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ORIGINAL RESEARCH

Vacation model for Markov machine repair problem with two heterogeneous unreliable servers and threshold recovery

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Abstract Markov model of multi-component machining system comprising two unreliable heterogeneous servers and mixed type of standby support has been studied. The repair job of broken down machines is done on the basis of bi-level threshold policy for the activation of the servers. The server returns back to render repair job when the pre-specified workload of failed machines is build up. The first (second) repairman turns on only when the work load of N_1 (N_2) failed machines is accumulated in the system. The both servers may go for vacation in case when all the machines are in good condition and there are no pending repair jobs for the repairmen. Runge–Kutta method is implemented to solve the set of governing equations used to formulate the Markov model. Various system metrics including the mean queue length, machine availability, throughput, etc., are derived to determine the performance of the machining system. To provide the computational tractability of the present investigation, a numerical illustration is provided. A cost function is also constructed to determine the optimal repair rate of the server by minimizing the expected cost incurred on the system. The hybrid soft computing method is considered to develop the adaptive neuro-fuzzy inference system (ANFIS). The validation of the numerical results obtained by Runge–Kutta approach is also facilitated by computational results generated by ANFIS.

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Introduction

In this industrial age, the machining system becomes the great boon for the human beings. In machining systems, the failure of its components is quite common phenomenon which causes adverse effect on the efficiency, quality, and output of the system. To overcome these problems, many queue theorists have paid attention towards the machine repair problems in different contexts. To enhance the performance and reliability of any machining system, there is need of backup support (standby) to working machines. The backup support in terms of redundancy is also helpful in enhancing the performance and smooth functioning of the machining system. The system with standby support plays a vital role in real-time machining systems due to its critical requirement in many computer embedded systems such as computer networks and telecommunication systems, industrial and information systems, and many more. In queueing and reliability literature, the notable research works on Markov modeling of machining system with standby support can be found [cf. Wang and Kuo (2000), Wang and Ke (2003), and Haque and Armstrong (2007)]. Shree et al. (2015) proposed a Markov model for the machining systems with hot spares. Jain (2016) presented the transient study of machining system by incorporating some realistic features, namely service interruption, priority and mixed standbys support. Recently, Jain et al. (2017) proposed a Markov model for the repairable system including the features of F-policy, working vacation, and server break down. In this study, they have derived system indices and steady-state probabilities using SOR method.

Newton-quasi method and heuristic approach to find the optimal parameters and minimum cost incurred to the system have also been implemented.

The threshold policy for starting of service can be used to overcome the waste of valuable resources, time and money of an industry, or company operating in machining environment. For any single-machine repair system, N-policy can be implemented for the economic utilization of the server. Threshold N-policy states that the server is turned on to render repair only when the workload of repair job of failed machines reaches to pre-defined threshold level *N*. To explore the performance of a Markovian machining system operating under N-policy, reneging, and the provision of warm standbys, Jain et al. (2004a, b) proposed a finite source queueing model. The matrix method is employed by Jain and Upadhyaya (2009) to evaluate the steady-state probabilities and other system indices of a degraded system by including the realistic concepts of threshold N-policy, multiple vacations, and multiple type spare support. Furthermore, Yang and Chang (2014) examined Markov machine repair model with threshold recovery policy to facilitate the performance analysis by taking some realistic factors into account. They also developed the queueing model for the cost analysis of multi-component machining system by particle swarm optimization. Kuo and Ke (2015) developed queueing model for a repairable multi-component machining system by including the concepts of spare provisioning, switching failure and unreliable server. Recently, a time shared Markov study of machine repair problem having some realistic features such as threshold policy, additional repairman, and mixed spares has been carried out by Jain et al. (2016).

In the recent past, the server vacation models have been studied to analyze the system performance in specific situations wherein the server becomes unavailable for some times. From the cost-economic point of view, it is beneficial to send the server on vacation as soon as he becomes idle or no repair job available in the system. Due to its critical applications, queueing model with server vacation can be applied in many systems operating in machining environment in different setup. In most of the machine repair queueing models, it is assumed that if any failed machine joins the queue, the server will be immediately activated for rendering the service. The important contributions on vacation queueing models in different contexts can be found in the article reported by Gupta (1997), Jain et al. (2004a, b), Ke and Wang (2007), Ke and Wu (2012), Wu and Ke (2014) and many more. Recently, Jain and Meena (2017) studied an unreliable server Markovian multi-component fault tolerant machining system with arrival controlling policy. They have included the feature of optional working vacation instead of complete vacation which makes the study quite interesting and applicable to realistic scenarios. In this study, they obtained the steady-state probabilities and system metrics to characterize the system behavior using SOR method.

The use of soft computing technique for the performance modeling plays a vital role due to its critical utility in decision analysis, automatic control, data classification, and many more. The combination of neural network and fuzzy logic presents an emerging soft computing technique ANFIS. The noticeable works on ANFIS has been done by Jang (1993). Bhargava and Jain (2014) explored the utilization of the hybrid ANFIS technique to provide the comparative study of queueing and reliability results obtained by matrix geometric method (MGM) of a Markov queueing model having an unreliable server operating under vacation policy. Recently, Jain and Meena (2016) used the ANFIS technique for the analysis of Markovian fault tolerant machine repair problem to compute some performance metrics and to compare those with the results obtained using Runge–Kutta approach.

