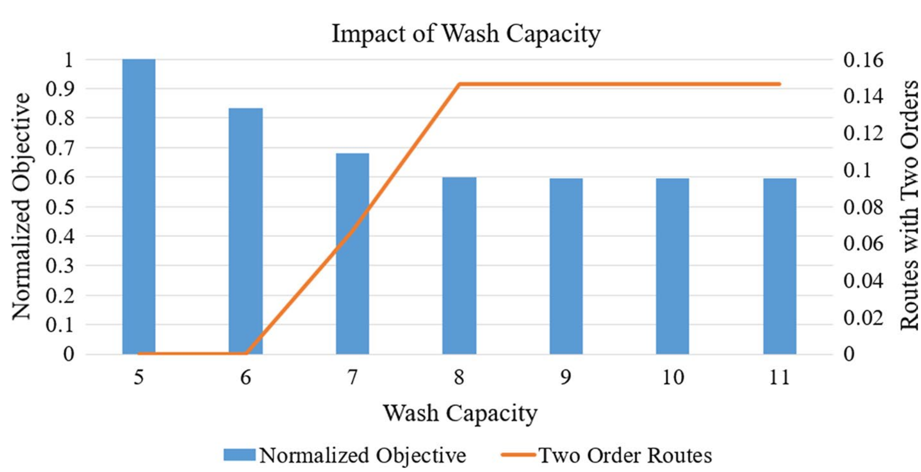
Impact of wash capacity

Wash constraints like capacity, location etc. need to be examined as they could create complexities while covering the orders. We perform sensitivity analysis to evaluate the impact of wash capacity on order coverage and normalized cost (see Fig. 8). Recall that the wash location better prepares the trailer for the next order. In Fig. 8, we observe that the optimal solution has a better chance to cover two orders in a route when the wash capacity is not limited. However, at low capacity (capacities of 5 and 6), the order coverage is limited by the number of trailers as well as wash capacity. The total cost is also impacted as the number of feasible route choices are non-increasing with the increase in wash capacity.

Conclusion

We analyze a rich tanker trailer routing problem with stochastic travel times for chemical orders where the chemical interaction properties of the orders create an additional complexity while determining the best strategies to dispatch trailers. For each chemical order, we maintain a list of up to three prior orders. To prevent the risk of chemical contamination, the set of orders assigned to a trailer route must comply with the prior order compatibility relationships. To address large-scale nature of the problem, we propose an integrated approach which incorporates the efficacy of stochastic models, column generation technique and parallel computation to generate the optimal candidate routes. Our initial set of experiments focus on determining the optimal number of threads needed to parallelize candidate route selection. Later, we conduct comparative analysis between the performances of the traditional approach and our proposed parallel computation method. Then, we perform a set of experiments to validate the performance of the proposed approach using two different hardware. Experimental results indicate that our approach results in a significant improvement in run time performance (around 85% run time improvement on workstation and an additional 65% improvement on the server). Although most of the performance improvement is due to the implementation of parallel computing framework for the route generation and evaluation process, we also observe significant improvement with balancing the

Fig. 8 Impact of wash capacity



tradeoffs between column generation step run time and the number of column generation iterations. We also observe that with the increase in problem size, the column generation process tends to converge in fewer iterations. Then, we analyze the impact of the percentage of prior order compatibility relationships on our solution approach and find out that increase in prior order incompatibilities result in the reduction of total run time due to the reduced number of route evaluations The next experiment analyzes the impact of wash constraints like capacity and location as they could potentially affect the coverage of subsequent orders. The results of the numerical investigation reveal that the order coverage is limited by the number of trailers as well as wash capacity. Less wash capacity leads to single order trailer routes in the optimal solution to reduce the number of trips to the wash locations. Future work will focus on developing efficient route-pruning approaches to minimize the candidate set and improving the parallel computation framework to further decrease the run time. We also intend to investigate the impact of uncertainty in wash capacity on the optimal decisions.

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