The provision of more servers is always helpful in reducing the work load and to facilitate the faster service. However, to keep the server active in case of less work load is costly affairs. In this article, we study the transient analysis of machine repair problem having mixed warm standby support and two heterogeneous unreliable servers. The first (second) server is activated only when workload of N_1 (N_2) failed machines is accumulated in the system. As soon as the server becomes idle, he goes for vacation. The remaining part of the paper is structured as follows. The system description to formulate the mathematical model is presented in Sect. 2. In Sect. 3, Chapman–Kolmogorov equations at transient state are constructed. To predict the performance of the developed model, some metrics have been constructed. Furthermore, total expected cost function is also constructed to evaluate the optimal service rate in Sect. 4. The architecture of ANFIS model is also briefly described. In Sect. 5, numerical illustration and cost analysis have been presented. Finally, the noble features of work done and concluding remarks are given in Sect. 6.

Model description

Notations

M : The number of operating machines.

S : The number of warm standbys machines i.e. $S^{(k)} = \sum_{j=1}^{k} S_j \ (j = 1, 2, 3, \ldots, k)$.

K : The sum of both operating and warm standby machines i.e. $K = M + S^{(k)}$.

λ : The failure rate of operating machines.

a_j : The failure rate of $jth (j = 1, 2, 3, \ldots, k)$ types of warm standby machines.

υ_i : Mean vacation rate by which the $ith (i = 1,\ 2)$ server returns from the vacation.

λ_{d} : Degraded failure rate of machines.

μ_i : Mean service rate of $ith (i = 1,\ 2)$ server

α_i : The life time of $ith (i = 1,\ 2)$server.

β_i : Repair rate of $ith (i = 1,\ 2)$server.

To study the machine repair problem (MRP), we develop Markov machining system with vacation. For the maintenance purpose, there is provision of two unreliable servers and mixed-type warm standbys. Markovian model is formulated by considering the following assumptions:

- The system consists of M operating and k-types of warm standbys machines having the dissimilar failure characteristic. At least M operating machines are required for the normal operation of the system; however, the system can operate in short mode with at least l ($<M$) operating machines. The operating machine may fail in Poisson pattern with failure rate λ. The *jth* type of warm standbys machines fail according to Poisson process with rates $a_j \ (j = 1, 2, 3, \ldots, k)$.
- The repair facility consists of two heterogeneous repairmen. The first (second) server becomes activate after taking exponentially distributed setup time $v_i^{-1} (i = 1,\ 2)$ to render repair of failed machines when the workload of $N_1 (N_2)$ failed machines has accumulated in the system.
- The repair job of the failed machines is done by the *ith* $(i = 1,\ 2)$ repairman according to exponential distribution with repair rate μ_i. The repairman follows first in first out (FIFO) discipline to render the repair to the failed machines and can repair only one failed machine at a time.
- The switchover time from standby state to operating or from repair to standby state is negligible and assumed to be perfect.
- While rendering the service to the failed machine, the *ith* $(i = 1, 2)$ server may fail following Poisson distribution with rate α_i.
- When the server fails during the busy period, the *ith* $(i = 1, 2)$ repair of the broken down server is done immediately by the repairman according to exponential distribution with rate $\beta_i \ (i = 1, 2)$.

The following indicator function $\xi(\tau)$ is used to define the server status at time epoch $'\tau'$:

$$\xi(\tau) = \begin{cases} 0, \text{when both the servers are on vacation.} \\ 1, \text{when the server 1 is in busy state and server 2 is on vacation.} \\ 2, \text{when the server 1 is brokendown and server 2 is on vacation.} \\ 3, \text{when both the servers are busy.} \\ 4, \text{when the server 1 is brokendown and server 2 is busy.} \\ 5, \text{when the server 2 is brokendown and server 1 is busy.} \\ 6, \text{when both the servers are brokendown.} \end{cases}$$

The transient state probabilities of the system states are defined as follows:

$P_{0,m}(\tau)$: The probability that at time τ, there are m $(0 \le m \le K)$ failed machines in the system and both the servers are unavailable due to vacation.

$P_{i,m}(\tau)$: The probability that at time τ, there are m $(1 \le m \le K)$ failed machines in the system and the server is in state $\xi(\tau) = i;\ 1 \le i \le 6$.

Model governing equations

To develop Markov model for the transient behavior of machining system described in the previous section, the state-dependent transition rates for all the system states are to be specified. Using these rates, the governing Chapman–Kolmogorov differential difference equations can be easily constructed to formulate the model using birth–death process.

For notational convenience, we shall use $\mu^{(2)} = \mu_1 + \mu_2$, $\alpha = \alpha_1 + \alpha_2$, and $S^{(k)} = \sum_{j=1}^{k} S_j$.

The failure rate of operating machines λ_m is given by

$$\lambda_m = \begin{cases} M\lambda + a_n, & 0 \le m < S_1 \\ M\lambda + a_n, & S^{(j-1)} \le m < S^{(j)},\ 2 \le j \le k \\ (K - n)\lambda_d, & S^{(k)} \le m < M + S^{(k)} = K \\ 0, & \text{otherwise} \end{cases}$$

where

$$a_m = \begin{cases} (S_1 - m)\delta_1 + \sum_{i=2}^{k} S_i \delta_i, & 0 \le m < S_1 \\ (S^{(j)} - m)\delta_j + \sum_{i=j+1}^{k} S_i \delta_i, & S^{(j-1)} \le m < S^{(j)},\ 2 \le j \le k. \end{cases}$$

The repair rate $\mu_{i,m}$ depends upon the server status $'\xi(\tau) = i'$, defined in the previous section. Now, we define

$$\mu_{i,m} = \begin{cases} \mu^{(2)}; 2 \le m \le K, \xi(\tau) = 3 \\ \mu_1; 1 \le m \le K, \xi(\tau) = 1, 5 \\ \mu_2; 1 \le m \le K, \xi(\tau) = 4. \end{cases}$$

The transient equations to frame the Markov model are constructed by the following flow conversation law. Now, we frame the equations using the appropriate transition rates for different level i ($0 \le i \le 6$) as follows:

(i) For $i = 0$: When both servers are on vacation:

$$\frac{dP_{0,0}(\tau)}{d\tau} = -\lambda_0 P_{0,0}(\tau) + \mu_1 P_{1,1}(\tau) \tag{1}$$

$$\frac{dP_{0,m}(\tau)}{d\tau} = -\lambda_n P_{0,m}(\tau) + \lambda_{m-1} P_{0,m-1}(\tau); \quad 1 \le m \le N_1 - 1 \tag{2}$$

$$\frac{dP_{0,m}(\tau)}{d\tau} = -(\lambda_m + \upsilon_1) P_{0,m}(\tau) + \lambda_{m-1} P_{0,m-1}(\tau); N_1 \le m \le K - 1 \tag{3}$$

$$\frac{dP_{0,K}(\tau)}{dt} = -\upsilon_1 P_{0,K}(\tau) + \lambda_{K-1} P_{0,K-1}(\tau). \tag{4}$$

(ii) For $i = 1$: Busy state for server 1, while the server 2 is on vacation:

$$\frac{dP_{1,1}(\tau)}{d\tau} = -(\lambda_1 + \mu_1 + \alpha_1) P_{1,1}(\tau) + \mu_1 P_{1,2}(\tau) + \beta_1 P_{2,1}(\tau) + \mu_2 P_{3,2}(\tau) \tag{5}$$

$$\frac{dP_{1,m}(\tau)}{d\tau} = -(\lambda_m + \mu_1 + \alpha_1) P_{1,m}(\tau) + \lambda_{m-1} P_{1,m-1}(\tau) + \mu_1 P_{1,m+1}(\tau) + \beta_1 P_{2,m}(\tau); \quad 2 \le m \le N_1 - 1 \tag{6}$$

$$\frac{dP_{1,m}(\tau)}{d\tau} = -(\lambda_m + \mu_1 + \alpha_1) P_{1,m}(\tau) + \lambda_{m-1} P_{1,m-1}(\tau) + \mu_1 P_{1,m+1}(\tau) + \beta_1 P_{2,m}(\tau) + \upsilon_1 P_{0,m}(\tau); N_1 \le m \le N_2 - 1 \tag{7}$$

$$\frac{dP_{1,m}(\tau)}{d\tau} = -(\lambda_m + \mu_1 + \alpha_1 + \upsilon_2) P_{1,m}(\tau) + \lambda_{m-1} P_{1,m-1}(\tau) + \mu_1 P_{1,m-3}(\tau) + \beta_1 P_{2,m}(\tau) + \upsilon_1 P_{0,m}(\tau); \; N_2 \le m \le K - 1 \tag{8}$$

$$\frac{dP_{1,K}(\tau)}{d\tau} = -(\mu_1 + \alpha_1 + \upsilon_2) P_{1,K}(\tau) + \lambda_{K-1} P_{1,K-1}(\tau) + \beta_1 P_{2,K}(\tau) + \upsilon_1 P_{0,K}(\tau). \tag{9}$$

(iii) For $i = 2$: Broken down state for server 1, while the server 2 is on vacation:

$$\frac{dP_{2,1}(\tau)}{d\tau} = -(\lambda_1 + \beta_1) P_{2,1}(\tau) + \alpha_1 P_{2,1}(\tau) \tag{10}$$

$$\frac{dP_{2,m}(\tau)}{d\tau} = -(\lambda_m + \beta_1) P_{2,m}(\tau) + \lambda_{m-1} P_{2,m-1}(\tau) + \alpha_1 P_{1,m}(\tau); \quad 2 \le m \le K - 1 \tag{11}$$

$$\frac{dP_{2,K}(\tau)}{d\tau} = -\beta_1 P_{2,K}(\tau) + \lambda_{K-1} P_{2,K-1}(\tau) + \alpha_1 P_{1,K}(\tau) \tag{12}$$

(iv) For $i = 3$: When both servers are busy:

$$\frac{dP_{3,2}(\tau)}{d\tau} = -(\lambda_2 + \alpha + \mu_2) P_{3,2}(\tau) + \mu^{(2)} P_{3,3}(\tau) + \beta_1 P_{4,2}(\tau) + \beta_2 P_{5,2}(\tau) \tag{13}$$

$$\frac{dP_{3,m}(\tau)}{d\tau} = -(\lambda_m + \alpha + \mu^{(2)}) P_{i,m}(\tau) + \lambda_{m-1} P_{i,m-1}(\tau) + \mu^{(2)} P_{i,m+1}(\tau) + \beta_1 P_{i+1,m}(\tau) + \beta_2 P_{i+2,m}(\tau); \quad 3 \le m \le N_2 - 1 \tag{14}$$

$$\frac{dP_{3,m}(\tau)}{d\tau} = -(\lambda_m + \alpha + \mu^{(2)}) P_{i,m}(\tau) + \lambda_{m-1} P_{i,m-1}(\tau) + \mu^{(2)} P_{i,m+1}(\tau) + \beta_1 P_{i+1,m}(\tau) + \beta_2 P_{i+2,m}(\tau) + \upsilon_2 P_{i-2,m}(\tau); \quad N_2 \le m \le K - 1 \tag{15}$$

$$\frac{dP_{3,K}(\tau)}{d\tau} = -(\alpha + \mu^{(2)}) P_{i,K}(\tau) + \lambda_{K-1} P_{i,K-1}(\tau) + \mu^{(2)} P_{i,K+1}(\tau) + \beta_1 P_{i+1,K}(\tau) + \beta_2 P_{i+2,K}(\tau) + \upsilon_2 P_{i-2,K}(\tau). \tag{16}$$

(v) For $i = 4$: When server 1 is broken down and server 2 is busy:

$$\frac{dP_{4,2}(\tau)}{d\tau} = -(\lambda_2 + \beta_1 + \alpha_2) P_{4,2}(\tau) + \mu_2 P_{4,3}(\tau) + \alpha_1 P_{3,2}(\tau) + \beta_2 P_{6,2}(\tau) \tag{17}$$

$$\frac{dP_{4,m}(\tau)}{d\tau} = -(\lambda_m + \beta_1 + \mu_2 + \alpha_2) P_{i,m}(\tau) + \lambda_{m-1} P_{i,m-1}(\tau) + \mu_2 P_{i,m+1}(\tau) + \alpha_1 P_{i-1,m}(\tau) + \beta_2 P_{i+2,m}(\tau); \quad N_1 \le m \le K - 1 \tag{18}$$

$$\frac{dP_{4,K}(\tau)}{d\tau} = -(\beta_1 + \mu_2 + \alpha_2) P_{4,K}(\tau) + \lambda_{K-1} P_{4,K-1}(\tau) + \alpha_1 P_{3,K}(\tau) + \beta_2 P_{6,K}(\tau) \tag{19}$$

(vi) For $i = 5$: When server 2 is broken down and server 1 is busy:

$$\frac{dP_{5,2}(\tau)}{d\tau} = -(\lambda_2 + \beta_2 + \alpha_1)P_{5,2}(\tau) + \mu_1 P_{5,3}(\tau) + \alpha_2 P_{3,2}(\tau) + \beta_1 P_{6,2}(\tau) \quad (20)$$

$$\frac{dP_{5,m}(\tau)}{d\tau} = -(\lambda_m + \beta_2 + \mu_1 + \alpha_1)P_{5,m}(\tau) + \lambda_{m-1}P_{5,m-1}(\tau) + \mu_1 P_{5,m+1}(\tau) + \alpha_2 P_{3,m}(\tau) + \beta_1 P_{6,m}(\tau); \quad N_1 \le m \le K-1 \quad (21)$$

$$\frac{dP_{5,K}(\tau)}{d\tau} = -(\beta_2 + \mu_1 + \alpha_1)P_{5,K}(\tau) + \lambda_{K-1}P_{5,K-1}(\tau) + \alpha_2 P_{3,K}(\tau) + \beta_1 P_{6,K}(\tau). \quad (22)$$

(vii) For $i = 6$: When both servers are broken down:

$$\frac{dP_{6,2}(\tau)}{d\tau} = -(\lambda_2 + \beta_1 + \beta_2)P_{6,2}(\tau) + \alpha_2 P_{4,2}(\tau) + \alpha_1 P_{5,2}(\tau) \quad (23)$$

$$\frac{dP_{6,m}(\tau)}{d\tau} = -(\lambda_m + \beta_1 + \beta_2)P_{6,m}(\tau) + \lambda_{m-1}P_{6,m-1}(\tau) + \alpha_2 P_{4,m}(\tau) + \alpha_1 P_{5,m}(\tau); \quad N_1 \le m \le K-1 \quad (24)$$

$$\frac{dP_{6,K}(\tau)}{d\tau} = -(\beta_1 + \beta_2)P_{6,K}(\tau) + \lambda_{K-1}P_{6,K-1}(\tau) + \alpha_2 P_{4,K}(\tau) + \alpha_1 P_{5,K}(\tau). \quad (25)$$

Performance measures

The performance of any real-time system can be assessed in terms of metrics which reveal the system's operating behavior in different scenarios.

Queueing indices

To predict and explore the behavior of the system, we formulate transient performance indices, viz., (i) expected number of broken down machines $E[N(\tau)]$, (ii) machine availability $\mathrm{MA}(\tau)$, (iii) carried load $\lambda_{\mathrm{eff}}(\tau)$, and (iv) throughput $\mathrm{Th}(\tau)$ at time epoch $'\tau'$ as follows:

$$(i) \quad E[N(\tau)] = \sum_{m=0}^{K} nP_{0,n}(\tau) + \sum_{i=1}^{2}\sum_{m=1}^{K} nP_{i,m}(\tau) + \sum_{i=3}^{6}\sum_{m=2}^{K} nP_{i,m}(\tau) \quad (26)$$

$$(ii) \quad \mathrm{MA}(\tau) = 1 - \frac{E[N(\tau)]}{M + S^{(k)}} \quad (27)$$

$$(iii) \quad \lambda_{\mathrm{eff}}(\tau) = \sum_{m=0}^{K} \lambda_m P_{0,m}(\tau) + \sum_{i=1}^{2}\sum_{m=1}^{K} \lambda_m P_{i,m}(\tau) + \sum_{i=3}^{6}\sum_{m=2}^{K} \lambda_m P_{i,m}(\tau) \quad (28)$$

$$(iv) \quad \mathrm{Th}(\tau) = \sum_{n=1}^{K} \mu_1 P_{1,m}(\tau) + \sum_{m=2}^{K} \mu^{(2)} P_{3,m}(\tau) + \sum_{m=2}^{K} \mu_2 P_{4,m}(\tau) + \sum_{m=2}^{K} \mu_1 P_{5,m}(\tau). \quad (29)$$

Long-run system states probabilities

The long-run probabilities of the server being in different states, i.e., (i) both servers being on vacation $P_v(\tau)$, (ii) only server 1 being busy $P_{B1}(\tau)$, (iii) only server 2 being busy $P_{B2}(\tau)$, (iv) both servers being busy $P_B(\tau)$, (v) only server 1 is under repair $P_{D1}(\tau)$, (vi) only server 2 is under repair $P_{D2}(\tau)$, and (vii) both servers are broken down $P_D(\tau)$, respectively, at time epoch $'\tau'$ are constructed as follows:

$$(i) \quad P_v(\tau) = \sum_{m=0}^{K} P_{0,m}(\tau) \quad (30)$$

$$(ii) \quad P_{B1}(\tau) = \sum_{m=1}^{K} P_{1,m}(\tau) + \sum_{m=2}^{K} P_{5,m}(\tau) \quad (31)$$

$$(iii) \quad P_{B2}(\tau) = \sum_{m=2}^{K} P_{4,m}(\tau) \quad (32)$$

$$(iv) \quad P_B(\tau) = \sum_{m=2}^{K} P_{3,m}(\tau) \quad (33)$$

$$(v) \quad P_{D1}(\tau) = \sum_{m=1}^{K} P_{2,m}(\tau) + \sum_{m=2}^{K} P_{4,m}(\tau) \quad (34)$$

$$(vi) \quad P_{D2}(\tau) = \sum_{m=2}^{K} P_{5,m}(\tau) \quad (35)$$

$$(vii) \quad P_D(\tau) = \sum_{m=2}^{K} P_{6,m}(\tau). \quad (36)$$

System cost

To determine the cost incurred for the system operation, we formulate a cost function involving some cost elements associated with different states of the machining system.

The various cost elements per unit time associated with the status of the system, are defined as follows:

- C_H Holding cost of one failed unit in the system
- C_V Cost spent on the system when both the servers are on vacation
- C_{B1} Cost spent on the system when the server 1 is busy
- C_{B2} Cost spent on the system when the server 2 is busy
- C_B Cost spent on the system when both the servers are busy
- C_{D1} Cost spent on the system, while only server 1 is under repair
- C_{D2} Cost spent on the system, while only server 2 is under repair
- C_D Cost spent on the system when both the servers are under repair

Now, we frame the cost function TC(t) which involves the total cost per unit time by considering the above cost elements and respective performance measures as follows:

$$\begin{aligned}\mathrm{TC}(\tau) = {} & C_H\,E[N(\tau)] + C_V\,P_V(\tau) + C_{B1}\,P_{B1}(\tau) \\ & + C_{B2}\,P_{B2}(\tau) + C_B\,P_B(\tau) + C_{D1}\,P_{D1}(\tau) \\ & + C_{D2}\,P_{D2}(\tau) + C_D\,P_D(\tau).\end{aligned} \tag{37}$$

Neuro-fuzzy-based ANFIS Model

Now, we outline a brief concept of ANFIS approach which is based on a neural network underlying the fuzzified parameters. The fuzzy rules employed in ANFIS can be formulated as

$$\begin{aligned}&\text{IF } (e_1 \text{is } E_1) \quad \text{AND} \quad (e_2 \text{ is } E_2)\ldots \quad \text{AND} \quad (e_n \text{ is } E_n) \\ &\text{THEN } G = F(e_1, e_1, \ldots, e_n\,).\end{aligned} \tag{38}$$

Here, F is a linear combination of the input variables $(e_1, e_2, \ldots, e_n\,)$, and E_i's are the respective fuzzy sets. Now, output is obtained using weighted average for the defuzzification method, given by

$$F(e_1, e_2, \ldots, e_n\,) = h_0 + h_1 e_1 + h_2 e_2 + \cdots + h_n e_n \tag{39}$$

where h_i represents the weights corresponding to input parameter $e_i\,(i = 0, 1, 2, \ldots, n)$.

For our FTS model, an adaptive neuro-fuzzy inference system is constructed by considering the input parameters λ and v and one output $E[N(\tau)]$.

Numerical simulation

The numerical results are presented to explore the sensitivity of the system descriptors for the various performance measures and to facilitate the cost analysis. The numerical computation has been done using Runge–Kutta method to provide the transient solution. For numerical simulation purpose, 4th order Runge–Kutta algorithm is implemented using the ode45 function of the MATLAB software. To characterize the system behavior for different system descriptors, the numerical results are displayed in Tables 1, 2 and Figs. 2, 3, 4, 5, 6, 7.

In Tables 1 and 2, we display the trends of the mean number of failed machines $E[N(\tau)]$, availability of machines MA(τ), throughput Th (τ), and system state long-run probabilities by varying different input parameters. The default parameters are chosen as $M = 5$, $l = 2$, $R = 2$, $k = 2$, $S = 2$, $\lambda = 0.5$, $a_1 = 0.02$, $a_2 = 0.04$, $\lambda_d = 0.5$, $\mu_1 = 2$, $\mu_2 = 3$, $\alpha_1 = 0.1$, $\alpha_2 = 0.3$, $\beta_1 = 10$, $\beta_2 = 8$.

As we expect, it is noted in Tables 1, 2 that as time increases, both $E[N(\tau)]$ and Th(τ) increase, whereas MA(τ) decreases.

To compute the ANFIS results, neuro-fuzzy tool in MATLAB is used by considering Gaussian membership function for the input parameters (λ and v). For fuzzification of λ and v, we opt the five members which are shown in Fig. 1. The members taken for each λ and v are (i) very low, (ii) low, (iii) average, (iv) high, and (v) very high.

In figs. 2, 3, 4, 5, 6, 7, to plot the results evaluated by R–K method, the continuous lines are used, whereas the numerical results obtained using ANFIS are depicted by

Table 1 Variations in different system indices by varying time for different values of α

α	τ	$E[N(\tau)]$	$Th(\tau)$	$MA(\tau)$	$P_B(\tau)$	$P_V(\tau)$	$TC(\tau)$
0.02	2	0.549268	0.001812	0.921533	7.50E − 07	0.998181	206.6814
	6	1.502964	0.057002	0.785291	3.36E − 04	0.94283	275.096
	10	2.029223	0.163365	0.710111	2.85E − 03	0.836967	302.3159
0.04	2	0.705696	0.003618	0.899186	2.91E − 06	0.996354	218.9347
	6	1.878615	0.096852	0.731626	1.10E − 03	0.902545	299.466
	10	2.400913	0.243451	0.657012	8.03E − 03	0.757294	321.1928
0.06	2	0.861606	0.006133	0.876913	8.06E − 06	0.993796	231.0421
	6	2.229894	0.141126	0.681444	2.59E − 03	0.857623	321.2991
	10	2.72298	0.318345	0.611003	1.65E − 02	0.683456	337.2844

Table 2 Variations in different system indices by varying time for different values of μ

μ	τ	$E[N(\tau)]$	$Th(\tau)$	$MA(\tau)$	$P_B(\tau)$	$P_V(\tau)$	$TC(\tau)$
1	1	1.647587	0.030931	0.76463	5.01E − 05	0.984513	312.5974
	3	3.971559	0.43216	0.432634	0.009167	0.787161	471.2221
	5	4.605148	0.827214	0.342122	0.042498	0.604173	499.1506
5	1	0.868953	0.012663	0.875864	2.00E − 06	0.996826	292.0626
	3	2.30164	0.247366	0.671194	4.90E − 04	0.938067	398.3246
	5	3.175741	0.609982	0.546323	5.43E − 03	0.849098	456.0356
9	1	0.867817	0.017336	0.876026	1.47E − 06	0.997103	332.0112
	3	2.26336	0.275525	0.676663	2.70E − 04	0.953961	437.5128
	5	2.916208	0.51288	0.583399	1.30E − 03	0.914607	484.2185

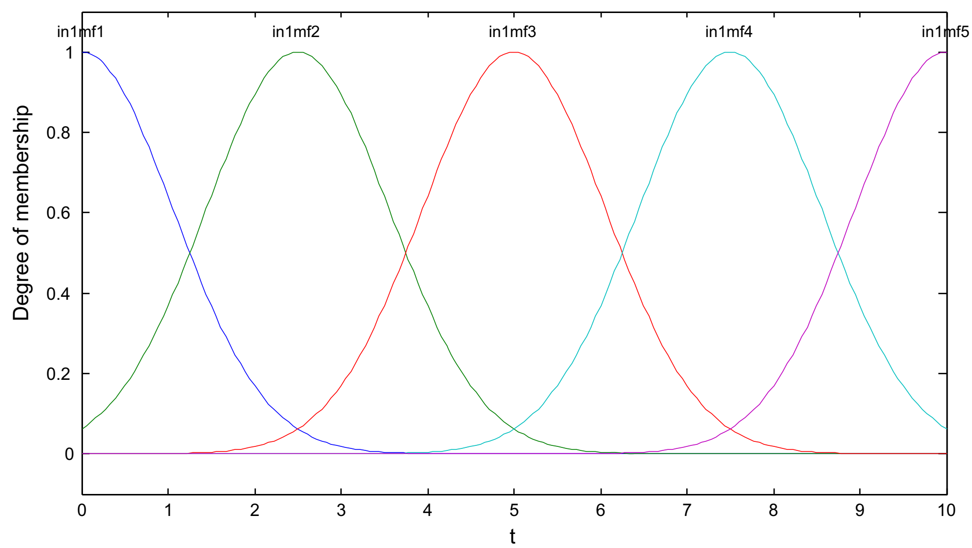

Fig. 1 Membership function for input variable λ and v

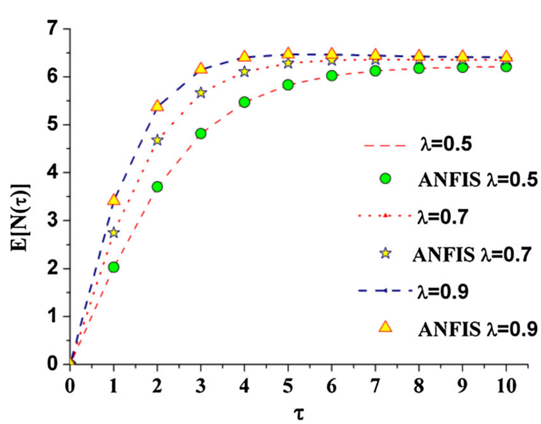

Fig. 2 $E[N(\tau)]$ vs time at various values of λ

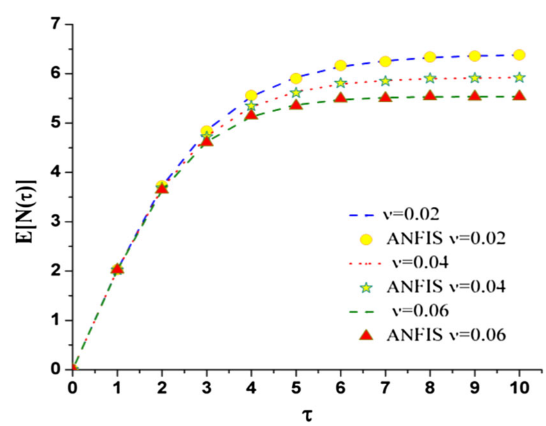

Fig. 3 $E[N(\tau)]$ vs time at various values of v

tick marks. From Figs. 2, 3, it is noticed that as time grows up, the number of failed machines $E[N(\tau)]$ increases; this trend matches with the realistic situation also. From Fig. 2, we see that as failure rate (λ) of machine increases, the number of failed machines $E[N(\tau)]$ also becomes higher. It is clear from Fig. 3 that the average number of failed machine lowers down as the value of vacation rate v increases; the trend for increasing the values of time τ is also observed in the figures. Figures 4, 5 show the decreasing trend for the system availability MA(τ) with respect to time τ. The system availability MA(τ) significantly decreases (increases) as value of λ (v) increases. The

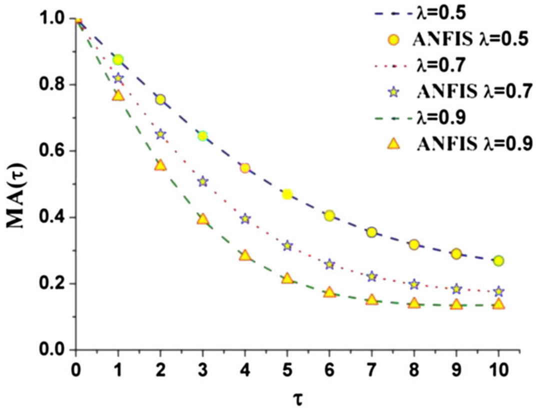

Fig. 4 $MA(\tau)$ vs time at various values of λ

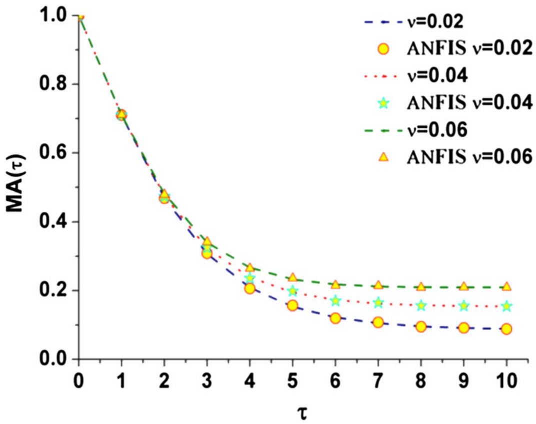

Fig. 5 $MA(\tau)$ vs time at various values of v

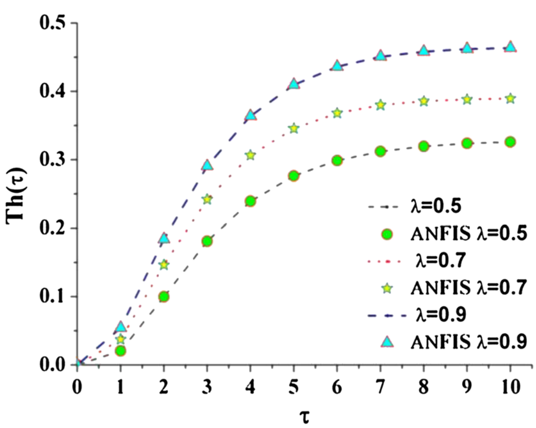

Fig. 6 $Th(\tau)$ vs time at various values of λ

throughput Th(τ) plotted in Figs. 6 and 7, is significantly increases as λ and v increase. In these figures, it is quite clear that the effects of parameters λ and v on throughput Th(τ) are much prevalent as time τ grows; however, after a certain time, the impact seems to be stabilized.

In Figs. 2, 3, 4, 5, 6, and 7, numerical results for $E[N(\tau)]$, MA(τ), and Th(τ), respectively, are plotted using

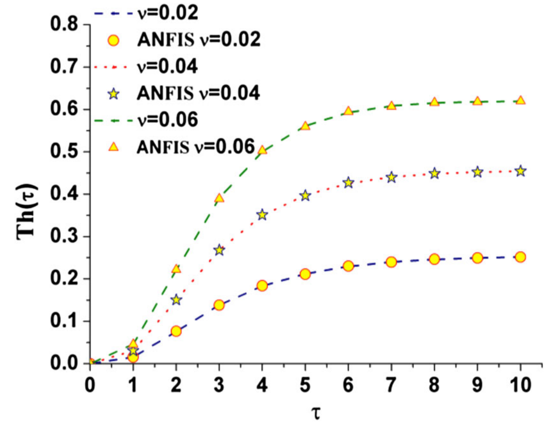

Fig. 7 $Th(\tau)$ vs time at various values of v

Table 3 Cost elements (in $) associated with various system indices

Cost set	C_H	C_V	C_{B1}	C_{B2}	C_B	C_{D1}	C_{D2}	C_D	C_m
I	170	70	50	60	70	80	90	130	30
II	120	70	50	60	70	80	90	130	25
III	80	70	50	60	70	80	90	130	20

both Runge–Kutta method (curve) and ANFIS (ticked marks) approach. From these figures, we can easily see that the ANFIS results are at par with the results obtained by the Runge–Kutta method. In addition, we conclude that the neuro-fuzzy controller can be developed for the quantitative assessment of metrics of unreliable machining system to track the system performance.

The total expected cost incurred on the system TC(τ) can be minimized with respect to the decision parameter repair rate (μ) of the failed machines using heuristic search approach. To search the optimal value of repair rate 'μ*', we choose three sets of cost elements (in $) as given in Table 3. To make the study more useful from the cost–benefit view point, the total cost function is plotted in Figs. 8, 9, and 10 for three cost sets I, II, and III, respectively, and varying values of μ and τ. It is noticed that the TC(μ*) is a convex function with respect to μ and τ both which can be seen in Figs. 8, 9, 10. The results obtained are quite interesting and can be applied to any real-time machining systems for upgrading the system by suitable choice of service/repair rate.

The minimum expected cost of the system is obtained as TC(μ*) = $190.59 at time $\tau = 1$ and the corresponding optimal repair rate is $\mu^* = 1.54485$ for cost set I. For cost set II, the minimum expected cost of the system obtained is TC(μ*) = $150.37 and the associated optimal repair rate is $\mu^* = 1.485456$ at time $\tau = 1$. The minimum expected cost of the system is TC(μ*) = $128.07 and the corresponding optimal repair rate is achieved $\mu^* = 1.24788$ at time $\tau = 1$ for the cost set III.

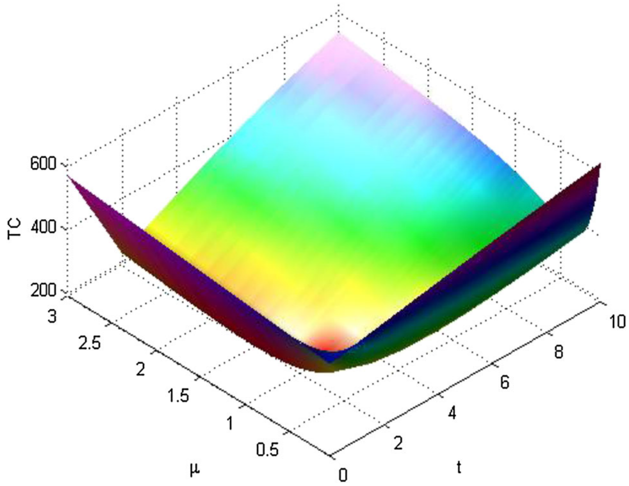

Fig. 8 TC(τ) at various values of μ

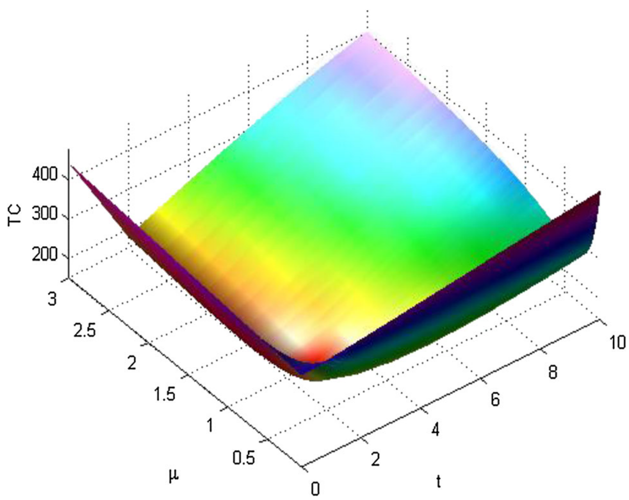

Fig. 9 TC(τ) at various values of μ

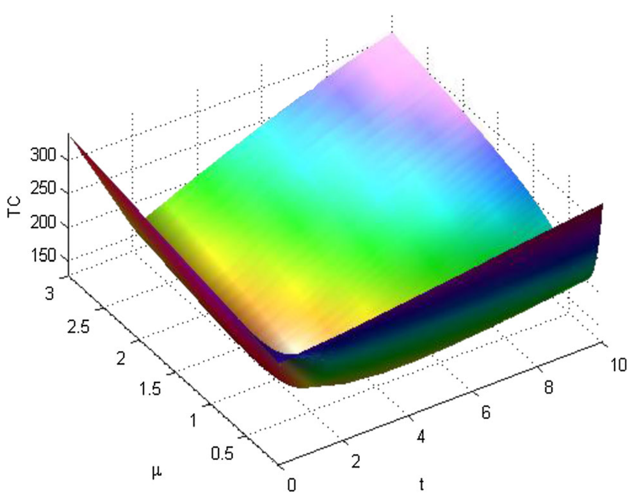

Fig. 10 TC(τ) at various values of μ

Conclusion

In this article, we have studied a Markov model by including the features of vacation, threshold policy, two unreliable heterogeneous servers, and mixed warm standbys which make our model generic and more versatile from application point of view. The transient study of the system has been carried out using the Runge–Kutta method to evaluate various system metrics in terms of transient probabilities. To determine the total cost of the system, a heuristic search approach is used so as to obtain the minimum cost and corresponding optimal repair rate of the server. The provision of unreliable servers which are allowed to take vacation can be noticed in many multi components redundant machining systems. In industrial scenario, the model developed can be used to provide the valuable insights for the fault tolerant embedded systems such as computers, power transmission lines, distributed data networks, telecommunications, and power plants, wherein the server as well as machining components are failure prone. The present work can be further extended by including the optimal threshold N-policy or F-policy. Furthermore, the realistic feature of bulk failure can be included, but in that case, the evaluation of system performance indices seems to be tedious.

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